Exercises for nonlocality, entanglement und geometry of quantum systems
Sheet 5

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Exercise 13

As we have seen in exercise 6, a pure state can be parametrized as follows:

$$|\,\psi\rangle_{pure} = \cos\frac{\theta}{2}\;|\!\uparrow\rangle + \sin\frac{\theta}{2}e^{-i\varphi}\;|\!\downarrow\rangle$$

Calculate the density matrix of this state.

The totally mixed density matrix can be obtained by mixing $|\!\uparrow\rangle$ and $|\!\downarrow\rangle$ with weights $\frac{1}{2}$.

$$\rho_{mix} = \frac{1}{2}(|\!\uparrow\rangle\langle\uparrow\!| + |\!\downarrow\rangle\langle\downarrow\!|)$$

Now consider this mixed density matrix and a pure density matrix with $\theta = 90°,\ \varphi = 0°$.

Calculate the expectation values ($P_\uparrow = |\!\uparrow\rangle\langle\uparrow\!|,\ P_\downarrow = |\!\downarrow\rangle\langle\downarrow\!|$):

$$\langle\sigma_z\rangle_{mix},\ \langle\sigma_z\rangle_{pure}$$

$$\langle P_\uparrow\rangle_{mix},\ \langle P_\uparrow\rangle_{pure},\ \langle P_\downarrow\rangle_{mix},\ \langle P_\downarrow\rangle_{pure}$$

Find the projector for which the pure state has the expectation value 1. Also find the projector for a general pure state, which gives the expectation value 1.

Exercise 14

Consider silver atoms which are evaporated from an oven. The spin of these particles has no prefered direction, so the state of those atoms is in statistical mixture of ρ_{pure}. To obtain the density matrix of such a state, one has to integrate over all directions.

Calculate this density matrix.

$$\rho_{mix} = \frac{1}{4}\int d\Omega\rho(\Omega) = \frac{1}{4}\int_0^{2\pi} d\varphi \int_0^{\pi} \sin\theta d\theta\rho(\theta, \phi)$$

Exercise 15

Find three pure states that, when mixed together with equal weights, create a totally mixed state.