

19. Standard model of particle physics

spontaneously broken gauge theory with gauge group

$$\underbrace{SU(3)_c}_{\text{strong}} \times \underbrace{SU(2)_L \times U(1)_Y}_{\text{electroweak}} \xrightarrow{\text{SSB}} SU(3)_c \times U(1)_{em}$$

$SU(3)_c$ (QCD) already discussed

→ electroweak interaction $G_{ew} = SU(2)_L \times U(1)_Y$

Glashow - Weinberg - Salam (Nobel prize 1979)

a) gauge group and multiplets

fundamental fields:

spin 1 four generators of $SU(2) \times U(1) \rightarrow W^\pm, Z^0, \gamma$

spin 1/2 leptons:

			Q_{em}	
$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}$	$\begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}$	$\begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$	0	
			-1	

3 generations

quarks:

$\begin{pmatrix} u \\ d \end{pmatrix}$	$\begin{pmatrix} c \\ s \end{pmatrix}$	$\begin{pmatrix} t \\ b \end{pmatrix}$	$2/3$	
			$-1/3$	

of quarks and leptons

V-A theory \rightarrow lefthanded SU(2) doublets
righthanded SU(2) singlets

\rightarrow mesons $q_1 \bar{q}_2$, baryons $q_1 q_2 q_3$

SSB \rightarrow Higgs field

$G_{ew} = SU(2)_L \times U(1)_Y \rightarrow$ weak hypercharge Y

W_μ^a B_μ gauge bosons
 $a=1,2,3$

L_j ℓ_{Rj} leptons
 $j=1,2,3$ (generation index) $f_{L,R} = \frac{1-\gamma_5}{2} f_{L,R}$

q_{Lj} u_{Rj}, d_{Rj} quarks
 $T_a = \tau_a/2$ 0

ϕ Higgs doublet

gauge transformations:

SU(2)_L: $\psi \rightarrow U\psi$ for doublets

$\psi \rightarrow \psi$ for singlets

$\phi \rightarrow U\phi \quad U \in SU(2)$

$\vec{W}_\mu \cdot \frac{\vec{\tau}}{2} \rightarrow U \vec{W}_\mu \cdot \frac{\vec{\tau}}{2} U^{-1} + \frac{i}{g} (\partial_\mu U) U^{-1}$

$B_\mu \rightarrow B_\mu$

U(1)_Y: $\psi \rightarrow e^{-i\alpha \frac{Y_\psi}{2}} \psi$

$\phi \rightarrow e^{-i\alpha \frac{Y_\phi}{2}} \phi$

$\vec{W}_\mu \rightarrow \vec{W}_\mu$

$B_\mu \rightarrow B_\mu + \frac{1}{g'} \partial_\mu \alpha$

Lagrange density:

$$\mathcal{L}_W = -\frac{1}{4} \sum_{a=1}^3 F_a^{\mu\nu} F_{a\mu\nu}$$

$$F_a^{\mu\nu} = \partial^\mu W_a^\nu - \partial^\nu W_a^\mu - g \epsilon_{abc} W_b^\mu W_c^\nu$$

$$\mathcal{L}_B = -\frac{1}{4} (\partial_\mu B_\nu - \partial_\nu B_\mu) (\partial^\mu B^\nu - \partial^\nu B^\mu)$$

$$\mathcal{L}_\psi = i \bar{\psi} \gamma^\mu D_\mu \psi, \quad D_\mu = \partial_\mu + ig \vec{T} \cdot \vec{W}_\mu + ig' \frac{Y}{2} B_\mu$$

$$\mathcal{L}_\phi = (D_\mu \phi)^\dagger D^\mu \phi \quad \vec{T} = \begin{cases} \vec{T}/2 & \text{doublets} \\ \vec{0} & \text{singlets} \end{cases}$$

$$- \mathcal{L}_Y = \bar{q}_{Li} \phi d_{Rj} \Gamma_{ij} + \bar{q}_{Li} \tilde{\phi} u_{Rj} \Delta_{ij} + \bar{l}_i \phi l_{Rj} \gamma_{ij} + h.c.$$

Yukawa couplings

$$\tilde{\phi} = i\tau^2 \phi^* \Rightarrow Y_{\tilde{\phi}} = -Y_\phi$$

$$\tilde{\phi} \rightarrow U \tilde{\phi} \quad \text{as } \tau_2 U^* = U \tau_2$$

(representations are equivalent)

$$V(\phi) = -r \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2, \quad r > 0 \rightarrow \text{SSB}$$

Higgs potential

$$\mathcal{L}_{\text{GWS}} = \mathcal{L}_W + \mathcal{L}_B + \sum_{\psi} \mathcal{L}_{\psi} + \mathcal{L}_{\phi} + \mathcal{L}_Y - V$$

remark: neutrinos massless in this minimal

version of the SM

experimental evidence for massive neutrinos

(neutrino oscillations) \rightarrow extension of \mathcal{L}_{GWS}

necessary (exercise: find correct extension of

SM \rightarrow receive Nobel prize)

b) SSB

$$r > 0 \Rightarrow \text{SSB}$$

$$SU(2) \times U(1) \text{ invariance} \rightarrow \langle 0 | \phi | 0 \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

($v > 0$) without loss of generality

$U(1)_{em}$ unbroken $\Rightarrow Q_{em} =$ group generator

which annihilates $\frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \langle 0 | \phi | 0 \rangle$

$$\underbrace{\left(\vec{c} \cdot \frac{\vec{T}}{2} + d \frac{Y_\phi}{2} \right)}_{Q_{em}^\phi} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = 0, \quad \vec{c} \in \mathbb{R}^3, d \in \mathbb{R}$$

$$\Rightarrow c_1 = c_2 = 0, \quad c_3 = d Y_\phi$$

$$\Rightarrow Q_{em}^\phi = c_3 \left(\frac{T_3}{2} + \frac{1}{2} \right) = c_3 \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

em. charge matrix
for Higgs doublet

general form (arbitrary representation):

$$Q_{em} = c_3 T_3 + d \frac{Y}{2}$$

charge difference within lepton or quark
doublets $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}, \begin{pmatrix} u \\ d \end{pmatrix} \rightarrow c_3 = +1$

$$\Rightarrow Q_{em}^{\Phi} = \frac{T_3}{2} + \frac{1}{2} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\Phi = \begin{pmatrix} \Phi^+ \\ \Phi^0 \end{pmatrix}$$

