Exercises for nonlocality, entanglement und geometry of quantum systems
Sheet 4

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Exercise 13

Start from the CHSH inequality

$$\left|E(\vec{a},\vec{b}) - E(\vec{a},\vec{b'})\right| + \left|E(\vec{a'},\vec{b'}) + E(\vec{a'},\vec{b})\right| \leq 2$$

and suppose that two angles are equal (e.g. $\vec{a'} = \vec{b'}$) and derive Bell's original inequality. At which angles is Bell's inequality maximally violated?

Exercise 14

The expectation value can be expressed by the probabilities:

$$E(\vec{a},\vec{b}) = P(\vec{a}\Uparrow,\vec{b}\Uparrow) + P(\vec{a}\Downarrow,\vec{b}\Downarrow) - P(\vec{a}\Uparrow,\vec{b}\Downarrow) - P(\vec{a}\Downarrow,\vec{b}\Uparrow)$$

where for instance $P(\vec{a}\Uparrow,\vec{b}\Uparrow)$ means that Alice measures spin up at direction $\vec{a}$ and Bob spin up at direction $\vec{b}$. Use the property that the sum of all probabilities is one to

- express the expectation value through probabilities
- rewrite Bell's inequality of expectation values into an inequality of probabilities called Wigner's inequality
- use the expression of the probability $P(\vec{a}\Uparrow,\vec{b}\Uparrow)$ of exercise 12 to show at which angles the Wigner inequality is maximally violated.

Exercise 15

Derive the so called Clauser Horne inequality which is an expression in terms of probabilities. Recall that the probabilities in terms of hidden variables are defined by

$$p_A(\vec{a}) = \int d\lambda \rho(\lambda) p_A(\lambda,\vec{a})$$

$$p_B(\vec{b}) = \int d\lambda \rho(\lambda) p_B(\lambda,\vec{b})$$

$$p_{AB}(\vec{a},\vec{b}) = \int d\lambda \rho(\lambda) p_{AB}(\lambda,\vec{a},\vec{b})$$

where $p_A(\vec{a})$ is the probability of Alice measuring spin in direction $\vec{a}$, analogously with $\vec{b}$ for Bob. $p(\vec{a},\vec{b})$ is the probability for measuring simultaneously the spins of Alice at $\vec{a}$ and Bob at $\vec{b}$. Recall that in hidden variable theories the probability $p(\vec{a},\vec{b})$ can be factorized by the single probabilites of Alice and Bob $p_A(\vec{a}), p_B(\vec{b})$ (Bell's locality hypothesis). Start from the pure algebraic inequality which is valid for any numbers $0 \leq x_1, x_2 \leq X, 0 \leq y_1, y_2 \leq Y$

$$-XY \leq x_1 y_1 - x_1 y_2 + x_2 y_1 + x_2 y_2 - Y x_2 - X y_1 \leq 0$$

- Since the probability intervals fit for $X, Y = 1$ you can derive an inequality for the probabilities, named after Clauser and Horne.
- Calculate the quantum mechanical probability of measurements at Alice and Bob for angles α, β for Photons in the Bell state $|\phi^+\rangle = \frac{1}{\sqrt{2}}(|H\rangle|H\rangle + |V\rangle|V\rangle)$.
- Insert the quantum mechanical probabilities and show at which angles the Clauser Horne inequality is maximally violated.