Chapter 1: Introduction

1.1 Historical Overview

- The beginning of quantum physics
- First essential steps at the end of the 19th and the beginning of the 20th century

(1) Particle Character of Light

Max Planck (1900)

- Introduction of the "Planck constant" $h$ to find correct theoretical description of the energy density of black-body radiation.

\[ u(v) dv = \frac{h v^3}{e^{h v/kT} - 1} \, dv \]

Planck's Radiation Law

( Planck's thermal equilibrium )

Derivation: (a) Density of $u$-radiation states: consider box of volume $V$.

\[ \rho \equiv \text{number of waves in frequency interval [v, v+dv]} \]

\[ \rho = \frac{\hbar v^3}{e^{h v/kT} - 1} \, dv \]

Planck's constant

\[ \hbar = \frac{E_n}{\lambda_n} \]

\[ n = 0, 1, 2, \ldots \]

(b) classical: each state can carry an arbitrary amount of energy $\rightarrow E_n(v, T) = n \hbar \omega$

- Planck's constant: probability for a state with energy $E$ is created at temperature $T$

\[ p(E) = e^{-E/kT} \]

\[ p(v) = e^{-h v/kT} (1 - e^{-h v/kT}) \]

\[ \text{Average energy} \quad \langle E(v) \rangle = \sum_{v} E(v) p(v) = \frac{h v}{e^{h v/kT} - 1} \]

Planck's constant

\[ \hbar = \frac{E_n}{\lambda_n} \]

\[ \text{Planck's constant} \]

\[ \hbar = \frac{6.582 \times 10^{-16}}{6.63 \times 10^{-34}} \text{ eV s} \]

\[ \text{Planck's constant} \]

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Albert Einstein (1905):

Light quantization hypothesis: light consists of particles with energy $E = h \nu$, $\nu = \frac{c}{\lambda}$, $\lambda = \frac{c}{\nu}$ wave length.

1. $E = \frac{c^2}{\lambda}$ (angular) wave number.

2. Light particle at freq $\nu$ has momentum $p = h \nu$. $p = \frac{h}{\lambda}$.

3. $E = mc^2$, $E = h \nu = \frac{h c}{\lambda}$.

4. $E = \frac{h c}{\lambda}$ is relation of massless particles.

- Light state of freq $\nu$ with energy $E = h \nu$ in light particles each having energy $E$.

- Photoelectric effect.

- Double Slit Experiment.

- Compton Effect.

(2) Atomic Structure

Spectroscopy: Line spectrum. Atoms absorb and emit light with (small) specific frequencies (spectral lines).

Joseph John Thomson (1903):

Planck’s Quantum Theory: $E = h \nu$.

- Planck’s quantum theory explains electron emission.

- Planck’s quantum theory explains photoelectric effect.

Heinrich Hertz, Ernest Rutherford (1909):

- Scattering of alpha particle off (known) gold atom.

- Scattering when $\theta > 2\theta_c$ (scattering angle) observed.

- Which is not compatible with the planck-predicted model.
Ernst Rutherford (1911) - Classical calculation of differential cross section of 2 point charges

\[ \frac{d\sigma}{d\Omega} = \frac{\pi}{3} \left( \frac{r_1 r_2}{2\sigma \hbar} \right)^2 \]

- Atomic nucleus
- Protons and neutrons

\[ e^+ \text{ and } e^- \text{ are emitted in pairs in very small numbers (-10^-11)} \]

- Atom - planetary system
- But stable atoms cannot be explained by classic mechanics + E-dynamics
- Electrons orbit photo, lose energy
- In explanation for spectral lines

Wolfgang Pauli (1918) - Spectral lines explained by transitions (with photon radiation) between discrete energy levels

\[ \text{Pauli's exclusion principle} \]

- Idea: electrons in hydrogen: classic mechanics + E-dynamics supplemented by " quantum " conditions for allowed energy levels

\[ E_n = -\frac{m_e c^2}{n^2} \]

\[ n = 1, 2, 3, \ldots \]

(3) Wave-Particle Duality of Matter Particles

Louis de Broglie (1923) - Every particle has wave properties (just as photons)

- Connection: Uncertainty - wave number vector: \( \vec{p} = h \vec{k} \)

- Connection: Energy & angular frequency: \( E = h \nu = h \omega \)

- Description of a particle with momentum \( \vec{p} \) & mass \( m \) by wave-particle plane wave

\[ \psi(x, t) = \exp \left[ i (\vec{p} \cdot \vec{x} - \omega t) \right] \]

- Double slit experiment

(4) Quantum Theory

Werner Heisenberg (1925) - Radical departure from concept of classical mechanics!

