11. Vector field

Massive vector field

$V_\mu(x)$ describes massive spin 1 field (vector boson)

$$\mathcal{L} = -\frac{1}{4} V_{\mu\nu} V^{\mu\nu} + \frac{M^2}{2} V_\mu V^\mu$$

$V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$, V_μ real

$\Rightarrow$ field equation $(\Box + M^2) V^\mu - \partial^\mu \partial_\nu V^\nu = 0$

$\overset{\partial_\mu \ldots}{\Longrightarrow}$ $(\Box + M^2)\partial_\mu V^\mu - \Box \partial_\nu V^\nu = M^2 \partial_\mu V^\mu = 0$

$\overset{M \neq 0}{\Longrightarrow}$ $\partial_\mu V^\mu = 0$ (eliminates spin 0 component contained in V^μ)

field equations:
$$\boxed{\begin{aligned} (\Box + M^2) V^\mu &= 0 \\ \partial_\mu V^\mu &= 0 \end{aligned}}$$

plane-wave solutions:

$\varepsilon^\mu e^{\pm ikx}$, $k^2 = M^2$, $k_\mu \varepsilon^\mu = 0$

rest frame: $k = \begin{pmatrix} M \\ \vec{0} \end{pmatrix} \Rightarrow \varepsilon = \begin{pmatrix} 0 \\ \vec{\varepsilon} \end{pmatrix}$

normalization $|\vec{\varepsilon}| = 1 \Rightarrow \varepsilon^2 = -1$

3 polarizations for given $\vec{k}$: $\varepsilon^\mu(k,\lambda)$, $\lambda = 1,2,3$

$$\varepsilon(k,1) = \begin{pmatrix} 0 \\ \vec{\varepsilon}(k,1) \end{pmatrix}, \quad \varepsilon(k,2) = \begin{pmatrix} 0 \\ \vec{\varepsilon}(k,2) \end{pmatrix}$$

$\vec{\varepsilon}(k,1) \cdot \vec{k} = \vec{\varepsilon}(k,2) \cdot \vec{k} = 0$ transversal

$\varepsilon(k,3) = \frac{1}{M} \begin{pmatrix} |\vec{k}| \\ \frac{k^0 \vec{k}}{|\vec{k}|} \end{pmatrix}$ longitudinal

$$\varepsilon^\mu(k,\lambda)\, \varepsilon_\mu(k,\sigma) = -\delta_{\lambda\sigma}, \quad \varepsilon^\mu(k,\lambda)\, k_\mu = 0$$

$$\sum_\lambda \varepsilon^\mu(k,\lambda)\, \varepsilon^\nu(k,\lambda) = -g^{\mu\nu} + k^\mu k^\nu / M^2$$

$\longrightarrow$ general solution of field equations:

$$V^\mu(x) = \sum_{\lambda=1}^{3} \int d\mu(k) \left[\varepsilon^\mu(k,\lambda)\, a(k,\lambda)\, e^{-ikx} + h.c. \right]$$

quantization: $[a(k,\lambda), a(k',\lambda')^\dagger] = \delta_{\lambda\lambda'}\, \delta(k,k')$

generating functional of free massive vector field (using the functional integral method):

$$Z[J] = \langle 0 | T e^{-i\int d^4x\, V^\mu(x) J_\mu(x)} | 0 \rangle$$

$$= \frac{1}{\mathcal{N}} \int [dV^\mu]\, e^{i\int d^4x \left(-\frac{1}{4} V_{\mu\nu} V^{\mu\nu} + \frac{M^2 - i\varepsilon}{2} V_\mu V^\mu - V^\mu J_\mu\right)}$$

external current $J_\mu(x)$, normalization $Z[0] = 1$

$$S = \int d^4x \left(-\frac{1}{4} V_{\mu\nu} V^{\mu\nu} + \frac{M^2 - i\varepsilon}{2} V_\mu V^\mu - J_\mu V^\mu\right)$$

