Exercises "Particle Physics II"

26. Using the result of the previous problem, discuss the pairing rule for the Green's function

$$\langle 0|T\phi(x_1)\ldots\phi(x_m)\phi(y_1)^\dagger\ldots\phi(y_n)^\dagger|0\rangle$$

of a non-hermitean scalar field $\phi(x)$.

27. Compute the Gaussian mean values

$$\langle\langle\varphi(x_1)\varphi(x_2)e^{iS_{\rm int}}\rangle\rangle\ , \qquad \langle\langle e^{iS_{\rm int}}\rangle\rangle$$

in φ^4 theory,

$$S_{\rm int} = -\frac{\lambda}{4!}\int d^dy\,\varphi(y)^4\ ,$$

including the contributions of order λ. Convince yourself that the contributions of graphs with vacuum bubbles cancel when the ratio of the two terms is taken.

28. Show the following formula in dimensional regularization:

$$\int\frac{d^dk}{(2\pi)^d}\frac{(k^2)^\beta}{(M^2-k^2-i\varepsilon)^\alpha} = \frac{(-1)^\beta i}{(4\pi)^{d/2}}\frac{\Gamma(\alpha-\beta-d/2)\Gamma(\beta+d/2)}{\Gamma(\alpha)\Gamma(d/2)}M^{d+2\beta-2\alpha}\ ,$$

where $\alpha,\beta\in\mathbb{N}$. Discuss the case $\alpha=0$ and the implication for $\delta^d(0)$ in dimensional regularization.

29. Write the finite one-loop function ($d=4$)

$$\bar{B}(p^2,m^2) = B(p^2,m^2) - B(0,m^2)$$

in the form

$$\bar{B}(p^2,m^2) = \int_0^1 d\alpha\, f(\alpha,p^2,m^2)\ .$$

30. Using the previous result, determine the imaginary part of $\bar{B}(p^2,m^2)$.

31. Consider the kinematics of the scattering process $\varphi(p_1)\varphi(p_2)\to\varphi(p_3)\varphi(p_4)$ in the center of mass system. Express the Mandelstam variables

$$t=(p_1-p_3)^2\ ,\quad u=(p_1-p_4)^2$$

in terms of $s=(p_1+p_2)^2$ and the scattering angle θ.