Introduction into
Standard Model and Precision Physics

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Overview

1 Electroweak phenomenology before the GSW model

Some phenomenological facts:

- discovery of the weak interaction via radioactive β-decay of nuclei:
  $\mathrm{n} \to \mathrm{p} + \mathrm{e}^- + \bar{\nu}_\mathrm{e}$, $\quad \mathrm{p} \to \mathrm{n} + \mathrm{e}^+ + \nu_\mathrm{e}$ (not possible for free protons)
- terminology "weak": interaction at low energy has very short range
  $\hookrightarrow$ long life time of weakly decaying particles:

strong int.:	$\rho \to 2\pi$,	$\tau \sim 10^{-22}$s
elmg. int.:	$\pi \to 2\gamma$,	$\tau \sim 10^{-16}$s
weak int.:	$\pi^- \to \mu^- + \bar{\nu}_\mu$	$\tau \sim 10^{-8}$s
	$\mu^- \to \mathrm{e}^- + \bar{\nu}_\mathrm{e} + \nu_\mu$,	$\tau \sim 10^{-6}$s

- lepton-number conservation: $\mu^- \not\to \mathrm{e}^- + \gamma$ (BR $\lesssim 10^{-11}$)
  $\Rightarrow$ $L_\mathrm{e}, L_\mu, L_\tau$ individually conserved:
  $L_\mathrm{e} = +1$ for $\mathrm{e}^-, \nu_\mathrm{e}$, $\quad L_\mathrm{e} = -1$ for $\mathrm{e}^+, \bar{\nu}_\mathrm{e}$, etc.

  (For massive ν's with different masses, only $L_\mathrm{e}+L_\mu+L_\tau$ is conserved.)
- parity violation (Wu et al. 1957):
  e.g.: $K^+ \to \underbrace{2\pi, 3\pi}$ final states of different parity

  ${}^{60}\mathrm{Co} \to {}^{60}\mathrm{Ni}^* + \mathrm{e}^- + \bar{\nu}_\mathrm{e}$
  $\hookrightarrow$ polarization inversion does not yield inversion of spectra

The Fermi model

(Fermi 1933, further developed by Feynman, Gell-Mann and others after 1958)

Lagrangian for "current–current interaction" of four fermions:

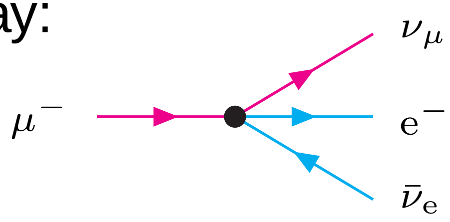
$$\mathcal{L}_{\mathrm{Fermi}}(x) = -2\sqrt{2} G_\mu J_\rho^\dagger(x) J^\rho(x), \qquad G_\mu = 1.16639 \times 10^{-5}\,\mathrm{GeV}^{-2}$$

$$\text{with} \quad J_\rho(x) = J_\rho^{\mathrm{lep}}(x) + J_\rho^{\mathrm{had}}(x) = \text{charged weak current}$$

- Leptonic part J_ρ^{lep} of J_ρ:

$$J_\rho^{\mathrm{lep}} = \overline{\psi_{\nu_\mathrm{e}}}\gamma_\rho\omega_-\psi_\mathrm{e} + \overline{\psi_{\nu_\mu}}\gamma_\rho\omega_-\psi_\mu \qquad \omega_\pm = \tfrac{1}{2}(1 \pm \gamma_5) = \text{chirality projectors}$$

  - ◇ only left-handed fermions $(\omega_-\psi)$, right-handed anti-fermions $(\overline{\psi}\omega_+)$ feel (charged-current) weak interactions $\Rightarrow$ maximal P-violation
  - ◇ doublet structure: $\begin{pmatrix}\nu_\mathrm{e}\\ \mathrm{e}^-\end{pmatrix}$, $\begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}$, later completed by $\begin{pmatrix}\nu_\tau\\ \tau^-\end{pmatrix}$
  - ◇ $(J^{\mathrm{lep},\rho})^\dagger J_\rho^{\mathrm{lep}}$ induces muon decay:



- Hadronic part $J_\rho^{\rm had}$ of J_ρ:

Relevant quarks for energies $\lesssim 1\,\mathrm{GeV}$: $\quad \mathrm{u,d,s,c}$

$\hookrightarrow$ meson ($q\bar{q}$) and baryon (qqq) spectra

Question: doublet structure $\begin{pmatrix} \mathrm{u} \\ \mathrm{d} \end{pmatrix}, \begin{pmatrix} \mathrm{c} \\ \mathrm{s} \end{pmatrix}$?

Problem: e.g. annihilation of $\mathrm{u\bar{s}}$ pair would not be allowed,
but is observed: $\underbrace{K^+}_{\mathrm{u\bar{s}}\text{ pair in quark model}} \to \mu^+\nu_\mu$

Solution (Cabibbo 1963):

u-c-mixing and d-s-mixing in weak interaction

$\hookrightarrow$ doublets $\begin{pmatrix} \mathrm{u} \\ \mathrm{d'} \end{pmatrix}, \begin{pmatrix} \mathrm{c} \\ \mathrm{s'} \end{pmatrix}$ with $\begin{pmatrix} \mathrm{d'} \\ \mathrm{s'} \end{pmatrix} = U_\mathrm{C} \begin{pmatrix} \mathrm{d} \\ \mathrm{s} \end{pmatrix}$,

orthogonal Cabbibo matrix $\quad U_\mathrm{C} = \begin{pmatrix} \cos\theta_\mathrm{C} & \sin\theta_\mathrm{C} \\ -\sin\theta_\mathrm{C} & \cos\theta_\mathrm{C} \end{pmatrix}$,

empirical result: $\quad \theta_\mathrm{C} \approx 13^\circ$

$$J_\rho^{\rm had} = \overline{\psi_\mathrm{u}}\gamma_\rho\omega_-\psi_{\mathrm{d'}} + \overline{\psi_\mathrm{c}}\gamma_\rho\omega_-\psi_{\mathrm{s'}}$$

Remarks on the Fermi model:

- universal coupling G_μ for all transitions
  ($U_C^\dagger U_C = \mathbf{1}$ is part of universality)
- no (pseudo-)scalar or tensor couplings, such as $(\overline{\psi}\psi)(\overline{\psi}\psi)$, $(\overline{\psi}\psi)(\overline{\psi}\gamma_5\psi)$, etc., necessary to describe low-energy experiments ($E \lesssim 1\,\mathrm{GeV}$)
- Problems:
  - ◇ cross sections for $\nu_\mu \mathrm{e} \to \nu_\mathrm{e}\mu$, etc., grow for energy $E \to \infty$ as E^2

    $\hookrightarrow$ unitarity violation !
  - ◇ no consistent evaluation of higher perturbative orders possible
    (no cancellation of UV divergences)

    $\hookrightarrow$ non-renormalizability !

“Intermediate-vector-boson (IVB) model”

Idea: “resolution” of four-fermion interaction by vector-boson exchange

Lagrangian:

$$\mathcal{L}_{\mathrm{IVB}} = \mathcal{L}_{0,\mathrm{ferm}} + \mathcal{L}_{0,\mathrm{W}} + \mathcal{L}_{\mathrm{int}},$$

$$\mathcal{L}_{0,\mathrm{ferm}} = \overline{\psi_f}(\mathrm{i}\partial\!\!\!/ - m_f)\psi_f, \qquad \text{(summation over } f \text{ assumed)}$$

$$\mathcal{L}_{0,\mathrm{W}} = -\frac{1}{2}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+)(\partial^\mu W^{-,\nu} - \partial^\nu W^{-,\mu}) + M_\mathrm{W}^2 W_\mu^+ W^{-,\mu},$$

$$\text{with } W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp \mathrm{i}W_\mu^2), \quad W_\mu^i \text{ real}$$

$\mathrm{W}^\pm$ are vector bosons with electric charge $\pm e$ and mass M_W.