$$= \int d^4x \left\{ \frac{1}{2} V^\mu \left[g_{\mu\nu} (\Box + M^2 - i\varepsilon) - \partial_\mu \partial_\nu \right] V^\nu - J_\mu V^\mu \right\}$$

usual trick: shift of integration variable

$$V^\mu = \underset{\substack{\uparrow}}{V'^\mu} + W^\mu$$

new integration variable in the functional integral

$[dV^\mu] = [dV'^\mu]$ translation invariance of the measure terms linear in V'^{μ} disappear, if

$$[g_{\mu\nu}(\Box + M^2 - i\varepsilon) - \partial_\mu \partial_\nu] W^\nu = J_\mu$$

<u>propagator</u> $\Delta^{\nu\rho}(x)$ = Green's function of the differential operator $g_{\mu\nu}(\Box + M^2 - i\varepsilon) - \partial_\mu \partial_\nu$:

$$[g_{\mu\nu}(\Box + M^2 - i\varepsilon) - \partial_\mu \partial_\nu] \Delta^{\nu\rho}(x) = \delta_\mu{}^\rho \, \delta^{(4)}(x)$$

Fourier representation $\Delta^{\nu\rho}(x) = \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \tilde{\Delta}^{\nu\rho}(k)$

→ equation in momentum space

$$[g_{\mu\nu}(-k^2 + M^2 - i\varepsilon) + k_\mu k_\nu] \tilde{\Delta}^{\nu\rho}(k) = \delta_\mu{}^\rho$$

we discuss the more <u>general</u> problem :

find the inverse of

$$T_{\mu\nu} = a(k^2)\, k_\mu k_\nu + b(k^2)\, g_{\mu\nu}$$

ansatz for $(T^{-1})^{\nu\rho}$:

$$(T^{-1})^{\nu\rho} = A(k^2)\, k^\nu k^\rho + B(k^2)\, g^{\nu\rho}$$

$$(a\, k_\mu k_\nu + b\, g_{\mu\nu})(A\, k^\nu k^\rho + B\, g^{\nu\rho}) = \delta_\mu{}^\rho$$

$$a A\, k^2 k_\mu k^\rho + a B\, k_\mu k^\rho + b A\, k_\mu k^\rho$$

$$+ b B\, g_\mu{}^\rho = \delta_\mu{}^\rho$$

$$\Rightarrow B = \frac{1}{b}\,, \quad a A k^2 + a B + b A = 0$$

$$A\,(a k^2 + b) = -\frac{a}{b} \Rightarrow A = -\frac{a}{b\,(a k^2 + b)}$$

$$\Rightarrow (T^{-1})^{\nu\rho} = \frac{g^{\nu\rho}}{b} - \frac{a\, k^\nu k^\rho}{b\,(a k^2 + b)}$$

$\Rightarrow$ T^{-1} exists if $b \neq 0$ and $a k^2 + b \neq 0$

in the case of the massive vector field we have

$$a = 1\,, \quad b = -k^2 + M^2 - i\varepsilon$$

$$\Rightarrow \quad \tilde{\Delta}^{\nu\rho}(k) = \frac{g^{\nu\rho} - k^\nu k^\rho / M^2}{M^2 - k^2 - i\varepsilon}$$

$$\Delta^{\nu\rho}(x) = \int \frac{d^4k}{(2\pi)^4}\, e^{-ikx}\, \frac{g^{\nu\rho} - k^\nu k^\rho / M^2}{M^2 - k^2 - i\varepsilon}$$

$$W^\mu(x) = \int d^4y\ \Delta^{\mu\nu}(x-y)\, J_\nu(y)$$

$$Z[J] = e^{-\frac{i}{2}\int d^4x\, J_\mu(x)\, W^\mu(x)}$$

$$= e^{-\frac{i}{2}\int d^4x\, d^4y\, J_\mu(x)\, \Delta^{\mu\nu}(x-y)\, J_\nu(y)}$$

$$\Rightarrow \quad -\frac{1}{2}\langle 0|\, T\, V^\mu(x)\, V^\nu(y)\, |0\rangle = -\frac{i}{2}\, \Delta^{\mu\nu}(x-y)$$

$$\langle 0|\, T\, V^\mu(x)\, V^\nu(y)\, |0\rangle = i\, \Delta^{\mu\nu}(x-y)$$

$$= i \int \frac{d^4k}{(2\pi)^4}\, e^{-ikx}\, \frac{g^{\mu\nu} - k^\mu k^\nu / M^2}{M^2 - k^2 - i\varepsilon}$$

