**Exercises "Particle Physics II"** (2010)

28. Compute:

    (a) $\gamma^\alpha\gamma^\mu\gamma_\alpha$

    (b) $\gamma^\alpha\gamma^\mu\gamma^\nu\gamma_\alpha$

    (c) $\gamma^\alpha\gamma^\mu\gamma^\nu\gamma^\rho\gamma_\alpha$.

29. Show:

    $\mathrm{Tr}(\not p_1 \ldots \not p_{2n}) = p_1{\cdot}p_2 \mathrm{Tr}(\not p_3 \ldots \not p_{2n}) - p_1{\cdot}p_3 \mathrm{Tr}(\not p_2 \ldots \not p_{2n}) + \ldots + p_1{\cdot}p_{2n} \mathrm{Tr}(\not p_2 \ldots \not p_{2n-1})$

30. Compute the two-body phase space integral

    $$\int \frac{d^3k_1}{2k_1^0}\frac{d^3k_2}{2k_2^0}\delta^{(4)}(P-k_1-k_2)f(k_1\cdot k_2),$$

    where $k_i^2 = m_i^2$. $P$ denotes a timelike 4-vector. Give your final answer in terms of the function $\lambda(x,y,z) = x^2 + y^2 + z^2 - 2(xy + xz + yz)$.

    Hint: The integral is a Lorentz scalar.

31. Compute

    $$I^\mu = \int \frac{d^3k_1}{2k_1^0}\frac{d^3k_2}{2k_2^0}\,\delta^{(4)}(P-k_1-k_2)\,k_2^\mu.$$

    Hint: The integral is a Lorentz vector and can be written in the form $I^\mu = JP^\mu$, where $J$ is a scalar function (why?).

32. Compute the tensor integral

    $$I^{\mu\nu} = \int \frac{d^3k_1}{2k_1^0}\frac{d^3k_2}{2k_2^0}\,\delta^{(4)}(P-k_1-k_2)\,k_1^\mu k_2^\nu.$$

33. Apply the formulae obtained in the previous problems to derive the total cross section of $e^-e^+ \to \mu^-\mu^+$, using the expression for

    $$\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}_{fi}|^2$$

    given in the lecture notes (p. 9/4).

34. Use Wick's theorem to compute the invariant amplitude of electron proton scattering ($e^-p \to e^-p$) at tree level. Describe the electromagnetic interaction of the proton by simply using the appropriate covariant derivative in the Dirac Lagrangean of the proton.

Remark: In contrast to the electron, the proton is not a pointlike particle. This reflects itself by the fact that, e.g., the magnetic moment of the proton is not correctly desribed by the Dirac Lagrangean alone. In a phenomenological description, an additional term $\sim \bar{\psi}\sigma_{\mu\nu}\psi F^{\mu\nu}$ is needed. In this exercise, we ignore these complications.

35. Compute the differential cross section of electron proton scattering in the rest frame of the proton, using the result of the previous problem. Consider relevant limiting cases.

In the following problems (36-39), you are supposed to investigate several QED processes to lowest order. Assume unpolarized beams in the initial state and polarization-blind detectors for the final-state particles. Compute the relevant traces of gamma-matrices using FORM. Work out the differential and total cross sections, discuss the physical content of your results and limiting cases of interest.

36. $e^-\gamma \to e^-\gamma$ (Compton scattering)

37. $e^-e^+ \to \gamma\gamma$ (pair annihilation)

38. $e^-e^+ \to e^-e^+$ (Bhabha scattering)

39. $e^-e^- \to e^-e^-$ (Møller scattering)

40. Consider a real scalar field with a $\phi^4$ self-interaction:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^2$$

Derive the invariant amplitude of the scattering process

$$\phi(p_1)\phi(p_2) \to \phi(k_1)\phi(k_2)$$

to lowest order.

41. Compute the differential and total cross section of the reaction of the previous problem.