16. Two-point function of an interacting theory

Heisenberg picture

basis of eigenstates of P^μ

$|\Omega\rangle$ vacuum of the interacting theory (to be distinguished from the vacuum of the free theory $|0\rangle$)

$$P^\mu |\Omega\rangle = 0$$

$$P^\mu \,|p_1, \dots, p_n \;{}^{in}_{out}\rangle = (p_1^\mu + \dots + p_2^\mu)\,|p_1, \dots, p_n \;{}^{in}_{out}\rangle$$

$$p_i^0 = \sqrt{\vec{p}_i^{\,2} + m_{ph}^2}$$

discussion for scalar theory (extension to higher spin straightforward, but more indices!)

Heisenberg operators!

$$\langle\Omega| T\, \phi(x)\, \phi(0)\, |\Omega\rangle =$$

$$= \Theta(x^0)\, \langle\Omega| \phi(x)\, \phi(0) |\Omega\rangle$$

$$+\ \Theta(-x^0)\, \langle\Omega| \phi(0)\, \phi(x) |\Omega\rangle$$

insert complete ONS of momentum eigenstates

$$= \Theta(x^0) \sum_\alpha \langle \Omega | \phi(x) | \alpha \rangle \langle \alpha | \phi(0) | \Omega \rangle$$

$$+ \Theta(-x^0) \sum_\alpha \langle \Omega | \phi(0) | \alpha \rangle \langle \alpha | \phi(x) | \Omega \rangle$$

$$= \Theta(x^0) \sum_\alpha \langle \Omega | e^{+iPx} \phi(0) e^{-iPx} | \alpha \rangle \langle \alpha | \phi(0) | \Omega \rangle$$

$$+ \Theta(-x^0) \sum_\alpha \langle \Omega | \phi(0) | \alpha \rangle \langle \alpha | e^{+iPx} \phi(0) e^{-iPx} | \Omega \rangle$$

$$= \Theta(x^0) \sum_\alpha e^{-ip_\alpha x} |\langle \Omega | \phi(0) | \alpha \rangle|^2$$

$$+ \Theta(-x^0) \sum_\alpha e^{ip_\alpha x} |\langle \Omega | \phi(0) | \alpha \rangle|^2$$

$$\sum_\alpha |\alpha\rangle\langle\alpha| = |\Omega\rangle\langle\Omega| + \int d\mu(p) \, |p\rangle\langle p|$$

$$+ \sum_{n=2}^{\infty} \frac{1}{n!} \int d\mu(p_1) \dots d\mu(p_n) \, |p_1, \dots, p_n \; {}^{in}_{out}\rangle\langle p_1, \dots, p_n \; {}^{in}_{out}|$$

assume: $\langle \Omega | \phi(0) | \Omega \rangle = 0$ (in theories with $\langle \Omega | \phi(0) | \Omega \rangle \neq 0$ take $\phi(x) - \langle \Omega | \phi(0) | \Omega \rangle$ instead of $\phi(x)$)

$$\Rightarrow \langle \Omega | T \phi(x) \phi(0) | \Omega \rangle$$

$$= \Theta(x^0) \int d\mu(p)\, e^{-ipx}\, |\langle \Omega | \phi(0) | p \rangle|^2$$

$$+ \Theta(-x^0) \int d\mu(p)\, e^{+ipx}\, |\langle \Omega | \phi(0) | p \rangle|^2$$

+ contributions from intermediate states with $n \geq 2$

$\langle \Omega | \phi(0) | p \rangle$ is <u>independent</u> of p because of Lorentz invariance:

$|p\rangle \equiv | \sqrt{m_{ph}^2 + \vec{p}^{\,2}}, \vec{p} \rangle$ can be obtained by a Lorentz transformation acting on $|m_{ph}, \vec{0}\rangle$:

$$|p\rangle = | \sqrt{m_{ph}^2 + \vec{p}^{\,2}}, \vec{p} \rangle = U(L)\, |m_{ph}, \vec{0}\rangle$$

where $L\,(m_{ph}, \vec{0}) = (\sqrt{m_{ph}^2 + \vec{p}^{\,2}}, \vec{p})$

$$\Rightarrow \langle \Omega | \phi(0) | p \rangle = \langle \Omega | \phi(0)\, U(L)\, | m_{ph}, \vec{0} \rangle$$

$$= \langle \Omega | U(L)^{-1} \, \phi(0) \, U(L) \, | m_{ph}, \vec{0} \rangle$$

$\uparrow$

$U(L)|\Omega\rangle = |\Omega\rangle$

$$= \langle \Omega | \phi(0) | m_{ph}, \vec{0} \rangle$$ independent of p

$\uparrow$

ϕ is a scalar field: $U(L)^{-1} \phi(x) U(L) = \phi(L^{-1}x)$

$$Z := |\langle \Omega | \phi(0) | p \rangle|^2$$

with a suitable definition of the phase of $|p\rangle$:

$$\sqrt{Z} = \langle \Omega | \phi(0) | p \rangle$$

$$\Rightarrow \langle \Omega | T \phi(x) \phi(0) | \Omega \rangle =$$

$$= Z \underbrace{\left\{ \Theta(x^0) \int d\mu(p) \, e^{-ipx} + \Theta(-x^0) \int d\mu(p) \, e^{ipx} \right\}}_{\frac{1}{i} \Delta(x; m_{ph})} + \ldots$$

$$= \frac{Z}{i} \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ikx}}{m_{ph}^2 - k^2 - i\varepsilon} + \ldots$$

pole of the two-point function in momentum space determines the physical mass m_{ph}

- fermions :

$$\langle \Omega | T\, \psi(x)\, \overline{\psi}(0) | \Omega \rangle = \frac{1}{i}\, S(x; m_{ph}) + \ldots$$

Heisenberg operators

$$= \frac{Z}{i} \int \frac{d^4k}{(2\pi)^4}\, \frac{e^{-ikx}}{m_{ph} - \not{k} - i\varepsilon} + \underbrace{\ldots\ldots}_{\text{continuum contributions}}$$

$$\langle \Omega | \psi(0) | p, s \rangle = \sqrt{Z}\, u(p, s; m_{ph})$$

$$\langle p, s | \overline{\psi}(0) | \Omega \rangle = \sqrt{Z}\, \overline{u}(p, s; m_{ph})$$

$$\langle \overline{p, s} | \psi(0) | \Omega \rangle = \sqrt{Z}\, v(p, s; m_{ph})$$

anti-particle state

$$\langle \Omega | \overline{\psi}(0) | \overline{p, s} \rangle = \sqrt{Z}\, \overline{v}(p, s; m_{ph})$$