19. Standard model of particle physics

spontaneously broken gauge theory with gauge group

$$\underbrace{SU(3)_c}_{\text{strong}} \times \underbrace{SU(2)_L \times U(1)_Y}_{\text{electroweak}} \xrightarrow{SSB} SU(3)_c \times U(1)_{em}$$

$SU(3)_c$ (QCD) already discussed

$\to$ electroweak interaction $\quad G_{ew} = SU(2)_L \times U(1)_Y$

Glashow – Weinberg – Salam (Nobel prize 1979)

a) gauge group and multiplets

fundamental fields:

spin 1 $\quad$ four generators of $SU(2) \times U(1) \to W^\pm, Z^0, \gamma$

spin 1/2 $\quad$ leptons:

$$\begin{array}{cccc} & & & Q_{em} \\ \begin{pmatrix} \nu_e \\ e^- \end{pmatrix} & \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} & \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix} & \begin{array}{c} 0 \\ -1 \end{array} \end{array}$$

quarks:

$$\begin{array}{cccc} \begin{pmatrix} u \\ d \end{pmatrix} & \begin{pmatrix} c \\ s \end{pmatrix} & \begin{pmatrix} t \\ b \end{pmatrix} & \begin{array}{c} 2/3 \\ -1/3 \end{array} \end{array}$$

3 generations of quarks and leptons

V-A theory → lefthanded SU(2) doublets
righthanded SU(2) singlets

→ mesons $q_1 \overline{q_2}$, baryons $q_1 q_2 q_3$

SSB → Higgs field

$$G_{ew} = SU(2)_L \times U(1)_Y$$

$U(1)_Y$ → weak hypercharge Y

W_μ^a $\quad$ B_μ $\quad$ gauge bosons

$a = 1,2,3$

L_j $\quad$ ℓ_{R_j} $\quad$ leptons

$j = 1,2,3$ (generation index) $\quad f_{L,R} = \frac{1 \mp \gamma_5}{2} f_{L,R}$

q_{L_j} $\quad$ $u_{R,j}$, $d_{R,j}$ $\quad$ quarks

$T_a = \tau_a/2$ $\quad$ 0

ϕ $\quad$ Higgs doublet

gauge transformations:

$SU(2)_L$:

$$\psi \to U\psi \quad \text{for doublets}$$

$$\psi \to \psi \quad \text{for singlets}$$

$$\phi \to U\phi \qquad U \in SU(2)$$

$$\vec{W}_\mu \cdot \frac{\vec{\tau}}{2} \to U\vec{W}_\mu \cdot \frac{\vec{\tau}}{2} U^{-1} + \frac{i}{g}(\partial_\mu U)U^{-1}$$

$$B_\mu \to B_\mu$$

$U(1)_Y$:

$$\psi \to e^{-i\alpha \frac{Y_\psi}{2}} \psi$$

$$\phi \to e^{-i\alpha \frac{Y_\phi}{2}} \phi$$

$$\vec{W}_\mu \to \vec{W}_\mu$$

$$B_\mu \to B_\mu + \frac{1}{g'} \partial_\mu \alpha$$

Lagrange density:

$$\mathcal{L}_W = -\frac{1}{4}\sum_{a=1}^{3} F_a^{\mu\nu} F_{a\,\mu\nu}$$

$$F_a^{\mu\nu} = \partial^\mu W_a^\nu - \partial^\nu W_a^\mu - g\,\varepsilon_{abc} W_b^\mu W_c^\nu$$

$$\mathcal{L}_B = -\frac{1}{4}\left(\partial_\mu B_\nu - \partial_\nu B_\mu\right)\left(\partial^\mu B^\nu - \partial^\nu B^\mu\right)$$

$$\mathcal{L}_\psi = i\,\overline{\psi}\gamma^\mu D_\mu \psi \quad , \quad D_\mu = \partial_\mu + ig\,\vec{T}\cdot\vec{W}_\mu + ig'\frac{Y}{2}B_\mu$$

$$\mathcal{L}_\phi = (D_\mu\phi)^\dagger D^\mu\phi \qquad \vec{T} = \begin{cases} \vec{\tau}/2 & \text{doublets} \\ \vec{0} & \text{singlets} \end{cases}$$

$$-\mathcal{L}_Y = \overline{q}_{Li}\,\phi\, d_{Rj}\,\Gamma_{ij} + \overline{q}_{Li}\,\tilde{\phi}\, u_{Rj}\,\Delta_{ij}$$

$$+ \overline{L}_i\,\phi\, l_{Rj}\,\gamma_{ij} + h.c.$$

Yukawa couplings

$$\tilde{\phi} = i\tau^2\phi^* \Rightarrow Y_{\tilde{\phi}} = -Y_\phi$$

$$\tilde{\phi} \rightarrow U\tilde{\phi} \qquad \text{as } \tau_2 U^* = U\tau_2$$

(representations are equivalent)

$$V(\phi) = -r\,\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2, \; r>0 \rightarrow SSB$$

Higgs potential

$$\mathcal{L}_{GWS} = \mathcal{L}_W + \mathcal{L}_B + \sum_\psi \mathcal{L}_\psi + \mathcal{L}_\phi + \mathcal{L}_y - V$$

remark: neutrinos massless in this minimal version of the SM

experimental evidence for massive neutrinos (neutrino oscillations) $\rightarrow$ extension of $\mathcal{L}_{GWS}$ necessary (exercise: find correct extension of SM $\longrightarrow$ receive Nobel prize)

b) SSB

$r>0 \Rightarrow SSB$

$SU(2)\times U(1)$ invariance $\longrightarrow \langle 0|\phi|0\rangle = \frac{v}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix}$

$(v>0)$ without loss of generality

$U(1)_{em}$ unbroken $\Rightarrow$ Q_{em} = group generator which annihilates $\frac{v}{\sqrt{2}}\binom{0}{1} = \langle 0|\phi|0\rangle$

$$\underbrace{\left(\vec{c}\cdot\frac{\vec{\tau}}{2} + d\,\frac{Y_\phi}{2}\right)}_{Q^\phi_{em}} \begin{pmatrix}0\\1\end{pmatrix} = 0 \quad , \quad \vec{c}\in\mathbb{R}^3,\ d\in\mathbb{R}$$

$$\Rightarrow \quad c_1 = c_2 = 0 \quad , \quad c_3 = d\,Y_\phi$$

$$\Rightarrow \quad Q^\phi_{em} = c_3\left(\frac{\tau_3}{2} + \frac{1}{2}\right) = c_3\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}$$

elm. charge matrix for Higgs doublet

general form (arbitrary representation):

$$Q_{em} = c_3\,T_3 + d\,\frac{Y}{2}$$

charge difference within lepton or quark doublets $\binom{\nu_e}{e^-}$, $\binom{u}{d}$ $\longrightarrow$ $c_3 = +1$

$$\Rightarrow \quad Q^{\phi}_{em} = \frac{T_3}{2} + \frac{1}{2} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

choice of "normalization" $d = 1$ (fixes Y)

$$\Rightarrow \quad Q = T_3 + \frac{Y}{2}$$

known charges of fundamental particles

$\rightarrow$ $Y(L) = -1$, $Y(\ell_R) = -2$

$\rightarrow$ $Y(q_L) = \frac{1}{3}$, $Y(u_R) = \frac{4}{3}$, $Y(d_R) = -\frac{2}{3}$