Complex vector field ($M \neq 0$)

$$\mathcal{L} = -\frac{1}{2} V_{\mu\nu}^{*} V^{\mu\nu} + M^2 V_\mu^* V^\mu$$

$V^\mu = (V_1^\mu + i V_2^\mu)/\sqrt{2}$, $V_{1,2}^\mu$ real

generating functional

$$Z[J, J^*] = \langle 0 | T \, e^{-i \int d^4x \, (V_\mu^\dagger(x) J^\mu(x) + V^\mu(x) J_\mu^*(x))} | 0 \rangle$$

can be obtained from the generating functional of a real vector field ($J^\mu = (J_1^\mu + i J_2^\mu)/\sqrt{2}$,

J_1, J_2 real)

$$\rightarrow Z[J, J^*] = e^{-i \int d^4x \, d^4y \, J_\mu^*(x) \Delta^{\mu\nu}(x-y) J_\nu(y)}$$

$$\Rightarrow \langle 0 | T \, V_\mu(x) \, V_\nu^\dagger(y) | 0 \rangle = i \, \Delta^{\mu\nu}(x-y)$$

$$\langle 0 | T \, V_\mu(x) \, V_\nu(y) | 0 \rangle = 0$$

Abelian gauge field

Quantum electrodynamics (QED) with a single charged Dirac field (e.g. $e^\pm$) described by

$$\mathcal{L}_{QED} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \overline{\psi}(i\not{\partial} - m)\psi - j^\mu A_\mu$$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ field strength tensor of <u>photon field</u> A_μ (massless!)

$j^\mu = q\,\overline{\psi}\gamma^\mu\psi$ <u>electromagnetic current</u> (density)

q = electromagnetic charge of Dirac field ψ ($q = -e$ for electron field)

<u>remark:</u> $\overline{\psi}(i\not{\partial} - m)\psi - j^\mu A_\mu =$

$$= \overline{\psi}\,[\,i\gamma^\mu \underbrace{(\partial_\mu + iqA_\mu)}_{\text{covariant derivative}} - m\,]\,\psi$$

$\mathcal{L}_{QED}$ is invariant under <u>local</u> $U(1)$ gauge transformation $\psi(x) \to e^{-iq\Lambda(x)}\psi(x)$

$$A_\mu(x) \to A_\mu(x) + \partial_\mu \Lambda(x)$$

$$\partial_\mu \psi(x) \to \partial_\mu \left(e^{-iq\Lambda(x)} \psi(x) \right) =$$

$$= e^{-iq\Lambda(x)} \left(\partial_\mu \psi(x) - iq\, \partial_\mu \Lambda(x) \;\psi(x) \right)$$

$$iq\, A_\mu(x)\, \psi(x) \to iq \left(A_\mu(x) + \partial_\mu \Lambda(x) \right) e^{-iq\Lambda(x)} \psi(x)$$

$$\Rightarrow D_\mu \psi := (\partial_\mu + iq A_\mu)\psi \to e^{-iq\Lambda(x)} D_\mu \psi$$

$\Rightarrow \quad \bar{\psi}\, i \not{D}\, \psi \to \bar{\psi}\, i \not{D}\, \psi \qquad$ invariant

$\qquad \bar{\psi}\psi \;,\; F_{\mu\nu}F^{\mu\nu} \qquad\qquad$ —"—

$\Rightarrow \quad \mathcal{L}_{QED}$ invariant (massterm $M^2 A_\mu A^\mu$ forbidden by gauge invariance!)

