Exercises for decoherence and open quantum systems
Sheet 4

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Exercise 13

Calculate the von Neumann entropy $S(\rho) = -Tr(\rho \, log_2 \rho)$ of the following quantum states:

- $\rho_\alpha = |\alpha\rangle\langle\alpha| \quad$ where $|\alpha\rangle = |\uparrow\rangle + e^{i\alpha}|\downarrow\rangle$
- $\rho_{mix} = \frac{1}{2}\mathbb{1}$

Exercise 14

Calculate the von Neumann entropy $S(\rho)$ and the Shannon entropy $H(p, 1-p)$ of the state

$$\rho = p|0\rangle\langle 0| + (1-p)|+\rangle\langle +| \quad \text{with } |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

and compare the results.

Exercise 15

Consider the following 2-qubit state:

$$|\Psi\rangle = \frac{1}{\sqrt{3}}(|0\rangle|0\rangle + |0\rangle|1\rangle + |1\rangle|1\rangle)$$

Calculate the mixedness (linear entropy) of the two subsystems ρ^A and ρ^B.

Exercise 16

Calculate the von Neumann entropy of the Bell state $\rho^- = |\psi^-\rangle\langle\psi^-|$, with $|\psi^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle)$, and of the reduced density matrix of Alice $\rho^A = Tr_B(\rho^-)$.