Propagator:

$$G_{\mu\nu}^{\mathrm{WW}}(k) = \frac{-\mathrm{i}}{k^2 - M_\mathrm{W}^2}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{M_\mathrm{W}^2}\right), \qquad k = \text{momentum}$$

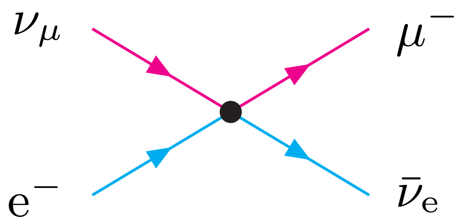
Interaction Lagrangian:

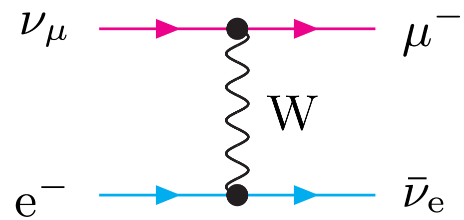
$$\mathcal{L}_{\mathrm{int}} = \frac{g_\mathrm{W}}{\sqrt{2}}\left(J^\rho W_\rho^+ + J^{\rho\dagger} W_\rho^-\right),$$

J^ρ = charged weak current as in Fermi model

Four-fermion interaction in process $\nu_\mu \mathrm{e}^- \to \mu^- \nu_\mathrm{e}$

Fermi model: | IVB model:





Fermi model:
$$-\mathrm{i}2\sqrt{2}G_\mu\, g_{\rho\sigma}$$
$$\times \left[\bar{u}_{\mu^-}\gamma^\rho\omega_- u_{\nu_\mu}\right]\,\left[\bar{u}_{\nu_\mathrm{e}}\gamma^\sigma\omega_- u_{\mathrm{e}^-}\right]$$

IVB model:
$$\tfrac{\mathrm{i}}{2}g_\mathrm{W}^2\,\frac{1}{k^2-M_\mathrm{W}^2}\left(g_{\rho\sigma}-\frac{k_\rho k_\sigma}{M_\mathrm{W}^2}\right)$$
$$\times \left[\bar{u}_{\mu^-}\gamma^\rho\omega_- u_{\nu_\mu}\right]\,\left[\bar{u}_{\nu_\mathrm{e}}\gamma^\sigma\omega_- u_{\mathrm{e}^-}\right]$$

$\Rightarrow$ identification for $|k| \ll M_\mathrm{W}$: $\quad 2\sqrt{2}G_\mu = \dfrac{g_\mathrm{W}^2}{2M_\mathrm{W}^2}$

Consequences for the high-energy behaviour:

- k^ρ terms: $\quad \bar{u}_{\nu_\mathrm{e}}\not{k}\omega_- u_{\mathrm{e}^-} = \bar{u}_{\nu_\mathrm{e}}(\not{p}_\mathrm{e} - \not{p}_{\nu_\mathrm{e}})\omega_- u_{\mathrm{e}^-} = m_\mathrm{e}\bar{u}_{\nu_\mathrm{e}}\omega_- u_{\mathrm{e}^-}$
  $\hookrightarrow$ no extra factors of scattering energy E
- propagator $1/(k^2 - M_\mathrm{W}^2) \;\sim\; 1/E^2$ for $|k| \sim E \gg M_\mathrm{W}$
  $\hookrightarrow$ damping of amplitude in high-energy limit by factor $1/E^2$

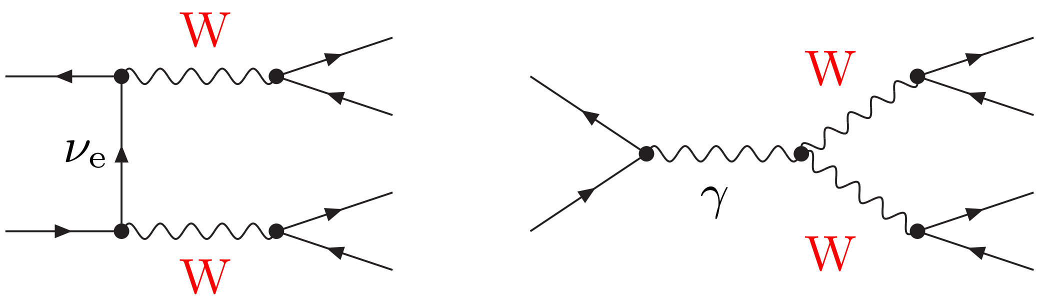
$\Rightarrow$ cross section $\underset{E\to\infty}{\sim}$ const$/E^2$, $\qquad \Rightarrow$ No unitarity violation !

Comments on the IVB model:

- Formal similiarity with QED interaction: $J^\rho W^+_\rho$ + h.c. $\longleftrightarrow$ $j^\rho_{\mathrm{elmg.}} A_\rho$
- Intermediate vector bosons can be produced, e.g.

$$\underbrace{\mathrm{u\bar{d}}}_{\text{in pp collision}} \longrightarrow \underbrace{\mathrm{W}^+ \to f\bar{f}'}_{\mathrm{W}^\pm \text{ unstable}}$$ (discovery 1983 at CERN)

- Problems:
  - ◇ unitarity violations in cross sections with longitudinal W bosons, e.g.



  - ◇ non-renormalizability
    (no consistent treatment of higher perturbative orders)

$\hookrightarrow$ Solution by spontaneously broken gauge theories !

2 The principle of local gauge invariance

QED as U(1) gauge theory:

Lagrangian $\mathcal{L}_{0,\mathrm{ferm}} = \overline{\psi_f}(\mathrm{i}\partial\!\!\!/ - m_f)\psi_f$ has global phase symmetry:

$$\psi_f \to \psi'_f = \exp\{-\mathrm{i}Q_f e\theta\}\psi_f, \quad \overline{\psi_f} \to \overline{\psi'_f} = \overline{\psi_f}\exp\{+\mathrm{i}Q_f e\theta\}$$

with space-time-independent group parameter θ

"Gauging the symmetry": demand local symmetry, $\theta \to \theta(x)$

To maintain local symmetry, extend theory by "minimal substitution":

$\partial^\mu \to D^\mu = \partial^\mu + \mathrm{i}Q_f e A^\mu(x)$ = "covariant derivative",

$A^\mu(x)$ = spin-1 gauge field (photon).

Transformation property of photon $A_\mu(x) \to A'_\mu(x) = A_\mu(x) + \partial_\mu\theta(x)$ ensures

- $D_\mu\psi_f \to (D_\mu\psi_f)' = D'_\mu\psi'_f = \exp\{-\mathrm{i}Q_f e\theta\}(D_\mu\psi_f)$
- gauge invariance of field-strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

Gauge-invariant Lagrangian of QED:

$$\mathcal{L}_{\mathrm{QED}} = \overline{\psi_f}(\mathrm{i}\partial\!\!\!/ - Q_f e A\!\!\!/ - m_f)\psi_f - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Non-Abelian gauge theory (Yang–Mills theory):

Starting point:

Lagrangian $\mathcal{L}_\Phi(\Phi, \partial_\mu \Phi)$ of free or self-interacting fields with "internal symmetry":

- $\Phi = \begin{pmatrix} \phi_1 \\ \vdots \\ \phi_n \end{pmatrix}$ = multiplet of a compact Lie group G:

  $\Phi \to \Phi' = U(\theta)\Phi, \quad U(\theta) = \exp\{-\mathrm{i}gT^a\theta^a\}$ = unitary,

  T^a = group generators, $\quad [T^a, T^b] = \mathrm{i}C^{abc}T^c$, $\quad$ Tr $\{T^aT^b\} = \frac{1}{2}\delta^{ab}$

- $\mathcal{L}_\Phi$ is invariant under G: $\quad \mathcal{L}_\Phi(\Phi, \partial_\mu\Phi) = \mathcal{L}_\Phi(\Phi', \partial_\mu\Phi')$

Example: self-interacting (complex) boson multiplet

$$\mathcal{L}_\Phi = (\partial_\mu\Phi)^\dagger(\partial^\mu\Phi) - m^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2$$

(m = common boson mass, λ = coupling strength)

Gauging the symmetry by minimal substitution:

$$\mathcal{L}_\Phi(\Phi, \partial_\mu\Phi) \to \mathcal{L}_\Phi(\Phi, D_\mu\Phi) \quad \text{with} \quad D_\mu = \partial_\mu + \mathrm{i}gT^aA^a_\mu(x),$$

g = gauge coupling, T^a = generator of G in Φ representation, $A^a_\mu(x)$ = gauge fields

Transformation property of gauge fields:

- $\mathcal{L}_\Phi(\Phi, D_\mu\Phi)$ local invariant if $\quad D_\mu\Phi \;\to\; (D_\mu\Phi)' = D'_\mu\Phi' = U(\theta)(D_\mu\Phi)$

  $\Rightarrow \; T^a A'^a_\mu \;=\; U T^a A^a_\mu U^\dagger - \frac{\mathrm{i}}{g} U(\partial_\mu U^\dagger), \qquad A^a_\mu A^{a,\mu}$ = not gauge invariant

  infinitesimal form: $\quad \delta A^a_\mu = g C^{abc}\delta\theta^b A^c_\mu + \partial_\mu\delta\theta^a$

- covariant definition of field strength: $\quad [D_\mu, D_\nu] = \mathrm{i} g T^a F^a_{\mu\nu}$