- Position and momentum of a particle related to non-commutable operators \( \hat{Q} \) and \( \hat{P} \), which lead to the outcome of a measurement at position and momentum. Pauli & Jordan:

\[ [\hat{Q}, \hat{P}] = i\hbar \]

- Development of matrix mechanics (Heisenberg, Max Born, Pascual Jordan)

Wolfgang Pauli (1925) - Solution of H-atom using matrix mechanics: binding energy binds E.
Erwin Schrödinger (1926):
- Formulation of a wave (diffusion) equation for de-Broglie waves for matter particles

\[ \text{Schrödinger Equation:} \quad \frac{\hbar}{2i} \frac{\partial}{\partial t} \psi(q,t) = H(q,p) \psi(q,t) \]

\[ H = \frac{p^2}{2m} + V(q) \]

\( \psi(q,t) \) is the wave function, \( H(q,p) \) is the Hamiltonian operator, \( q \) is location, \( p = -i \hbar \frac{\partial}{\partial q} \) is the momentum operator.

- Solution of 1-dimensional energy levels \( E_n \)
- Equivalence to matrix mechanics

Max Born (1926):
- Probability interpretation of the wave function \( \psi(q,t) \): \( |\psi(q,t)|^2 \) is probability to find a particle in a region of the spatial coordinate \( [q, q + dq] \).

- \( \psi(q,t) \) is the complex valued probability amplitude.

- \( \psi(q,t) \) uniquely describes the state of the particle at time \( t \). This state is in general not associated by a unique position, but a unique probability distribution of the particle's position for a measurement of the position at time \( t \).

Heisenberg's uncertainty principle (1927):
- Relation links together the standard deviations for the incorrect value of location and momentum for a particle

\[ \Delta q \Delta p \geq \frac{\hbar}{2} \]

\[ \Delta x = \sqrt{\langle (x-x_0)^2 \rangle} \]

- Location and momentum cannot be measured arbitrarily precise at the same time.
1.2. Main Principles of Quantum Physics

We consider a "Gratwine" scattering experiment to illustrate some essential rules of quantum physics.

(A) Basics of a scattering experiment

- Source Q produces neutrons with fixed momentum $\mathbf{p}$ (beam) at large distance from the crystal K that shall be analyzed.

- Detector D located at large distance from crystal K, detects scattered neutrons in an ideal way:
  - does not miss a scattered particle (giving $\mathbf{p}'$)
  - can measure $\mathbf{p}'$ (including the scattering angle $\theta$) very precisely
  - the whole solid angle 4$\pi$ is covered by D (e.g. with solid angle covered by detector)

- We are only measuring scattering events with $\theta = 0$.

- The incoherent transfer $\mathbf{p} = \mathbf{p}'$ is absorbed by the crystal.

We assume the case of elastic scattering of an infinitely heavy scattering center, i.e.

(a) Crystal is infinitely heavy, so that it does not absorb any energy. Also, can absorb any momentum transfer $\mathbf{p}$.

(b) Scattering process does not modify (e.g. altering charge) one of the atoms in the crystal. Assumption given such that one cannot tell (even in principle) at which atom the interaction took place. Up in [solidified] X.

For pedagogical reasons we simplify the setup even further:

(c) Crystal contains 2 formic chains (naphthalene) $A_1$ and $A_2$ with distance $d$ at lattice points 1 and 2.

- Interaction of neutrons with regular crystal atoms is negligibly small, only consider scattering off $A_2$.

- The resulting setup is equivalent to the double slit experiment.

(d) De Broglie wave length $\lambda = \frac{h}{p}$ of neutrons $\gg$ size of atom $A_{\lambda}$ (A$_2$ are protons).

- Importance of "any distance" (one shouldn't assume for "scattering")

1. The process is Q (particle production), at the crystal (scattering of crystal atoms) and in D (particle detection and measurement) are independent and not influenced by each other.

2. Particles in the beam are all parallel and have nearly the same direction of $\mathbf{p}$. 

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One makes the following general observations (for any scattering process):

1. For each incoming electron which is scattered (θ + φ) exactly one detector D detects a scattered particle (and nothing in momentum p′).

