Exercises for nonlocality, entanglement und geometry of quantum systems
Sheet 5

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Exercise 16

Quantum teleportation:
Imagine the following setup in picture one on page 2 where path 1 is a photon beam in a 1-qubit state $|\eta\rangle_1$ and paths 2 and 3 are an entangled pair of qubits in the $|\psi^-\rangle_{23}$ Bell state.

$$|\eta\rangle_1 = a|H\rangle_1 + b|V\rangle_1\ ,\ |a|^2 + |b|^2 = 1$$

$$|\psi^-\rangle_{23} = \frac{1}{\sqrt{2}}(|H\rangle_2 \otimes |V\rangle_3 - |V\rangle_2 \otimes |H\rangle_3)$$

- Combine these states to a state $|\psi\rangle_{123}$ in the Hilbert space $\mathcal{H}^1 \otimes \mathcal{H}^2 \otimes \mathcal{H}^3$.
- Expand the subset of states $\mathcal{H}^1 \otimes \mathcal{H}^2$ into Bell states. (e.g. $|\ \rangle_1 |\ \rangle_2 = |\psi^-\rangle_{12} + \ldots$)
- Rewrite the result to the form: $[|\psi^-\rangle_{12}(a|\ \rangle_3 + b|\ \rangle_3) + \ldots]$
- Calculate the result of Alice performing a Bell state measurement e.g. Alice projects onto a Bell state. Do this for all four Bell states. ($|\psi^-\rangle_{12}\langle\psi^-|\psi\rangle_{123} = \ldots$)
- From this result calculate the unitary matrices which Bob has to apply in order to achieve the initial state.

Exercise 17

Entanglement swapping:
Imagine the following situation in picture two on page 2 where you have two sources for entangled states which produce the following states.

$$|\psi^-\rangle_{12} = \frac{1}{\sqrt{2}}(|H\rangle_1 \otimes |V\rangle_2 - |V\rangle_1 \otimes |H\rangle_2)$$

$$|\psi^-\rangle_{34} = \frac{1}{\sqrt{2}}(|H\rangle_3 \otimes |V\rangle_4 - |V\rangle_3 \otimes |H\rangle_4)$$

- Combine these states to a state $|\psi\rangle_{1234}$ in the Hilbert space $\mathcal{H}^1 \otimes \mathcal{H}^2 \otimes \mathcal{H}^3 \otimes \mathcal{H}^4$.
- Expand the subset of states $\mathcal{H}^2 \otimes \mathcal{H}^3$ and $\mathcal{H}^1 \otimes \mathcal{H}^4$ into Bell states. (e.g. $|\ \rangle_1 |\ \rangle_4 = |\psi^-\rangle_{14} + \ldots$)
- Rewrite the result to a sum of tensor products of Bell states: $[|\psi\rangle_{1234} = (|\psi^+\rangle_{14}|\psi^+\rangle_{23}) + \ldots]$
- What is the result, if you project the whole state onto a Bell state of the subset $\mathcal{H}^2 \otimes \mathcal{H}^3$?

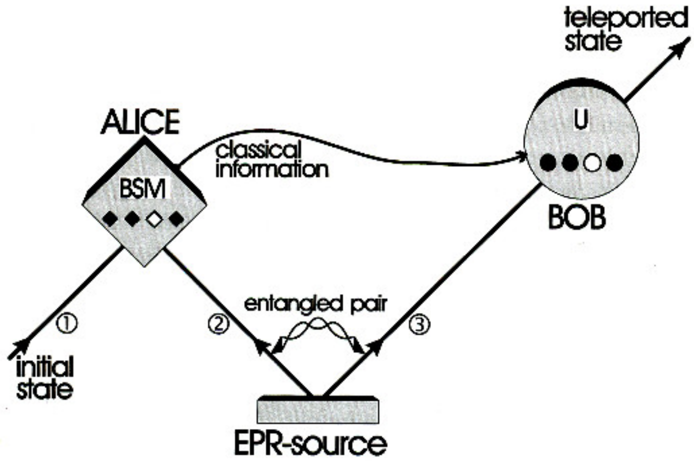

Figure 3.1: *Scheme showing principle of quantum teleportation. Alice has a quantum system, particle 1, in an initial state which she wants to teleport to Bob. Alice and Bob also share an ancillary entangled pair of particles 2 and 3 emitted by an Einstein-Podolsky-Rosen(EPR) source. Alice then performs a joint Bell-state measurement (BSM) on the initial particle and one of the ancillaries, projecting them also onto an entangled state. After she has sent the result of her measurement as classical information to Bob, he can perform a unitary transformation (U) on the other ancillary particle resulting in it being in the state of the original particle.*

Figure 1: Quantum teleportation

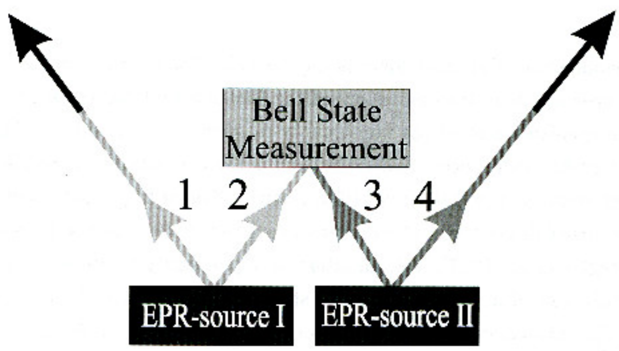

Figure 4.1: *Principle of entanglement swapping. Two EPR sources produce two pairs of entangled photons, pair 1-2 and pair 3-4. One photon from each pair (photon 2 and 3) is subjected to a Bell-state measurement (BSM). This results in projecting the other two outgoing photons 1 and 4 into an entangled state. Change of the shading of the lines indicates a change in the set of possible predictions that can be made.*

Figure 2: Entanglement swapping