Exercises for decoherence and open quantum systems
Sheet 8

Prof. Reinhold A. Bertlmann, Philipp Köhler & Veronika Baumann

12.06.2013

Exercise 27

The Jaynes Cummings Hamiltonian is given by

$$H_{JC} = H_{atom} + H_{field} + H_I = \hbar\omega_0 S^+ S^- + \hbar\omega_L (a^\dagger a + \frac{1}{2}) + g(S^+ a + S^- a^\dagger)$$

where $S^+ = |e\rangle\langle g|, S^- = |g\rangle\langle e|$ are the ladder operators for the atom and $a, a^\dagger$ the ladder operators for the light field.

Calculate the energy eigenvalues.

Exercise 28

Discuss the properties of the dressed states in the Jaynes Cummings model, namely the energy versus the field frequency, the Rabi frequency and the possible transitions (anticrossing,...).

Exercise 29

Calculate the time evolution of the probability for the atom to be in the ground- or excited state, being in the ground state at time $t = 0$ and no detuning ($\delta = 0$).

Exercise 30

Prove that the operator representation

$$\begin{aligned} W(p,q) &= \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} d\sigma d\tau \; e^{i(\tau p + \sigma q)} \mathrm{Tr}\; e^{-i(\tau P + \sigma Q)} \rho = \\ &= \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} d\sigma d\tau \; \mathrm{Tr}\; \rho e^{-i\tau(P-p)} e^{-i\sigma(Q-q)} e^{-\frac{i}{2}\sigma\tau} \end{aligned}$$

coincides with the usual definition

$$W(p,q) = \frac{1}{\pi} \int_{-\infty}^{\infty} dy \; e^{-2\tau p y} \langle q - y | \rho | q + y \rangle$$

of the Wigner function.