If the particle is not scattered, it goes along the beam along the original path and can in principle be detected as well.

2. For each single incoming particle one cannot say in which detector the scattered particle will be detected. → In this sense, the process is indeterministic.

3. In a large number of repetitions (with equal initial setup set to Q and K) of the scattering process one obtains (within statistical fluctuations) a unique measurement of an angular distribution of the scattered particles. → In this case the process is deterministic.

1st Rule of Quantum Physics

There is a well-defined and unique probability that a particle emitted from source Q is being observed at a detector D (at φ + θ).

1st Rule. There exists a complex-valued (probability) amplitude for that to happen which we write as:

\[ \langle D | \Phi_n | Q \rangle \]

Overlap of the state of an incoming particle (produced with probability \( |\Phi_n| \)) with the state of the outgoing particle (with \( |\Phi_n| \)) being detected at the corresponding detector.

Standard notation: In meaning:
- Outgoing "left" LHS: "in"
- Ingoing "right" RHS: "out" below.

From this amplitude one obtains the probability for the event to happen by taking the modulus squared:

\[ \text{Probability} = |\langle D | \Phi_n | Q \rangle|^2 = \langle D | \Phi_n | Q \rangle \langle Q | \Phi_n | D \rangle \]

2nd Rule of Quantum Physics

We assume that the scattering is a new phase and that multiple scattering of a single particle is negligible.

The incoming particle has 2 possibilities to get from Q to D:
1. From Q to point 1 (with interaction with A1) to D → amplitude \( \langle D | Q \rangle \)
2. From Q to point 2 (with interaction with A2) to D → amplitude \( \langle D | Q \rangle \).

We do not leave any possibility (according to our assumptions) to leave which way the particle took.

2nd Rule. If there is no principle way to know or assume which process path the particle took to get from Q to D, one has to sum over the amplitudes for all possible processes to obtain the complete amplitude for the process to happen:

\[ \langle D | Q \rangle_{\text{tot}} = \langle D | Q \rangle_1 + \langle D | Q \rangle_2 \]

\[ \Rightarrow \langle D | Q \rangle_{\text{tot}} = \Phi_1 + \Phi_2 \]

If e.g. A1 could not be there, the complete amplitude would be just \( \langle D | Q \rangle_1 \).
- The process pathways related to amplitudes \( \phi_1 \) and \( \phi_2 \) are related to a sequence of independent processes

\[
\text{Eg.} \quad \phi_1 = \begin{align*}
1 & \quad \text{Partially propagated from A to point 1} \\
2 & \quad \text{Interaction between particle and A}_1 \\
3 & \quad \text{Scattered particle propagated from point 1 to D}
\end{align*}
\rightarrow \text{amplitude } K(D, A)
\]

- 3rd Rule: IF several independent processes take place within a considered process, one has to multiply the amplitudes of the subprocesses to obtain the complete amplitude.

\[
\phi_i = \langle D | Q \rangle_c = K(D, i) \cdot \psi_i K(i, A) \quad i = 1, 2
\]

\[\text{(E) Interference}\]

- \( \phi_1 \) and \( \phi_2 \) are complex-valued and can combine \( \phi_1 = 10.1 e^{i \theta_1}, \quad \phi_2 = 10.3 e^{i \theta_2} \)

- If e.g. only \( A_1 \) could be present, \( \psi = 10.1 e^{i \theta} \) could be the probability that the scattering and the phase \( \theta \) would be irrelevant. However, \( A_2 \) exist and the probability for the scattering process is

\[
\text{probability}(D = 2) = \psi_1^* \psi_2 = |10.1 e^{i \theta_1} |^2 + |10.3 e^{i \theta_2} |^2 = 2 \cdot 10.1 \cdot 10.3 \cdot \cos(\theta_1 - \theta_2) \]

\[\text{interference term}\]

The probability for a process that can take place via different processes for which there is no principle way to tell/measure which is being realised is in general not the sum of probabilities of the different processes due to possible phase differences.

