Exercises for T2, Summer term 2016, Sheet 6

1) H-atom in three spatial dimensions

Calculate the expectation values of $\vec{P}$ und $\vec{P}^2$ for the ground state wave function of the H-atom. See exercise (2) on sheet 4.

2) Time evolution in momentum space

Let a free particle with mass m be described at $t = 0$ by the wave function calculated in exercise (5.5) (also see the hand-written lecture notes, chap. 2.8.). Determine the time-dependent momentum space wave function $\tilde{\psi}(p, t)$ as well as mean and mean square deviations of X and P at time t.

3) Time evolution in configuration space

Use the result of exercise (2) for $x_0 = 0$, and determine the time-dependent configuration space wave function $\psi(x, t)$. Write down $|\psi(x, t)|^2$ as well (also see the hand-written lecture notes, chap. 2.9. for the discussion of the Gaussian wave packet).

Hint: Let $c, d \in \mathbb{C}$ (!) where $\mathrm{Re}\, c > 0$. Then the formula

$$\int_{-\infty}^{+\infty} dx\, e^{-c(x-d)^2} = \sqrt{\frac{\pi}{c}}$$

holds as in the real case! Check this by simply performing some numerical tests (e.g. with Mathematica).

4*) Video-clip

Visualize $|\psi(x, t)|^2$ from exercise (3) as a function of t in a video-clip by using for example the `(List)Animate`-function of Mathematica. Try to think about how to choose the parameters so that one gets a nice-looking visualization.

This star exercise is not mandatory and acutally represents a small project that tries to encourage you to look beyond the scope of this lecture. Discuss your results with other students.