Exercises for T2, Summer term 2017, Sheet 3

1) Photoelectric effect

Light with sufficient energy is able to trigger ejection of electrons from the atomic bonds in a metal via the photoelectric effect. Assuming that electrons start being emitted if the wavelength of the incident light falls below 500 nm, calculate the work function and the corresponding voltage.

2) Natural units

Express the following quantities in units of eV (with respective power) by using natural units $\hbar = c = k = 1$: typical atomic radius (1 $\mathring{A}$), typical radius of nucleons (1 fm), Compton wavelength of the electron, gravitational acceleration g on the surface of the earth, temperature inside the ITER Tokamak that is necessary to initiate nuclear fusion. Use the internet to find numbers that you dont know by heart.

3) Wave function

Let the wave function of a particle in one dimension be given by

$$\psi(x) = \begin{cases} \mathcal{N} & \text{if} \quad x \in [a, b], \\ 0 & \text{else} \end{cases}$$

Calculate the normalization constant $\mathcal{N}$, the expectation value $\langle x \rangle$ for the position of the particle, the expectation value $\langle x^2 \rangle$ and the mean square deviation (variance) $(\Delta x)^2 = \langle x^2 \rangle - \langle x \rangle^2$. Assuming $a = 0$, $b = 3$, calculate the probability to find the particle in the interval $[-2, 10.5]$.

4) Binomial distribution

Let a (pure) state of a spinless particle in 3 dimensions be represented by the wave-function $\psi(\vec{x})$. A detector D shall be able to determine if the particle is located within the region $V \subset \mathbb{R}^3$, while a second detector D' shall be able to register the particle if its position is outside of V. We now conduct a measurement on N equally prepared copies of the system. For a certain copy of the system **exactly one** of the detectors triggers. The probability p that the particle is detected within the region V is given by $p = \int_V d^3\vec{x}\, |\psi(\vec{x})|^2$. The probability to register the particle outside V is therefore $1 - p$. The probability w_n that the particle is detected inside V for exactly n of the N copies is given by the binomial distribution

$$w_n = \binom{N}{n} p^n (1-p)^{N-n} = \frac{N!}{n!(N-n)!}\, p^n (1-p)^{N-n}$$

The mean $\bar{n}$ and the standard deviation Δn (as well as higher moments) of n can be easiliy calculated with the characterisic function

$$\phi(x) = \sum_{n=0}^{N} w_n e^{nx}$$

Start by calculating[1] $\phi(x)$ and use the characteristic function to subsequently compute $\bar{n}/N$ and $\Delta n/N$. Discuss the behaviour for $N \to \infty$!

What is the conceptual connection of these results to those you calculated in problem 3?

4) Dirac δ-Function in 1 Dimension

The Dirac δ function is a distribution whose properties are defined with its action on test functions $f(x)$ upon integrations:

$$\int_{-\infty}^{\infty} d\bar{x}\, f(\bar{x})\, \delta(x - \bar{x}) = f(x)\,,$$

where $\bar{x}$ be real, $\delta(x) = \delta(-x)$ and $\int_{-\infty}^{\infty} dx\, \delta(x - \bar{x}) = 1$, and f be (at least at x) be a continuous function. Show the following properties of the Dirac δ function for integrals over arbitrary test functions:

(a) $x\,\delta(x) = 0 \qquad x\,\delta(x - \bar{x}) = \bar{x}\,\delta(x - \bar{x})\,,$

(b) $f(x)\,\delta(x - \bar{x}) = f(\bar{x})\,\delta(x - \bar{x})\,,$

(c) $\delta(ax) = \frac{1}{|a|}\delta(x)\,,$ a real and finite,

(d) $\delta((x - a)(x - b)) = \frac{1}{|a-b|}[\delta(x - a) + \delta(x - b)]\,,$ a, b real and different.

Note: Because these properties are valid for integrations over arbitrary test functions, you can view them as direct properties of the δ function itself.

[1] Just write everything down and take your time!