We determine the interference effect for our scattering example (see figure where arrangement of \( A_1, A_2, \) \( K(D, A) = K(D, A_2) - K(D, A_1) \))

\[
\phi_i = (K(D, i) + K(D, i)) \cdot \psi_{i, c}(D)
\]

\[r_{W_i} \text{ = distance of } A_i \text{ to } D\]

We call later convince ourselves that \( K(D, A) = e^{i p v \lambda} \text{ with } K(D, A_2) - K(D, A_1) \quad \text{or} \quad p = 10.1 + 10.3\)

Because \( r_{W_1} \Rightarrow A_1 = r_1 = r_2 \Rightarrow r_{1, c} = r_{2, c} = \lambda \text{ and } r_{1} - r_{2} = \Delta \text{ with } \phi_{1, c} - \phi_{2, c} = \frac{\rho (r_1 - r_2)}{\Delta} = \frac{p \Delta}{\Delta} \)

and thus \( U_{1, c} = 2 \cdot 10.1 \cdot \cos \left( \frac{p \Delta}{\Delta} \theta \right) = 2 \cdot 10.1 \cdot \cos \left( \frac{p \Delta}{\Delta} \theta \right) = 4 \cdot 10.1 \cdot \cos^2 \left( \frac{p \Delta}{\Delta} \theta \right) \lambda = \frac{\lambda}{p} \quad k = \frac{1}{\lambda} \)

\[\text{The smaller } k \text{ the more interference minimum exist}
\]

There are n interference minimum for

\[
\frac{2n}{2\pi n} < k < \frac{2n}{2\pi n}
\]

\[1 + \cos(x) = 2 \cdot \cos^2 \left( \frac{x}{2} \right) \]

\[\text{in minimum valid for } 0 \times n \times \pi \]

\[
\frac{2n - 1}{2n} \pi < \pi < \frac{2n}{2n} \pi
\]
Comments:

(1) In a more complete treatment we have to account for the scattering off all atoms in the crystal. We then have to sum over the functions of all lattice points:
\[ \langle \Omega_1|Q_1|\Omega_2 \rangle = \sum_{i} K(i;\Omega_1) \hat{\Omega}_i K(i;\Omega_2) \]

(2) Likewise in a more complete treatment we also have to account for multiple scattering contributions. In this context the previous treatment represents the first term of a series of terms accounting for increasing number of scatterings, i.e.
\[ \langle \Omega_1|Q_1|\Omega_2 \rangle = \sum_{i} K(i;\Omega_1) \hat{\Omega}_i K(i;\Omega_2) + \sum_{i,j} K(i;\Omega_1) \hat{\Omega}_i K(j;\Omega_2) \hat{\Omega}_j K(i;\Omega_2) + O(\mu^3) \]

For scattering with Spin-Dependence:

(3) Up to now we have assumed that the neutrons and the crystal atoms do not have any internal structure that might be changed in the scattering process.

However, neutrons carry and atoms carry a Spin, which is an intrinsic form of angular momentum that is quite similar to the polarization freedom of light (photon).

The total spin of all particles is a conserved quantity (as in the initial state and the final state of the scattering process), but spin can be exchanged between the particles involved in the scattering process ("spin flip")

- Neutrons can carry 2 spin states:
  - "spin up": \( n \uparrow \)
  - "spin down": \( n \downarrow \)

For our gedanken experiment we assume that each of the above \( A_i \) can carry also 2 spin states:

- "spin up": \( A_i \uparrow \)
- "spin down": \( A_i \downarrow \)

If the above \( A_i \) would carry no spin ("scalar" "spin-0"), the considerations of the previous section remain unchanged due to angular momentum conservation up to the fact that there are two different neutron states (instead of a single one).

- Neutrons spin state could be \( \uparrow \quad \downarrow \)

Case 1: The spin of neutron and atom \( A_{i1} \) are either all up or all down

- Due to angular momentum conservation: no spin flip possible

\[ \Phi_{12} = \langle D_{\uparrow} n_{\uparrow}; A_{i1} \uparrow, A_{i2} \uparrow | Q_{\uparrow} n_{\uparrow}; A_{i1} \uparrow, A_{i2} \uparrow \rangle \quad \text{or} \quad \Phi_{12} = \langle D_{\uparrow} n_{\downarrow}; A_{i1} \downarrow, A_{i2} \downarrow | Q_{\downarrow} n_{\downarrow}; A_{i1} \downarrow, A_{i2} \downarrow \rangle \]

These are no differences to the previous considerations described in (3) to (6) up to the fact that the subamplitudes in the calculation of \( \Phi_{12} \) are different for the up (f) and the down (i) case.

\[ 03/04/2016 (4) \]
Case 2: The spin of the incoming neutron is down and the spins of $A_1$'s are initially both up.

