Exercises for nonlocality, entanglement und geometry of quantum systems
Sheet 6

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Again we want to look at Kaon systems. The Kaon or K-meson is a particle consisting of a strange quark and an antidown quark ($s\bar{d}$). Kaons have a property, which is similar to spin, the strangeness. So the Kaon state is defined by

$$S \mid K^0\rangle = \mid K^0\rangle$$

$$S \mid \bar{K^0}\rangle = - \mid \bar{K^0}\rangle$$

where $\mid \bar{K^0}\rangle$ is the anti Kaon ($\bar{s}d$). However, the Kaon state is also decaying in time.
The states describing the decay process ($\mid K_S\rangle, \mid K_L\rangle$) are connected with the strangeness eigenstates.

$$\mid K_S\rangle = \frac{1}{|p|^2 + |q|^2} \left\{ p \mid K^0\rangle - q \mid \bar{K^0}\rangle \right\}$$

$$\mid K_L\rangle = \frac{1}{|p|^2 + |q|^2} \left\{ p \mid K^0\rangle + q \mid \bar{K^0}\rangle \right\}$$

The time evolution of these states is given by (see exercise 11) the Hamiltonian $H = M - \frac{i}{2}\Gamma$ with the eigenvalues $\lambda_{S,L} = m_{S,L} - \frac{i}{2}\Gamma_{S,L}$. So the time evolution is described by

$$\mid K_{S,L}(t)\rangle = e^{i\lambda_{S,L}t} \mid K_{S,L}\rangle$$

Exercise 16

Calculate the time evolution of $\mid K^0(t)\rangle$ and $\mid \bar{K^0}(t)\rangle$ by using the relation between $K^0, \bar{K^0}$ and $K_{S,L}$.

Exercise 17

Starting with a K^0, calculate the probability to find a K^0 after a certain amount of time t. Also calculate the probability to find a $\bar{K^0}$ after a time t, when you start with a K^0.

Exercise 18

Imagine an entangled pair of Kaons where the subsystems are measured at different times.

$$\mid \psi(t_l, t_r)\rangle = \frac{1}{\sqrt{2}} \left\{ \mid K^0(t_l)\rangle \otimes \mid \bar{K^0}(t_r)\rangle - \mid \bar{K^0}(t_l)\rangle \otimes \mid K^0(t_r)\rangle \right\}$$

Calculate the probability $P(K^0, t_l; K^0, t_r)$, which means that a K^0 is measured on the left side at time t_l and K^0 is measured on the right side at time t_r ($\mid K^0\rangle_l \otimes \mid K^0\rangle_r$).