  $\Rightarrow \; T^a F^a_{\mu\nu} \;\to\; T^a F'^a_{\mu\nu} = U T^a F^a_{\mu\nu} U^\dagger, \qquad F^a_{\mu\nu}F^{a,\mu\nu}$ = gauge invariant

  explicit form: $\quad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g C^{abc} A^b_\mu A^c_\nu$

Yang–Mills Lagrangian for gauge and matter fields:

$$\mathcal{L}_{\mathrm{YM}} = -\frac{1}{4} F^a_{\mu\nu}F^{a,\mu\nu} + \mathcal{L}_\Phi(\Phi, D_\mu\Phi)$$

- Lagrangian contains terms of order $(\partial A)A^2$, A^4 in F^2 part

  $\hookrightarrow$ cubic and quartic gauge-boson self-interactions

- gauge coupling determines gauge-boson–matter and gauge-boson self-interaction $\to$ unification of interactions

- mass term $M^2(A^a_\mu A^{a,\mu})$ for gauge bosons forbidden by gauge invariance

  $\hookrightarrow$ gauge bosons of unbroken Yang–Mills theory are massless

Quantum chromodynamics — gauge theory of strong interactions

- **Gauge group:** SU(3)$_c$, dim. $= 8$

  structure constants f^{abc}, gauge coupling g_s, $\alpha_\mathrm{s} = \dfrac{g_\mathrm{s}^2}{4\pi}$

- **Gauge bosons:** 8 massless gluons g with fields $A_\mu^a(x)$, $a = 1, \dots, 8$

- **Matter fermions:** quarks q (spin-$\frac{1}{2}$) with flavours $q = \mathrm{d, u, s, c, b, t}$

  in fundamental representation:

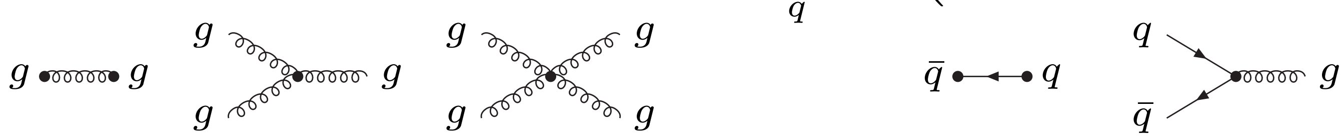
$$\psi_q(x) \equiv q(x) = \begin{pmatrix} q_r(x) \\ q_g(x) \\ q_b(x) \end{pmatrix} = \text{colour triplet}$$

$$T^a = \frac{\lambda^a}{2}, \quad \text{Gell-Mann matrices } \lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \text{ etc.}$$

- **Lagrangian:**

$$\mathcal{L}_\mathrm{QCD} = -\frac{1}{4} F^a_{\mu\nu} F^{a,\mu\nu} + \sum_q \overline{\psi}_q (\mathrm{i}\not{D} - m_q) \psi_q$$

$$= -\frac{1}{4} \left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_\mathrm{s} f^{abc} A^b_\mu A^c_\nu \right)^2 + \sum_q \overline{\psi}_q \left(\mathrm{i}\not{\partial} - g_\mathrm{s} \frac{\lambda^a}{2} \not{A}^a - m_q \right) \psi_q$$



3 The Standard Model of electroweak interaction (Glashow–Salam–Weinberg model)

3.1 The gauge group for electroweak interaction

Why unification of weak and elmg. interaction ?

- similiarity: spin-1 fields couple to matter currents formed by spin-$\frac{1}{2}$ fields
- elmg. coupling of charged $\mathrm{W}^\pm$ bosons

$\gamma, \mathrm{W}^+, \mathrm{W}^-$ as gauge bosons of group SU(2) ? – No!

Reason: charge operator Q cannot be SU(2) generator, since Tr $\{Q\} \neq 0$

for fermion doublets: $Q = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}$ for $\begin{pmatrix} \nu_\mathrm{e} \\ \mathrm{e}^- \end{pmatrix}$, etc.

Possible way out: additional heavy fermions like E^+ as partner to e^- ?
$\hookrightarrow$ no experimental confirmation !

Minimal solution: SU(2)$_{\mathrm{I}}$ $\times$ U(1)$_{\mathrm{Y}}$

- SU(2)$_{\mathrm{I}}$ $\rightarrow$ weak isospin group with gauge bosons $\mathrm{W}^+, \mathrm{W}^-, \mathrm{W}^0$
- U(1)$_{\mathrm{Y}}$ $\rightarrow$ weak hypercharge with gauge boson B

W^0 and B carry identical quantum numbers

$\hookrightarrow$ two neutral gauge bosons γ, Z as mixed states

Experiment: 1973 discovery of neutral weak currents at CERN

$\hookrightarrow$ indirect confirmation of Z exchange

1983 discovery of $\mathrm{W}^\pm$ and Z bosons at CERN

3.2 Fermion sector and minimal substitution

Multiplet structure:

Distinguish between left-/right-handed parts of fermions: $\psi^{\mathrm{L}} = \omega_- \psi$, $\psi^{\mathrm{R}} = \omega_+ \psi$

- ψ^{L} couple to $\mathrm{W}^\pm$ $\quad \to$ group ψ^{L} into SU(2)$_{\mathrm{I}}$ doublets, weak isospin $T^a_{\mathrm{I}} = \frac{\sigma^a}{2}$
- ψ^{R} do not couple to $\mathrm{W}^\pm$ $\quad \to$ ψ^{R} are SU(2)$_{\mathrm{I}}$ singlets, weak isospin $T^a_{\mathrm{I}} = 0$
- $\psi^{\mathrm{L/R}}$ couple to γ in the same way

  $\hookrightarrow$ adjust coupling to U(1)$_{\mathrm{Y}}$ (i.e. fix weak hypercharges $Y^{\mathrm{L/R}}$ for $\psi^{\mathrm{L/R}}$)
  such that elmg. coupling results: $\mathcal{L}_{\mathrm{int,QED}} = -Q_f e \overline{\psi_f} \not{A} \psi_f$

Fermion content of the SM:
(ignoring possible right-handed neutrinos)

					T^3_{I}	Q
leptons:	$\Psi^{\mathrm{L}}_L =$	$\begin{pmatrix} \nu^{\mathrm{L}}_{\mathrm{e}} \\ \mathrm{e}^{\mathrm{L}} \end{pmatrix}$,	$\begin{pmatrix} \nu^{\mathrm{L}}_{\mu} \\ \mu^{\mathrm{L}} \end{pmatrix}$,	$\begin{pmatrix} \nu^{\mathrm{L}}_{\tau} \\ \tau^{\mathrm{L}} \end{pmatrix}$,	$+\frac{1}{2}$ / $-\frac{1}{2}$	0 / -1
	$\psi^{\mathrm{R}}_l =$	e^{R},	μ^{R},	τ^{R},	0	-1
quarks: (Each quark exists in 3 colours!)	$\Psi^{\mathrm{L}}_Q =$	$\begin{pmatrix} \mathrm{u}^{\mathrm{L}} \\ \mathrm{d}^{\mathrm{L}} \end{pmatrix}$,	$\begin{pmatrix} \mathrm{c}^{\mathrm{L}} \\ \mathrm{s}^{\mathrm{L}} \end{pmatrix}$,	$\begin{pmatrix} \mathrm{t}^{\mathrm{L}} \\ \mathrm{b}^{\mathrm{L}} \end{pmatrix}$,	$+\frac{1}{2}$ / $-\frac{1}{2}$	$+\frac{2}{3}$ / $-\frac{1}{3}$
	$\psi^{\mathrm{R}}_u =$	u^{R},	c^{R},	t^{R},	0	$+\frac{2}{3}$
	$\psi^{\mathrm{R}}_d =$	d^{R},	s^{R},	b^{R},	0	$-\frac{1}{3}$

Free Lagrangian of (still massless) fermions:

$$\mathcal{L}_{0,\text{ferm}} = \mathrm{i}\overline{\psi_f}\not{\partial}\psi_f = \mathrm{i}\overline{\Psi_L^{\mathrm{L}}}\not{\partial}\Psi_L^{\mathrm{L}} + \mathrm{i}\overline{\Psi_Q^{\mathrm{L}}}\not{\partial}\Psi_Q^{\mathrm{L}} + \mathrm{i}\overline{\psi_l^{\mathrm{R}}}\not{\partial}\psi_l^{\mathrm{R}} + \mathrm{i}\overline{\psi_u^{\mathrm{R}}}\not{\partial}\psi_u^{\mathrm{R}} + \mathrm{i}\overline{\psi_d^{\mathrm{R}}}\not{\partial}\psi_d^{\mathrm{R}}$$