1. The no spin flip case: Then, in no principle way to tell which, when the neutron interacted, so we have to add the amplitudes for interaction with $A_1$ and $A_2$ -> Interference.

$$ A_{y_1} = \langle D, m_t, A_1, A_2 | Q, m_t, A_1, A_2 \rangle = K(2) \bar{D}_1(k_1, 0) + K(2) D_2(k_2, 0) $$

Amplitudes for propagation of the neutron:

we can assume that $\bar{D}_1$ is in no different in $A_1$'s for spin up ($u_1$) or spin down ($d_1$) neutron.

2. The spins of the neutron and of $A_1$ flip: After the scattering, the spin of $A_1$ has changed, so that it is clear that $A_2$ but not participated in the process.

$$ A_{y_2} = \langle D, m_t, A_1, A_2 | Q, m_t, A_1, A_2 \rangle = K(2) \bar{D}_1(k_1, 0) $$

Amplitudes for interaction with spin flip.

3. The spins of the neutron and of $A_2$ flip: After the scattering, the spin of $A_2$ has changed, so that it is clear that $A_1$ but not participated in the process.

$$ A_{y_3} = \langle D, m_t, A_1, A_2 | Q, m_t, A_1, A_2 \rangle = K(2) \bar{D}_1(k_1, 0) $$

Amplitudes for interaction with spin flip.

Outcomes 2 and 3 are physically distinguishable, because the final states are to $A_1$ and $A_2$ are different. Even if one chooses not to measure/observe the spin of $A_1$ and $A_2$.

In general: Processes for which initial or final states are physically distinguishable - even if one decides not to measure/observe the corresponding observables - do not interfere, i.e., their amplitudes shall never be added.

One can ask for the probability that the neutron being detected at $D$ without specifying (i.e., restricting) the momenta or the spins final state spin. One then has to add the probabilities of outcomes 1-3.

$2 \times \text{probability}(D, u_1, A_1, \text{any spin}) = Q_{u_1}(A_1, A_2) = 14_{1}^{0} + 14_{1}^{1} + 14_{1}^{2}$
We assume the in the initial state $A_e$ and $A_e$ both carry "spin-up", but that $D$ produces neutrons with
50% spin-up and 50% spin-down. We say that in the initial state the above $A_e$ are 100% potential spin-up
and the two neutrons are unpolarized.

2. $x_0$, $y_0$, $y_l$ are the amplitudes for the processes 1-3 for $u \uparrow$ in the initial state. We call $x_0$ the amplitudes for scattering into $D$ for $u \uparrow$ in the initial state, it is of the interference type because there is no possible spin-flip and thus the amplitudes for scattering off $A_e$ and $A_e$ have to be added to get $x_0$.

2. $\text{probability} (D, u, A_e, any \ spin \rightarrow Q, u(\text{unpolarized}), A_e, A_e) = \frac{1}{2} \left( |x_0|^2 + |y_0|^2 + |y_l|^2 \right) + \frac{1}{2} |x_0|^2$

2. If the neutrons have an arbitrary polarization (e.g. $q$-polarized spin up) we have $(0 < q < 1)$

2. $\text{probability} (D, u, A_e, any \ spin \rightarrow Q, u(\text{polarized}), A_e, A_e) = (1-q) \left( |x_0|^2 + |y_0|^2 + |y_l|^2 \right) + q |x_0|^2 \quad (0 < q < 1)$

2. A state with non-trivial polarization $(0 < q < 1)$ is called a mixed state.
A state that is to 100% in a unique quantum mechanical state (100% polarized) is called a pure state.
1.3. Units, Scales, and Fundamental Constants of Nature

In 1.2(E) we learned that an interference pattern is only visible if the de-Broglie wave length \( \lambda \) of the probing particle is smaller than the distance between the slits; i.e.
\[ \lambda = \frac{h}{p} < d \]

In general, the resolution one has is limited by the de-Broglie wave length \( \lambda \) of the probing particle.

We see that through the fundamental relation \( h = \lambda p \), there is a unique connection between the apparently different quantities "distance" and "momentum". We can - in fact - consider them as quantities that are dependent (and not independent as we might naturally feel).

- Frequently used energy unit: electron volt (eV)

1 eV = kinetic energy a particle with one elementary charge \( e \) (\( \approx \) proton \( e^+ \), electron \( e^- \)) receives when traversing a potential difference of 1 V (wrt).