$U(1)$ = abelian gauge group

QED = abelian gauge theory

still to be determined: photon propagator

consider elm. field in the presence of a classical current density J^μ ($\partial_\mu J^\mu = 0$)

→ generating functional

$$Z[J] = \frac{1}{\mathcal{N}} \int [dA^\mu]\, e^{i\int d^4x \left(-\frac{1}{4} F_{\mu\nu}F^{\mu\nu} - J_\mu A^\mu\right)}$$

$$S = \int d^4x \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - J_\mu A^\mu\right)$$

$$= \int d^4x \left[-\frac{1}{2} A^\mu \left(-g_{\mu\nu} (\Box - i\varepsilon) + \partial_\mu \partial_\nu\right) A^\nu - J_\mu A^\mu\right]$$

partial integration

we could try again the usual trick: shift of variables $A^\mu = A'^\mu + B^\mu$, where B^μ should fulfil

$$\left(g_{\mu\nu} (\Box - i\varepsilon) - \partial_\mu \partial_\nu\right) B^\nu = J_\mu$$

→ propagator $D^{\nu\rho}(x)$ = Green function of the differential operator $g_{\mu\nu}(\Box - i\varepsilon) - \partial_\mu\partial_\nu$:

$$\left(g_{\mu\nu} (\Box - i\varepsilon) - \partial_\mu \partial_\nu\right) D^{\nu\rho}(x) = \delta_\mu{}^\rho\, \delta^{(4)}(x)$$

Fourier representation $D^{\nu\rho}(x) = \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \tilde{D}^{\nu\rho}(k)$

$\rightarrow$ in momentum space, we get the equation

$$\left(-g_{\mu\nu} k^2 + k_\mu k_\nu\right) \tilde{D}^{\nu\rho}(k) = \delta_\mu{}^\rho$$

$\Rightarrow \quad a = 1 \ , \ b = -k^2 \qquad$ (see 11/4)

$\Rightarrow \quad a k^2 + b = 0$

$\Rightarrow$ inverse of $\left(-g_{\mu\nu} k^2 + k_\mu k_\nu\right)$ does <u>not</u> exist!

indeed: $\quad \left(-g_{\mu\nu} k^2 + k_\mu k_\nu\right) k^\nu = 0 \qquad (k^\nu \neq 0)$

this can already be seen in the space-time representation:

$$\left(g_{\mu\nu} \Box - \partial_\mu \partial_\nu\right) \partial^\nu \Lambda =$$

$$= \Box \partial_\mu \Lambda - \partial_\mu \Box \Lambda = 0$$

for arbitrary $\Lambda \qquad (\partial^\nu \Lambda \neq 0$ in general$)$

way out: gauge fixing term in the Lagrangean

(Fermi's trick)

gauge parameter

$$\mathcal{L} \to \mathcal{L}_{eff} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \underbrace{- \frac{\xi}{2} (\partial_\mu A^\mu)^2}_{\text{gauge fixing term}} - J_\mu A^\mu$$

the action can now be written in the form

$$S = \int d^4x \left\{ \frac{1}{2} A^\mu \left[g_{\mu\nu} \Box - (1-\xi) \partial_\mu \partial_\nu \right] A^\nu - J_\mu A^\mu \right\}$$

the equation $[g_{\mu\nu} \Box - (1-\xi) \partial_\mu \partial_\nu] B^\nu = J_\mu$

can now be inverted:

$$T_{\mu\nu} = -k^2 g_{\mu\nu} + (1-\xi) k_\mu k_\nu$$

$$a = 1-\xi \, , \quad b = -k^2$$

$$\Rightarrow \quad (T^{-1})^{\nu\rho} = -\frac{1}{k^2} \left[g^{\nu\rho} - \left(1-\frac{1}{\xi}\right) \frac{k^\nu k^\rho}{k^2} \right]$$

