37. Using Itô’s formula one can show that the rule for the change of variables in a Stratonovich stochastic differential equation is exactly the same as in ordinary calculus: Start with a general Stratonovich stochastic differential equation for $dx$ and convert it into the Itô form. According to the Itô formula change to the new variable $y = f(x)$ (with the inverse $x = g(y)$) and transform the result back into its Stratonovich form.

38. Transform

\[ dx_1 = -x_1 dt + dW_1 \]
\[ dx_2 = -x_2 dt + dW_2 \]

from cartesian into polar coordinates, and show that

\[ dx = (-r + \frac{1}{r}) dt + dW_r \]
\[ d\varphi = \frac{1}{r} dW_\varphi \]

where

\[ dW_1 dW_1 = dW_2 dW_2 = dW_r dW_r = dW_r dW_\varphi = dt \]

and all other quadratic $dW$ correlations are vanishing.

39. In the spirit of the “slick proof” of the Feynman-Kac formula show that for

\[ d\bar{X} = \bar{a} dt + d\bar{W} \]

one has

\[ E(g(\bar{X}(t))) = \left( e^{-(t \Delta - \bar{a} \nabla)} g \right) (\bar{0}) \]

40. Similarly, let

\[ d\bar{X} = d\bar{W} \]

and derive the Cameron-Martin formula

\[ E(e^{-\frac{1}{2} \int_0^t \sigma^2 dt' + \int_0^t \int \sigma d\bar{W}} g(\bar{X}(t))) = \left( e^{-t \Delta - \bar{a} \nabla} g \right) (\bar{0}) \]