$Y(\phi) = 1$

c) bosonic sector

mass term of vector bosons:

$$D_\mu = \partial_\mu + i g \frac{\vec{\tau}}{2} \vec{W}_\mu + i g' \frac{1}{2} B_\mu$$ for Higgs doublet

adjoint representation of SU(2)

$$T_3^{ad} = \left(-i\,\varepsilon_{3ab}\right) = -i \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} W_1 \\ W_2 \\ W_3 \\ B \end{pmatrix} \qquad \rightarrow \quad Y = 0 \quad \text{for all gauge bosons}$$

$$\rightarrow \quad Q_{W,B} = -i \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

eigenvalues $\underset{W_{1,2}}{\uparrow}\!\!\pm 1$, $\underset{W_3-B}{\uparrow}\!\!0^{(2)}$

subspace

$$Q_{W,B}\begin{pmatrix} 1 \\ i \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ i \\ 0 \\ 0 \end{pmatrix} \qquad \text{eigenvector (eigenvalue } +1)$$

$$Q_{W,B}\begin{pmatrix} 1 \\ -i \\ 0 \\ 0 \end{pmatrix} = -\begin{pmatrix} 1 \\ -i \\ 0 \\ 0 \end{pmatrix} \qquad -\!\!\shortparallel\!\!- \quad (-\!\!\shortparallel\!\!- \quad -1)$$

$$\Rightarrow \quad W_1 = \frac{W^+ + W^-}{\sqrt{2}} \, , \quad W_2 = i\,\frac{W^+ - iW^-}{\sqrt{2}}$$

$$\Updownarrow$$

$$W^+ = \frac{W_1 - iW_2}{\sqrt{2}} \, , \quad W^- = \frac{W_1 + iW_2}{\sqrt{2}}$$

$$\frac{1}{2}\left(\tau_1 W_1 + \tau_2 W_2\right) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & W^+ \\ W^- & 0 \end{pmatrix}$$

$$D_\mu \underbrace{\langle 0|\phi|0\rangle}_{\frac{v}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix}} = \frac{iv}{\sqrt{2}}\left[\frac{g}{\sqrt{2}}\,W_\mu^+\begin{pmatrix}1\\0\end{pmatrix} + \frac{1}{2}\left(-g\,W_\mu^3 + g'B_\mu\right)\begin{pmatrix}0\\1\end{pmatrix}\right]$$

mass term:

$$\left(D_\mu \langle 0|\phi|0\rangle\right)^\dagger D^\mu \langle 0|\phi|0\rangle =$$

$$= \frac{g^2 v^2}{4}\, W_\mu^- W^{+\mu}$$

$$+ \frac{v^2}{8}\underbrace{\left(-g\,W_\mu^3 + g'B_\mu\right)}_{\text{linear combination of neutral VBs with mass}}\left(-g\,W_3^\mu + g'B^\mu\right)$$

diagonalization of mass matrix in the neutral sector $\rightarrow$ mass eigenfields (Z°, γ)

$$\begin{pmatrix} Z \\ A \end{pmatrix} = \begin{pmatrix} \cos\Theta_W & -\sin\Theta_W \\ \sin\Theta_W & \cos\Theta_W \end{pmatrix} \begin{pmatrix} W_3 \\ B \end{pmatrix}$$

$$\cos\Theta_W = \frac{g}{\sqrt{g^2+g'^2}} \quad , \quad \sin\Theta_W = \frac{g'}{\sqrt{g^2+g'^2}}$$

Θ_W = Weinberg angle

mass term : $\quad M_W^2 \, W_\mu^+ W^{-\mu} + \frac{1}{2} M_Z^2 \, Z_\mu Z^\mu$

$$M_W^2 = \frac{g^2 v^2}{4} \quad , \quad M_Z^2 = \frac{(g^2+g'^2)v^2}{4}$$

$$\frac{M_W^2}{M_Z^2} = \frac{g^2}{g^2+g'^2} = \cos^2\Theta_W$$

$$\frac{M_W}{M_Z} = \cos\Theta_W$$

general form of the covariant derivative expressed in terms of mass eigenfields:

$$D_\mu = \partial_\mu + \frac{ig}{\sqrt{2}}\left(T_+ W^+_\mu + T_- W^-_\mu\right) +$$

$$+ ig\, T_3 \left(\cos\Theta_W Z_\mu + \sin\Theta_W A_\mu\right)$$

$$+ ig' \frac{Y}{2}\left(-\sin\Theta_W Z_\mu + \cos\Theta_W A_\mu\right)$$

$$T_\pm = T_1 \pm i\, T_2 = \begin{cases} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} & \text{for } SU(2) \text{ doublets} \\ 0 & \text{-"- -"- singlets} \end{cases}$$

coefficient of A_μ:

$$g\, T_3 \sin\Theta_W + g' \cos\Theta_W \frac{1}{2} Y$$

$$= g \sin\Theta_W T_3 + g \frac{\sin\Theta_W}{\cancel{\cos\Theta_W}} \cancel{\cos\Theta_W} \frac{Y}{2}$$

$$= g \sin\Theta_W \left(T_3 + \frac{Y}{2}\right) = g \sin\Theta_W\, Q_{em}$$

$$g \sin\Theta_W = e$$

coefficient of Z_μ:

$$g\cos\Theta_W\, T_3 - g'\sin\Theta_W \frac{Y}{2}$$

$$= g\cos\Theta_W\, T_3 - g\frac{\sin^2\Theta_W}{\cos\Theta_W}\frac{Y}{2}$$

$$= g\cos\Theta_W\, T_3 - g\frac{\sin^2\Theta_W}{\cos\Theta_W}\left(Q_{em} - T_3\right)$$

$$= \frac{g}{\cos\Theta_W}\left(T_3 - \sin^2\Theta_W\, Q_{em}\right)$$

$$\Rightarrow \quad D_\mu = \partial_\mu + \frac{ig}{\sqrt{2}}\left(T_+ W_\mu^+ + T_- W_\mu^-\right)$$

