Exercises for nonlocality, entanglement und geometry of quantum systems

Sheet 6

Prof. Reinhold A. Bertlmann & Philipp Köhler
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Exercise 18
Show that for mixed states the following properties are fulfilled:

$\rho^2 \neq \rho$

$Tr(\rho^2) < 1$

Exercise 19
Consider the Hamiltonian

$H = -\vec{\mu} \cdot \vec{B}$ with $\vec{\mu} = \frac{g\mu_B}{2} \vec{\sigma} = \frac{g\mu_B}{2} \sigma_z$ (if $\vec{B} || z$-axis)

What is the time evolution of a general density matrix $\rho(t)$?

Calculate the expectation values $\langle \vec{\sigma} \rangle(t)$.

Exercise 20
A pure qubit state can be written as

$|\psi\rangle_{\text{pure}} = \cos \frac{\theta}{2} |\uparrow\rangle + \sin \frac{\theta}{2} e^{-i\varphi} |\downarrow\rangle$

Find three pure states that, when mixed together with equal weights, create a totally mixed state.
(Hint: set $\varphi$ to zero).

Exercise 21
Write down the explicit density matrices for the Bell states

$\rho^\pm = |\psi^\pm\rangle \langle \psi^\pm |$

$\omega^\pm = |\phi^\pm\rangle \langle \phi^\pm |$

where

$|\psi^\pm\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle |\downarrow\rangle \pm |\downarrow\rangle |\uparrow\rangle)$ and $|\phi^\pm\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle |\uparrow\rangle \pm |\downarrow\rangle |\downarrow\rangle)$

Exercise 22
Write down the Bell states in Bloch representation (in terms of Pauli matrices).