choice of "normalization" $d=1$ (fixes Y)

$$\Rightarrow Q = T_3 + \frac{Y}{2}$$

known charges of fundamental particles

$$\rightarrow Y(L) = -1, \quad Y(l_R) = -2$$

$$\rightarrow Y(q_L) = \frac{1}{3}, \quad Y(u_R) = \frac{4}{3}, \quad Y(d_R) = -\frac{2}{3}$$

$$Y(\Phi) = 1$$

c) bosonic sector

mass term of vector bosons:

$$D_{\mu} = \partial_{\mu} + ig \frac{\vec{T}}{2} \vec{W}_{\mu} + ig' \frac{1}{2} B_{\mu} \quad \text{for Higgs doublet}$$

$$\Rightarrow W_1 = \frac{W^+ + W^-}{\sqrt{2}}, \quad W_2 = i \frac{W^+ - W^-}{\sqrt{2}}$$



$$W^+ = \frac{W_1 - iW_2}{\sqrt{2}}, \quad W^- = \frac{W_1 + iW_2}{\sqrt{2}}$$

$$\frac{1}{2} (\tau_1 W_1 + \tau_2 W_2) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & W^+ \\ W^- & 0 \end{pmatrix}$$

$$D_\mu \underbrace{\langle 0 | \phi | 0 \rangle}_{\frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}} = \frac{iv}{\sqrt{2}} \left[\frac{g}{\sqrt{2}} W_\mu^+ \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{1}{2} (-g W_\mu^3 + g' B_\mu) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right]$$

mass term:

$$(D_\mu \langle 0 | \phi | 0 \rangle)^\dagger D^\mu \langle 0 | \phi | 0 \rangle =$$

$$= \frac{g^2 v^2}{4} W_\mu^- W^{+\mu}$$

$$+ \frac{v^2}{8} \underbrace{(-g W_\mu^3 + g' B_\mu)}_{\text{linear combination of neutral VBs with mass}} (-g W_3^\mu + g' B^\mu)$$

linear combination of neutral VBs with mass

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diagonalization of mass matrix in the neutral sector \rightarrow mass eigenfields (Z^0, γ)

$$\begin{pmatrix} Z \\ A \end{pmatrix} = \begin{pmatrix} \cos \Theta_W & -\sin \Theta_W \\ \sin \Theta_W & \cos \Theta_W \end{pmatrix} \begin{pmatrix} W_3 \\ B \end{pmatrix}$$

$$\cos \Theta_W = \frac{g}{\sqrt{g^2 + g'^2}}, \quad \sin \Theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}$$

$\Theta_W =$ Weinberg angle

mass term: $M_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} M_Z^2 Z_\mu Z^\mu$

$$M_W^2 = \frac{g^2 v^2}{4}, \quad M_Z^2 = \frac{(g^2 + g'^2) v^2}{4}$$

$$\frac{M_W^2}{M_Z^2} = \frac{g^2}{g^2 + g'^2} = \cos^2 \Theta_W$$

$$\frac{M_W}{M_Z} = \cos \Theta_W$$

general form of the covariant derivative
expressed in terms of mass eigenfields:

$$D_\mu = \partial_\mu + \frac{ig}{\sqrt{2}} (T_+ W_\mu^+ + T_- W_\mu^-) +$$

$$+ ig T_3 (\cos \Theta_w Z_\mu + \sin \Theta_w A_\mu)$$

$$+ ig' \frac{Y}{2} (-\sin \Theta_w Z_\mu + \cos \Theta_w A_\mu)$$

$$T_\pm = T_1 \pm iT_2 = \begin{cases} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} & \text{for } SU(2) \text{ doublets} \\ 0 & \text{---} \text{---} \text{ singlets} \end{cases}$$

coefficient of A_μ :

$$g T_3 \sin \Theta_w + g' \cos \Theta_w \frac{Y}{2}$$

$$= g \sin \Theta_w T_3 + g \frac{\sin \Theta_w}{\cos \Theta_w} \cancel{\cos \Theta_w} \frac{Y}{2}$$

$$= g \sin \Theta_w \left(T_3 + \frac{Y}{2} \right) = g \sin \Theta_w Q_{em}$$

$$g \sin \Theta_w = e$$

coefficient of Z_μ :

$$g \cos \theta_w T_3 - g' \sin \theta_w \frac{Y}{2}$$

$$= g \cos \theta_w T_3 - g \frac{\sin^2 \theta_w}{\cos \theta_w} \frac{Y}{2}$$

$$= g \cos \theta_w T_3 - g \frac{\sin^2 \theta_w}{\cos \theta_w} (Q_{em} - T_3)$$

$$= \frac{g}{\cos \theta_w} (T_3 - \sin^2 \theta_w Q_{em})$$

$$\rightarrow D_\mu = \partial_\mu + \frac{ig}{\sqrt{2}} (T_+ W_\mu^+ + T_- W_\mu^-)$$

$$+ \frac{ig}{\cos \theta_w} (T_3 - \sin^2 \theta_w Q_{em}) Z_\mu$$

$$+ ie Q_{em} A_\mu$$

unitary gauge: $\phi(x) \rightarrow \frac{1}{\sqrt{2}} (v + h(x)) \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$\langle 0 | h(x) | 0 \rangle = 0$$

$h(x)$ = Higgs boson of SM

Higgs - vector boson coupling:

$$\begin{aligned}
 D_\mu \phi &\xrightarrow{\text{unit. gauge}} \frac{1}{\sqrt{2}} D_\mu (v+h) \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \\
 &= \frac{1}{\sqrt{2}} \left[\partial_\mu h \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \frac{ig}{\sqrt{2}} W_\mu^+ \begin{pmatrix} 1 \\ 0 \end{pmatrix} (v+h) \right. \\
 &\quad \left. - \frac{ig}{2 \cos \Theta_w} Z_\mu \begin{pmatrix} 0 \\ 1 \end{pmatrix} (v+h) \right]
 \end{aligned}$$

$$L_\phi = (D_\mu \phi)^\dagger D^\mu \phi \rightarrow$$

$$\rightarrow \frac{1}{2} \partial_\mu h \partial^\mu h + M_W^2 W_\mu^+ W^{-\mu} \left(1 + \frac{h}{v}\right)^2$$

$$+ \frac{1}{2} M_Z^2 Z_\mu Z^\mu \left(1 + \frac{h}{v}\right)^2$$

Higgs search at LEP:

