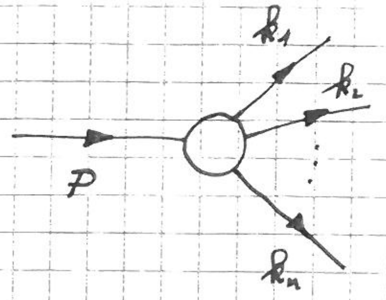
20. Decay width



decay $\quad P = k_1 + k_2 + \dots + k_n$

in rest frame of decaying particle: $\quad P = (M, \vec{0})$

time evolution operator in IP

$$\int_B d\mu(k_1) \dots d\mu(k_n) \; \left| \langle k_1, \dots, k_n | \, U(t_f, t_i) \, | \psi \rangle \right|^2$$

= probability for decay of a particle (described by state vector $|\psi\rangle$) into n particles in phase-space region B within the time interval $[t_i, t_f]$

remarks:

i) discussion for spin 0 particles (gen. to higher spin trivial)

ii) final state particles assumed to be distinguishable (generalisation: $\times \frac{1}{n_1!} \frac{1}{n_2!} \dots \frac{1}{n_p!}$)

iii) momentum distribution of initial-state wave function peaked around $\vec{p} = 0$

iv) time interval $T = t_f - t_i$ microscopically large, but macroscopically small

iv) $\Rightarrow$ study first "limit" $T \to \infty$, come back to (realistic) finite T later

$$\int_B d\mu(k_1) \dots d\mu(k_n) \, |\langle k_1, \dots, k_n | S | \psi \rangle|^2$$

$$= \int_B d\mu(k_1) \dots d\mu(k_n) \, \langle \psi | S^\dagger | k_1, \dots, k_n \rangle \langle k_1, \dots k_n | S | \psi \rangle$$

$$= \int_B d\mu(k_1) \dots d\mu(k_n) \int d\mu(p) \, d\mu(p') \, \overbrace{\langle \psi | p' \rangle}^{\psi(p')^*} \langle p' | S^\dagger | k_1, \dots, k_n \rangle$$

$$\langle k_1, \dots, k_n | S | p \rangle \underbrace{\langle p | \psi \rangle}_{\psi(p)}$$

$$= \int_B d\mu(k_1) \dots d\mu(k_n) \int d\mu(p) \, d\mu(p') \, \psi(p')^* \, \psi(p)$$

$$(2\pi)^4 \, \delta^{(4)}(p' - k_1 - \dots - k_n) \; \mathcal{M}(p' \to k_1, \dots, k_n)^*$$

$$(2\pi)^4 \, \delta^{(4)}(p - k_1 - \dots - k_n) \; \mathcal{M}(p \to k_1, \dots, k_n)$$

$$= \int_B d\mu(k_1) \ldots d\mu(k_n) \int d\mu(p) \; |\psi(p)|^2 \; \frac{1}{(2\pi)^3 2p^0}$$

$$(2\pi)^4 \; \delta(p^0 - k_1^0 - \ldots - k_n^0)$$

$$(2\pi)^4 \; \delta^{(4)}(p - k_1 - \ldots - k_n) \; |M(p \to k_1, \ldots, k_n)|^2 = (*)$$

$$\delta(p^0 - k_1^0 - \ldots - k_n^0) = \int_{-\infty}^{+\infty} \frac{dt}{2\pi} \, e^{it(p^0 - k_1^0 - \ldots - k_n^0)}$$

$$= \int_{t_i}^{t_f} \frac{dt}{2\pi} \, e^{it \times 0} = \frac{t_f - t_i}{2\pi} = \frac{T}{2\pi}$$

$$\Rightarrow (*) \simeq T \int_B d\mu(k_1) \ldots d\mu(k_n) \underbrace{\int d\mu(p) \; |\psi(p)|^2}_{1} \; \frac{1}{2M}$$

$$\times \; (2\pi)^4 \; \delta^{(4)}(P - k_1 - \ldots - k_n) \; |M(P \to k_1, \ldots, k_n)|^2$$

assumption in last step: $M(p \to k_1, \ldots, k_n)$ varying slowly in region $p \simeq P$ where $\psi(p)$ gives nonvanishing contribution result:

$$\Gamma(P \to B) = \frac{\text{transition probability}}{T}$$

$$= \int d\mu(k_1) \dots d\mu(k_n) \frac{1}{2M} (2\pi)^4 \delta^{(4)}(P - k_1 - \dots - k_n) \left| \mathcal{M}(P \to k_1, \dots, k_n) \right|^2$$

(partial) decay width

total decay width $\Gamma = \sum_f \Gamma_f$

sum over all decay modes

exponential decay $N(t) = N(0)\, e^{-\Gamma t}$

life time $\tau = 1/\Gamma$