29. The real space RG calculation for the Ising model in $d = 2$ on a triangular lattice in a nonzero magnetic field leads to the approximate result (see exercise 25)

$$K' = K \frac{1}{2} \left[ \phi(K + \frac{1}{2} h) + \phi(K - \frac{1}{2} h) \right]^2$$

$$h' = h + \frac{1}{2} \log \left( 1 + \frac{3 e^{-4K - 2h}}{1 + 3 e^{-4K + 2h}} \right) + 3K \left[ \phi(K + \frac{1}{2} h)^2 - \phi(K - \frac{1}{2} h)^2 \right]$$

where $\phi(x) = \frac{1 + e^{-4x}}{1 + 3 e^{-4x}}$. Find the 5 fixed points of the renormalization group transformations, linearize around them (study one after the other) and discuss the nature of the fixed points. Perform a numerical simulation.

30. We study now the mapping of quantum Ising models to classical Ising models. We consider first the case of the single spin quantum Ising model

$$\hat{H} = \Gamma \sigma^x$$

and demonstrate that its partition function $Z = Tr e^{-\beta \hat{H}}$ is represented as the partition function of the classical 1d Ising chain.

In the spirit of the Feynman path integral formulation write $e^{-\beta \hat{H}} = e^{-\frac{\beta}{2} \hat{H}} e^{-\frac{\beta}{2} \hat{H}} \ldots e^{-\frac{\beta}{2} \hat{H}}$ and insert complete sets of $|S_z>$ eigenstates of $\sigma_z^i$. Insert additionally also complete sets of $|S_x>$ eigenstates of $\sigma_x^i$, noting that $<S_x|S_z> = \frac{1}{\sqrt{2}} e^{i \pi \frac{1-S_x}{2} - \frac{1-S_z}{2}}$. Verify that $<S_x'|e^{\frac{\beta}{2} \Gamma \sigma^x}|S_z> = e^{J_x S_x' S_z}$. Here $J_x = \text{artanh}(e^{-2\delta \tau \Gamma})$, $\delta \tau = \frac{\beta}{\Gamma}$ and $c$ is an unimportant constant. Demonstrate the equivalence of the single spin quantum Ising model to the Ising chain.

31. Generalize the above arguments to the quantum Ising chain

$$\hat{H} = -J \sum_i \sigma^z_i \sigma^z_{i+1} - \Gamma \sum_i \sigma^x_i$$

showing that it is mapped to the 2d classical Ising model of size $\beta = \frac{1}{\Gamma}$ in the imaginary time direction. In the limit $T \to 0$ the 2d classical Ising model has infinite spatial and temporal directions. The coupling constants in the classical model are anisotropic and depend on the discretization step $\delta \tau = \frac{\beta}{\Gamma}$

$$J_{\text{spat}} = J \delta \tau, \quad J_\tau = \text{artanh}(e^{-2\delta \tau \Gamma})$$