$$+ \frac{ig}{\cos\Theta_W}\left(T_3 - \sin^2\Theta_W\, Q_{em}\right) Z_\mu$$

$$+ ie\, Q_{em} A_\mu$$

unitary gauge: $\quad \Phi(x) \longrightarrow \frac{1}{\sqrt{2}}\left(v + h(x)\right)\begin{pmatrix}0\\1\end{pmatrix}$

$$\langle 0|h(x)|0\rangle = 0$$

$h(x)$ = Higgs boson of SM

Higgs – vector boson coupling:

$$D_\mu \phi \xrightarrow{\text{unit. gauge}} \frac{1}{\sqrt{2}} D_\mu (v+h) \begin{pmatrix} 0 \\ 1 \end{pmatrix} =$$

$$= \frac{1}{\sqrt{2}} \left[\partial_\mu h \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \frac{ig}{\sqrt{2}} W_\mu^+ \begin{pmatrix} 1 \\ 0 \end{pmatrix} (v+h) \right.$$

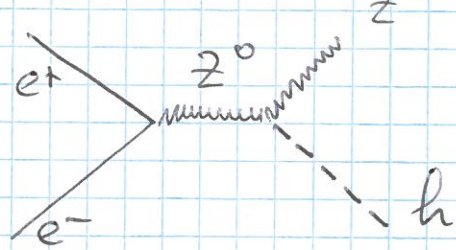
$$\left. - \frac{ig}{2\cos\Theta_W} Z_\mu \begin{pmatrix} 0 \\ 1 \end{pmatrix} (v+h) \right]$$

$$\mathcal{L}_\phi = (D_\mu \phi)^\dagger D^\mu \phi \longrightarrow$$

$$\rightarrow \frac{1}{2} \partial_\mu h \, \partial^\mu h + M_W^2 W_\mu^+ W^{-\mu} \left(1 + \frac{h}{v}\right)^2$$

$$+ \frac{1}{2} M_Z^2 Z_\mu Z^\mu \left(1 + \frac{h}{v}\right)^2$$

Higgs search at LEP:



Higgs potential:

$$V(\phi) = -r\,\phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 , \quad r, \lambda > 0$$

minima determined by $-r + 2\lambda\phi^\dagger\phi = 0$

$$\langle 0|\phi|0\rangle = \frac{v}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix} \Rightarrow v^2 = \frac{r}{\lambda}$$

$$V(\phi) = \lambda\, \phi^\dagger\phi\, (-v^2 + \phi^\dagger\phi)$$

$$V\left(\frac{v+h}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix}\right) = \lambda \frac{(v+h)^2}{2}\left(-v^2 + \frac{(v+h)^2}{2}\right)$$

$$= \frac{\lambda}{4}(v^2+2vh+h^2)(-v^2+2vh+h^2)$$

$$= \underbrace{-\frac{\lambda v^4}{4}}_{V_0} + \frac{\lambda}{4}(2vh+h^2)^2$$

$$= V_0 + \underbrace{\lambda v^2}_{M_h^2/2} h^2\left(1+\frac{h}{2v}\right)^2$$

$M_h^2 = 2\lambda v^2$

mass term and cubic and quartic self couplings of the Higgs field

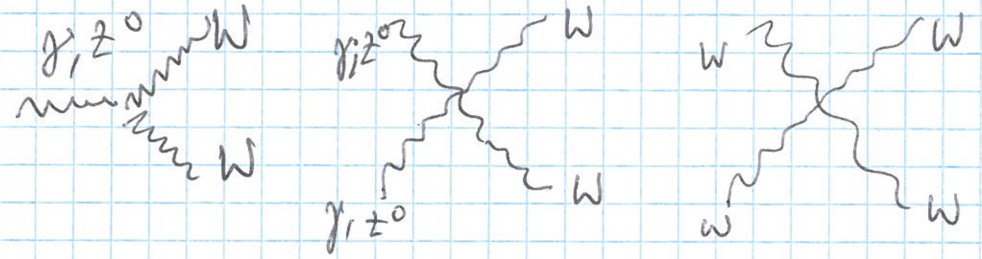
$M_h = 125.09 \pm 0.21 \pm 0.11$ GeV (PDG, 2016) self couplings of the vector bosons ($W^\pm, Z^0, \gamma$)

from

$$-\frac{1}{4} F_{a\mu\nu} F_a^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \,, \quad F_a^{\mu\nu} = \partial^\mu W_a^\nu - \partial^\nu W_a^\mu - g\,\varepsilon_{abc} W_b^\mu W_c^\nu$$

substitions : $W_1 = \dfrac{W^+ + W^-}{\sqrt{2}}$, $W_2 = i\,\dfrac{W^+ - W^-}{\sqrt{2}}$,

$W_3 = \cos\Theta_W\, Z + \sin\Theta_W\, A$, $B = -\sin\Theta_W\, Z + \cos\Theta_W\, A$

$\rightarrow$ 

d) fermion masses

Yukawa couplings:

$$-\mathcal{L}_Y = \bar{q}_{Li}\, \phi\, d_{Rj}\, \Gamma_{ij} + \bar{q}_{Li}\, \tilde{\phi}\, u_{Rj}\, \Delta_{ij} + \bar{L}_i\, \phi\, l_{Rj}\, g_{ij} + h.c.$$

$$\tilde{\phi} = i\tau_2 \phi^* = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} (\phi^+)^* \\ (\phi^0)^* \end{pmatrix} = \begin{pmatrix} (\phi^0)^* \\ -\phi^- \end{pmatrix}$$

$$\Rightarrow \quad \langle 0|\tilde{\phi}|0\rangle = \frac{v}{\sqrt{2}}\begin{pmatrix}1\\0\end{pmatrix}$$

$$\tilde{\phi} = \frac{v+h}{\sqrt{2}}\begin{pmatrix}1\\0\end{pmatrix} \qquad \text{(unitary gauge)}$$

$$\bar{q}_{Li}\frac{v}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix} d_{Rj}\,\Gamma_{ij} = \frac{v}{\sqrt{2}}\Gamma_{ij}\,(\bar{u}_{Li},\bar{d}_{Li})\begin{pmatrix}0\\1\end{pmatrix} d_{Rj} =$$

$$= \bar{d}_{Li}\,\underbrace{\frac{v}{\sqrt{2}}\Gamma_{ij}}_{(M_d)_{ij}}\,d_{Rj}$$

