Exercises for nonlocality, entanglement und geometry of quantum systems
Sheet 6

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Exercise 18

Show that for mixed states the following properties are fulfilled:

$$\rho^2 \neq \rho$$
$$Tr\left(\rho^2\right) < 1$$

Exercise 19

Consider the Hamiltonian

$$H = -\vec{\mu} \cdot \vec{B} \quad \text{with} \quad \vec{\mu} = \frac{g\mu_B}{2}\vec{\sigma} = \frac{g\mu_B}{2}\sigma_z \quad (\text{if } \vec{B} \text{ || z-axis})$$

What is the time evolution of a general density matrix $\rho(t)$?

Calculate the expectation values $\langle\vec{\sigma}\rangle(t)$.

Exercise 20

A pure qubit state can be written as

$$|\psi\rangle_{pure} = \cos\frac{\theta}{2}|\uparrow\rangle + \sin\frac{\theta}{2}e^{-i\varphi}|\downarrow\rangle$$

Find three pure states that, when mixed together with equal weights, create a totally mixed state. (Hint: set φ to zero).

Exercise 21

Write down the explicit density matrices for the Bell states

$$\rho^{\pm} = |\psi^{\pm}\rangle\langle\psi^{\pm}|$$
$$\omega^{\pm} = |\phi^{\pm}\rangle\langle\phi^{\pm}|$$

where

$$|\psi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle|\downarrow\rangle \pm |\downarrow\rangle|\uparrow\rangle\right) \quad \text{and} \quad |\phi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle|\uparrow\rangle \pm |\downarrow\rangle|\downarrow\rangle\right)$$

Exercise 22

Write down the Bell states in Bloch representation (in terms of Pauli matrices).