Higgs potential:

$$V(\phi) = -r \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2, \quad r, \lambda > 0$$

minima determined by $-r + 2\lambda\phi^\dagger\phi = 0$

$$\langle 0|\phi|0\rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \Rightarrow v^2 = \frac{r}{\lambda}$$

$$V(\phi) = \lambda \phi^\dagger\phi (-v^2 + \phi^\dagger\phi)$$

$$V\left(\frac{v+h}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right) = \lambda \frac{(v+h)^2}{2} \left(-v^2 + \frac{(v+h)^2}{2}\right)$$

$$= \frac{\lambda}{4} (v^2 + 2vh + h^2) (-v^2 + 2vh + h^2)$$

$$= \underbrace{-\frac{\lambda v^4}{4}}_{V_0} + \frac{\lambda}{4} (2vh + h^2)^2$$

$$= V_0 + \underbrace{\lambda v^2}_{M_h^2/2} h^2 \left(1 + \frac{h}{2v}\right)^2$$

$$M_h^2 = 2\lambda v^2$$

mass term and cubic and quartic self couplings of the Higgs field

$$M_h = 125.09 \pm 0.21 \pm 0.11 \text{ GeV}$$

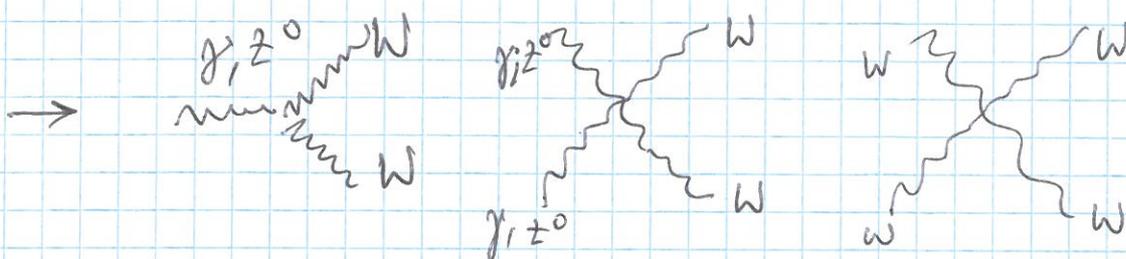
(PDG, 2016)

self couplings of the vector bosons (W^\pm, Z^0, γ)
from

$$-\frac{1}{4} F_{a\mu\nu} F_a^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}, \quad F_a^{\mu\nu} = \partial^\mu W_a^\nu - \partial^\nu W_a^\mu - g \varepsilon_{abc} W_b^\mu W_c^\nu$$

substitutions: $W_1 = \frac{W^+ + W^-}{\sqrt{2}}, \quad W_2 = i \frac{W^+ - W^-}{\sqrt{2}},$

$$W_3 = \cos\theta_w Z + \sin\theta_w A, \quad B = -\sin\theta_w Z + \cos\theta_w A$$



d) fermion masses

Yukawa couplings:

$$-\mathcal{L}_Y = \bar{q}_{Li} \phi d_{Rj} \Gamma_{ij} + \bar{q}_{Li} \tilde{\phi} u_{Rj} \Delta_{ij} + \bar{l}_i \phi l_{Rj} \gamma_{ij} + \text{h.c.}$$

$$\tilde{\phi} = i\tau_2 \phi^* = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} (\phi^+)^* \\ (\phi^0)^* \end{pmatrix} = \begin{pmatrix} (\phi^0)^* \\ -\phi^- \end{pmatrix}$$

$$\Rightarrow \langle 0 | \tilde{\Phi} | 0 \rangle = \frac{\nu}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\tilde{\Phi} = \frac{\nu + h}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (\text{unitary gauge})$$

$$\begin{aligned} \bar{e}_{Li} \frac{\nu}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} d_{Rj} \Gamma_{ij} &= \frac{\nu}{\sqrt{2}} \Gamma_{ij} (\bar{u}_{Li}, \bar{d}_{Li}) \begin{pmatrix} 0 \\ 1 \end{pmatrix} d_{Rj} = \\ &= \bar{d}_{Li} \underbrace{\frac{\nu}{\sqrt{2}} \Gamma_{ij}}_{(M_d)_{ij}} d_{Rj} \end{aligned}$$

$$M_d = \frac{\nu}{\sqrt{2}} \Gamma \quad \text{mass matrix of down-type quarks} \\ (d, s, b)$$

$$\begin{aligned} \bar{e}_{Li} \frac{\nu}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} u_{Rj} \Delta_{ij} &= (\bar{u}_{Li}, \bar{d}_{Li}) \begin{pmatrix} 1 \\ 0 \end{pmatrix} u_{Rj} \frac{\nu}{\sqrt{2}} \Delta_{ij} \\ &= \bar{u}_{Li} \underbrace{\frac{\nu}{\sqrt{2}} \Delta_{ij}}_{(M_u)_{ij}} u_{Rj} \end{aligned}$$

$$M_u = \frac{\nu}{\sqrt{2}} \Delta \quad \text{mass matrix of up-type quarks} \\ (u, c, t)$$

$$\bar{l}_i \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} l_{Rj} \gamma_{ij} = (\bar{\nu}_{Li}, \bar{l}_{Li}) \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} l_{Rj} \gamma_{ij}$$

$$= \bar{l}_{Li} \underbrace{\frac{v}{\sqrt{2}} \gamma_{ij}}_{(M_e)_{ij}} l_{Rj}$$

$M_e = \frac{v}{\sqrt{2}} \gamma$ mass matrix of (charged) leptons

M arbitrary complex $n \times n$ matrix $\Rightarrow \exists$ unitary $n \times n$ matrices U_L, U_R such that $U_L^\dagger M U_R = \hat{M}$ diagonal and nonnegative (proof: $M = R U$,

$$R^\dagger = R, R \geq 0, U^\dagger = U^{-1} \Rightarrow R = U_L^\dagger \hat{M} U_L$$

$$\Rightarrow M = U_L^\dagger \hat{M} \underbrace{U_L U}_U$$

$$u_{L,R} = U_{L,R}^u u_{L,R}', \quad d_{L,R} = U_{L,R}^d d_{L,R}', \quad l_{L,R} = U_{L,R}^l l_{L,R}'$$

$$- \mathcal{L}_Y = \bar{d}'_L \hat{M}_d d'_R \left(1 + \frac{h}{v}\right) + \bar{u}'_R \hat{M}_u u'_R \left(1 + \frac{h}{v}\right) + \bar{l}'_L \hat{M}_e l'_R \left(1 + \frac{h}{v}\right) + h.c.$$

$$\begin{aligned} \bar{f}_L \hat{M} f_R + \text{h.c.} &= \bar{f}_L \hat{M} f_R + \bar{f}_R \hat{M} f_L = \\ &= \overline{(f_L + f_R)} \hat{M} \underbrace{(f_L + f_R)}_{=: f} = \bar{f} \hat{M} f \end{aligned}$$

$$(\bar{f}_L f_R = 0)$$

$$\Rightarrow -\mathcal{L}_Y = -\sum_f m_f \bar{f} f \left(1 + \frac{h}{v}\right)$$