$M_d = \frac{v}{\sqrt{2}}\Gamma$ <u>mass matrix</u> of down-type quarks (d, s, b)

$$\bar{q}_{Li}\frac{v}{\sqrt{2}}\begin{pmatrix}1\\0\end{pmatrix} u_{Rj}\,\Delta_{ij} = (\bar{u}_{Li},\bar{d}_{Li})\begin{pmatrix}1\\0\end{pmatrix} u_{Rj}\frac{v}{\sqrt{2}}\Delta_{ij}$$

$$= \bar{u}_{Li}\,\underbrace{\frac{v}{\sqrt{2}}\Delta_{ij}}_{(M_u)_{ij}}\,u_{Rj}$$

$M_u = \frac{v}{\sqrt{2}}\Delta$ <u>mass matrix</u> of up-type quarks (u, c, t)

$$\overline{L}_i \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} l_{R_j} y_{ij} = (\overline{\nu}_{Li}, \overline{l}_{Li}) \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} l_{R_j} y_{ij}$$

$$= \overline{l}_{Li} \underbrace{\frac{v}{\sqrt{2}} y_{ij}}_{(M_l)_{ij}} l_{R_j}$$

$M_l = \frac{v}{\sqrt{2}} y$ <u>mass matrix</u> of (charged) leptons

M arbitrary complex $n \times n$ matrix $\Rightarrow \exists$ unitary $n \times n$ matrices U_L, U_R such that $U_L^\dagger M U_R = \hat{M}$ <u>diagonal</u> and <u>nonnegative</u> (proof: $M = RU$, $R^\dagger = R$, $R \geq 0$, $U^\dagger = U^{-1} \Rightarrow R = U_L^\dagger \hat{M} U_L$

$\Rightarrow M = U_L^\dagger \hat{M} \underbrace{U_L U}_{U_R}$)

$$u_{\substack{L\\R}} = U^u_{\substack{L\\R}} u'_{\substack{L\\R}}, \quad d_{\substack{L\\R}} = U^d_{\substack{L\\R}} d'_{\substack{L\\R}}, \quad l_{\substack{L\\R}} = U^l_{\substack{L\\R}} l'_{\substack{L\\R}}$$

$$-\mathcal{L}_Y = \overline{d}'_L \hat{M}_d d'_R \left(1 + \frac{h}{v}\right) + \overline{u}'_R \hat{M}_u u'_R \left(1 + \frac{h}{v}\right)$$
$$+ \overline{l}_L \hat{M}_l l_R \left(1 + \frac{h}{v}\right) + h.c.$$

$$\overline{f_L}\,\hat{M} f_R + h.c. = \overline{f_L}\,\hat{M} f_R + \overline{f_R}\,\hat{M} f_L =$$

$$= \overline{(f_L + f_R)}\,\hat{M}\,\underbrace{(f_L + f_R)}_{=:f} = \overline{f}\,\hat{M} f$$

$$\left(\overline{f_{\substack{L\\R}}}\, f_{\substack{L\\R}} = 0\right)$$

$$\Rightarrow \quad -\mathcal{L}_y = -\sum_f m_f \overline{f} f \left(1 + \frac{h}{v}\right)$$

$f = u_i', d_i', l_i' \quad (i = 1, 2, 3)$ <u>physical</u> fields

<u>remark:</u> chiral fields = fundamental fermionic building blocks (left- and right-handed fields in <u>different</u> multiplets); but: after SSB and diagonalization of mass matrices → → Dirac fields for physical particles

<u>remark:</u> neutrinos massless in minimal version of standard model → clearly incomplete description of the real world → extension of SM necessary

e) charged current

$$\mathcal{L}_{cc} = -\frac{g}{\sqrt{2}} \sum_{\psi} \bar{\psi} \gamma^{\mu} \left(T_+ W_{\mu}^+ + T_- W_{\mu}^- \right) \psi$$

sum runs over all fermion multiplets (singlets do not contribute)

$$-\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} \left(\bar{u}_L \gamma^{\mu} d_L + \bar{\nu}_L \gamma^{\mu} \ell_L \right) W_{\mu}^+ + h.c.$$

$\rightarrow$ physical fields $\quad u_L = U_L^u u_L'$, $d_L = U_L^d d_L'$

$$\rightarrow \quad \bar{u}_L \gamma^{\mu} d_L = \bar{u}_L' \gamma^{\mu} \underbrace{U_L^{u\dagger} U_L^d}_{V} d_L'$$

$V = U_L^{u\dagger} U_L^d$ <u>Cabibbo - Kobayashi - Maskawa matrix</u>

(quark mixing matrix)

<u>leptons</u>: $\quad \ell_L = U_L^{\ell} \ell_L'$

for <u>massless</u> neutrinos one defines $\nu_L = U_L^{\ell} \nu_L'$

$\Rightarrow \bar{\nu}_L \gamma^{\mu} \ell_L = \bar{\nu}_L' \gamma^{\mu} \ell_L' \qquad$ no mixing matrix for massless νs

<u>experiment</u>: ν - oscillations observed $\rightarrow$ νs have <u>different</u> masses

$$\rightarrow \; -\mathcal{L}_{cc} = \frac{g}{2\sqrt{2}} \left[\bar{u}' \gamma^\mu (1-\gamma_5) V d' + \right.$$

$$\left. + \bar{\nu}' \gamma^\mu (1-\gamma_5) \ell' \right] W^+_\mu + h.c.$$

f) neutral current

$$-\mathcal{L}_{NC} = \frac{g}{\cos\Theta_W} \sum_\psi \bar{\psi} \gamma^\mu (T_3 - \sin^2\Theta_W Q_{em}) \psi Z_\mu$$

(now <u>also</u> contributions from SU(2) singlets)

$$-\mathcal{L}_{NC} = \frac{g}{\cos\Theta_W} \left[\bar{u}_L \gamma^\mu u_L \left(\tfrac{1}{2} - \sin^2\Theta_W \tfrac{2}{3}\right) + \right.$$

$$+ \bar{d}_L \gamma^\mu d_L \left(-\tfrac{1}{2} + \sin^2\Theta_W \tfrac{1}{3}\right) + \bar{\ell}_L \gamma^\mu \ell_L \left(-\tfrac{1}{2} + \sin^2\Theta_W\right)$$

$$+ \bar{\nu}_L \gamma^\mu \nu_L \tfrac{1}{2} + \bar{u}_R \gamma^\mu u_R \left(-\sin^2\Theta_W \tfrac{2}{3}\right)$$

$$\left. + \bar{d}_R \gamma^\mu d_R \sin^2\Theta_W \tfrac{1}{3}) + \bar{\ell}_R \gamma^\mu \ell_R \sin^2\Theta_W \right] Z_\mu$$

$\rightarrow$ physical fields : $\bar{u}_L \gamma^\mu u_L = \bar{u}'_L \gamma^\mu u'_L$, etc.

