Exercises to QM2, Summer Term 2018, Sheet 3

1) Optical theorem and Neumann series

Consider the scattering of a spinless particle off the stationary potential V. Show that when one applies the optical theorem to the Neuman series, one obtains the relation

$$\sigma_{\text{tot}}^{\text{Born approx.}} = \frac{4\pi}{k} \,\text{Im} \left[f(\theta = 0)|^{2^{\text{nd}}\text{order in V}} \right] .$$

This means that the optical theorem relates different orders of the perturbation seires to each other.

Hint: Use the momentum space Feynman rules discussed in class and the relation from exercise (3) on sheet 2.

2) Scattering off the Yukawa potential

Determine the Borm approximation of the differential and total cross section for the scattering of a spinless particle off the so-called Yukawa potential, which has the form

$$V(r) = \frac{V_0}{r}\, e^{-mr} .$$

(Which physical interpretation may the parameter m have?) Derive from the result the Rutherford's differential cross section formula for Coulomb potential scattering ($V(r) = 1/(4\pi)\,(q_1 q_2)/r$). Compare to the classical scattering formula.

3) Form factors

Consider elastic scattering of a spinless particle in Born approximation off a target that actually consists of n separate fixed scattering centres at locations $\mathbf{x}_n$ each having the same potential V. These are, however, so close to each other that one can consider them as one single target. Show that the resulting differential cross section has the form

$$\frac{d\sigma}{d\Omega}(\theta, \phi) = \frac{d\sigma}{d\Omega}(\theta, \phi)\bigg|_0 \, |F(\mathbf{q})|^2 ,$$

where $d\sigma/d\Omega|_0$ is the differential cross section of a single scattering center and $\mathbf{q} = \mathbf{k}_f - \mathbf{k}_i$ is the momentum transfer of the scattered particle. Determine the *form factor* $F(\mathbf{q})$ for the case that that particle beam is going in positive z direction and the target consists of two scattering centres located at $\mathbf{x}_1 = (r_0/2, 0, 0)$ und $\mathbf{x}_2 = (-r_0/2, 0, 0)$. Compare the result to the case where the scattering centres located at $\mathbf{x}_1 = (0, 0, r_0/2)$ und $\mathbf{x}_2 = (0, 0, -r_0/2)$.

4) "Minus one" row method**

The following Hermitan matrix

$$a = \begin{pmatrix} 1 & i & 1 & i & 1 \\ -i & 2 & i & 1 & i \\ 1 & -i & 1 & i & 1 \\ -i & 1 & -i & 0 & i \\ 1 & -i & 1 & -i & 0 \end{pmatrix} \tag{1}$$

has the eivenvalues -2, -1, $2-\sqrt{2}$, $2+\sqrt{2}$, 3. Determine the eivenvectors by hand.

Hint: This exercise is **not** meant that you start doing this brute force solving for 5 linear equations for each eivenvalue. Of course you may do so, but this is the best way to kill off anyone. The"minus one" row method is based on matrix manipulation and the most efficient method to get the solutions in a clean and swift manner. It will teach you that, sometimes, using the right notation, can make a huge difference in efficiency. We will discuss the method in class.