Exercises for nonlocality, entanglement und geometry of quantum systems
Sheet 2
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Exercise 6
Show that
\[ \sigma_x \otimes \sigma_x \otimes \sigma_x = -(\sigma_x \otimes \sigma_y \otimes \sigma_y) \cdot (\sigma_y \otimes \sigma_z \otimes \sigma_y) \cdot (\sigma_y \otimes \sigma_y \otimes \sigma_x) \]

Exercise 7

The GHZ state is
\[ |GHZ\rangle = \frac{1}{\sqrt{2}} (|0\rangle \otimes |0\rangle \otimes |0\rangle - |1\rangle \otimes |1\rangle \otimes |1\rangle) \]
Construct all states that are orthogonal to this state.

Exercise 8

Is it possible to create a similar argument as the GHZ argument for two qubits? What would be the elements to consider?

Exercise 9

Entangled entanglement:
The GHZ state in the \(|L\rangle = \frac{1}{\sqrt{2}}(|H\rangle + i|V\rangle), |R\rangle = \frac{1}{\sqrt{2}}(|H\rangle - i|V\rangle)\) basis is:
\[ |GHZ^1\rangle_{123} = \frac{1}{\sqrt{2}} (|R\rangle_1 |R\rangle_2 |R\rangle_3 + |L\rangle_1 |L\rangle_2 |L\rangle_3) \]
Show that this is equal to writing:
\[ |GHZ^1\rangle_{123} = \frac{1}{\sqrt{2}} (|H\rangle_1 |\phi^-\rangle_{23} - |V\rangle_1 |\psi^+\rangle_{23}) \]
which is two entangled states which are entangled.

The GHZ states can be seen as basis elements for the \(2 \times 2 \times 2\) dimensional Hilbert space. Write down the other missing basis elements as entangled entanglement states.