Exercises for nonlocality, entanglement und geometry of quantum systems
Sheet 2

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Exercise 6

Show that

$$\sigma_x \otimes \sigma_x \otimes \sigma_x = -(\sigma_x \otimes \sigma_y \otimes \sigma_y) \cdot (\sigma_y \otimes \sigma_x \otimes \sigma_y) \cdot (\sigma_y \otimes \sigma_y \otimes \sigma_x)$$

Exercise 7

The GHZ state is

$$|GHZ\rangle = \frac{1}{\sqrt{2}}(|0\rangle \otimes |0\rangle \otimes |0\rangle - |1\rangle \otimes |1\rangle \otimes |1\rangle)$$

Construct all states that are orthogonal to this state.

Exercise 8

Is it possible to create a similar argument as the GHZ argument for two qubits? What would be the elements to consider?

Exercise 9

Entangled entanglement:
The GHZ state in the $|L\rangle = \frac{1}{\sqrt{2}}(|H\rangle + i|V\rangle), |R\rangle = \frac{1}{\sqrt{2}}(|H\rangle - i|V\rangle)$ basis is:

$$|GHZ^1\rangle_{123} = \frac{1}{\sqrt{2}}\left(|R\rangle_1|R\rangle_2|R\rangle_3 + |L\rangle_1|L\rangle_2|L\rangle_3\right)$$

Show that this is equal to writing:

$$|GHZ^1\rangle_{123} = \frac{1}{\sqrt{2}}\left(|H\rangle_1|\phi^-\rangle_{23} - |V\rangle_1|\psi^+\rangle_{23}\right)$$

which is two entangled states which are entangled.

The GHZ states can be seen as basis elements for the $2 \times 2 \times 2$ dimensional Hilbert space. Write down the other missing basis elements as entangled entanglement states.