Minimal substitution:

$$\partial_\mu \to D_\mu = \partial_\mu - \mathrm{i}g_2 T_{\mathrm{I}}^a W_\mu^a + \mathrm{i}g_1 \tfrac{1}{2} Y B_\mu = D_\mu^{\mathrm{L}}\omega_- + D_\mu^{\mathrm{R}}\omega_+,$$

$$D_\mu^{\mathrm{L}} = \partial_\mu - \frac{\mathrm{i}g_2}{\sqrt{2}}\begin{pmatrix} 0 & W_\mu^+ \\ W_\mu^- & 0 \end{pmatrix} - \frac{\mathrm{i}}{2}\begin{pmatrix} g_2 W_\mu^3 - g_1 Y^{\mathrm{L}} B_\mu & 0 \\ 0 & -g_2 W_\mu^3 - g_1 Y^{\mathrm{L}} B_\mu \end{pmatrix},$$

$$D_\mu^{\mathrm{R}} = \partial_\mu + \mathrm{i}g_1 \tfrac{1}{2} Y^{\mathrm{R}} B_\mu$$

Photon identification:

"Weinberg rotation":

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c_{\mathrm{W}} & s_{\mathrm{W}} \\ -s_{\mathrm{W}} & c_{\mathrm{W}} \end{pmatrix}\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix},$$

$c_{\mathrm{W}} = \cos\theta_{\mathrm{W}}$, $s_{\mathrm{W}} = \sin\theta_{\mathrm{W}}$,
$\theta_{\mathrm{W}} =$ weak mixing angle

$$D_\mu^{\mathrm{L}}\Big|_{A_\mu} = -\frac{\mathrm{i}}{2} A_\mu \begin{pmatrix} -g_2 s_{\mathrm{W}} - g_1 c_{\mathrm{W}} Y^{\mathrm{L}} & 0 \\ 0 & g_2 s_{\mathrm{W}} - g_1 c_{\mathrm{W}} Y^{\mathrm{L}} \end{pmatrix} \stackrel{!}{=} \mathrm{i}eA_\mu \begin{pmatrix} Q_1 & 0 \\ 0 & Q_2 \end{pmatrix}$$

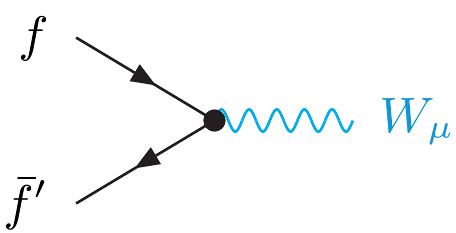
- charged difference in doublet $Q_1 - Q_2 = 1 \quad \to \quad g_2 = \dfrac{e}{s_{\mathrm{W}}}$
- normalize $Y^{\mathrm{L/R}}$ such that $g_1 = \dfrac{e}{c_{\mathrm{W}}}$

$\hookrightarrow$ Y fixed by "Gell-Mann–Nishijima relation": $Q = T_{\mathrm{I}}^3 + \dfrac{Y}{2}$

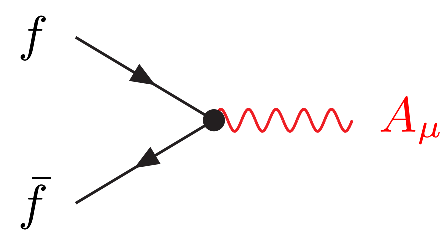
Fermion–gauge-boson interaction:

$$\mathcal{L}_{\mathrm{ferm,YM}} = \frac{e}{\sqrt{2}s_{\mathrm{W}}}\overline{\Psi_F^{\mathrm{L}}}\begin{pmatrix} 0 & \not{W}^+ \\ \not{W}^- & 0 \end{pmatrix}\Psi_F^{\mathrm{L}} + \frac{e}{2c_{\mathrm{W}}s_{\mathrm{W}}}\overline{\Psi_F^{\mathrm{L}}}\sigma^3\not{Z}\Psi_F^{\mathrm{L}}$$
$$- e\frac{s_{\mathrm{W}}}{c_{\mathrm{W}}}Q_f\overline{\psi_f}\not{Z}\psi_f - eQ_f\overline{\psi_f}\not{A}\psi_f \qquad (f\text{=all fermions, } F\text{= all doublets})$$

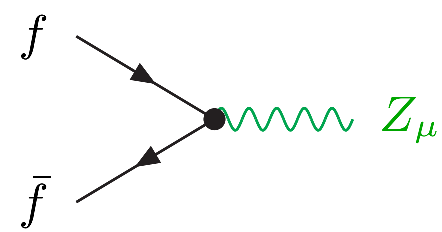
Feynman rules:



$$\frac{\mathrm{i}e}{\sqrt{2}s_{\mathrm{W}}}\gamma_\mu\omega_-$$



$$-\mathrm{i}Q_f e\gamma_\mu$$



$$\mathrm{i}e\gamma_\mu(g_f^+\omega_+ + g_f^-\omega_-) = \mathrm{i}e\gamma_\mu(v_f - a_f\gamma_5)$$

with $g_f^+ = -\frac{s_{\mathrm{W}}}{c_{\mathrm{W}}}Q_f, \quad g_f^- = -\frac{s_{\mathrm{W}}}{c_{\mathrm{W}}}Q_f + \frac{T^3_{\mathrm{I},f}}{c_{\mathrm{W}}s_{\mathrm{W}}},$

$$v_f = -\frac{s_{\mathrm{W}}}{c_{\mathrm{W}}}Q_f + \frac{T^3_{\mathrm{I},f}}{2c_{\mathrm{W}}s_{\mathrm{W}}}, \quad a_f = \frac{T^3_{\mathrm{I},f}}{2c_{\mathrm{W}}s_{\mathrm{W}}}$$

3.3 Gauge-boson sector

Yang–Mills Lagrangian for gauge fields:

$$\mathcal{L}_{\mathrm{YM}} = -\frac{1}{4} W^a_{\mu\nu} W^{a,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

Field-strength tensors:

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g_2 \epsilon^{abc} W^b_\mu W^c_\nu, \qquad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

Lagrangian in terms of “physical” fields:

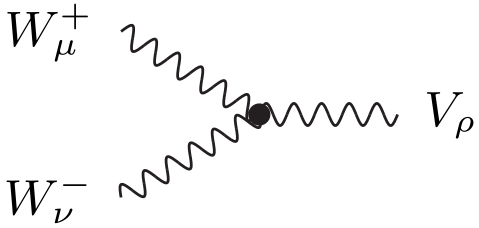
$$\begin{aligned}
\mathcal{L}_{\mathrm{YM}} = & -\frac{1}{2}(\partial_\mu W^+_\nu - \partial_\nu W^+_\mu)(\partial^\mu W^{-,\nu} - \partial^\nu W^{-,\mu}) \\
& - \frac{1}{4}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)(\partial^\mu Z^\nu - \partial^\nu Z^\mu) - \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu)
\end{aligned}$$

$+$ (trilinear interaction terms involving AW^+W^-, ZW^+W^-)

$+$ (quadrilinear interaction terms involving AAW^+W^-, AZW^+W^-, ZZW^+W^-, $W^+W^-W^+W^-$)

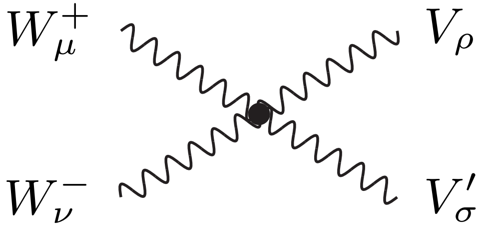
Feynman rules for gauge-boson self-interactions:

(fields and momenta incoming)



$$\mathrm{i}eC_{WWV}\Big[g_{\mu\nu}(k_+ - k_-)_\rho + g_{\nu\rho}(k_- - k_V)_\mu + g_{\rho\mu}(k_V - k_+)_\nu\Big]$$

with $C_{WW\gamma} = 1, \quad C_{WWZ} = -\frac{c_\mathrm{W}}{s_\mathrm{W}}$



$$\mathrm{i}e^2C_{WWVV'}\Big[2g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\sigma\nu} - g_{\mu\sigma}g_{\nu\rho}\Big]$$

with $C_{WW\gamma\gamma} = -1, \qquad C_{WW\gamma Z} = \frac{c_\mathrm{W}}{s_\mathrm{W}},$

$$C_{WWZZ} = -\frac{c^2_\mathrm{W}}{s^2_\mathrm{W}}, \quad C_{WWWW} = \frac{1}{s^2_\mathrm{W}}$$