$f = u_i', d_i', l_i' \quad (i=1,2,3)$ physical fields

remark: chiral fields = fundamental fermionic building blocks (left- and right-handed fields in different multiplets); but: after SSB and diagonalization of mass matrices \rightarrow
 \rightarrow Dirac fields for physical particles

remark: neutrinos massless in minimal version of standard model \rightarrow clearly incomplete description of the real world \rightarrow extension of SM necessary

e) charged current

$$\mathcal{L}_{cc} = - \frac{g}{\sqrt{2}} \sum_{\psi} \bar{\psi} \gamma^{\mu} (T_{+} W_{\mu}^{+} + T_{-} W_{\mu}^{-}) \psi$$

sum runs over all fermion multiplets (singlets do not contribute)

$$- \mathcal{L}_{cc} = \frac{g}{\sqrt{2}} (\bar{u}_L \gamma^{\mu} d_L + \bar{\nu}_L \gamma^{\mu} \ell_L) W_{\mu}^{+} + \text{h.c.}$$

→ physical fields $u_L = U_L^u u_L'$, $d_L = U_L^d d_L'$

$$\rightarrow \bar{u}_L \gamma^{\mu} d_L = \bar{u}_L' \gamma^{\mu} \underbrace{U_L^{u\dagger} U_L^d}_{V} d_L'$$

$V = U_L^{u\dagger} U_L^d$ Cabibbo - Kobayashi - Maskawa matrix

(quark mixing matrix)

leptons: $\ell_L = U_L^{\ell} \ell_L'$

for massless neutrinos one defines $\nu_L = U_L^{\nu} \nu_L'$

⇒ $\bar{\nu}_L \gamma^{\mu} \ell_L = \bar{\nu}_L' \gamma^{\mu} \ell_L'$ no mixing matrix for massless ν_s

experiment: ν -oscillations observed → ν_s have different masses

$$\rightarrow -\mathcal{L}_{cc} = \frac{g}{2\sqrt{2}} \left[\bar{u}' \gamma^\mu (1-\gamma_5) V d' + \bar{\nu}' \gamma^\mu (1-\gamma_5) l' \right] W_\mu^+ + \text{h.c.}$$

f) neutral current

$$-\mathcal{L}_{NC} = \frac{g}{\cos\theta_W} \sum_\Psi \bar{\Psi} \gamma^\mu (T_3 - \sin^2\theta_W Q_{em}) \Psi Z_\mu$$

(now also contributions from SU(2) singlets)

$$\begin{aligned} -\mathcal{L}_{NC} = & \frac{g}{\cos\theta_W} \left[\bar{u}_L \gamma^\mu u_L \left(\frac{1}{2} - \sin^2\theta_W \frac{2}{3} \right) + \right. \\ & + \bar{d}_L \gamma^\mu d_L \left(-\frac{1}{2} + \sin^2\theta_W \frac{1}{3} \right) + \bar{l}_L \gamma^\mu l_L \left(-\frac{1}{2} + \sin^2\theta_W \right) \\ & + \bar{\nu}_L \gamma^\mu \nu_L \frac{1}{2} + \bar{u}_R \gamma^\mu u_R \left(-\sin^2\theta_W \frac{2}{3} \right) \\ & \left. + \bar{d}_R \gamma^\mu d_R \left(\sin^2\theta_W \frac{1}{3} \right) + \bar{l}_R \gamma^\mu l_R \sin^2\theta_W \right] Z_\mu \end{aligned}$$

\rightarrow physical fields : $\bar{u}_L \gamma^\mu u_L = \bar{u}'_L \gamma^\mu u'_L$, etc.

GIM mechanism (Glashow - Iliopoulos - Maiani)

neutral currents do not change flavour

experiment: no flavour changing neutral currents (FCNC) observed (so far)

$$- \mathcal{L}_{NC} = \frac{g}{2 \cos \Theta_W} \sum_f \bar{f} \gamma^\mu (a_f - b_f \gamma_5) f Z_\mu$$

$$a_f = t_3^L f - 2 \sin^2 \Theta_W Q_f$$

$$b_f = t_3^L f$$

$t_3^L f$ = eigenvalue of T_3 in $SU(2)$ doublet

Q_f = charge of f

	a_f	b_f
u	$\frac{1}{2} - \frac{4}{3} \sin^2 \Theta_W$	$\frac{1}{2}$
d	$-\frac{1}{2} + \frac{2}{3} \sin^2 \Theta_W$	$-\frac{1}{2}$
ν	$\frac{1}{2}$	$\frac{1}{2}$
l	$-\frac{1}{2} + 2 \sin^2 \Theta_W$	$-\frac{1}{2}$

g) electromagnetic current

$$- \mathcal{L}_{em} = e \sum_f Q_f \bar{f} \gamma^\mu f A_\mu$$

remark: $U_{L,R}^{u,d,l}$ cancel (like in \mathcal{L}_{NC})

h) V-A theory

$$- \mathcal{L}_{cc} = \frac{g}{2\sqrt{2}} (\bar{u}' \gamma^\mu (1-\gamma_5) V d' + \bar{\nu}' \gamma^\mu (1-\gamma_5) l') W_\mu^+ + \text{h.c.}$$

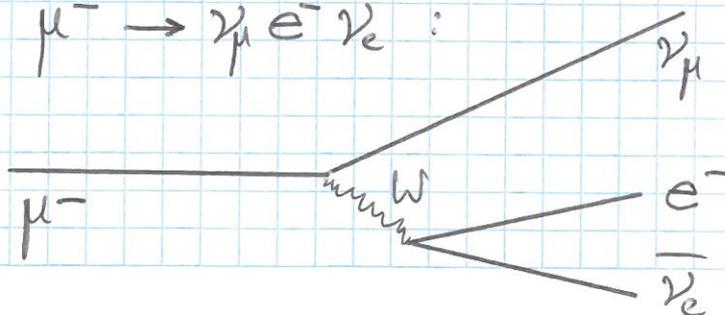
$V^\mu := \bar{u}' \gamma^\mu V d' + \bar{\nu}' \gamma^\mu l'$ (charged) vector current

