Exercises for decoherence and open quantum systems
Sheet 6

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08.05.2013

Exercise 17

Calculate the mixedness and the concurrence for the Gisin state (Exercise 17,18) before and after the filtering for the values:

$$\alpha = \sqrt{0.3} \quad \beta = \sqrt{0.7} \quad \lambda = 0.9$$

Exercise 18

Bob knows that Alice wears only red and green socks and always socks of different colour. Moreover he knows that she prefers a red sock on the left foot according to the following probability distributions:

$$p_{left}(red) = \frac{3}{4} \quad ; \quad p_{left}(green) = \frac{1}{4}$$
$$p_{right}(red) = \frac{1}{4} \quad ; \quad p_{right}(green) = \frac{3}{4}$$

What are the probabilities for all possible pairs of socks on Alice's feet?

Calculate the joint entropy $H(left, right)$, the mutual information $I(left : right)$ and the conditional entropy $H(left|right)$.
This corresponds to the mean uncertainty and information Bob already has and the remaining uncertainty of Bob after looking at Alice's left foot.

Exercise 19

Consider a 2-qubit system, where each of the subsystems, A and B, has maximal entropy ($S(A) = S(B) = 1$).
Calculate the joint entropy $S(A, B)$, the conditional entropy $S(A|B)$, the mutual information $I(A : B)$ and draw a Venn- diagram for each of the following cases:

a. Separabel state: $\rho^{AB} = \rho^A \otimes \rho^B$

b. Mixed state: $\rho^{AB}_{mix} = \frac{1}{2}|01\rangle\langle 01| + \frac{1}{2}|10\rangle\langle 10|$

c. Maximally entangled state: $\rho^{AB} = \rho^- = |\psi^-\rangle\langle\psi^-|$