3.4 Higgs sector and spontaneous symmetry breaking

Idea: spontaneous breakdown of $\mathrm{SU(2)_I}\times\mathrm{U(1)_Y}$ symmetry $\to$ $\mathrm{U(1)_{elmg}}$ symmetry

$\hookrightarrow$ masses for $\mathrm{W}^{\pm}$ and Z bosons, but γ remains massless

Note: choice of scalar extension of massless model involves freedom

GSW model:

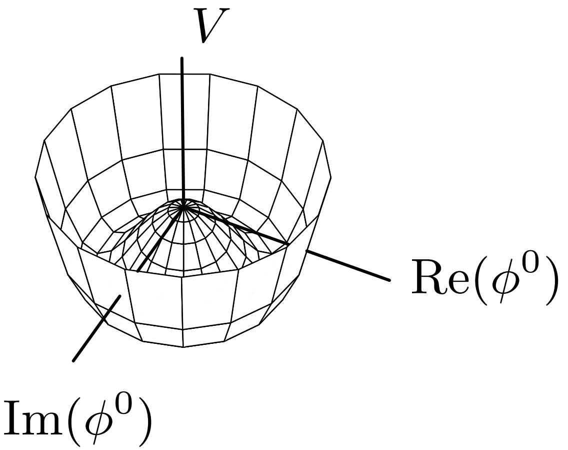
Minimal scalar sector with complex scalar doublet $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$, $Y_\Phi = 1$

Scalar self-interaction via Higgs potential:

$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \frac{\lambda}{4}(\Phi^\dagger \Phi)^2, \quad \mu^2, \lambda > 0,$$

$$= \mathrm{SU(2)_I}\times\mathrm{U(1)_Y} \text{ symmetric}$$

$$V(\Phi) = \text{minimal for} \quad |\Phi| = \sqrt{\frac{2\mu^2}{\lambda}} \equiv \frac{v}{\sqrt{2}} > 0$$



ground state Φ_0 (=vacuum expectation value of Φ) not unique

specific choice $\Phi_0 = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$ not gauge invariant $\Rightarrow$ spontaneous symmetry breaking

elmg. gauge invariance unbroken, since $\quad Q\Phi_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \Phi_0 = 0$

Field excitations in Φ:

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}\Big(v + H(x) + \mathrm{i}\chi(x)\Big) \end{pmatrix}$$

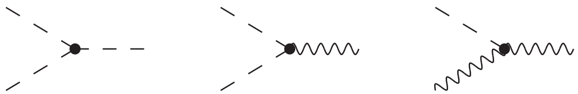
Gauge-invariant Lagrangian of Higgs sector: $(\phi^- = (\phi^+)^\dagger)$

$$\mathcal{L}_\mathrm{H} = (D_\mu\Phi)^\dagger(D^\mu\Phi) - V(\Phi) \qquad \text{with } D_\mu = \partial_\mu - \mathrm{i}g_2\frac{\sigma^a}{2}W^a_\mu + \mathrm{i}\frac{g_1}{2}B_\mu$$

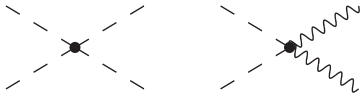
$$= (\partial_\mu\phi^+)(\partial^\mu\phi^-) - \frac{\mathrm{i}ev}{2s_\mathrm{W}}(W^+_\mu\partial^\mu\phi^- - W^-_\mu\partial^\mu\phi^+) + \frac{e^2v^2}{4s^2_\mathrm{W}}W^+_\mu W^{-,\mu}$$

$$+\frac{1}{2}(\partial\chi)^2 + \frac{ev}{2c_\mathrm{W}s_\mathrm{W}}Z_\mu\partial^\mu\chi + \frac{e^2v^2}{4c^2_\mathrm{W}s^2_\mathrm{W}}Z^2 + \frac{1}{2}(\partial H)^2 - \mu^2H^2$$

$+$ (trilinear SSS, SSV, SVV interactions)



$+$ (quadrilinear $SSSS$, $SSVV$ interactions)



Implications:

- gauge-boson masses: $M_\mathrm{W} = \frac{ev}{2s_\mathrm{W}}$, $M_\mathrm{Z} = \frac{ev}{2c_\mathrm{W}s_\mathrm{W}} = \frac{M_\mathrm{W}}{c_\mathrm{W}}$, $M_\gamma = 0$
- physical Higgs boson H: $M_\mathrm{H} = \sqrt{2\mu^2}$ = free parameter
- would-be Goldstone bosons $\phi^\pm$, χ: unphysical degrees of freedom

3.5 Fermion masses and Yukawa couplings

Ordinary Dirac mass terms $m_f \overline{\psi_f} \psi_f = m_f (\overline{\psi_f^{\mathrm{L}}} \psi_f^{\mathrm{R}} + \overline{\psi_f^{\mathrm{R}}} \psi_f^{\mathrm{L}})$ not gauge invariant

$\hookrightarrow$ introduce fermion masses by (gauge-invariant) Yukawa interaction

Lagrangian for Yukawa couplings:

$$\mathcal{L}_{\mathrm{Yuk}} = -\overline{\Psi_L^{\mathrm{L}}} G_l \psi_l^{\mathrm{R}} \Phi - \overline{\Psi_Q^{\mathrm{L}}} G_u \psi_u^{\mathrm{R}} \tilde{\Phi} - \overline{\Psi_Q^{\mathrm{L}}} G_d \psi_d^{\mathrm{R}} \Phi + \text{h.c.}$$

- G_l, G_u, G_d = 3×3 matrices in 3-dim. space of generations (ν masses ignored)
- $\tilde{\Phi} = i\sigma^2 \Phi^* = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix}$ = charge conjugate Higgs doublet, $Y_{\tilde{\Phi}} = -1$

Fermion mass terms:

mass terms = bilinear terms in $\mathcal{L}_{\mathrm{Yuk}}$, obtained by setting $\Phi \to \Phi_0$:

$$\mathcal{L}_{m_f} = -\frac{v}{\sqrt{2}} \overline{\psi_l^{\mathrm{L}}} G_l \psi_l^{\mathrm{R}} - \frac{v}{\sqrt{2}} \overline{\psi_u^{\mathrm{L}}} G_u \psi_u^{\mathrm{R}} - \frac{v}{\sqrt{2}} \overline{\psi_d^{\mathrm{L}}} G_d \psi_d^{\mathrm{R}} + \text{h.c.}$$

$\hookrightarrow$ diagonalization by unitary field transformations ($f = l, u, d$)

$\hat{\psi}_f^{\mathrm{L/R}} \equiv U_f^{\mathrm{L/R}} \psi_f^{\mathrm{L/R}}$ such that $\frac{v}{\sqrt{2}} U_f^{\mathrm{L}} G_f (U_f^{\mathrm{R}})^\dagger = \mathrm{diag}(m_f)$

$\Rightarrow$ standard form: $\mathcal{L}_{m_f} = -m_f \overline{\hat{\psi}_f^{\mathrm{L}}} \hat{\psi}_f^{\mathrm{R}} + \text{h.c.} = -m_f \overline{\hat{\psi}_f} \hat{\psi}_f$

Quark mixing:

- ψ_f correspond to eigenstates of the gauge interaction
- $\hat{\psi}_f$ correspond to mass eigenstates,
  for massless neutrinos define $\quad \hat{\psi}^{\mathrm{L}}_\nu \equiv U^{\mathrm{L}}_l \psi^{\mathrm{L}}_\nu \quad \rightarrow$ no lepton-flavour changing

