37. Using Ito's formula one can show that the rule for the change of variables in a Stratonovich stochastic differential equation is exactly the same as in ordinary calculus: Start with a general Stratonovich stochastic differential equation for dx and convert it into the Ito form. According to the Ito formula change to the new variable $y = f(x)$ (with the inverse $x = g(y)$) and transform the result back into its Stratonovich form.

38. Transform

$$dx_1 = -x_1 dt + dW_1$$

$$dx_2 = -x_2 dt + dW_2$$

from cartesian into into polar coordinates, and show that

$$dr = (-r + \frac{1}{r})dt + dW_r$$

$$d\varphi = \frac{1}{r} dW_\varphi$$

where

$$dW_1 dW_1 = dW_2 dW_2 = dW_r dW_r = dW_\varphi dW_\varphi = dt$$

and all other quadratic dW correlations are vanishing.

39. In the spirit of the "slick proof" of the Feynman-Kac formula show that for

$$d\vec{X} = \vec{a} dt + d\vec{W}$$

one has

$$E(g(\vec{X}(t)) = \left(e^{-t(-\frac{1}{2}\triangle - \vec{a}\vec{\nabla})} g \right) (\vec{0})$$

40. Similarly, let

$$d\vec{X} = d\vec{W}$$

and derive the Cameron-Martin formula

$$E(e^{-\frac{1}{2}\int_0^t \vec{a}^2 dt' + \int_0^t \vec{a} d\vec{W}} g(\vec{X}(t)) = \left(e^{-t(-\frac{1}{2}\triangle - \vec{a}\vec{\nabla})} g \right) (\vec{0})$$