19. Photon two-point function at one-loop

$$\langle \Omega | T A_\mu(x) A_\nu(y) | \Omega \rangle = \frac{\langle\!\langle e^{iS_{int}} A_\mu(x) A_\nu(y) \rangle\!\rangle}{\langle\!\langle e^{iS_{int}} \rangle\!\rangle}$$

$$\langle\!\langle e^{iS_{int}} A_\mu(x) A_\nu(y) \rangle\!\rangle =$$

$$= \langle\!\langle \left(1 + i S_{int} + \frac{i^2}{2!} S_{int}^2 + \dots\right) A_\mu(x) A_\nu(y) \rangle\!\rangle$$

$$= \langle\!\langle A_\mu(x) A_\nu(y) \rangle\!\rangle + \frac{i^2}{2!} \langle\!\langle S_{int}^2 A_\mu(x) A_\nu(y) \rangle\!\rangle$$

$$+ \dots$$

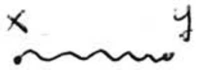
$$= i D_{\mu\nu}(x-y)$$

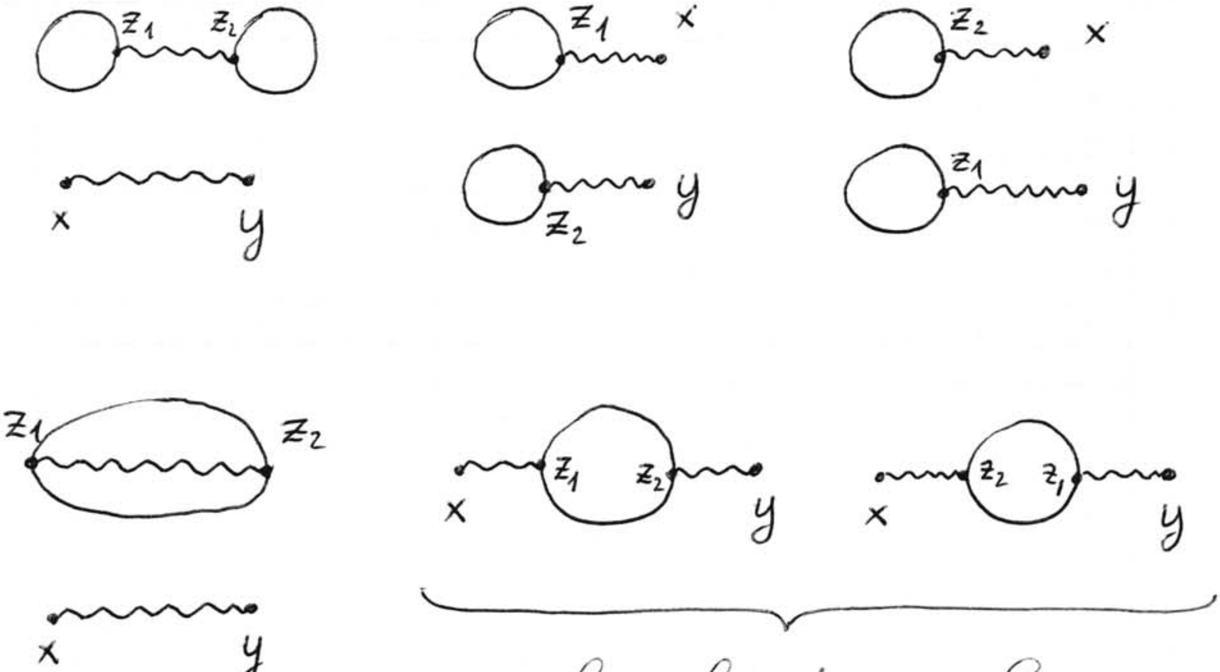
$$+ \frac{(ie)^2}{2!} \int d^4 z_1 \, d^4 z_2 \, \langle\!\langle \overline{\psi}(z_1) \not{A}(z_1) \psi(z_1)$$

$$\overline{\psi}(z_2) \not{A}(z_2) \psi(z_2) A_\mu(x) A_\nu(y) \rangle\!\rangle$$

$$+ \dots$$

graphs :





only relevant graphs

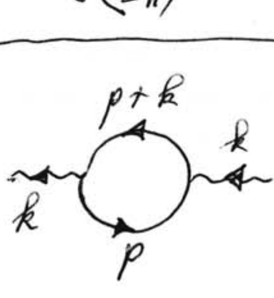
a straightforward calculation yields

$$\langle\Omega|\,T\,A_\mu(x)\,A_\nu(y)\,|\Omega\rangle\Big|_{\text{one-loop}} =$$

$$= \int\frac{d^4k}{(2\pi)^4}\, e^{-ik(x-y)}\left\{ \frac{-i g_{\mu\nu}}{k^2+i\varepsilon} + \right.$$

$$+ \frac{-i g_{\mu\alpha}}{k^2+i\varepsilon}\underbrace{\left[-i\int\frac{d^4p}{(2\pi)^4}\,\mathrm{Tr}\left(\frac{i}{\not p-m+i\varepsilon}\, ie\gamma^\alpha\,\frac{i}{\not p+\not k-m+i\varepsilon}\, ie\gamma^\beta\right)\right]}_{i\Pi^{\alpha\beta}(k)}$$