<u>GIM mechanism</u> (Glashow - Iliopoulos - Maiani)

neutral currents do <u>not</u> change flavour experiment: no flavour changing neutral currents (FCNC) observed (so far)

$$-\mathcal{L}_{NC} = \frac{g}{2\cos\Theta_W} \sum_f \bar{f} \gamma^\mu (a_f - b_f \gamma_5) f Z_\mu$$

$$a_f = t^L_{3f} - 2\sin^2\Theta_W \, Q_f$$

$$b_f = t^L_{3f}$$

t^L_{3f} = eigenvalue of T_3 in $SU(2)$ doublet

Q_f = charge of f

	a_f	b_f
u	$\frac{1}{2} - \frac{4}{3}\sin^2\Theta_W$	$\frac{1}{2}$
d	$-\frac{1}{2} + \frac{2}{3}\sin^2\Theta_W$	$-\frac{1}{2}$
ν	$\frac{1}{2}$	$\frac{1}{2}$
ℓ	$-\frac{1}{2} + 2\sin^2\Theta_W$	$-\frac{1}{2}$

g) electromagnetic current

$$-\mathcal{L}_{em} = e \sum_f Q_f \, \bar{f} \gamma^\mu f \, A_\mu$$

remark: $U^{u,d,\ell}_{L,R}$ cancel (like in $\mathcal{L}_{NC}$)

h) V-A theory

$$-\mathcal{L}_{cc} = \frac{g}{2\sqrt{2}} \left(\bar{u}' \gamma^\mu (1-\gamma_5) V d' + \bar{\nu}' \gamma^\mu (1-\gamma_5) \ell' \right) W^+_\mu$$

$$+ \text{h.c.}$$

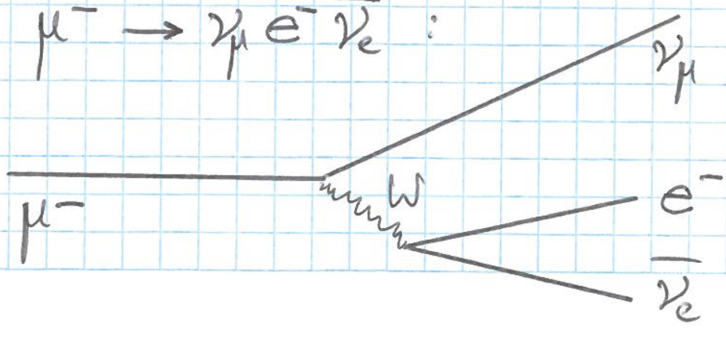
$V^\mu := \bar{u}' \gamma^\mu V d' + \bar{\nu}' \gamma^\mu \ell'$ (charged) vector current

$A^\mu := \bar{u}' \gamma^\mu \gamma_5 V d' + \bar{\nu}' \gamma^\mu \gamma_5 \ell'$ (charged) axial vector current

$$\Rightarrow \; -\mathcal{L}_{cc} = \frac{g}{2\sqrt{2}} \left[(V^\mu - A^\mu) W^+_\mu + (V^\mu - A^\mu)^\dagger W^-_\mu \right]$$

describes for instance $\mu^- \to \nu_\mu e^- \bar{\nu}_e$:

muon decay



low-energy effective theory (energies $\ll M_W$)

"integrate out" W (heavy degree of freedom)

W propagator $$\int \frac{d^4k}{(2\pi)^4}\, e^{-ik(x-y)}\, i\, \frac{-g_{\mu\nu} + \frac{k_\mu k_\nu}{M_W^2}}{k^2 - M_W^2}$$

$$\xrightarrow[|k| \ll M_W]{} \frac{i g_{\mu\nu}}{M_W^2}\, \delta^{(4)}(x-y)$$ point interaction

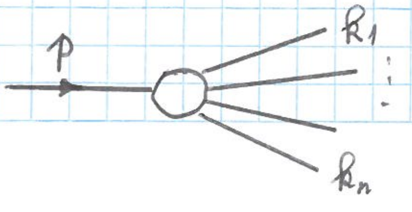
$$\longrightarrow \quad \mathcal{L}_{V-A} = -\frac{G_F}{\sqrt{2}} (V^\mu - A^\mu)(V_\mu - A_\mu)^\dagger$$

Fermi coupling constant $$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W^2}$$

$$G_F = 1.166\,378\,7(6) \times 10^{-5}\ \text{GeV}^{-2}$$ (PDG 2016)

remark: computation of decay width

$$d\Gamma = \frac{S}{2M} |M_{fi}|^2 \frac{d^3k_1}{(2\pi)^3 2k_1^0} \cdots \frac{d^3k_n}{(2\pi)^3 2k_n^0} (2\pi)^4 \delta^{(4)}(p - \sum_{i=1}^{n} k_i)$$

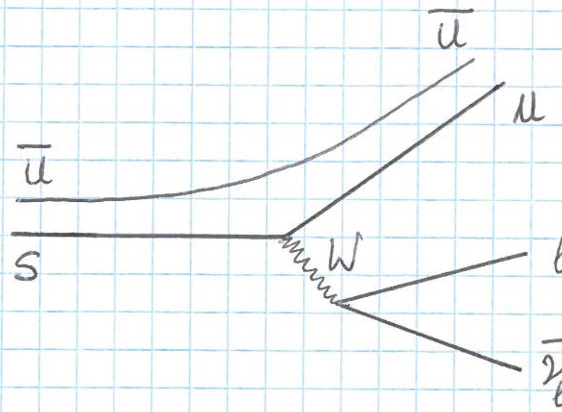


$p^2 = M^2$, S = statistical factor

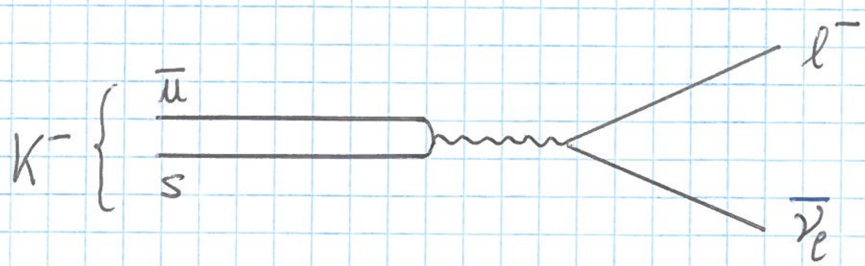
$\mathcal{L}_{V-A} \rightarrow$ weak decays of μ, τ, quarks