Yukawa and gauge interactions in terms of mass eigenstates:

$$\begin{aligned}
\mathcal{L}_{\mathrm{Yuk}} = &-\frac{\sqrt{2}m_l}{v}\left(\phi^+\overline{\hat{\psi}^{\mathrm{L}}_{\nu_l}}\hat{\psi}^{\mathrm{R}}_l + \phi^-\overline{\hat{\psi}^{\mathrm{R}}_l}\hat{\psi}^{\mathrm{L}}_{\nu_l}\right) + \frac{\sqrt{2}m_u}{v}\left(\phi^+\overline{\hat{\psi}^{\mathrm{R}}_u}V\hat{\psi}^{\mathrm{L}}_d + \phi^-\overline{\hat{\psi}^{\mathrm{L}}_d}V^\dagger\hat{\psi}^{\mathrm{R}}_u\right) \\
&-\frac{\sqrt{2}m_d}{v}\left(\phi^+\overline{\hat{\psi}^{\mathrm{L}}_u}V\hat{\psi}^{\mathrm{R}}_d + \phi^-\overline{\hat{\psi}^{\mathrm{R}}_d}V^\dagger\hat{\psi}^{\mathrm{L}}_u\right) - \frac{m_f}{v}\mathrm{i}\,\mathrm{sgn}(T^3_{\mathrm{I},f})\chi\,\overline{\hat{\psi}_f}\gamma_5\hat{\psi}_f \\
&-\frac{m_f}{v}(v+H)\,\overline{\hat{\psi}_f}\hat{\psi}_f,
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{\mathrm{ferm,YM}} = &\frac{e}{\sqrt{2}s_{\mathrm{W}}}\overline{\hat{\Psi}^{\mathrm{L}}_L}\begin{pmatrix} 0 & W\!\!\!/^+ \\ W\!\!\!/^- & 0\end{pmatrix}\hat{\psi}^{\mathrm{L}}_L + \frac{e}{\sqrt{2}s_{\mathrm{W}}}\overline{\hat{\Psi}^{\mathrm{L}}_Q}\begin{pmatrix} 0 & VW\!\!\!/^+ \\ V^\dagger W\!\!\!/^- & 0\end{pmatrix}\hat{\psi}^{\mathrm{L}}_Q \\
&+\frac{e}{2c_{\mathrm{W}}s_{\mathrm{W}}}\overline{\hat{\Psi}^{\mathrm{L}}_F}\sigma^3 Z\!\!\!/\hat{\Psi}^{\mathrm{L}}_F - e\frac{s_{\mathrm{W}}}{c_{\mathrm{W}}}Q_f\overline{\hat{\psi}_f}Z\!\!\!/\hat{\psi}_f - eQ_f\overline{\hat{\psi}_f}A\!\!\!/\hat{\psi}_f
\end{aligned}$$

- only charged-current coupling of quarks modified by $V = U^{\mathrm{L}}_u(U^{\mathrm{L}}_d)^\dagger$ = unitary
  (Cabibbo–Kobayashi–Maskawa (CKM) matrix)
- Higgs–fermion coupling strength = $\frac{m_f}{v}$

Features of the CKM mixing:

- V = 3-dim. generalization of Cabibbo matrix U_{C}
- V is parametrized by 4 free parameters: 3 real angles, 1 complex phase
  $\hookrightarrow$ complex phase is the only source of CP violation in SM

  counting:

  $$\begin{pmatrix}\text{\#real d.o.f.}\\ \text{in } V\end{pmatrix} - \begin{pmatrix}\text{\#unitarity}\\ \text{relations}\end{pmatrix} - \begin{pmatrix}\text{\#phase diffs. of}\\ u\text{-type quarks}\end{pmatrix} - \begin{pmatrix}\text{\#phase diffs. of}\\ d\text{-type quarks}\end{pmatrix} - \begin{pmatrix}\text{\#phase diff. between}\\ u\text{- and } d\text{-type quarks}\end{pmatrix}$$

  $$= 18 - 9 - 2 - 2 - 1 = 4$$

- no flavour-changing neutral currents in lowest order,
  flavour-changing suppressed by factors $G_\mu(m_{q_1}^2 - m_{q_2}^2)$ in higher orders
  ("Glashow–Iliopoulos–Maiani mechanism")

3.6 Quantization — gauge fixing and Faddeev–Popov sector

Gauge fields contain unphysical degrees of freedom that must not be quantized.

Consequences:

- gauge-boson propagators ill-defined without gauge fixing,
  e.g. for photon the (singular) operator $(g_{\mu\nu}\Box - \partial_\mu\partial_\nu)$ would have to be inverted
- in path integral $\int \mathcal{D}A^a_\mu \exp\{\mathrm{i}\int \mathrm{d}x\,\mathcal{L}\}$:
  only one representative of each gauge orbit should contribute,
  otherwise integral over gauge-equivalent fields diverges
  $\hookrightarrow$ fix gauge by δ-functions $\delta(F^a[A^a_\mu] - C^a)$ in path integral (C^a = const.)
  $\hookrightarrow$ by averaging over C^a, gauge fixing can be cast in terms of a
  gauge-fixing Lagrangian $\mathcal{L}_{\mathrm{gf}}$

Gauge-fixing Lagrangian of general R_ξ gauge:

$$\mathcal{L}_{\mathrm{gf}} = -\frac{1}{\xi_{\mathrm{W}}}F^+F^- - \frac{1}{2\xi_{\mathrm{Z}}}(F^{\mathrm{Z}})^2 - \frac{1}{2\xi_\gamma}(F^\gamma)^2$$

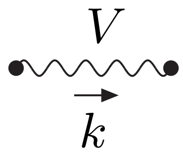
with the gauge-fixing functionals F^a: (ξ_V = arbitrary gauge-fixing parameters)

$$F^\pm = \partial W^\pm \mp \mathrm{i}\xi_{\mathrm{W}} M_{\mathrm{W}}\phi^\pm, \qquad F^{\mathrm{Z}} = \partial Z - \xi_{\mathrm{Z}} M_{\mathrm{Z}}\chi, \qquad F^\gamma = \partial A$$

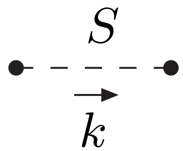
Features of the R_ξ gauge fixing:

- elimination of mixing terms $(W^{\pm}_{\mu}\partial^{\mu}\phi^{\mp})$, $(Z_\mu\partial^\mu\chi)$ in Lagrangian
  $\hookrightarrow$ decoupling of gauge and would-be Goldstone fields (no mix propagators)

- boson propagators:



$$G^{VV}_{\mu\nu}(k) = -\mathrm{i}\left[\frac{g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}}{k^2-M_V^2}+\frac{k_\mu k_\nu}{k^2}\frac{\xi_V}{k^2-\xi_V M_V^2}\right], \qquad V=W,Z,\gamma$$



$$G^{SS}(k) = \frac{\mathrm{i}}{k^2-\xi_V M_V^2}, \qquad S=\phi,\chi$$

- important special cases:
  - $\xi_V=1$: 't Hooft–Feynman gauge
    $\hookrightarrow$ convenient gauge-boson propagators $\frac{-\mathrm{i}g_{\mu\nu}}{k^2-M_V^2}$
  - $\xi_\mathrm{W},\xi_\mathrm{Z}\to\infty$: "unitary gauge"
    $\hookrightarrow$ elimination of would-be Goldstone bosons

Faddeev–Popov ghosts

Consistent use of gauge fixing in path integral: Faddeev–Popov ansatz

$$\int \mathcal{D}A_\mu^a \; \exp\left\{ \mathrm{i} \int \mathrm{d}x \, \mathcal{L} \right\} \; \delta(F^a[A_\mu^a] - C^a) \; \det\left(\frac{\delta F^a}{\delta \theta^b} \right)$$

with the gauge variation of the functionals F^a:

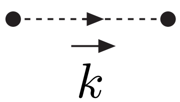
$$\left(\frac{\delta F^a(x)}{\delta \theta^b(y)} \right) = M^{ab}(x)\, \delta(x-y),$$

$$M^{VV'}(x) = \delta^{VV'}(\Box_x + \xi_V M_V^2) + \text{terms linear in vector and scalar fields}$$

Functional determinant can be written as path integral over
Grassmann-valued auxiliary fields $u^a(x), \bar{u}^a(x)$: (Faddeev–Popov ghost fields)

$$\det\left(\frac{\delta F^a}{\delta \theta^b} \right) \propto \int \mathcal{D}u^a \int \mathcal{D}\bar{u}^a \; \exp\left\{ \mathrm{i} \int \mathrm{d}x \, \mathcal{L}_{\mathrm{FP}} \right\}$$

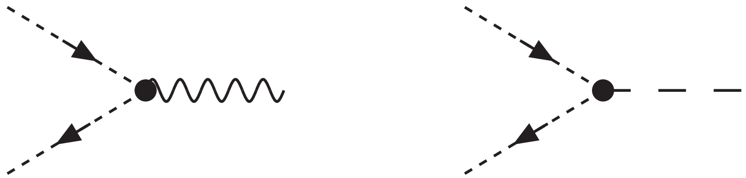
$$\mathcal{L}_{\mathrm{FP}}(x) = -\bar{u}^a(x) M^{ab}(x) u^b(x) \; = \; -\bar{u}^V(\Box + \xi_V M_V^2) u^V + \dots$$

$\hookrightarrow$ ghost propagators: u^V 

$\bar{u}^V$ $\qquad \dfrac{\mathrm{i}}{k^2 - \xi_V M_V^2}$

Features of the Faddeev–Popov ghost fields:

- ghosts do not correspond to physical states
  (ghost propagators have poles at unphysical mass values $\xi_V M_V^2$)
  $\hookrightarrow$ appear only inside loops in diagrams for physical processes
- ghost fields have spin 0, but are anti-commuting
  (would violate spin-statistics theorem as physical states)
  $\hookrightarrow$ signs as for fermions in Feynman rules
- ghost fields couple to gauge and scalar fields (not to fermions):



4 Electroweak phenomenology

4.1 Brief overview

Features of the electroweak Standard Model

- Higgs boson not yet found, particle content verified otherwise
- No really significant contradictions of GSW model with experiment
- Input parameters:

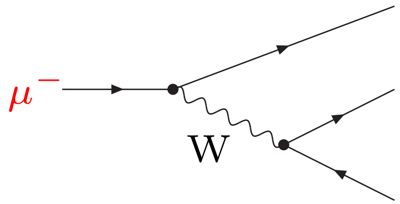
$$\alpha = \frac{e^2}{4\pi} \approx 1/137, \ \ M_\mathrm{W} \approx 80\,\mathrm{GeV}, \ \ M_\mathrm{Z} \approx 91\,\mathrm{GeV}, \ \ M_\mathrm{H} \gtrsim 100\,\mathrm{GeV}, \ \ m_f, \ \ V$$

- GSW model = consistent quantum field theory
  - ◇ matrix elements respect unitarity
  - ◇ renormalizability

$\Rightarrow$ evaluation of higher perturbative orders possible
(and phenomenologically necessary !)

