## Exercises for T2, Summer term 2016, Sheet 8

### 1) Spatial translation

Show that the one-dimensional spatial translation operator $T_a$ from exercise (4.6) has the explicit form $T_a = \exp(-a\frac{d}{dx})$. Also write this expression as a function of the momentum operator $P$.

### 2) Particle scattering on the potential barrier I

Let there be a one-dimensional system of a particle with mass $m$ in the potential $V(x) = V_0\Theta(x)\Theta(a-x)$. Take now the eigenfunctions $\phi_k(x)$ of the eigenvalue $E(k) = \hbar^2k^2/2m > V_0$ that correspond to a particle entering from the left, which have been discussed in the lecture (english lecture notes, chapter 3.4).

(a) Determine the amplitudes $A$, $B$ und $T$ by using continuity of the wave function at the points $x = 0$ and $x = a$.

(b) Determine the probability current in the three regions $x < 0$, $0 < x < a$ and $x > a$ as a function of $A$, $B$ and $T$. Interpret the individual contributions.

(c) Show that conservation of particle number is valid in all regions.

### 3) Particle scattering on the potential barrier II

Work through exercise (2) for the case $E(k) = \hbar^2k^2/2m < V_0$.

### 4) Distributions

Calculate the first and second derivative of the following functions in the distributional sense ($\theta(x)$ is the Heaviside step function).

$$\theta(x),\ \theta(-x),\ |x| = -x\theta(-x) + x\theta(x),\ e^{-a|x|} = e^{ax}\theta(-x) + e^{-ax}\theta(x),\ a > 0.$$

### 5) Particle in the delta-potential

Let the wave function of a particle with one degree of freedom be given by

$$\psi(x) = \mathcal{N}\exp(-a|x|), \quad a > 0.$$

(a) Convince yourself with the help of results from exercise (4) that the wave function $\psi(x)$ is, for a suitable choice of the parameter $a$, an energy eigenfunction of the Hamilton operator

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} - \lambda\,\delta(x), \quad (\lambda > 0)$$

What is the necessary choice for $a$ and what is the result for the energy eigenvalue $E$?

(b) Determine the probability current for $|x| > 0$ and argue that the wave function has to be interpreted as a bound state. What is the interpretation of the corresponding energy eigenvalue?

(c) Argue why there cannot be any further bound states.