Feynman gauge ($\xi = 1$) : $(T^{-1})^{\nu\rho} = -\frac{g^{\nu\rho}}{k^2}$

$$D^{\nu\rho}(x) = \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \underbrace{\frac{-1}{k^2+i\varepsilon}\left[g^{\nu\rho} - \left(1-\frac{1}{\xi}\right)\frac{k^\nu k^\rho}{k^2}\right]}_{\tilde{D}^{\nu\rho}(k)}$$

$$S'_{eff} = \int d^4x \left\{ \frac{1}{2} A^\mu \left[g_{\mu\nu}(\Box - i\varepsilon) - (1-\xi)\partial_\mu \partial_\nu \right] A^\nu - J_\mu A^\mu \right\}$$

$$\stackrel{A_\mu = A'_\mu + B_\mu}{=} \int d^4x \left\{ \frac{1}{2} A'^\mu \left[g_{\mu\nu}(\Box - i\varepsilon) - (1-\xi)\partial_\mu \partial_\nu \right] A'^\nu \right.$$

$$+ A'^\mu \left[g_{\mu\nu}(\Box - i\varepsilon) - (1-\xi)\partial_\mu \partial_\nu \right] B^\nu - J_\mu A'^\mu$$

$$\left. + \frac{1}{2} B^\mu \left[g_{\mu\nu}(\Box - i\varepsilon) - (1-\xi)\partial_\mu \partial_\nu \right] B^\nu - J_\mu B^\mu \right\}$$

$$\left[g_{\mu\nu}(\Box - i\varepsilon) - (1-\xi)\partial_\mu \partial_\nu \right] B^\nu = J_\mu$$

i.e. $B^\mu(x) = \int d^4y \, D^{\mu\nu}(x-y) J_\nu(y)$

$\rightarrow$ terms linear in A'^μ disappear

$$\Rightarrow \quad S_{eff} = \int d^4x \left\{ \frac{1}{2} A'^{\mu} \left[g_{\mu\nu} (\Box - i\varepsilon) - (1-\xi) \partial_\nu \partial_\nu \right] A'^{\nu} - \frac{1}{2} J_\mu B^\mu \right\}$$

$$= \int d^4x \; \frac{1}{2} A'^{\mu} \left[g_{\mu\nu} (\Box - i\varepsilon) - (1-\xi) \partial_\mu \partial_\nu \right] A'^{\nu}$$

$$- \frac{1}{2} \int d^4x \, d^4y \; D^{\mu\nu}(x-y) \, J_\mu(x) \, J_\nu(y)$$

$$\Rightarrow \quad Z[J] = e^{-\frac{i}{2} \int d^4x \, d^4y \; D^{\mu\nu}(x-y) J_\mu(x) J_\nu(y)}$$

<u>remark:</u> $Z[J]$ is independent of the gauge parameter ξ for a <u>conserved</u> current ($\partial^\mu J_\mu = 0$): to see this, we compute

$$\int d^4y \; D^{\mu\nu}(x-y) \, J_\nu(y)$$

and convince ourselves that this expression is independent of ξ if $\partial^\mu J_\mu = 0$

$$J_\nu(y) = \int \frac{d^4p}{(2\pi)^4}\, e^{-ipy}\, \tilde{J}_\nu(p)$$

$$\partial^\nu J_\nu(y) = \int \frac{d^4p}{(2\pi)^4}\, e^{-ipy}\, (-ip^\nu)\, \tilde{J}_\nu(p)$$

$$\partial^\nu J_\nu = 0 \quad \Leftrightarrow \quad p^\nu \tilde{J}_\nu(p) = 0$$

$$\Rightarrow \int d^4y\, D^{\mu\nu}(x-y)\, J_\nu(y) =$$

$$= \int d^4y \int \frac{d^4k}{(2\pi)^4}\, e^{-ik(x-y)}\, \frac{-1}{k^2+i\varepsilon}\left[g^{\mu\nu} - \left(1-\frac{1}{\xi}\right)\frac{k^\mu k^\nu}{k^2}\right]$$