$$\left.\times\ \frac{-i g_{\nu\beta}}{k^2+i\varepsilon}\right\}$$



loop integral UV-divergent in $d=4$

$\rightarrow$ dimensional regularization:

$$i\Pi^{\alpha\beta}(k) = (-1)\,(ie)^2 \int \frac{d^dp}{(2\pi)^d} \frac{i}{p^2-m^2+i\varepsilon} \frac{i}{(p+k)^2-m^2+i\varepsilon}$$

$$\times \mathrm{Tr}\left[(\not{p}+m)\,\gamma^\alpha\,(\not{p}+\not{k}+m)\,\gamma^\beta\right]$$

$$= -4e^2 \int \frac{d^dp}{(2\pi)^d} \frac{1}{p^2-m^2+i\varepsilon} \frac{1}{(p+k)^2-m^2+i\varepsilon}$$

$$\times \left\{2p^\alpha p^\beta + p^\alpha k^\beta + k^\alpha p^\beta + (m^2-p^2-pk)\,g^{\alpha\beta}\right\}$$

$\rightarrow$ Feynman parametrization:

$$i\Pi^{\alpha\beta}(k) = -4e^2 \int_0^1 dx \int \frac{d^dp}{(2\pi)^d}$$

$$\times \frac{\left(\frac{2}{d}-1\right) g^{\alpha\beta} p^2 + \left[m^2 + x(1-x)k^2\right] g^{\alpha\beta} - 2x(1-x)k^\alpha k^\beta}{\left[p^2 + k^2 x(1-x) - m^2 + i\varepsilon\right]^2}$$

$$i\,\Pi^{\alpha\beta}(k) = i\,\overbrace{\frac{-8e^2\,\Gamma\left(2-\frac{d}{2}\right)}{(4\pi)^{d/2}}\int_0^1 dx\,\left[m^2-k^2x(1-x)-i\varepsilon\right]^{\frac{d}{2}-2} x(1-x)}^{\Pi(k^2)}$$

$$\times\left(g^{\alpha\beta}k^2 - k^\alpha k^\beta\right)$$

structure of $\Pi^{\alpha\beta}(k)$:

$$\Pi^{\alpha\beta}(k) = \Pi(k^2)\left(g^{\alpha\beta}k^2 - k^\alpha k^\beta\right)$$

$$\Pi(k^2) = \frac{8e^2}{(4\pi)^2}\,\mu^{d-4}\left\{\frac{1}{3}\Lambda_d + \int_0^1 dx\,x(1-x)\ln\frac{m^2-k^2x(1-x)-i\varepsilon}{\mu^2}\right\}$$

$$\left.\langle\Omega|T A_\mu(x)A_\nu(y)|\Omega\rangle\right|_{\text{one-loop}} = \int\frac{d^4k}{(2\pi)^4}\,e^{-ik(x-y)}$$

$$\left\{\frac{-i g_{\mu\nu}}{k^2+i\varepsilon} + \right.$$

$$\left. + \frac{-i g_{\mu\alpha}}{k^2+i\varepsilon}\; i\left(k^2 g^{\alpha\beta} - k^\alpha k^\beta\right)\Pi(k^2)\,\frac{-i g_{\beta\nu}}{k^2-i\varepsilon}\right\}$$

momentum space:

$$\frac{-i g_{\mu\nu}}{k^2 + i\varepsilon} + \frac{-i g_{\mu\nu} \Pi(k^2)}{k^2 + i\varepsilon} + k_\mu k_\nu\text{-terms}$$

$$= \frac{-i g_{\mu\nu}}{(k^2 + i\varepsilon)(1 - \Pi(k^2))} + k_\mu k_\nu\text{-terms}$$

pole stays at $k^2 = 0$ with residue

$$Z_3 := \frac{1}{1 - \Pi(0)} = 1 + \Pi(0) + \ldots$$

wave function renormalization constant

(field renormalization constant)

of the photon

$$\Pi(0) = \frac{8e^2}{(4\pi)^2} \mu^{d-4} \left\{ \frac{1}{3} \Lambda_d + \frac{1}{6} \ln \frac{m^2}{\mu^2} \right\}$$

$$\frac{1}{1-\Pi(k^2)} = \frac{1}{1-\Pi(k^2)+\Pi(0)-\Pi(0)} =$$

$$= \frac{1}{(1-\Pi(0))\,(1-\Pi(k^2)+\Pi(0))} = \frac{Z_3}{1-[\Pi(k^2)-\Pi(0)]}$$

$$\hat{\Pi}(k^2) := \Pi(k^2)-\Pi(0)$$

$$= \frac{8e^2}{(4\pi)^2}\,\mu^{d-4}\int_0^1 dx\, x\,(1-x)\ln\frac{m^2-k^2x(1-x)-i\varepsilon}{m^2}$$

<u>finite</u> for $d \to 4$

$\to$ photon two-point function in momentum space:

$$\frac{-i\,Z_3\,g_{\mu\nu}}{k^2\,(1-\hat{\Pi}(k^2))+i\varepsilon}$$