21. Renormalized perturbation theory

$$\mathcal{L}_{QED} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\not{\partial} - m)\psi + e\bar{\psi}\gamma^\mu\psi$$

ψ, A <u>bare</u> (unrenormalized) fields

m, e bare (—‖—) parameters

$$\psi = \sqrt{Z_2}\,\psi_r \quad , \quad A^\mu = \sqrt{Z_3}\,A^\mu_r$$

ψ_r, A^μ_r <u>renormalized</u> fields

m_{phys}, e_{phys} <u>physical</u> parameters

$$\mathcal{L}_{QED} = -\frac{1}{4} Z_3 F_{r\mu\nu} F_r^{\mu\nu} + Z_2 \bar{\psi}_r (i\not{\partial} - m)\psi_r$$

$$+ \underbrace{e Z_2 \sqrt{Z_3}}_{e_{phys} Z_1} \bar{\psi}_r \gamma_\mu \psi_r A^\mu_r$$

$$-Z_2 m \bar{\psi}_r \psi_r = -m_{phys} \bar{\psi}_r \psi_r - \underbrace{(Z_2 m - m_{phys})}_{\delta_m} \bar{\psi}_r \psi_r$$

$$\mathcal{L}_{QED} = -\frac{1}{4} F_{r\mu\nu} F_r^{\mu\nu} - \frac{1}{4} \underbrace{(Z_3 - 1)}_{\delta_3} F_{r\mu\nu} F_r^{\mu\nu}$$

$$+ \overline{\Psi}_r (i\not{\partial} - m_{phys}) \Psi_r + \underbrace{(Z_2 - 1)}_{\delta_2} \overline{\Psi}_r i\not{\partial} \Psi_r - \delta_m \overline{\Psi}_r \Psi_r$$

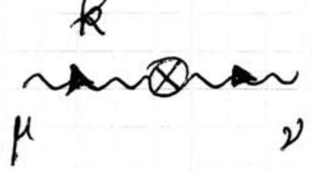
$$+ e_{phys} \overline{\Psi}_r \gamma_\mu \Psi_r A_r^\mu + e_{phys} \underbrace{(Z_1 - 1)}_{\delta_1} \overline{\Psi}_r \gamma_\mu \Psi_r A_r^\mu$$

$$\mathcal{L}_{QED} = \overbrace{-\frac{1}{4} F_{r\mu\nu} F_r^{\mu\nu} + \overline{\Psi}_r (i\not{\partial} - m_{phys}) \Psi_r}^{\mathcal{L}_0}$$

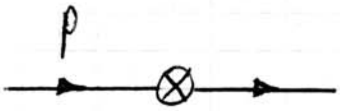
$$\mathcal{L}_{int} \begin{cases} + e_{phys} \overline{\Psi}_r \gamma_\mu \Psi_r A_r^\mu \\ \underbrace{-\frac{1}{4}\delta_3 F_{r\mu\nu} F_r^{\mu\nu} + \overline{\Psi}_r (i\delta_2 \not{\partial} - \delta_m) \Psi_r - e_{phys} \delta_1 \overline{\Psi}_r \gamma_\mu \Psi_r A_r^\mu}_{\text{counterterms}} \end{cases}$$

counterterm coefficients fixed by <u>renormalization conditions</u>

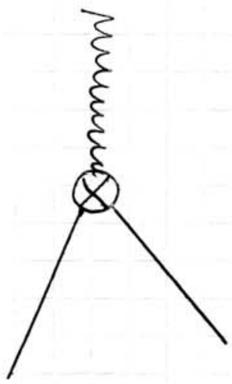
<u>additional</u> Feynman rules:



$$-i\left(g^{\mu\nu}k^2 - k^\mu k^\nu\right)\delta_3$$

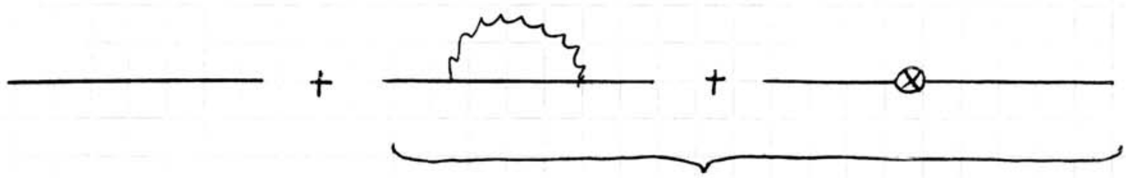


$$i\left(\not{p}\,\delta_2 - \delta_m\right)$$



$$i\, e_{phys}\, \gamma^\mu\, \delta_1$$

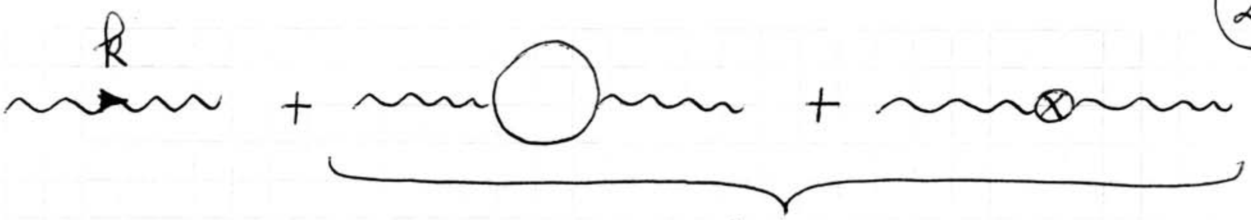
renormalization conditions:



$$\rightarrow \tilde{\Sigma}(\not{p})$$

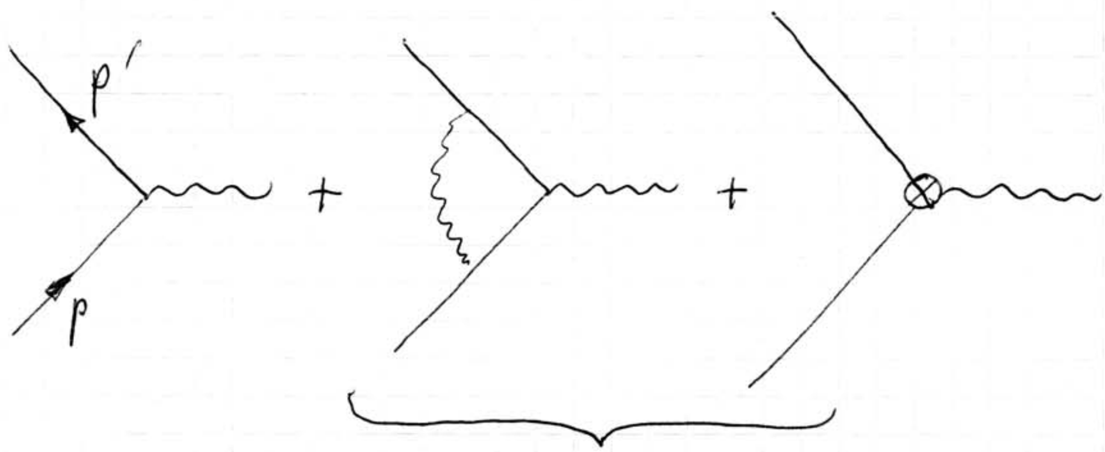
$$\tilde{\Sigma}(\not{p} = m_{phys}) = 0$$

$$\frac{\partial}{\partial \not{p}}\tilde{\Sigma}(\not{p})\Big|_{\not{p}=m_{phys}} = 0$$



$$\rightarrow \hat{\Pi}(k^2)$$

$$\hat{\Pi}(0) = 0$$



$$\tilde{\Lambda}(p', p)$$

$$\tilde{\Lambda}(p, p)\Big|_{p^2 = m_{ph}^2} = 0$$

<u>remark:</u> in the literature, the bare (unrenormalized) quantities are often denoted by ψ_0, A_0^μ, m_0, e_0 and the physical (renormalized) ones by ψ, A^μ, m, e (instead of ψ_r, A_r^μ, m_{phys}, e_{phys})

in the following: $m \equiv m_{phys}$, $e \equiv e_{phys}$

$$-i\tilde{\Sigma}(p) = \overbrace{-e^2\int\frac{d^dk}{(2\pi)^d}\,\gamma_\mu\,\frac{\not p-\not k+m}{(p-k)^2-m^2+i\varepsilon}\,\gamma^\mu\,\frac{1}{k^2-m_\gamma^2+i\varepsilon}}^{-i\Sigma(p)\quad p.\ 18/10}$$

$$+\,i\,(\not p\,\delta_2-\delta_m)$$

p. 18/17:

$$\Sigma(p) = \frac{e^2\,\Gamma(2-\frac{d}{2})}{(4\pi)^{d/2}}\int_0^1 dx\,\frac{(2-d)\,x\,\not p+d\,m}{[-x(1-x)p^2+(1-x)m^2+x\,m_\gamma^2-i\varepsilon]^{2-\frac{d}{2}}}$$

$$\tilde{\Sigma}\Big|_{\not p=m}=0 \Rightarrow\ -m\delta_2+\delta_m = -\frac{e^2 m\,\Gamma(2-\frac{d}{2})}{(4\pi)^{d/2}}\int_0^1 dx\,\frac{(2-d)x+d}{[(1-x)^2m^2+x\,m_\gamma^2]^{2-\frac{d}{2}}}$$

$$\frac{\partial\tilde{\Sigma}}{\partial\not p}\Big|_{\not p=m}=0\ \Rightarrow$$

$$\Rightarrow\ \delta_2 = \frac{e^2\,\Gamma(2-\frac{d}{2})}{(4\pi)^{d/2}}\int_0^1 dx\,\frac{1}{[(1-x)^2m^2+x\,m_\gamma^2-i\varepsilon]^{2-\frac{d}{2}}}\ \times$$

$$\times\left\{(2-d)\,x+\frac{-2x(1-x)\,m^2\,[(2-d)x+d]}{(1-x)^2m^2+x\,m_\gamma^2-i\varepsilon}\right\}$$

<u>vertex</u>:

$$ie\gamma^\mu + \underbrace{ie\,\Lambda^\mu(p',p) + ie\,\delta_1\gamma^\mu}_{ie\tilde{\Lambda}^\mu(p',p)} =: ie\,\Gamma^\mu(p',p)$$

$$\tilde{\Lambda}^\mu(p,p)\Big|_{p^2=m^2} \Rightarrow \gamma^\mu\delta_1 = -\Lambda^\mu(p,p)\Big|_{p^2=m^2}$$

$$\Rightarrow\ ie\Gamma^\mu(p',p) = ie\Big[\gamma^\mu + \underbrace{\Lambda^\mu(p',p) - \Lambda^\mu(p,p)\Big|_{p^2=m^2}}_{\tilde{\Lambda}^\mu(p',p)}\Big]$$

p. 20/1:

$$i\Lambda^\mu(p',p) = e^2\int\frac{d^dk}{(2\pi)^d}\,\frac{\gamma^\alpha(\not{p}'+\not{k}+m)\gamma^\mu(\not{p}+\not{k}+m)\gamma_\alpha}{(k^2-m_\gamma^2+i\varepsilon)\left[(k+p)^2-m^2+i\varepsilon\right]\left[(k+p')^2-m^2+i\varepsilon\right]}$$