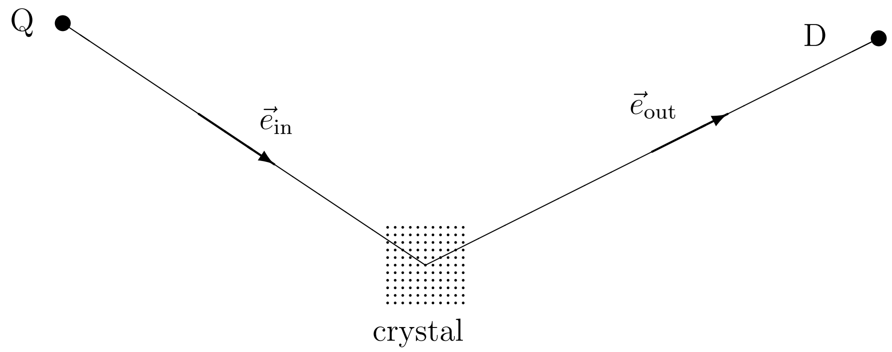
Exercises for T2, Summer term 2016, Sheet 2

1) De Broglie wavelength
Calculate the de Broglie wavelength of a proton with a kinetic energy of 1 eV, 100 eV, 100 keV ($m_p \simeq 938\,\mathrm{MeV}/c^2$). What is the de Broglie-wavelength of a human with a mass of 70 kg, who is moving at $1\,\mathrm{m\,s^{-1}}$? Compare the obtained results with the size of a proton and the size of a human. What is your conclusion?

2) De Broglie wavelength of nonrelativistic particles
Write down the general (relativistic) relation between the de Broglie wavelength and the **kinetic energy** $T = E - mc^2$ of a massive particle ($m \neq 0$). Find an approximation for the nonrelativistic case ($T \ll mc^2$).

3) Scattering of neutrons



The atoms of a crystal lattice are located at the points $\vec{x}_{\vec{n}} = a\vec{n}$, $\vec{n} \in \mathbb{Z}^3$, $n_i = -N, -N+1, \dots, N$ ($i = 1,2,3$). Furthermore there is a source Q ($R_{\mathrm{Q}} \gg Na$) at the point $\vec{x}_{\mathrm{Q}} = -R_{\mathrm{Q}}\,\vec{e}_{\mathrm{in}}$ ($|\vec{e}_{\mathrm{in}}| = 1$) which emits neutrons with momentum p. In addition a neutron-detector ($R_{\mathrm{D}} \gg Na$) is located at $\vec{x}_{\mathrm{D}} = R_{\mathrm{D}}\,\vec{e}_{\mathrm{out}}$ ($|\vec{e}_{\mathrm{out}}| = 1$). The amplitude $\langle \mathrm{D\,out}|\mathrm{Q\,in}\rangle$, that the detector D detects a neutron which originated from Q is of the form

$$\langle \mathrm{D\,out}|\mathrm{Q\,in}\rangle \sim \sum_{\vec{n}} \frac{e^{ip|\vec{x}_{\mathrm{Q}}-\vec{x}_{\vec{n}}|/\hbar}}{|\vec{x}_{\mathrm{Q}}-\vec{x}_{\vec{n}}|} W_{\vec{n}} \frac{e^{ip|\vec{x}_{\mathrm{D}}-\vec{x}_{\vec{n}}|/\hbar}}{|\vec{x}_{\mathrm{D}}-\vec{x}_{\vec{n}}|}.$$

(multiple scattering is omitted). Calculate this expression assuming that $W_{\vec{n}}$ is the same for all atoms. Use a suitable approximation for $|\vec{x}_{\mathrm{Q}} - \vec{x}_{\vec{n}}| = \sqrt{(\vec{x}_{\mathrm{Q}} - \vec{x}_{\vec{n}})^2}$ which should refect that $|\vec{x}_{\mathrm{Q}}| = R_{\mathrm{Q}} \gg |\vec{x}_{\vec{n}}|$. (analogous for $|\vec{x}_{\mathrm{D}} - \vec{x}_{\vec{n}}|$.)

By following that strategy you should get an expression like

$$\langle \text{D out}|\text{Q in}\rangle \sim \sum_{\vec{n}} e^{ipa\vec{\Delta}\cdot\vec{n}/\hbar} = \prod_{i=1}^{3} \sum_{n_i=-N}^{N} e^{ipa\Delta_i n_i/\hbar}, \quad \vec{\Delta} = \vec{e}_{\text{in}} - \vec{e}_{\text{out}}$$

This means it is necessary to calculate a geometric series of the type

$$s(\alpha) = \sum_{n=-N}^{N} e^{i\alpha n}$$

Now show that this equals

$$s(\alpha) = \frac{\sin \alpha(N+\frac{1}{2})}{\sin \frac{\alpha}{2}}$$

Discuss the behavior of this function. For which values of α are there distinct extrema? Show that in the case of neutron scattering

$$\frac{pa}{2\pi\hbar}\vec{\Delta} = \vec{\nu} \in \mathbb{Z}^3$$

leads to distinct maxima of the interference pattern. This is Laue's condition for interference for the simple cubic lattice. It is also possible to write it in the form

$$\frac{a}{2\pi}\left(\vec{k}_{\text{in}} - \vec{k}_{\text{out}}\right) = \vec{\nu} \in \mathbb{Z}^3$$