Exercises for QM2, Summer term 2018, Sheet 1

1) Natural units Express the following quantities in the elementary particle physics natural units (i.e. in proper eV units using $\hbar = c = 1$): atomic radius (1 $\mathring{A}$), nucleon radius (1 fm = typical size of atomic nuclei), graviational acceleration at the earths surface.
Determine, in natural units the inverse life times (also called decay width $\Gamma = \tau^{-1}$) of the neutron (n) and the muon (μ^-). Compare the results with the masses of these particles in natural units.

2) Hermitian operators
Construct the quantum mechanical operator to the classic observable $\vec{\mathbf{x}} \cdot \vec{\mathbf{p}}$ (location times momentum). Construct the quantum mechanical operator to the spatial angular momentum $\vec{\mathbf{L}} = \vec{\mathbf{x}} \times \vec{\mathbf{p}}$. What is different in the second case?

3) Repeated measurements
Given is a system with the time-independent Hamilton operators H, where H has only eigenstates with non-degenerate energy eigenvalues:

$$H|\nu\rangle = \omega_\nu |\nu\rangle \, . \tag{1}$$

Given is also the observable A defined in the same Hilbert space which also has only eigenstates with non-degenerate eigenvalues:

$$A|n\rangle = a_n |n\rangle \, . \tag{2}$$

The system is at the beginning in the state $|\nu\rangle$ and one measures observables A. What is the expectation value for the measurement of A, when you make the measurements on many equivalent systems being in the state $|\nu\rangle$? What is the probability to get the value a_m?

Assume that the measurement has yielded the value a_m. In which state is the system immediately after the measurement? What is the probablility that the energy of the particle at this time is still ω_ν (if you would make an energy measurement)? For which case would this probability be exactly 100%? Assume that the probability would not be 100%: Is there a problem with energy conservation?

Let the system after the measurement of value a_m now evolve freely for the time span t and then make another measurment of A. With which probablity do you obtain once again the value a_m. For which case is this probability 100%?

4) Schrödinger equation in momentum space representation
Start from the abstract (i.e. representation independent) Schrödinger equation for a spinless particle with mass μ in a Coulomb potential $V_c(r) = -\frac{\alpha}{r}$ and derive the corresponding momentum space Schrödinger equation.

Hint: Use the Dirac notation involving the eigenstates of the $\vec{X}$ and $\vec{P}$ operators to derive the ansatz for the required calculations. For the determination of the Coulomb potential V_c in momentum space it is useful to do the calculation with the Yukawa potential $V(r) = -\frac{\alpha}{r}e^{-mr}$ $(m > 0)$ and take the limit $m \to 0$ afterwards.

5) Classic potential scattering*

Consider classic potential scattering of a particle with kinetic energy E and mass m off a repulsive potential $V = V(r)$ that only dependes on the radius r. Use the notations and polar coordinates where the origin is the center of the potential as a given in the sketch attached. Use energy and angular momentum conservation to show that the relation between the angle of closest approach ϕ_m and the closest distance to the center r_m reads

$$\phi_m = b \int_0^{1/r_m} \frac{\mathrm{d}u}{\sqrt{1 - b^2u^2 - \frac{V}{E}}}\,, \tag{3}$$

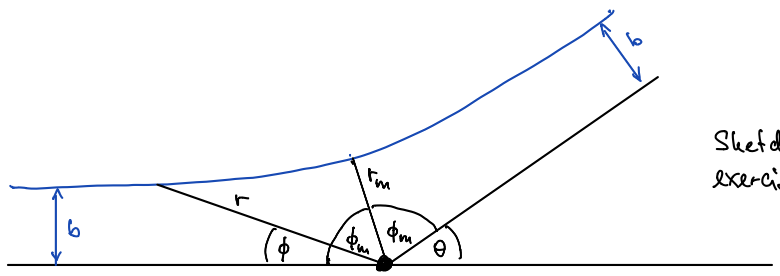
with $u = 1/r$. Which equation does r_m have to satisfy and explain the reason. This calculation from classical mechanics is a classic and can be found in many books.

Hint: To start one should realize that the scattering trajectory of the particle is fully embedded in the so-called scattering plane which also contains the center of the potential. So it is best to use cylindrical coordinates in the scattering plane w.r.t. to the center of the potential using ϕ as polar angle. Write down the velocity of the scattered particle that is located at the point $\vec{x}(r(t), \phi(t)) = r(t)\,\hat{n}_r$ noting that its differential has the form $d\vec{x}(r, \phi) = dr\,\hat{n}_r + rd\phi\,\hat{n}_\phi$, where $\hat{n}_r$ and $\hat{n}_\phi$ are the unit vectors in r and ϕ directions, respectively, at the point $\vec{x}$. One can now write down the expressions for the concerved angular momentum $\vec{L}$ and the total energy E.

6) Rutherford scattering

Derive from the integral in Eq. (3) the relation between the scattering angle θ and the impact parameter b for Coulomb scattering discussed in class. Use the sketch to relate ϕ_m to the scattering angle θ and determine r_m from the constraint that the denominator in the integrant vanishes.

The star exercise will be discussed during the lecture by the instructor and does not have to be prepared.



Sketch to exercise (5)