# Exercises for decoherence and open quantum systems
# Sheet 3

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## Exercise 9

Calculate the subsystems of the Werner state, by calculating the partial trace. The partial trace is defined as

$$(Tr_A(\rho))_{ij} = \sum_k \rho_{ij,kk}$$

where the indices of $\rho_{ij,kl}$ are denoting the subsystems (see explanation of PPT on the last sheet).

## Exercise 10

Global unitary operations can change a separable state into an entangled one and vice versa. Check this with the following example:

$$\rho_N = \frac{1}{2}\left(\rho^+ + \omega^+\right) = \frac{1}{4}\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

$$U = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & \sqrt{2} & 0 & 0 \\ 0 & 0 & \sqrt{2} & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix}$$

In order to do so, calculate the PPT criterion for the state before and after applying the transformation.

## Exercise 11

Does the separability of state $\rho_N$ (Ex.10) change, when the when the following unitary operation is applied?

$$V = \sigma_z \otimes \mathbf{1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Check it via the PPT criterion. Show geometrically the position of the transformed state. What can you say in general for local unitary operations?

## Exercise 12

The Schmidt decomposition of a state is defined as follows:

$$|\Psi\rangle = \sum_{ij} \beta_{ij} |\psi_i\rangle \otimes |\phi_j\rangle = \sum_i \alpha_i |\chi_i\rangle \otimes |\varphi_i\rangle$$

The general way of calculating this decomposition is to use the singular value theorem.

$$\beta_{ij} = U_{ik}\alpha_{kk}V^{\dagger}_{kj}$$

where $U, V$ are unitary matrices which transform the basis vectors $|\psi_i\rangle \to |\chi_i\rangle$, $\phi_j \to |\varphi_i\rangle$ and $\beta$ is a $m \times n$ matrix which consists of the coefficients in the Schmidt decomposition.
Consider the following example:

$$|\Psi\rangle = \frac{1}{\sqrt{3}}\left(|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |0\rangle_B + |2\rangle_A \otimes |1\rangle_B\right)$$

where $|0\rangle_A = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $|1\rangle_A = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $|2\rangle_A = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ and $|0\rangle_B = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $|1\rangle_B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

**a)**

What is the form of $\beta$? How many coefficients do you need in this case for the Schmidt decomposition? Calculate these coefficients by calculating the eigenvalues of $\beta\beta^{\dagger}$. How are these eigenvalues related to the coefficients?

**b)**

Calculate the new basis vectors in this case. Try to do so by rewriting the state $|\Psi\rangle$ as to have two orthogonal states for each subsystem.