<u>example:</u> $s \rightarrow u\, \ell^- \bar{\nu}_\ell \quad (\ell = e, \mu)$

amplitude $\sim V_{us}$



$K^- \rightarrow \pi^0\, \ell^- \bar{\nu}_\ell \quad (K_{\ell 3}$ decay$)$



$K^- \rightarrow \ell^- \bar{\nu}_\ell \quad (K_{\ell 2}$ decay$)$

<u>"semileptonic"</u> decays

$c \to \begin{matrix} d \\ s \end{matrix} \; \ell^+ \nu_\ell \quad (u\bar{d}, u\bar{s})$

$\ell = e, \mu, \tau$

$b \to \begin{matrix} c \\ u \end{matrix} \; \ell^- \bar{\nu}_\ell \quad (\bar{u}d, \bar{u}s, \bar{c}d, \bar{c}s)$

amplitude $(b \to c \, \ell \bar{\nu}_\ell) \sim V_{cb}$

— II — $(b \to c \, \bar{u} s) \sim V_{cb} V_{us}^*$

remark: $$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W^2} \overset{e = g \sin\Theta_W}{=} \frac{\pi \alpha}{2 \sin^2\Theta_W \, M_W^2}$$

$$\longrightarrow \quad M_W = \frac{37.3 \text{ GeV}}{\sin\Theta_W}$$

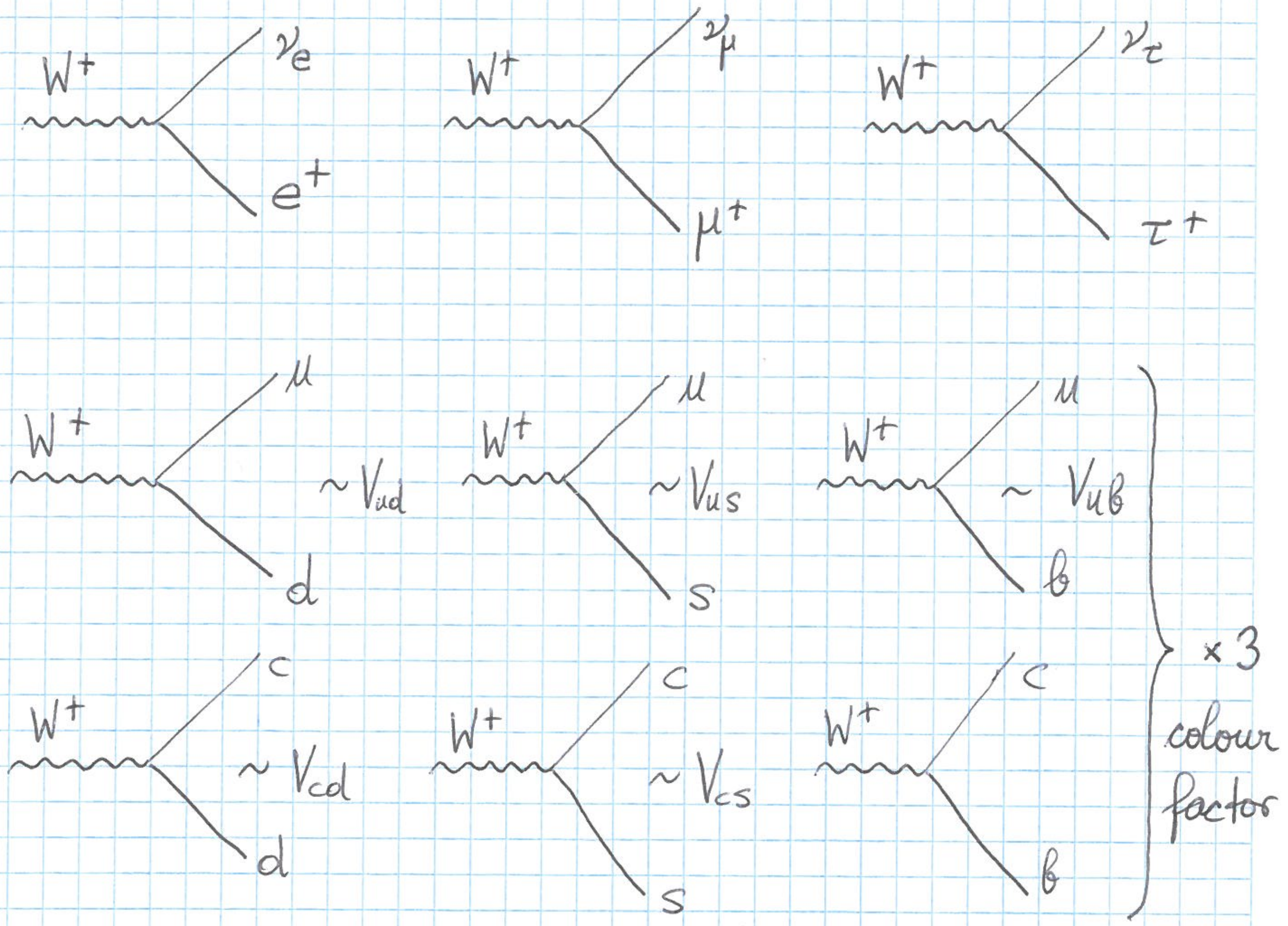
NC experiments before discovery of $W^\pm$: $\sin^2\Theta_W \simeq 0.23$

$\longrightarrow$ prediction $M_W \simeq 78$ GeV

current value (PDG 2016): $M_W = 80.385 \pm 0.015$ GeV

(radiative corrections!)

__exercise:__ compute the widths of the following decay modes of the W boson:



__remark:__ $$\sum_{\lambda=1}^{3} \varepsilon^{\mu}(\vec{k},\lambda)\,\varepsilon^{\nu}(\vec{k},\lambda) = -g^{\mu\nu} + \frac{k^{\mu}k^{\nu}}{M^2}$$

for massive vector bosons

i) Cabibbo – Kobayashi – Maskawa mixing matrix

number of independent (observable) parameters in V:

n_G = number of generations

V : n_G - dimensional unitary matrix $\rightarrow n_G^2$ real parameters (write $V = \exp(iA)$, $A = A^\dagger \rightarrow$ A has $n_G + \frac{2(n_G^2 - n_G)}{2} = n_G^2$ real parameters)

$\binom{n_G}{2} = \frac{n_G(n_G-1)}{2}$ of these parameters are

<u>angles</u> (generalized Euler angles parametrizing $SO(n_G)$: $SO(n_G)$ matrix $= \mathbb{1} + \Omega + \ldots$, $\Omega^T = -\Omega$ $\rightarrow \frac{n_G^2 - n_G}{2}$ real parameters)