$A^\mu := \bar{u}' \gamma^\mu \gamma_5 V d' + \bar{\nu}' \gamma^\mu \gamma_5 l'$ (charged) axial vector current

$$\Rightarrow - \mathcal{L}_{cc} = \frac{g}{2\sqrt{2}} [(V^\mu - A^\mu) W_\mu^+ + (V^\mu - A^\mu)^\dagger W_\mu^-]$$

describes for instance $\mu^- \rightarrow \nu_\mu e^- \bar{\nu}_e$:

muon decay



low-energy effective theory (energies $\ll M_W$)

"integrate out" W (heavy degree of freedom)

$$W \text{ propagator} \quad \int \frac{d^4 k}{(2\pi)^4} e^{-ik(x-y)} i \frac{-g_{\mu\nu} + \frac{k_\mu k_\nu}{M_W^2}}{k^2 - M_W^2}$$

$$\xrightarrow{|R| \ll M_W} \frac{i g_{\mu\nu}}{M_W^2} \delta^{(4)}(x-y) \quad \text{point interaction}$$

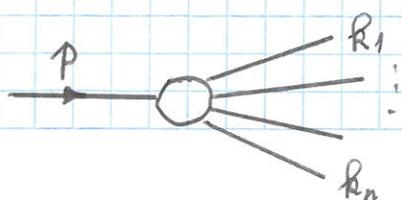
$$\xrightarrow{} \mathcal{L}_{V-A} = - \frac{G_F}{\sqrt{2}} (V^\mu - A^\mu) (V_\mu - A_\mu)^\dagger$$

$$\text{Fermi coupling constant} \quad \frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

$$G_F = 1.166\,378\,7(6) \times 10^{-5} \text{ GeV}^{-2} \quad (\text{PDG 2016})$$

remark: computation of decay width

$$d\Gamma = \frac{S}{2M} |M_{fi}|^2 \frac{d^3 k_1}{(2\pi)^3 2E_1} \dots \frac{d^3 k_n}{(2\pi)^3 2E_n} (2\pi)^4 \delta^{(4)}\left(p - \sum_{i=1}^n k_i\right)$$

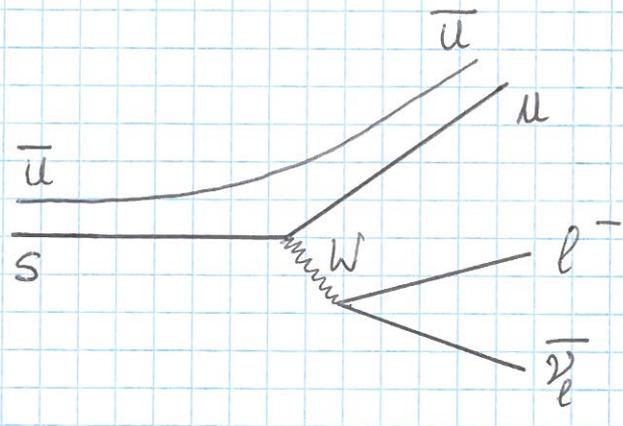


$$p^2 = M^2, \quad S = \text{statistical factor}$$

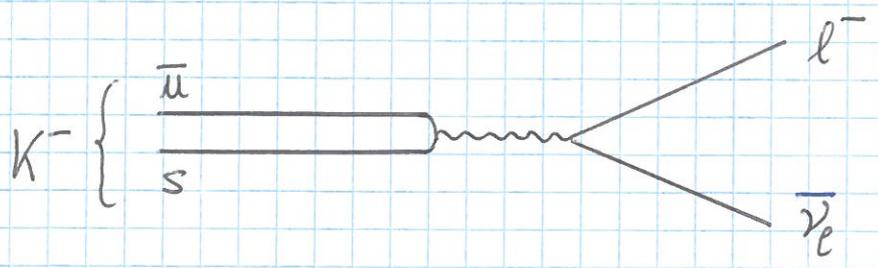
$L_{V-A} \rightarrow$ weak decays of μ, τ, quarks

example: $s \rightarrow u l^- \bar{\nu}_l$ ($l=e, \mu$)

amplitude $\sim V_{us}$



$K^- \rightarrow \pi^0 l^- \bar{\nu}_l$ (K_{l3} decay)



$K^- \rightarrow l^- \bar{\nu}_l$ (K_{l2} decay)

"semileptonic" decays

$$c \rightarrow \begin{matrix} d \\ s \end{matrix} l^+ \nu_l \quad (u\bar{d}, u\bar{s})$$

$$l = e, \mu, \tau$$

$$b \rightarrow \begin{matrix} c \\ u \end{matrix} l^- \bar{\nu}_l \quad (\bar{u}d, \bar{u}s, \bar{c}d, \bar{c}s)$$

amplitude $(b \rightarrow c l^- \bar{\nu}_l) \sim V_{cb}$

-||- $(b \rightarrow c \bar{u} s) \sim V_{cb} V_{us}^*$

remark: $\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} \stackrel{e=g\sin\theta_W}{=} \frac{\pi\alpha}{2\sin^2\theta_W M_W^2}$

