Exercises "Particle Physics II"

41. Compute the two-body phase space integral

$$\int \frac{d^3k_1}{2k_1^0}\frac{d^3k_2}{2k_2^0}\delta^{(4)}(P-k_1-k_2)f(k_1\dot{k}_2),$$

where $k_i^2 = m_i^2$. P denotes a timelike 4-vector. Give your final answer in terms of the function $\lambda(x,y,z) = x^2+y^2+z^2-2(xy+xz+yz)$.

Hint: The integral is a Lorentz scalar.

42. Compute

$$I^\mu = \int \frac{d^3k_1}{2k_1^0}\frac{d^3k_2}{2k_2^0}\,\delta^{(4)}(P-k_1-k_2)\,k_2^\mu.$$

Hint: The integral is a Lorentz vector and can be written in the form $I^\mu = JP^\mu$, where J is a scalar function (why?).

43. Compute the tensor integral

$$I^{\mu\nu} = \int \frac{d^3k_1}{2k_1^0}\frac{d^3k_2}{2k_2^0}\,\delta^{(4)}(P-k_1-k_2)\,k_1^\mu k_2^\nu.$$

44. Compute the invariant amplitude of electron proton scattering ($e^-p \to e^-p$) at tree level. Describe the electromagnetic interaction of the proton by simply using the appropriate covariant derivative in the Dirac Lagrangean of the proton.

Remark: In contrast to the electron, the proton is not a pointlike partice. This reflects itself by the fact that e.g. the magnetic moment of the proton is not correctly desribed by the Dirac Lagrangean alone. In a phenomenological description, an additional term $\sim \bar{\psi}\sigma_{\mu\nu}\psi F^{\mu\nu}$ is needed. In this exercise, we ignore these complications.

45. Compute the differential cross section of electron proton scattering in the rest frame of the proton, using the result of the previous problem. Determine also the total cross section of this reaction.

46. Verify the Gordon decomposition:

$$\bar{u}(p',s')\gamma^\mu u(p,s) = \bar{u}(p',s')\left[p^\mu + p'^\mu + i\sigma^{\mu\nu}(p'-p)_\nu\right]u(p,s)/(2m).$$

47. Compute $\gamma^\alpha\gamma^\mu\gamma_\alpha$, $\gamma^\alpha\gamma^\mu\gamma^\nu\gamma_\alpha$ and $\gamma^\alpha\gamma^\mu\gamma^\nu\gamma^\rho\gamma_\alpha$ ($d=4$).

48. The standard theory of electroweak interactions postulates the existence of a neutral scalar $h(x)$ (called Higgs field) with Yukawa couplings to the elementary fermions:

$$\mathcal{L} = -\sum_f g_f \bar{f}(x) f(x) h(x),$$

where $g_f = m_f/v$ ($v = 246\,\text{GeV}$). Compute the contribution of the virtual Higgs boson to the anomalous magnetic moment of the electron. The present experimental bound (CL = 95%) for the Higgs mass is $M_h > 114.4\,\text{GeV}$.

49. The structure constants f_{abc} of a Lie algebra $\mathcal{L}$ with generators T_a are defined by

$$[T_a, T_b] = i f_{abc} T_c.$$

Show that f_{abc} is totally antisymmetric, if the T_a form an orthogonal basis of $\mathcal{L}$ ($\text{Tr}(T_a T_b) = c\,\delta_{ab}$).

50. Show that $(t_a)_{bc} = -i f_{abc}$ defines a representation of $\mathcal{L}$ (adjoint representation).

51. The transformation formula for a nonabelian gauge field $A_\mu = A_\mu^a T_a$ under a local gauge transformation is given by

$$A'_\mu = U A_\mu U^{-1} + \frac{i}{g}(\partial_\mu U) U^{-1}.$$

Show that the generalized field strength tensor

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$$

transforms as

$$F'_{\mu\nu} = U F_{\mu\nu} U^{-1}.$$

52. The positive frequency part of the massless scalar propagator is given by

$$\Delta_0^+(x) := i \int \frac{d^3p}{(2\pi)^3 2p^0}\, e^{-ip\cdot x}, \quad x^0 \to x^0 - i\varepsilon, \quad p^0 = |\vec{p}\,|.$$

Show that this function can be written in the form

$$\Delta_0^+(x) = \frac{1}{4\pi^2 i[(x^0 - i\varepsilon)^2 - \vec{x}^{\,2}]}.$$

53. Determine the propagator of a real spin 1 field described by the Lagrangean

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{M^2}{2} A_\mu A^\mu,$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.