Exercises for nonlocality, entanglement und geometry of quantum systems
Sheet 1

Prof. Reinhold A. Bertlmann & Philipp Köhler

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Exercise 1

Show that the tensor product of vectors $|\psi\rangle = \sum_{i,j} a_i b_j |i\rangle \otimes |j\rangle$ is independent of the basis choice.

Exercise 2

Calculate the expectation value of $\langle \sigma_i \rangle_{\psi^-} = \langle \psi^- | \sigma_i^A \otimes \mathbb{1}^B | \psi^- \rangle$

where $\psi^- = \frac{1}{\sqrt{2}}(| \uparrow\rangle \otimes | \downarrow\rangle - | \downarrow\rangle \otimes | \uparrow\rangle)$.

Remark:
Before using the explicit vectors and matrices for the ψ^- state and the Pauli matrices, make use of the rules of the tensor product. (e.g. $(A \otimes B) \cdot (C \otimes D) = (A \cdot C) \otimes (B \cdot D)$ etc.) These rules are especially helpful for later examples.

Exercise 3

Calculate explicitly, that $\sigma_y^A \otimes \mathbb{1}^B$ is not the same as $\mathbb{1}^A \otimes \sigma_y^B$.

Also, calculate the following commutators: $[\sigma_x \otimes \sigma_y, \sigma_y \otimes \sigma_x]$, $[\sigma_x \otimes \sigma_y, \sigma_y \otimes \sigma_y]$

Exercise 4

Show, that the expectation value of $\langle \sigma_i^A \otimes \sigma_j^B \rangle_{\psi^-}$ fulfils the relation $\langle \sigma_i^A \otimes \sigma_j^B \rangle_{\psi^-} = -\delta_{ij}$

Exercise 5

Calculate the expectation value of $\langle \vec{\sigma} \cdot \vec{a} \otimes \vec{\sigma} \cdot \vec{b} \rangle_{\psi^-}$

Exercise 6

Calculate the Eigenstates of $\vec{n} \cdot \vec{\sigma}$, where $\vec{n}$ is a normalized vector.

$$\vec{n} \cdot \vec{\sigma} | \pm \vec{n}\rangle = \pm | \pm \vec{n}\rangle$$

Remark:
Use spherical coordinates for vector $\vec{n}$:

$$\vec{n} = \begin{pmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{pmatrix}$$