$$\times \int \frac{d^4p}{(2\pi)^4}\, e^{-ipy}\, \tilde{J}_\nu(p)$$

$$= \int \frac{d^4k}{(2\pi)^4}\, e^{-ikx}\, \frac{-1}{k^2+i\varepsilon}\left[g^{\mu\nu} - \left(1-\frac{1}{\xi}\right)\frac{k^\mu k^\nu}{k^2}\right]\tilde{J}_\nu(k)$$

$$\underset{\substack{\uparrow \\ k^\nu \tilde{J}_\nu(k)=0}}{=} \int \frac{d^4k}{(2\pi)^4}\, e^{-ikx}\, \frac{-g^{\mu\nu}}{k^2+i\varepsilon}\, \tilde{J}_\nu(k)$$

$$\Rightarrow \int d^4x\, d^4y\, D^{\mu\nu}(x-y)\, J_\mu(x)\, J_\nu(y)$$

$$= \int d^4x \int \frac{d^4k}{(2\pi)^4}\, e^{-ikx}\, \frac{-\tilde{J}^\mu(k)}{k^2+i\varepsilon} \int \frac{d^4p}{(2\pi)^4}\, e^{-ipx}\, \tilde{J}_\mu(p)$$

$$= -\int \frac{d^4k}{(2\pi)^4}\, \frac{\tilde{J}^\mu(k)\, \tilde{J}_\mu(-k)}{k^2+i\varepsilon}$$

$J^\mu(x)$ real $\Rightarrow$ $\tilde{J}^\mu(-k) = \tilde{J}^\mu(k)^*$

$$\Rightarrow \int d^4x\, d^4y\, D^{\mu\nu}(x-y)\, J_\mu(x)\, J_\nu(y)$$

$$= -\int \frac{d^4k}{(2\pi)^4}\, \frac{\tilde{J}^\mu(k)\, \tilde{J}_\mu(k)^*}{k^2+i\varepsilon}$$

$$\Rightarrow Z[J] = \langle 0\, \mathrm{out} | 0\, \mathrm{in} \rangle_J = e^{\frac{i}{2}\int \frac{d^4k}{(2\pi)^4}\, \frac{\tilde{J}^\mu(k)\, \tilde{J}_\mu(k)^*}{k^2+i\varepsilon}}$$

path integral quantization of an abelian gauge field

reminder: $S[A] = \int d^4x \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu}\right)$

$$= \frac{1}{2} \int d^4x \, A^\mu \underbrace{\left(g_{\mu\nu} \Box - \partial_\mu \partial_\nu\right)}_{\text{\underline{not} invertible}} A^\nu$$

→ usual "trick" (shift of integration variables) <u>not</u> applicable

reason: $S[A_\mu + \partial_\mu \Lambda] = S[A_\mu]$

(action invariant under abelian gauge transformations)

in the expression

$$\langle 0 | T\, \underbrace{\mathcal{O}[A]}_{\text{gauge invariant observable}} | 0 \rangle = \frac{\int [dA]\, e^{iS[A]}\, \mathcal{O}[A]}{\int [dA]\, e^{iS[A]}}$$

integration performed also over gauge-equivalent field configurations

goal: we want to factorize out the redundant part of the integration measure

→ Faddeev – Popov Quantization

possible procedure:

$$\Delta[A,\omega] \int [d\lambda]\, \delta(\partial_\mu A^\mu + \Box \lambda - \omega) = 1$$

this equation defines $\Delta[A,\omega]$

remark: $\Box \to \Box + i\varepsilon$ tacitly assumed to ensure uniqueness of the solution of

$$\partial^\mu A_\mu + (\Box + i\varepsilon)\lambda - \omega = 0$$

$$\to \quad \lambda = \frac{1}{\Box + i\varepsilon}(\omega - \partial_\mu A^\mu)$$

$$\Delta[A,\omega]^{-1} = \int [d\lambda]\, \delta(\partial . A + \Box \lambda - \omega)$$

$$= \Delta[0, \omega - \partial . A]^{-1}$$

transformation of the integration variable (shift) : $\lambda = \lambda' + \mu$

new variable of integration

$$\rightarrow \Delta[A,\omega]^{-1} = \int [d\lambda']\, \delta(\partial . A + \Box \lambda' + \Box \mu - \omega)$$

$$= \Delta[A, 0]^{-1}$$ (by an appropriate choice of μ)