$$\longrightarrow M_W = \frac{37.3 \text{ GeV}}{\sin\theta_W}$$

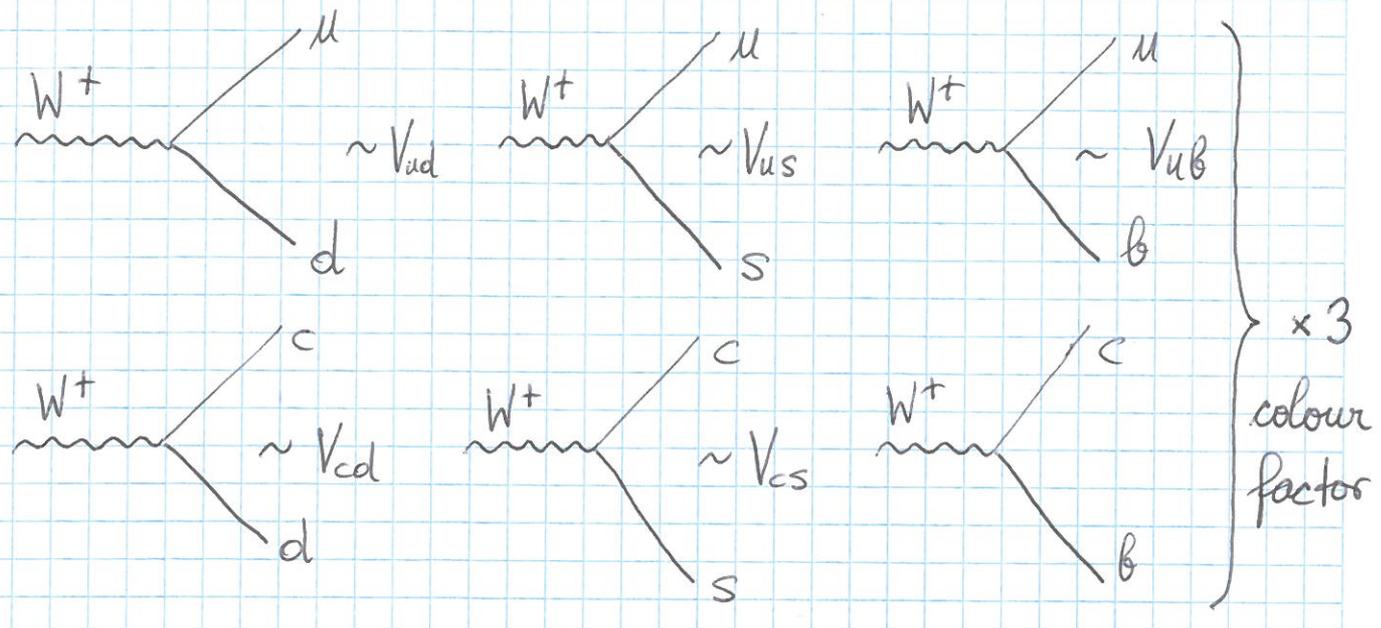
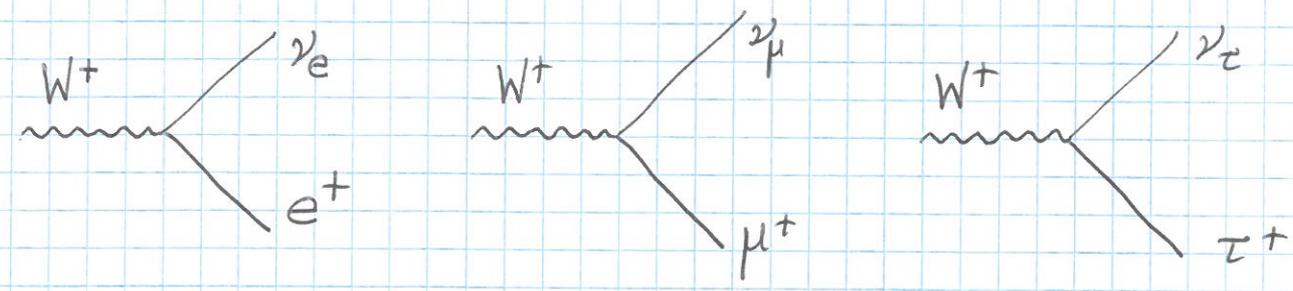
NC experiments before discovery of W^\pm : $\sin^2\theta_W \approx 0.23$

\longrightarrow prediction $M_W \approx 78 \text{ GeV}$

current value (PDG 2016): $M_W = 80.385 \pm 0.015 \text{ GeV}$

(radiative corrections!)

exercise: compute the widths of the following decay modes of the W boson:



remark:
$$\sum_{\lambda=1}^3 \epsilon^\mu(\vec{k}, \lambda) \epsilon^\nu(\vec{k}, \lambda) = -g^{\mu\nu} + \frac{k^\mu k^\nu}{M^2}$$

for massive vector bosons

i) Cabibbo - Kobayashi - Maskawa mixing matrix

number of independent (observable) parameters in V :

n_G = number of generations

V : n_G -dimensional unitary matrix $\rightarrow n_G^2$ real parameters (write $V = \exp(iA)$, $A = A^\dagger \rightarrow$

A has $n_G + \frac{2(n_G^2 - n_G)}{2} = n_G^2$ real parameters)

$\binom{n_G}{2} = \frac{n_G(n_G-1)}{2}$ of these parameters are

angles (generalized Euler angles parametrizing

$SO(n_G)$: $SO(n_G)$ matrix = $\mathbb{1} + \Omega + \dots$, $\Omega^T = -\Omega$

$\rightarrow \frac{n_G^2 - n_G}{2}$ real parameters)

\rightarrow remaining $n_G^2 - \frac{n_G(n_G-1)}{2} = \frac{n_G(n_G+1)}{2}$

parameters are phases

however: define diagonal matrices

$$e^{i\alpha} = \begin{pmatrix} e^{i\alpha_1} & & \\ & e^{i\alpha_2} & \\ & & e^{i\alpha_3} \end{pmatrix}, \quad e^{i\beta} = \begin{pmatrix} e^{i\beta_1} & & \\ & e^{i\beta_2} & \\ & & e^{i\beta_3} \end{pmatrix}$$

$$u' \rightarrow e^{i\alpha} u', \quad d' \rightarrow e^{i\beta} d'$$

(same transformation for L, R) \rightarrow L_Y, L_{NC}, L_{em}
remain unchanged, but

$$V \rightarrow e^{-i\alpha} V e^{i\beta} \quad \text{in } L_{CC}$$

\rightarrow $2n_G - 1$ phases can be absorbed by

this transformation \rightarrow

$$\rightarrow \frac{n_G(n_G+1)}{2} - (2n_G - 1) = \frac{(n_G-1)(n_G-2)}{2}$$

measurable phases

$n_G = 2$:

$$V = \begin{pmatrix} \cos \theta_c & \sin \theta_c \\ -\sin \theta_c & \cos \theta_c \end{pmatrix}$$

 $\theta_c =$ Cabibbo angleno observable phase $n_G = 3$:3 angles, 1 phase \rightarrow only

mechanism of (minimal) SM for CP violation

 \rightarrow all CP violating observables $\sim \sin \delta_{KM}$
(Kobayashi - Maskawa 1973)exercise:

study the possible parametrizations

of the CKM matrix shown in PDG 2014, p.214

remark:"hard" CP violation in the SM

(in contrast to spontaneous CP violation)

want to show: mixing matrix V can be source of
CP-violation for $n_G \geq 3$:

