Exercise 9
Calculate

$$|\langle \vec{a} \otimes \vec{b} | \psi^- \rangle|^2$$

where

$$|\vec{a}\rangle = \cos \left( \frac{\alpha}{2} \right) |0\rangle + \sin \left( \frac{\alpha}{2} \right) |1\rangle$$

$$|\vec{b}\rangle = \cos \left( \frac{\beta}{2} \right) |0\rangle + \sin \left( \frac{\beta}{2} \right) |1\rangle$$

Exercise 10
The Bell states are usually expressed in the z-basis (e.g. $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ are eigenvectors of $\sigma_z$).

Rewrite the Bell states $\psi^-$ and $\phi^+$ in the y- and x-basis (also called LR-basis and ±-basis). What is the difference between these two states when written in a different basis? What about the other two Bell states?

Think about the following questions:

How can you determine different Bell states in an experiment? Imagine you have two beams of light, which are maximally entangled in their polarization. If you now place two polarizing filters in your beams 1 and 2, what relative angle between the polarization directions do you need to detect anticorrelations? If you measure anticorrelations, can you be sure, that you have a $\psi^-$-state? If not, what could you do to determine if it is a $\psi^-$-state?

Figure 1: Preparation of an entangled state with a nonlinear crystal.