$\Rightarrow \Delta[A,\omega] = \Delta[0,0]$ is <u>independent</u> of A_μ, ω (this is <u>not</u> the case for <u>nonabelian</u> gauge theories) this result can also be seen in the following way:

$$\int [d\varphi]\, \delta(\varphi) = 1$$

variable transformation $\varphi = \partial . A + (\Box + i\varepsilon)\alpha - \omega$

$\rightarrow$ unique mapping $\alpha = \frac{1}{\Box + i\varepsilon}(\varphi + \omega - \partial . A)$

$$\int [d\alpha] \left(\det \frac{\delta\varphi(x)}{\delta\alpha(y)}\right) \delta\left(\partial . A + (\Box + i\varepsilon)\alpha - \omega\right) = 1$$

$$\frac{\delta\varphi(x)}{\delta\alpha(y)} = (\Box + i\varepsilon)\,\delta(x - y)$$

$\Rightarrow \Delta[A, \omega] = \det(\Box + i\varepsilon)$ <u>independent of A, ω</u>

$$\rightarrow \quad \langle 0 | T\, \mathcal{O}[A] | 0 \rangle$$

$$= \frac{\int [dA\, d\lambda]\, \Delta[0,0]\, \delta(\partial . A + \Box\lambda - \omega)\, e^{iS[A]}\, \mathcal{O}[A]}{\int [dA\, d\lambda]\, \underbrace{\Delta[0,0]\, \delta(\partial . A + \Box\lambda - \omega)}_{1}\, e^{iS[A]}}$$

$$= \frac{\int [dA\, d\lambda]\, \delta(\partial . A + \Box\lambda - \omega)\, e^{iS[A]}\, \mathcal{O}[A]}{\int [dA\, d\lambda]\, \delta(\partial . A + \Box\lambda - \omega)\, e^{iS[A]}}$$

variable transformation $A_\mu = A_\mu' + \partial_\mu \Lambda$

(gauge transformation)

$$S[A] = S[A'] \ , \quad \mathcal{O}[A] = \mathcal{O}[A']$$

(gauge invariant quantities)

$\rightarrow \lambda$ disapper by an appropriate choice of Λ

$$\rightarrow \langle 0 | T \mathcal{O}[A] | 0 \rangle = \frac{\cancel{\int [d\lambda]} \int [dA]\, \delta(\partial . A - \omega)\, e^{iS[A]}\, \mathcal{O}[A]}{\cancel{\int [d\lambda]} \int [dA]\, \delta(\partial . A - \omega)\, e^{iS[A]}}$$

desired factorization

numerator and denominator <u>independent</u> of ω $\rightarrow$ integrate these expressions over a <u>weight function</u>

$$\langle 0 | T\, \mathcal{O}[A] | 0 \rangle =$$

$$= \frac{\int [d\omega]\, e^{-\frac{i\xi}{2}\int d^4x\, \omega(x)^2} \int [dA]\, \delta(\partial.A-\omega)\, e^{iS[A]}\, \mathcal{O}[A]}{\int [d\omega]\, e^{-\frac{i\xi}{2}\int d^4x\, \omega(x)^2} \int [dA]\, \delta(\partial.A-\omega)\, e^{iS[A]}}$$

$$= \frac{\int [dA]\, e^{i\left[S[A]-\int d^4x \frac{\xi}{2} (\partial.A)^2\right]}\, \mathcal{O}[A]}{\int [dA]\, e^{i\left[S[A]-\int d^4x \frac{\xi}{2} (\partial.A)^2\right]}}$$

$$\rightarrow \quad S_{eff}[A] = S[A] - \frac{\xi}{2}\int d^4x\, (\partial.A)^2$$