$\rightarrow$ remaining $n_G^2 - \frac{n_G(n_G-1)}{2} = \frac{n_G(n_G+1)}{2}$

parameters are <u>phases</u>

<u>however:</u> define diagonal matrices

$$e^{i\alpha} = \begin{pmatrix} e^{i\alpha_1} & & \\ & e^{i\alpha_2} & \\ & & e^{i\alpha_3} \end{pmatrix}, \quad e^{i\beta} = \begin{pmatrix} e^{i\beta_1} & & \\ & e^{i\beta_2} & \\ & & e^{i\beta_3} \end{pmatrix}$$

$$u' \to e^{i\alpha} u', \quad d' \to e^{i\beta} d'$$

(<u>same</u> transformation for L, R) $\to \mathcal{L}_Y, \mathcal{L}_{NC}, \mathcal{L}_{em}$ remain <u>unchanged</u>, <u>but</u>

$$V \to e^{-i\alpha} V e^{i\beta} \quad \text{in } \mathcal{L}_{CC}$$

$\to$ $2n_G - 1$ phases can be absorbed by this transformation $\to$

$$\to \frac{n_G(n_G+1)}{2} - (2n_G - 1) = \frac{(n_G-1)(n_G-2)}{2}$$

<u>measurable phases</u>

$\underline{n_G = 2}$:
$$V = \begin{pmatrix} \cos\Theta_c & \sin\Theta_c \\ -\sin\Theta_c & \cos\Theta_c \end{pmatrix}$$

Θ_c = <u>Cabibbo angle</u>

<u>no</u> observable phase

$\underline{n_G = 3}$: 3 angles, 1 phase → only mechanism of (minimal) SM for CP violation

→ all CP violating observables $\sim \sin\delta_{KM}$
(Kobayashi – Maskawa 1973)

<u>exercise</u>: study the possible parametrizations of the CKM matrix shown in PDG 2014, p.214

<u>remark</u>: <u>"hard"</u> CP violation in the SM
(in contrast to spontaneous CP violation)

want to show: mixing matrix V can be source of CP-violation for $n_G \geq 3$:

Particle Physics I: $\psi(x^0, \vec{x}) \xrightarrow{P} \gamma^0 \psi(x^0, -\vec{x})$

$$\psi(x^0, \vec{x}) \xrightarrow{C} C\gamma^{0T}\psi^*(x^0, \vec{x})$$

$$C = -i\gamma^2\gamma^0$$

properties of charge-conjugation matrix C:

$C\gamma_\mu^T C^{-1} = -\gamma_\mu$, $C^T = -C$, $C^\dagger = C^{-1}$

$(\Rightarrow C\gamma_5^T C^{-1} = \gamma_5)$

$$\Rightarrow \psi(x^0, \vec{x}) \xrightarrow{CP} C\underbrace{\gamma^{0T}\gamma^{0*}}_{\mathbb{1}}\psi^*(x^0, -\vec{x}) = C\psi^*(x^0, -\vec{x})$$

(up to a phase factor)

$\rightarrow$ we define: $u'(x) \xrightarrow{CP} e^{i\alpha_u} C u'^*(\tilde{x})$

$$d'(x) \xrightarrow{CP} e^{i\alpha_d} C d'^*(\tilde{x})$$

$\tilde{x} = (x^0, -\vec{x})$, $e^{i\alpha_u}$, $e^{i\alpha_d}$ diagonal phase matrices

$\rightarrow$ mass terms and kinetic terms <u>invariant</u>

$$u'\gamma^\mu(1-\gamma_5)V d' \xrightarrow{CP}$$

$$(e^{i\alpha_u} C u'^*)^\dagger \gamma^0 \gamma^\mu (1-\gamma_5) V e^{i\alpha_d} C d'^*$$

$$= u'^T C^\dagger e^{-i\alpha_u} \gamma^0 \gamma^\mu (1-\gamma_5) V e^{i\alpha_d} C d'^*$$

$$= u'^T \gamma^0 \gamma^{\mu T} (1-\gamma_5^T) e^{-i\alpha_u} V e^{i\alpha_d} d'^*$$

$$= u'_{i\alpha} [(1-\gamma_5)\gamma^\mu\gamma^0]_{\beta\alpha} (e^{-i\alpha_u} V e^{i\alpha_d})_{ij} d'^*_{j\beta}$$

$$= - d'^*_{j\beta} [\underbrace{\gamma^\mu\gamma^0}_{\varepsilon(\mu)\gamma^0\gamma^\mu} (1-\gamma_5)]_{\beta\alpha} (e^{-i\alpha_u} V e^{i\alpha_d})^T_{ji} u'_{i\alpha}$$

anti-comm. fields!

$\varepsilon(0) = 1, \ \varepsilon(i) = -1$

$$= - \varepsilon(\mu)\, \overline{d}'\gamma^\mu(1-\gamma_5)\, (e^{-i\alpha_u} V e^{i\alpha_d})^T u'$$

remember:

$$-\mathcal{L}^{quarks}_{V-A} = \frac{G_F}{\sqrt{2}} [\overline{u}'\gamma^\mu(1-\gamma_5)V d'][\overline{d}'\gamma^\mu(1-\gamma_5)V^\dagger u']$$

CP invariance of $\mathcal{L}_{V-A}^{\text{quarks}} \Leftrightarrow \exists\ \alpha_u, \alpha_d$

with $\left(e^{-i\alpha_u} V e^{i\alpha_d}\right)^T = V^\dagger$

$$\Leftrightarrow \quad e^{-i\alpha_u} V e^{i\alpha_d} = V^*$$

$\Rightarrow \quad \exists$ phase-convention in which CKM matrix <u>real</u>:

$$e^{-i\frac{\alpha_u}{2}} V e^{i\frac{\alpha_d}{2}} = e^{i\frac{\alpha_u}{2}} V^* e^{-i\frac{\alpha_d}{2}}$$

$$= \left(e^{-i\frac{\alpha_u}{2}} V e^{i\frac{\alpha_d}{2}}\right)^*$$

however, for arbitrary complex Yukawa couplings $\Gamma, \Delta \longrightarrow \exists$ physical phase for $n_G \geq 3 \implies \cancel{CP}$ for $n_G \geq 3$

j) effective four-Fermi-interaction for neutral currents

$$-\mathcal{L}_{NC} = \frac{g}{2\cos\Theta_W} \underbrace{\sum_f \bar{f}\gamma^\mu (a_f - b_f\gamma_5) f}_{J^\mu_{NC}} Z_\mu$$

