Exercises for decoherence and open quantum systems
Sheet 5

Prof. Reinhold A. Bertlmann, Philipp Köhler & Veronika Baumann

24.04.2013

Exercise 17

The Gisin state is defined as

$$\rho(\lambda,\alpha) = \lambda \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \alpha^2 & -\alpha\beta & 0 \\ 0 & -\alpha\beta & \beta^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \frac{1}{2}(1-\lambda) \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Calculate the mixedness for this state $(1 - Tr(\rho^2))$. Apply the following filtering operation:

$$F = \begin{pmatrix} \sqrt{\frac{\beta}{\alpha}} & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{\frac{\beta}{\alpha}} \end{pmatrix}$$

Calculate the mixedness for the resulting state and answer the following questions:

- Is filtering a unitary operation?
- Is filtering a local or nonlocal operation?
- Is filtering a deterministic or stochastic operation? Why one or the other?
- What is the difference between local and nonlocal operations?

Exercise 18

Concurrence is an entanglement measure and defined as folows:

$$C(\rho) = max(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4})$$

$$\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4$$

$$\lambda_i \ldots \text{Eigenvalues of } \rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$$

Calculate the concurrence for the Gisin state before and after filtering and compare the results to that of the previous example.