Important electroweak experiments

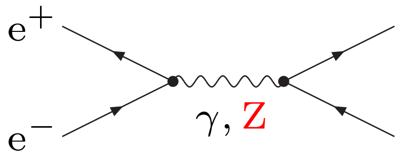
- Muon decay: $\mu^- \to \nu_\mu e^- \bar{\nu}_e$



determination of the Fermi constant

$$G_\mu = \frac{\pi \alpha M_Z^2}{\sqrt{2} M_W^2 (M_Z^2 - M_W^2)} + \ldots$$

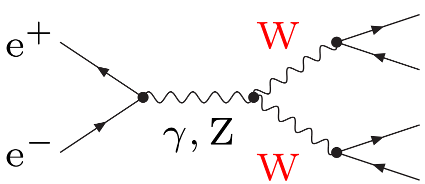
- Z production (LEP1/SLC): $e^+e^- \to Z \to f\bar{f}$

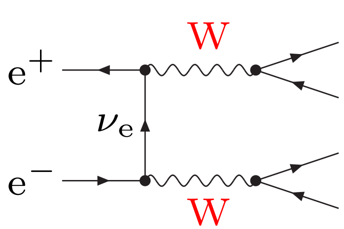


various precision measurements at the Z resonance: $M_Z, \Gamma_Z, \sigma_{had}, A_{FB}, A_{LR}$, etc.

$\Rightarrow$ good knowledge of the $Zf\bar{f}$ sector

- W-pair production (LEP2/ILC): $e^+e^- \to WW \to 4f(+\gamma)$





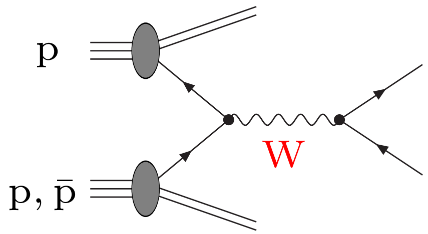
– measurement of M_W

– $\gamma WW/ZWW$ couplings

– quartic couplings: $\gamma\gamma WW$, γZWW

Important electroweak experiments (continued)

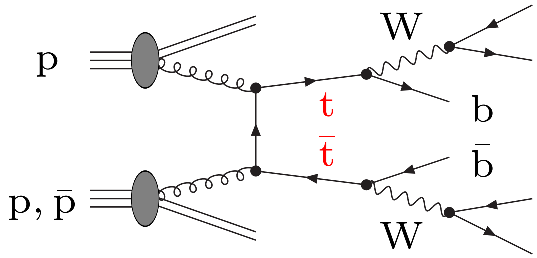
- W production (Tevatron/LHC): $\mathrm{pp}, \mathrm{p\bar{p}} \to \mathrm{W} \to l\nu_l(+\gamma)$



  – measurement of M_W

  – bounds on $\gamma\mathrm{WW}$ coupling

- top-quark production (Tevatron/LHC): $\mathrm{pp}, \mathrm{p\bar{p}} \to \mathrm{t\bar{t}} \to 6f$



  – measurement of m_t

Theoretical predictions

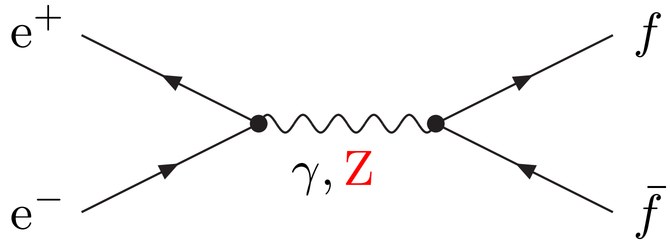
parametrized by $\alpha(M_\mathrm{Z})$, M_W, M_Z, m_t, m_f, $\alpha_\mathrm{s}(M_\mathrm{Z})$ and M_H

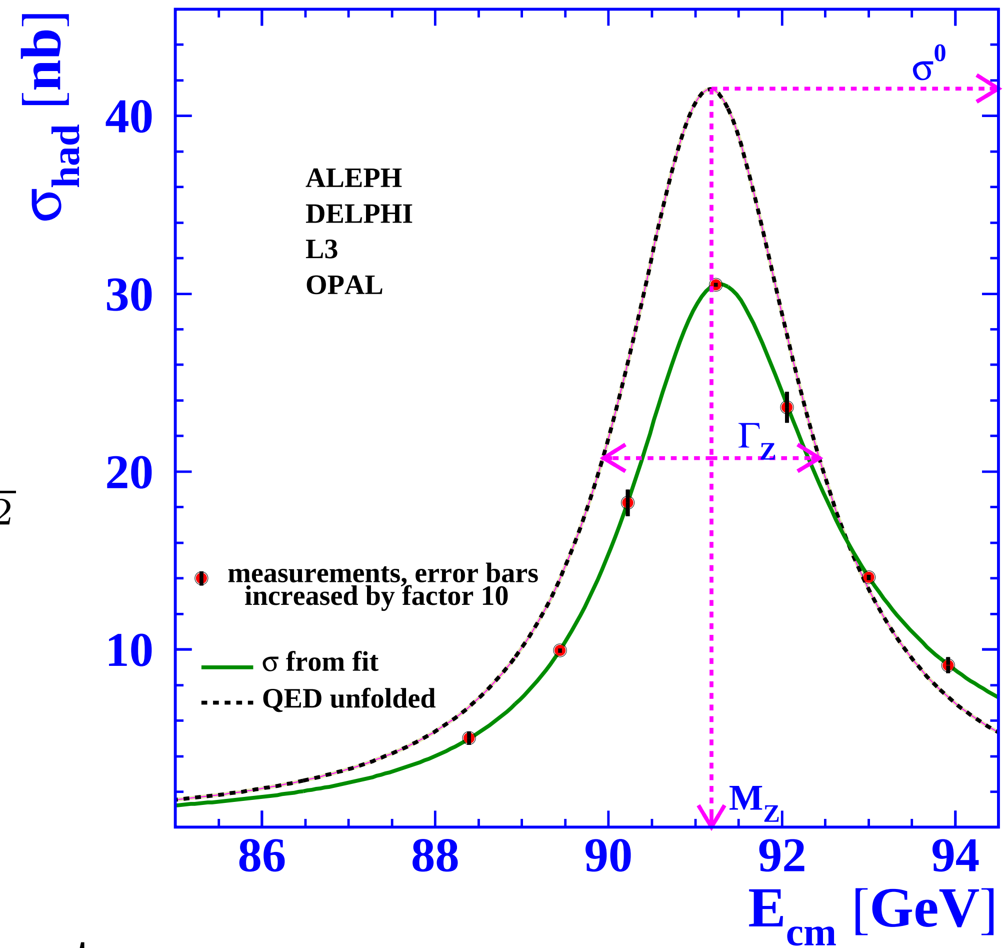
$\hookrightarrow$ global fit of SM to data yields bounds on M_H

But: high precision necessary,
since M_H sensitivity weak $\sim \frac{\alpha}{\pi}\log(M_\mathrm{H}/M_\mathrm{W})$

