Exercises for T2, Summer term 2016, Sheet 12

1) Spin-1

A spin-1 system is prepared in the eigenstate of S_3 with eigenvalue 0. What are the probabilities to get the results $+\hbar$, 0 and $-\hbar$ for a measurement of S_1?

2) Spin operator

Take the spin operator $\vec{S}$ in the fundamental (spin-1/2) representation of SU(2). This is a so-called vector operator, since it has three components for the three Cartesian coordinates. Calculate the vector operator $\vec{S} \times \vec{S}$.

3) Spin-1 representation and adjoint representation of SU(2)

Determine the unitary matrix U, which gives the transformation between the basis of the spin-1 and the basis of the adjoint representation, i.e. $T_k = U t_k U^\dagger$, where T_k are the angular momentum operators of the spin-1 representation and t_k the angular momentum operators in the adjoint representation ($k = 1, 2, 3$), which has been discussed in the lecture. To do so, find the eigenvectors of T_3 and t_3 and think about how they enter the matrix $U^\dagger$. Is U unique?

4) Spherical harmonics

Find all five spherical harmonics for angular momentum $\ell = 2$, by applying L_- sufficiently many times to $Y_{\ell\ell}$

5) Parity

The parity operator Π is defined (in position space) by $(\Pi\psi)(\vec{x}) = \psi(-\vec{x})$. Show that Π is self-adjoint and that $\Pi^2 = 1$. How does Π act on the wave function $\psi(\vec{x}) = u(r)\, Y_{\ell m}(\theta, \varphi)$ in spherical coordinates? Show, by using the form of the spherical harmonics discussed in the lecture and the properties of the associated Legendre polynomials, that this wave function is an eigenfunction of the observable "parity" with eigenvalue $(-1)^\ell$.

6) Parity

Write down how parity acts on the vector operators $\vec{P}$ and $\vec{X}$. Show that parity is conserved ($d\Pi/dt = 0$) if the Hamiltonian is given as $H = \vec{P}^{\,2}/2m + V(|\vec{X}|)$. How does parity act on the angular momentum operator $\vec{L} = \vec{X} \times \vec{P}$?