Particle Physics I: $\psi(x^0, \vec{x}) \xrightarrow{P} \gamma^0 \psi(x^0, -\vec{x})$

$$\psi(x^0, \vec{x}) \xrightarrow{C} C \gamma^{0T} \psi^*(x^0, \vec{x})$$

$$C = -i \gamma^2 \gamma^0$$

properties of charge-conjugation matrix C :

$$C \gamma_\mu^T C^{-1} = -\gamma_\mu, \quad C^T = -C, \quad C^\dagger = C^{-1}$$

$$(\Rightarrow C \gamma_5^T C^{-1} = \gamma_5)$$

$$\Rightarrow \psi(x^0, \vec{x}) \xrightarrow{CP} \underbrace{C \gamma^{0T} \gamma^{0*}}_{\mathbb{1}} \psi^*(x^0, -\vec{x}) = C \psi^*(x^0, -\vec{x})$$

(up to a phase factor)

$$\rightarrow \text{we define: } u'(x) \xrightarrow{CP} e^{i\alpha_u} C u'^*(\tilde{x})$$

$$d'(x) \xrightarrow{CP} e^{i\alpha_d} C d'^*(\tilde{x})$$

$\tilde{x} = (x^0, -\vec{x})$, $e^{i\alpha_u}$, $e^{i\alpha_d}$ diagonal phase matrices

\rightarrow mass terms and kinetic terms invariant

$$u' \gamma^\mu (1-\gamma_5) V d' \xrightarrow{CP}$$

$$(e^{i\alpha_u} C u'^*)^\dagger \gamma^0 \gamma^\mu (1-\gamma_5) V e^{i\alpha_d} C d'^*$$

$$= u'^T C^\dagger e^{-i\alpha_u} \gamma^0 \gamma^\mu (1-\gamma_5) V e^{i\alpha_d} C d'^*$$

$$= u'^T \gamma^0 \gamma^{\mu T} (1-\gamma_5^T) e^{-i\alpha_u} V e^{i\alpha_d} d'^*$$

$$= u'_\alpha [(1-\gamma_5) \gamma^\mu \gamma^0]_{\beta\alpha} (e^{-i\alpha_u} V e^{i\alpha_d})_{ij} d'_{j\beta}{}^*$$

$$= - d'_{j\beta}{}^* [\underbrace{\gamma^\mu \gamma^0 (1-\gamma_5)}_{\varepsilon(\mu) \gamma^0 \gamma^\mu}]_{\beta\alpha} (e^{-i\alpha_u} V e^{i\alpha_d})_{ji}^T u'_\alpha$$

↑
anti-comm. fields!

$\varepsilon(0) = 1, \varepsilon(i) = -1$

$$= - \varepsilon(\mu) \bar{d}' \gamma^\mu (1-\gamma_5) (e^{-i\alpha_u} V e^{i\alpha_d})^T u'$$

remember:

$$- \mathcal{L}_{V-A}^{\text{quarks}} = \frac{G_F}{\sqrt{2}} [\bar{u}' \gamma^\mu (1-\gamma_5) V d'] [\bar{d}' \gamma^\mu (1-\gamma_5) V^T u']$$

CP invariance of $L_{V-A}^{\text{quarks}} \Leftrightarrow \exists \alpha_u, \alpha_d$

$$\text{with } (e^{-i\alpha_u} V e^{i\alpha_d})^T = V^\dagger$$

$$\Leftrightarrow e^{-i\alpha_u} V e^{i\alpha_d} = V^*$$

$\Rightarrow \exists$ phase-convention in which
CKM matrix real:

$$\begin{aligned} e^{-i\frac{\alpha_u}{2}} V e^{i\frac{\alpha_d}{2}} &= e^{i\frac{\alpha_u}{2}} V^* e^{-i\frac{\alpha_d}{2}} \\ &= (e^{-i\frac{\alpha_u}{2}} V e^{i\frac{\alpha_d}{2}})^* \end{aligned}$$

however, for arbitrary complex Yukawa
couplings $\Gamma, \Delta \rightarrow \exists$ physical phase
for $n_g \geq 3 \Rightarrow \cancel{CP}$ for $n_g \geq 3$

j) effective four-Fermi-interaction for neutral currents

$$- \mathcal{L}_{NC} = \frac{g}{2 \cos \Theta_W} \underbrace{\sum_f \bar{f} \gamma^\mu (a_f - b_f \gamma_5) f}_{J_{NC}^\mu} Z_\mu$$

"integrate" out Z^0 :

$$\rightarrow \mathcal{L}_{NC}^{\text{eff}} = - \frac{g^2}{8 M_Z^2 \cos^2 \Theta_W} J_{NC}^\mu J_\mu^{NC}$$

$$M_W = M_Z \cos \Theta_W$$

$$\rightarrow \mathcal{L}_{NC}^{\text{eff}} = - \underbrace{\frac{g^2}{8 M_W^2}}_{\frac{G_F}{\sqrt{2}}} J_{NC}^\mu J_\mu^{NC}$$

$$M_Z = \frac{M_W}{\cos \Theta_W} = \frac{37.3 \text{ GeV}}{\sin \Theta_W \cos \Theta_W} \approx 89 \text{ GeV}$$

current experimental value (PDG 2016): $M_Z = 91.1876 \pm 0.0021 \text{ GeV}$

(loop corrections!)

decay modes of the Z^0 :

visible: $Z^0 \rightarrow l^+ l^-, q \bar{q}$ (jets)

invisible: $Z^0 \rightarrow \nu_l \bar{\nu}_l$ ($l = e, \mu, \tau$)

exercise: $\Gamma(Z^0 \rightarrow \nu_e \bar{\nu}_e) = \frac{G_F M_Z^3}{12\sqrt{2}\pi} = 166 \text{ MeV}$

$$\Gamma_{\text{tot}} - \Gamma_{\text{visible}} = \Gamma_{\text{invisible}} = 499.0 \pm 1.5 \text{ MeV} \quad (\text{PDG 2008})$$

$\rightarrow N_\nu = 2.984 \pm 0.008$ light neutrino types

(PDG 2008)

(with SM couplings to Z^0)

R) number of parameters of the (minimal) SM

$\hat{M}_{u,d,l}$ 9 fermion masses (electroweak sector only)

V 3 angles, 1 phase

g, g' 2 gauge couplings (alternatively: α, Θ_w)

v VEV

M_h Higgs mass

→ 17 parameters in the electroweak sector
of the minimal SM

extension of the SM: additional (free?)

parameters: neutrino masses, neutrino mixing angles,

CP-violation in the lepton sector (?), ... (?)

