

## Exercises for T2, Summer term 2017, Sheet 2

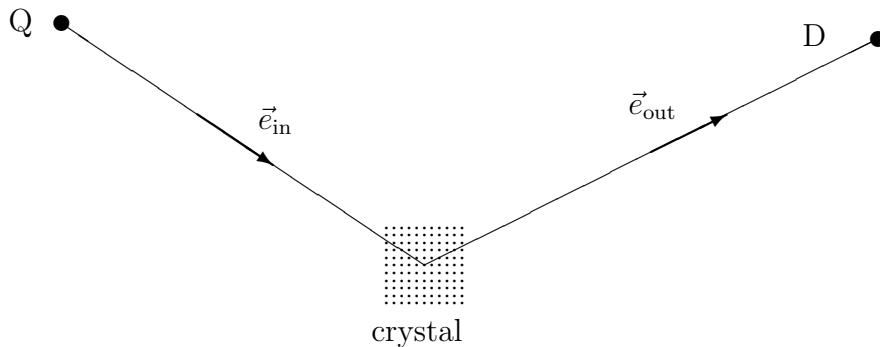
### 1) De Broglie wavelength

Calculate the de Broglie wavelength of a proton with a kinetic energy of 1 eV, 100 eV, 100 keV ( $m_p \simeq 938 \text{ MeV}/c^2$ ). What is the de Broglie-wavelength of a human with a mass of 70 kg, who is moving at 1 m/s? Compare the obtained results with the size of a proton and the size of a human. What is your conclusion concerning the wave character of a human being, compared to that of a proton?

### 2) De Broglie wavelength of nonrelativistic particles

Write down the general (relativistic) relation between the de Broglie wavelength and the **kinetic energy**  $T = E - mc^2$  of a massive particle ( $m \neq 0$ ). Find an approximation for the nonrelativistic case ( $T \ll mc^2$ ).

### 3) Scattering of neutrons



The atoms of a crystal lattice are located at the points  $\vec{x}_{\vec{n}} = a\vec{n}$ ,  $\vec{n} \in \mathbb{Z}^3$ ,  $n_i = -N, -N + 1, \dots, N$  ( $i = 1, 2, 3$ ). Furthermore there is a source Q ( $R_Q \gg Na$ ) at the point  $\vec{x}_Q = -R_Q \vec{e}_{\text{in}}$  ( $|\vec{e}_{\text{in}}| = 1$ ) which emits neutrons with momentum  $p$ . In addition a neutron-detector ( $R_D \gg Na$ ) is located at  $\vec{x}_D = R_D \vec{e}_{\text{out}}$  ( $|\vec{e}_{\text{out}}| = 1$ ). The amplitude  $\langle D \text{ out} | Q \text{ in} \rangle$ , that the detector D detects a neutron which originated from Q is of the form

$$\langle D \text{ out} | Q \text{ in} \rangle \sim \sum_{\vec{n}} \frac{e^{ip|\vec{x}_Q - \vec{x}_{\vec{n}}|/\hbar}}{|\vec{x}_Q - \vec{x}_{\vec{n}}|} W_{\vec{n}} \frac{e^{ip|\vec{x}_D - \vec{x}_{\vec{n}}|/\hbar}}{|\vec{x}_D - \vec{x}_{\vec{n}}|}.$$

(multiple scattering is omitted). Calculate this expression assuming that  $W_{\vec{n}}$  is the same for all atoms. Use a suitable approximation for  $|\vec{x}_Q - \vec{x}_{\vec{n}}| = \sqrt{(\vec{x}_Q - \vec{x}_{\vec{n}})^2}$  which should reflect that  $|\vec{x}_Q| = R_Q \gg |\vec{x}_{\vec{n}}|$ . (analogous for  $|\vec{x}_D - \vec{x}_{\vec{n}}|$ .)

By following that strategy you should get an expression like

$$\langle \text{D out} | \text{Q in} \rangle \sim \sum_{\vec{n}} e^{ipa\vec{\Delta} \cdot \vec{n} / \hbar} = \prod_{i=1}^3 \sum_{n_i=-N}^N e^{ipa\Delta_i n_i / \hbar}, \quad \vec{\Delta} = \vec{e}_{\text{in}} - \vec{e}_{\text{out}}$$

This means it is necessary to calculate a geometric series of the type

$$s(\alpha) = \sum_{n=-N}^N e^{i\alpha n}$$

Now show that this equals

$$s(\alpha) = \frac{\sin \alpha(N + \frac{1}{2})}{\sin \frac{\alpha}{2}}$$

Discuss the behavior of this function. For which values of  $\alpha$  are there distinct extrema? Show that in the case of neutron scattering

$$\frac{pa}{2\pi\hbar} \vec{\Delta} = \vec{\nu} \in \mathbb{Z}^3$$

leads to distinct maxima of the interference pattern. This is Laue's condition for interference for the simple cubic lattice. It is also possible to write it in the form

$$\frac{a}{2\pi} (\vec{k}_{\text{in}} - \vec{k}_{\text{out}}) = \vec{\nu} \in \mathbb{Z}^3$$