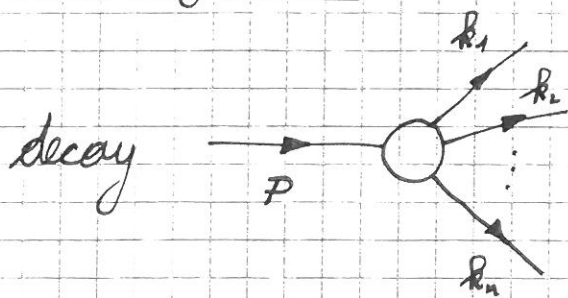


20. Decay width

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$$P = k_1 + k_2 + \dots + k_n$$

in rest frame of decaying particle: $P = (M, \vec{0})$

time evolution operator in IP

$$\int_B d\mu(k_1) \dots d\mu(k_n) \left| \langle k_1, \dots, k_n | U(t_f, t_i) | \psi \rangle \right|^2$$

= probability for decay of a particle (described by state vector $|\psi\rangle$) into n particles in phase-space region B within the time interval $[t_i, t_f]$

remarks:

- i) discussion for spin 0 particles (gen. to higher spin trivial)
- ii) final state particles assumed to be distinguishable
(generalisation: $\times \frac{1}{n_1!} \frac{1}{n_2!} \dots \frac{1}{n_p!}$)
- iii) momentum distribution of initial-state wave function peaked around $\vec{p} = 0$
- iv) time interval $T = t_f - t_i$ microscopically large, but macroscopically small

iv) $\Rightarrow$ study first "limit" $T \rightarrow \infty$, come back to (realistic) finite T later

$$\int_B d\mu(k_1) \dots d\mu(k_n) |\langle k_1, \dots, k_n | S | \psi \rangle|^2$$

$$= \int_B d\mu(k_1) \dots d\mu(k_n) \langle \psi | S^\dagger | k_1, \dots, k_n \rangle \langle k_1, \dots, k_n | S | \psi \rangle$$

$$= \int_B d\mu(k_1) \dots d\mu(k_n) \int d\mu(p) d\mu(p') \overbrace{\langle \psi | p' \rangle}^{\psi(p')^*} \langle p' | S^\dagger | k_1, \dots, k_n \rangle$$

$$\langle k_1, \dots, k_n | S | p \rangle \underbrace{\langle p | \psi \rangle}_{\psi(p)}$$

$$= \int_B d\mu(k_1) \dots d\mu(k_n) \int d\mu(p) d\mu(p') \psi(p')^* \psi(p)$$

$$(2\pi)^4 \delta^{(4)}(p' - k_1 - \dots - k_n) M(p' \rightarrow k_1, \dots, k_n)^*$$

$$(2\pi)^4 \delta^{(4)}(p - k_1 - \dots - k_n) M(p \rightarrow k_1, \dots, k_n)$$

$$= \int_B d\mu(k_1) \dots d\mu(k_n) \int d\mu(p) |\psi(p)|^2 \frac{1}{(2\pi)^3 2p^0}$$

$$(2\pi)^4 \delta(p^0 - k_1^0 - \dots - k_n^0)$$

$$(2\pi)^4 \delta^{(4)}(p - k_1 - \dots - k_n) |M(p \rightarrow k_1, \dots, k_n)|^2 = (*)$$

$$\begin{aligned} \delta(p^0 - k_1^0 - \dots - k_n^0) &= \int_{-\infty}^{+\infty} \frac{dt}{2\pi} e^{it(p^0 - k_1^0 - \dots - k_n^0)} \\ &= \int_{t_i}^{t_f} \frac{dt}{2\pi} e^{it \times 0} = \frac{t_f - t_i}{2\pi} = \frac{T}{2\pi} \end{aligned}$$

$$\begin{aligned} \Rightarrow (*) &\approx T \int_B d\mu(k_1) \dots d\mu(k_n) \underbrace{\int d\mu(p) |\psi(p)|^2}_1 \frac{1}{2M} \\ &\times (2\pi)^4 \delta^{(4)}(P - k_1 - \dots - k_n) |M(P \rightarrow k_1, \dots, k_n)|^2 \end{aligned}$$

assumption in last step: $M(p \rightarrow k_1, \dots, k_n)$ varying slowly in region $p \simeq P$ where $\psi(p)$ gives nonvanishing contribution

result:

$$\Gamma(P \rightarrow B) = \frac{\text{transition probability}}{T}$$

$$= \int d\mu(k_1) \dots d\mu(k_n) \frac{1}{2M} (2\pi)^4 \delta^{(4)}(P - k_1 - \dots - k_n) |N(P \rightarrow k_1, \dots, k_n)|^2$$

(partial) decay width

total decay width $\Gamma = \sum_f \Gamma_f$

sum over all decay modes

exponential decay $N(t) = N(0) e^{-\Gamma t}$

life time $\tau = 1/\Gamma$