2. Quantum Chromodynamics (QCD)

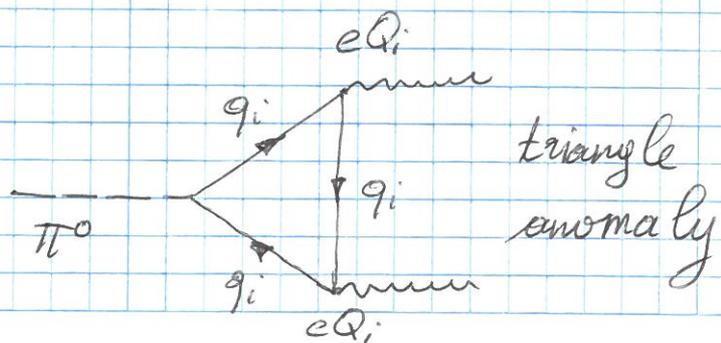
a few arguments in favour of three colour
degrees of freedom

$$a) \quad R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \rightarrow N_c = 3$$

(see p. 9/11)

$$b) \quad \pi^0 \rightarrow 2\gamma$$

chiral anomaly



$$\Gamma(\pi^0 \rightarrow 2\gamma) = \frac{\alpha^2 M_\pi^3}{16\pi^3 F_\pi^3} N_c^2 S$$

F_π = pion decay constant = 92.2 MeV

from $\pi^\pm \rightarrow l^\pm \bar{\nu}_l$

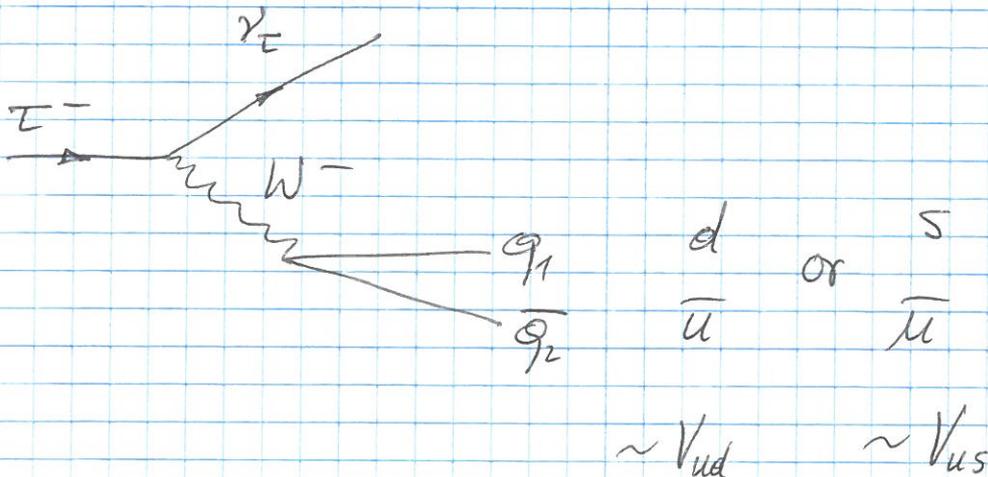
$$S = \sum_{q \in \pi^0} (I_3)_q Q_q^2 = \frac{1}{2} \left(\frac{2}{3}\right)^2 - \frac{1}{2} \left(-\frac{1}{3}\right)^2 = \frac{1}{6}$$

$$\Gamma(\pi^0 \rightarrow 2\gamma) = 0.859 N_c^2 \text{ eV}$$

experiment: $\Gamma(\pi^0 \rightarrow 2\gamma) = (7.74 \pm 0.60) \text{ eV}$

$$\rightarrow N_c^2 = 9.0 \pm 0.7$$

c) $\tau \rightarrow \nu_\tau + \text{hadrons}$



remark: c cannot be produced

$$(m_{\tau} = 1776.82 \text{ MeV}, \quad m_{D^{\pm}} = 1869.61 \text{ MeV})$$

(PDG 2014)

$$m_{D^0} = 1864.84 \text{ MeV})$$

$$D^+ = c\bar{d}, \quad D^0 = c\bar{u}$$

$$R_{\tau} = \frac{\Gamma(\tau^- \rightarrow \nu_{\tau} + \text{hadrons})}{\Gamma(\tau^- \rightarrow \nu_{\tau} \ell^- \bar{\nu}_{\ell})} \simeq N_c$$

$$\text{exp.: } R_{\tau} = 3.645 \pm 0.020 \quad (\text{PDG 2008})$$

$$\text{QCD corrections: } R_{\tau} = N_c (1 + \mathcal{O}(g_s^2))$$

→ allows determination of g_s

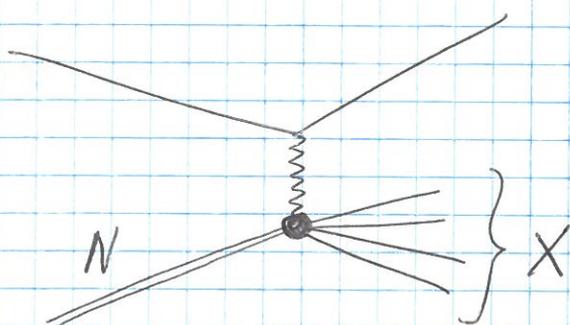
d) consistent quantization of SM

absence of anomalies → $N_c = 3$

(cancellation of quark- and lepton-contributions to the anomaly)

e) energy-momentum balance of "partons" in deep-inelastic scattering at high energies

$l + N \rightarrow l + X$	elm./neutral	} current	γ, Z^0
$\nu_e + N \rightarrow l + X$	charged weak		W^\pm
$\nu_e + N \rightarrow \nu_e + X$	neutral weak		Z^0



exp.: $\sim 1/2$ of partons in nucleon N does not feel electroweak interaction \rightarrow gluons

gauging of colour degree of freedom \rightarrow QCD

\rightarrow standard model $L_{SM} = L_{ew} + L_{QCD}$

in addition to the 17 parameters of $L_{ew} \rightarrow$

$\rightarrow g_s$ (strong coupling constant)

θ_{QCD} (strong CP violation)

\rightarrow 19 parameters in the minimal SM