Exercises for nonlocality, entanglement und geometry of quantum systems
Sheet 1

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Exercise 1

Show that the tensor product of vectors $|\psi\rangle = \sum_{i,j} a_i b_j |i\rangle \otimes |j\rangle$ is independent of the basis choice.
Show explicitly the equality of $|\psi^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle \otimes |\downarrow\rangle - |\downarrow\rangle \otimes |\uparrow\rangle)$ in σ_z and σ_x basis.

Exercise 2

Calculate the expectation value of $\langle\sigma_i\rangle_{\phi^+} = \langle\phi^+|\sigma_i^A \otimes \mathbb{1}^B|\phi^+\rangle$
where $|\phi^+\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle \otimes |\uparrow\rangle + |\downarrow\rangle \otimes |\downarrow\rangle)$.

Exercise 3

Calculate explicitly, that $\sigma_y^A \otimes \mathbb{1}^B$ is not the same as $\mathbb{1}^A \otimes \sigma_y^B$.

Exercise 4

Calculate the following commutators: $[\sigma_x \otimes \sigma_y, \sigma_y \otimes \sigma_x]$, $[\sigma_y \otimes \sigma_x, \sigma_y \otimes \sigma_y]$

Exercise 5

Show, that the expectation value of $\langle\sigma_i^A \otimes \sigma_j^B\rangle_{\psi^-}$ fulfils the relation $\langle\sigma_i^A \otimes \sigma_j^B\rangle_{\psi^-} = -\delta_{ij}$