Exercises for decoherence and open quantum systems
Sheet 2

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Exercise 5

A pure qubit state can be written as

$$|\psi\rangle_{pure} = \cos\frac{\theta}{2}|\uparrow\rangle + \sin\frac{\theta}{2}e^{-i\varphi}|\downarrow\rangle$$

Find three pure states that, when mixed together with equal weights, create a totally mixed state. (Hint: set φ to zero).

Exercise 6

Write down the explicit density matrices for the Bell states

$$\rho^{\pm} = |\psi^{\pm}\rangle\langle\psi^{\pm}|$$
$$\omega^{\pm} = |\phi^{\pm}\rangle\langle\phi^{\pm}|$$

Exercise 7

Write down the Bell states in Bloch representation.

Exercise 8

The following state

$$\rho_w = p\rho^{-} + (1-p)\frac{1}{4}\mathbb{1} \quad 0 \leq p \leq 1$$

is called the Werner state. Calculate the the p-value for which the state becomes entangled. Use the PPT criterion as entanglement criterion.

PPT-Criterion

A state ρ acting on $\mathcal{H}^2 \otimes \mathcal{H}^2$ is separable if and only if its partial transposition is a positive operator (all eigenvalues are positive),

$$\rho^{T_B} = (\mathbb{1} \otimes T)\rho \geq 0.$$

It does the following to a matrix:

$$\rho_{ij,kl} = \langle i \otimes k|\rho|j \otimes l\rangle = \begin{pmatrix} \rho_{\mathrm{11,kl}} & \rho_{\mathrm{12,kl}} \\ \rho_{\mathrm{21,kl}} & \rho_{\mathrm{22,kl}} \end{pmatrix}$$

$$\text{PPT} \longrightarrow (\mathbb{1} \otimes T)\rho_{ij,kl} = \rho_{ij,lk} = \begin{pmatrix} \rho_{\mathrm{11,lk}} & \rho_{\mathrm{12,lk}} \\ \rho_{\mathrm{21,lk}} & \rho_{\mathrm{22,lk}} \end{pmatrix}$$

That means the PPT criterion applies the following changes to a density matrix:

$$\rho^{T_B} = \begin{pmatrix} \rho_{11,11} & \rho_{11,12} & \rho_{12,11} & \rho_{12,12} \\ & \swarrow\!\!\nearrow & & \swarrow\!\!\nearrow \\ \rho_{11,21} & \rho_{11,22} & \rho_{12,21} & \rho_{12,22} \\ & & & \\ \rho_{21,11} & \rho_{21,12} & \rho_{22,11} & \rho_{22,12} \\ & \swarrow\!\!\nearrow & & \swarrow\!\!\nearrow \\ \rho_{21,21} & \rho_{21,22} & \rho_{22,21} & \rho_{22,22} \end{pmatrix}$$

Now one has to check if the eigenvalues of this new density matrix are positive or not.