"integrate" out Z^0:

$$\rightarrow \quad \mathcal{L}^{eff}_{NC} = -\frac{g^2}{8M_Z^2\cos^2\Theta_W} J^\mu_{NC} J_\mu^{NC}$$

$$M_W = M_Z \cos\Theta_W$$

$$\rightarrow \quad \mathcal{L}^{eff}_{NC} = -\underbrace{\frac{g^2}{8M_W^2}}_{\frac{G_F}{\sqrt{2}}} J^\mu_{NC} J_\mu^{NC}$$

$$M_Z = \frac{M_W}{\cos\Theta_W} = \frac{37.3\text{ GeV}}{\sin\Theta_W\cos\Theta_W} \simeq 89\text{ GeV}$$

current experimental value (PDG 2016): $M_Z = 91.1876 \pm 0.0021$ GeV

(loop corrections!)

decay modes of the Z^0:

visible: $Z^0 \to \ell^+ \ell^-$, $q\bar{q}$ (jets)

invisible: $Z^0 \to \nu_\ell \bar{\nu}_\ell$ $\quad (\ell = e, \mu, \tau)$

exercise: $\Gamma(Z^0 \to \nu_\ell \bar{\nu}_\ell) = \dfrac{G_F M_Z^3}{12\sqrt{2}\,\pi} = 166\,\text{MeV}$

$\Gamma_{tot} - \Gamma_{visible} = \Gamma_{invisible} = 499.0 \pm 1.5\,\text{MeV}$ (PDG 2008)

$\to N_\nu = 2.984 \pm 0.008$ light neutrino types

(PDG 2008) (with SM couplings to Z^0)

k) number of parameters of the (minimal) SM

(electroweak sector only)

$\hat{M}_{u,d,\ell}$	9 fermion masses
V	3 angles, 1 phase
g, g'	2 gauge couplings (alternatively: α, Θ_W)
v	VEV
M_h	Higgs mass

→ 17 parameters in the electroweak sector of the <u>minimal</u> SM

<u>extension of the SM</u> : <u>additional</u> (free ?) parameters : neutrino masses, neutrino mixing angles, CP-violation in the lepton sector (?), ... (?)

<u>2. Quantum chromodynamics (QCD)</u>

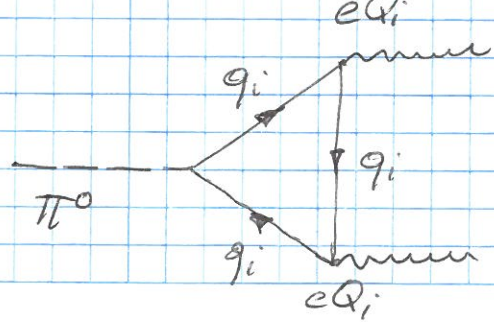
a few arguments in favour of three colour degrees of freedom

a) $$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \to N_c = 3$$

(see p. 9/11)

b) $\pi^0 \to 2\gamma$

<u>chiral anomaly</u>



triangle anomaly

$$\Gamma(\pi^0 \to 2\gamma) = \frac{\alpha^2 M_\pi^3}{16\pi^3 F_\pi^3} N_c^2 \, S$$

F_π = pion decay constant = 92.2 MeV

from $\pi^\pm \to \ell^\pm \overset{(-)}{\nu}_\ell$

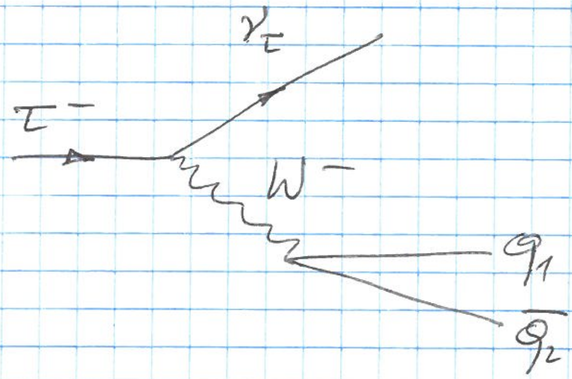
$$S = \sum_{q \in \pi^0} (I_3)_q \, Q_q^2 = \frac{1}{2}\left(\frac{2}{3}\right)^2 - \frac{1}{2}\left(-\frac{1}{3}\right)^2 = \frac{1}{6}$$

$$\Gamma(\pi^0 \to 2\gamma) = 0.859 \, N_c^2 \text{ eV}$$

experiment : $\Gamma(\pi^0 \to 2\gamma) = (7.74 \pm 0.60) \text{ eV}$

$\to N_c^2 = 9.0 \pm 0.7$

c) $\tau \to \nu_\tau + \text{hadrons}$



d	or	s
$\overline{u}$		$\overline{u}$
$\sim V_{ud}$		$\sim V_{us}$

remark: c cannot be produced

($m_\tau = 1776.82\,\text{MeV}$, $m_{D^\pm} = 1869.61\,\text{MeV}$

(PDG 2014) $m_{D^0} = 1864.84\,\text{MeV}$)

$D^+ = c\bar{d}$, $D^0 = c\bar{u}$

$$R_\tau = \frac{\Gamma(\tau^- \to \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \to \nu_\tau\, \ell^- \bar{\nu}_\ell)} \simeq N_c$$

exp.: $R_\tau = 3.645 \pm 0.020$ (PDG 2008)

QCD corrections: $R_\tau = N_c\,(1 + \mathcal{O}(g_s^2))$

$\rightarrow$ allows determination of g_s

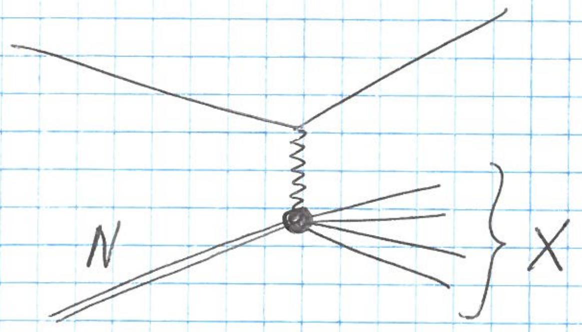
d) consistent quantization of SM

absence of anomalies $\rightarrow N_c = 3$

(cancellation of quark- and lepton-contributions to the anomaly)

e) energy-momentum balance of "partons" in deep-inelastic scattering at high energies

$\ell + N \to \ell + X$	elm./neutral	current	γ, Z^0
$\nu_\ell + N \to \ell + X$	charged weak	current	$W^\pm$
$\nu_\ell + N \to \nu_\ell + X$	neutral weak	current	Z^0



exp.: ~ 1/2 of partons in nucleon N does not feel electroweak interaction → gluons

gauging of colour degree of freedom → QCD

→ standard model $\mathcal{L}_{SM} = \mathcal{L}_{ew} + \mathcal{L}_{QCD}$

in addition to the 17 parameters of $\mathcal{L}_{ew}$ →

→ g_s (strong coupling constant)

θ_{QCD} (strong CP violation)

→ 19 parameters in the <u>minimal</u> SM