Using the relation \( E = \frac{1}{2} m v^2 \) we see that we can conversely use

\[ \frac{1 \text{ eV}}{c^2} \text{ as a unit for momentum and } \frac{1 \text{ eV}}{c^2} \text{ as a unit for mass} \]

Examples for masses of particles:

- \( m_{\text{e+}} \approx 0.5 \text{ MeV}/c^2 \)
- \( m_{\text{e-}} \approx 0.5 \text{ MeV}/c^2 \)
- \( m_{\text{nucleon}} \approx 940 \text{ MeV}/c^2 \)

\[ \Rightarrow \text{It is possible to use the unit convention} \ C = 1 \text{ as such:} \]

\[ \begin{align*}
[\text{energy}]: & \quad [\text{mass}] = [\text{momentum}] = [\text{eV}] \\
[\text{time}]: & \quad [\text{distance}] = [\text{m}] \\
& \text{from } C = 2.998 \times 10^8 \text{ m/s} \Rightarrow 1 \text{ m} = 2.998 \times 10^8 \text{ cm} 
\end{align*} \]

- Using the relation \( h = \lambda p \) which is valid for photons/light, one can easily identify the resolution capacity of light at a given energy.

\[
\begin{align*}
\ h & = (1.240 \times 10^{-10} \text{ m})(1 \text{ eV/c}) \\
& = (1.240 \times 10^{-10} \text{ m})(10^8 \text{ eV/c}) = (1.240 \times 10^{-10} \text{ m})(10 \text{ keV/c}) \\
& = (1.240 \times 10^{-10} \text{ m})(10^9 \text{ eV/c}) = (1.240 \times 10^{-10} \text{ m})(1 \text{ GeV/c}) \\
& = (1.240 \times 10^{-10} \text{ m})(10^8 \text{ eV/c}) = (1.240 \times 10^{-10} \text{ m})(1 \text{ TeV/c}) 
\end{align*}
\]

Photons/particles (i.e. when \( E = c(2.998 \times 10^8 \text{ m/s}) \)) have the same resolution capacity as light and their masses can be neglected.

Visible light: \( 3.6 \times 10^{-6} \text{ m} < \lambda < 7.8 \times 10^{-6} \text{ m} \), has energies \( E_l \)

\[ 3.4 \text{ eV} \approx E_l \ll 1.6 \text{ eV} \]

One can use \( \frac{h c}{eV} \) as a unit for distance.
LEP experiment (1989-2000 at CERN, Geneva): electron (e) - positron (e⁺) collision

with e⁺ and e each having 100 GeV energy → E_{had} = 200 GeV

2D resolution capacity = 6.2 \times 10^{-4} \mu m

→ allows range of the strong and weak interactions

LHC experiment (since 2009): 2 running modes

p⁺ - p⁺: E_{beam} = 3.5 TeV + 3.5 TeV = 7 TeV

2D resolution capacity = 1.8 \times 10^{-4} \mu m

→ allows range of the strong and weak interactions

and maybe some new physics, i.e. new structures

→ (We see that the fundamental constants of nature, such as

\[ c = 2.998 \times 10^8 \text{ m/s} \]  

(speed of light)

\[ h = 6.626 \times 10^{-27} \text{ eV s} \]  

(reduced Planck constant)

\[ k = 8.66 \times 10^{-12} \text{ eV K} \]  

(Boltzmann constant),

intrinsically connect the quantities distance, time, energy, momentum, mass and temperature.

It is thus possible to use units such that \[ c = h = k = 1. \] This is the "natural units" convention that is used e.g. in high-energy physics and many other branches of physics.

 public \[ \begin{align*}
  \text{[energy]} & \equiv \text{[length]} \cdot \text{[mass]} \cdot \text{[momentum]} = \text{[displacement]} \cdot \text{[force]} \\
  \text{[time]} & = 1 \text{ s} = 1.619 \times 10^{-10} \text{ eV} \\
  \text{[energy]} & = 1 \text{ J} = 6.564 \times 10^{20} \text{ eV} \\
  \text{[temperature]} & = 1 \text{ K} = 8.66 \times 10^{3} \text{ eV}
\end{align*} \]

One reason why using natural units is useful in physics is that non-trivial functions for require arguments x, which are dimensionless (i.e. x must be a number), and that typical functions for that appear in physics change typically for multiplicative \( \Delta x \) of order unity.