4.2 Z-boson physics at LEP1 and SLC

Precision study of the Z line shape





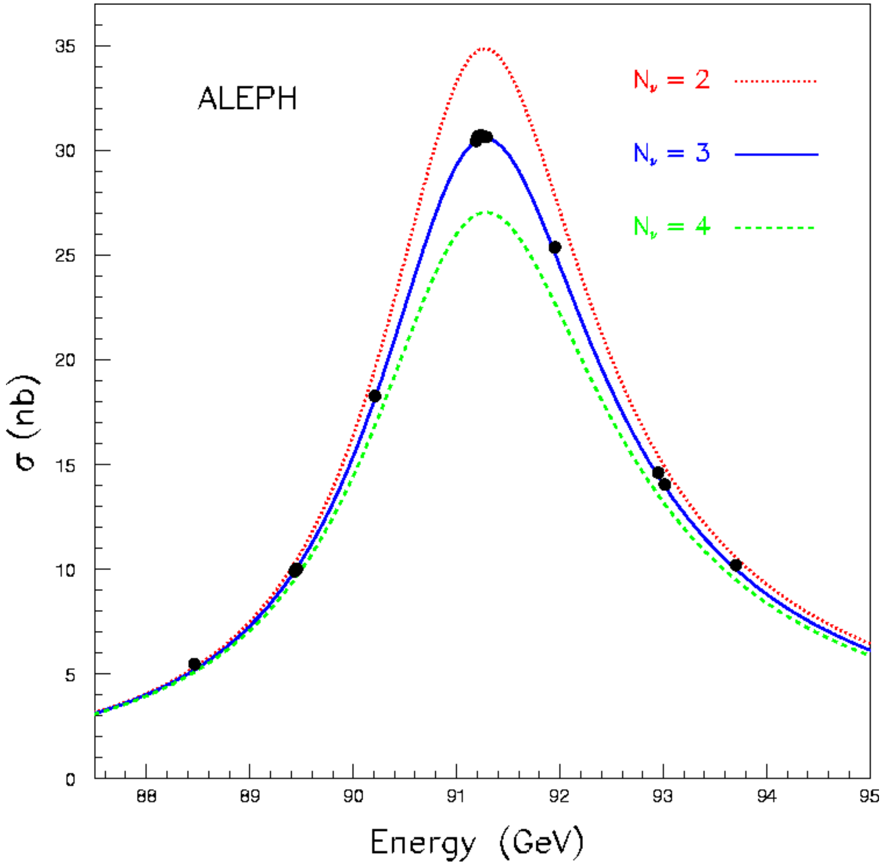
Unfolded resonance:

$$\sigma_{\mathrm{res}}(s) = \sigma^0 \, \frac{s\,\Gamma_{\mathrm{Z}}^2}{\left| s - M_{\mathrm{Z}}^2 + \mathrm{i} M_{\mathrm{Z}} \Gamma_{\mathrm{Z}} \frac{s}{M_{\mathrm{Z}}^2} \right|^2}$$

Resonance observables:

- Z mass and width: $M_{\mathrm{Z}}, \Gamma_{\mathrm{Z}}$
- peak cross section: σ^0_{had}
- various asymmetries: $A_{\mathrm{FB}}, A_{\mathrm{LR}}$, etc.
- ratios of decay widths: $R_l = \frac{\Gamma_{\mathrm{had}}}{\Gamma_l}$, etc.

Number of light neutrinos



$$\Gamma_Z = \Gamma_{had} + \Gamma_e + \Gamma_\mu + \Gamma_\tau + \Gamma_{inv}$$

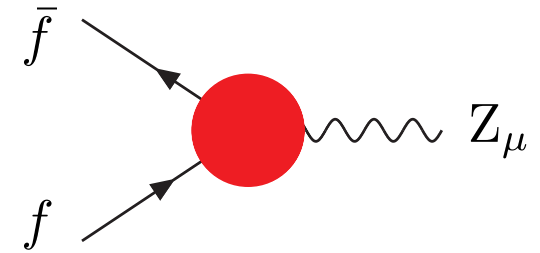
- Γ_Z measured from Z line shape
- Γ_{had} and $\Gamma_{l=e,\mu,\tau}$ from

$$R_l = \frac{\Gamma_{had}}{\Gamma_l} \quad \text{and} \quad \sigma^0_{had} = \frac{12\pi}{M_Z^2} \frac{\Gamma_e \Gamma_{had}}{\Gamma_Z^2}$$

Fit of Γ_Z, R_l, and σ^0_{had} yields invisible Z-decay width: $\quad \Gamma_{inv} = N_\nu \, \Gamma^{theory}_{Z\to\nu\bar{\nu}}$

$$\hookrightarrow \quad N_\nu = 2.9840 \pm 0.0082$$

Effective Z-boson–fermion couplings



$$= \mathrm{i}e\gamma_\mu(g_{Vf} - g_{Af}\gamma_5)$$

Leptonic couplings from LEP1 asymmetry measurements, e.g.:

$$A_{\mathrm{FB}}^{0,f} = \frac{\sigma_{f,\mathrm{F}}^0 - \sigma_{f,\mathrm{B}}^0}{\sigma_{f,\mathrm{F}}^0 + \sigma_{f,\mathrm{B}}^0} = \frac{3}{4}\mathcal{A}_{\mathrm{e}}\mathcal{A}_f$$

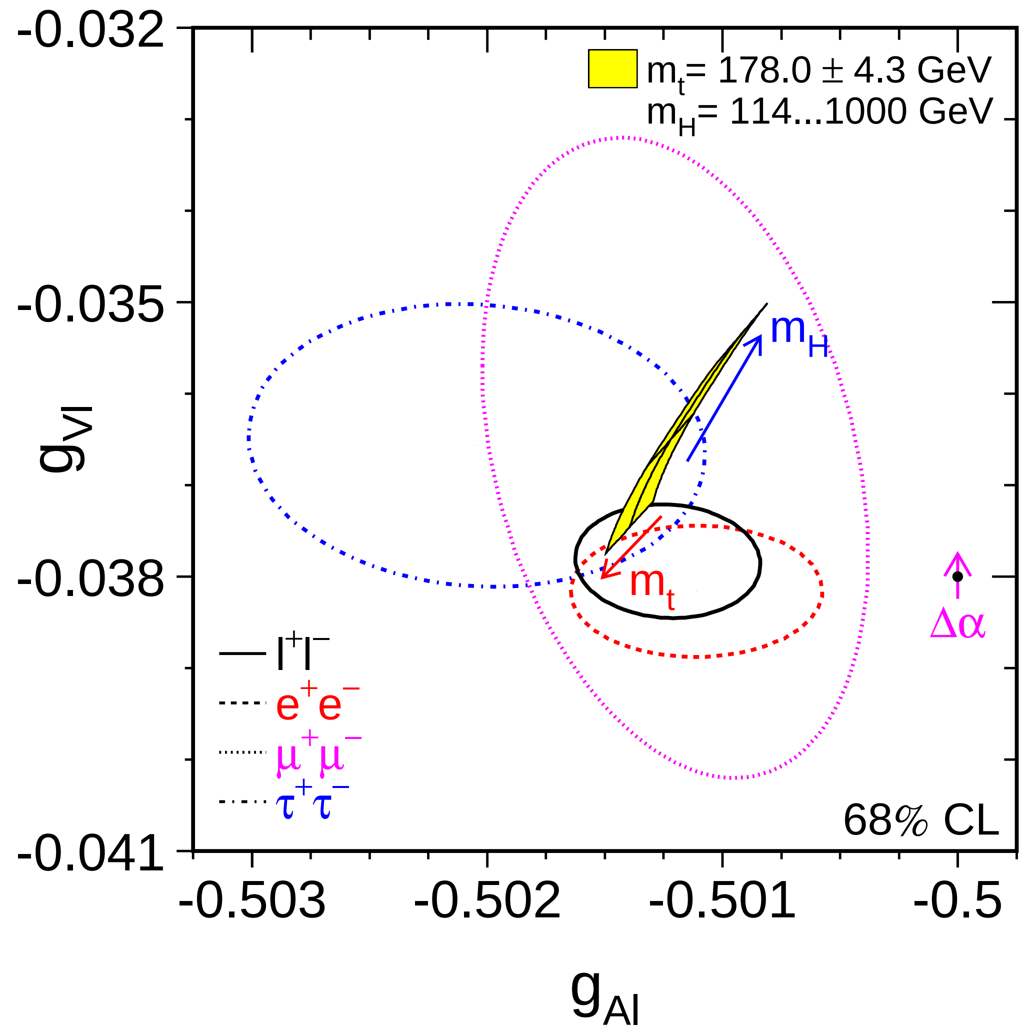
$(\mathrm{F/B} =$ For/Backward hemisphere)

with $\mathcal{A}_f = \dfrac{2g_{Vf}g_{Af}}{g_{Vf}^2 + g_{Af}^2}$

Good agreement with SM

- lepton universality confirmed
- constraints on m_t and M_H

LEPEWWG '05



Translation of effective couplings into effective weak mixing angle

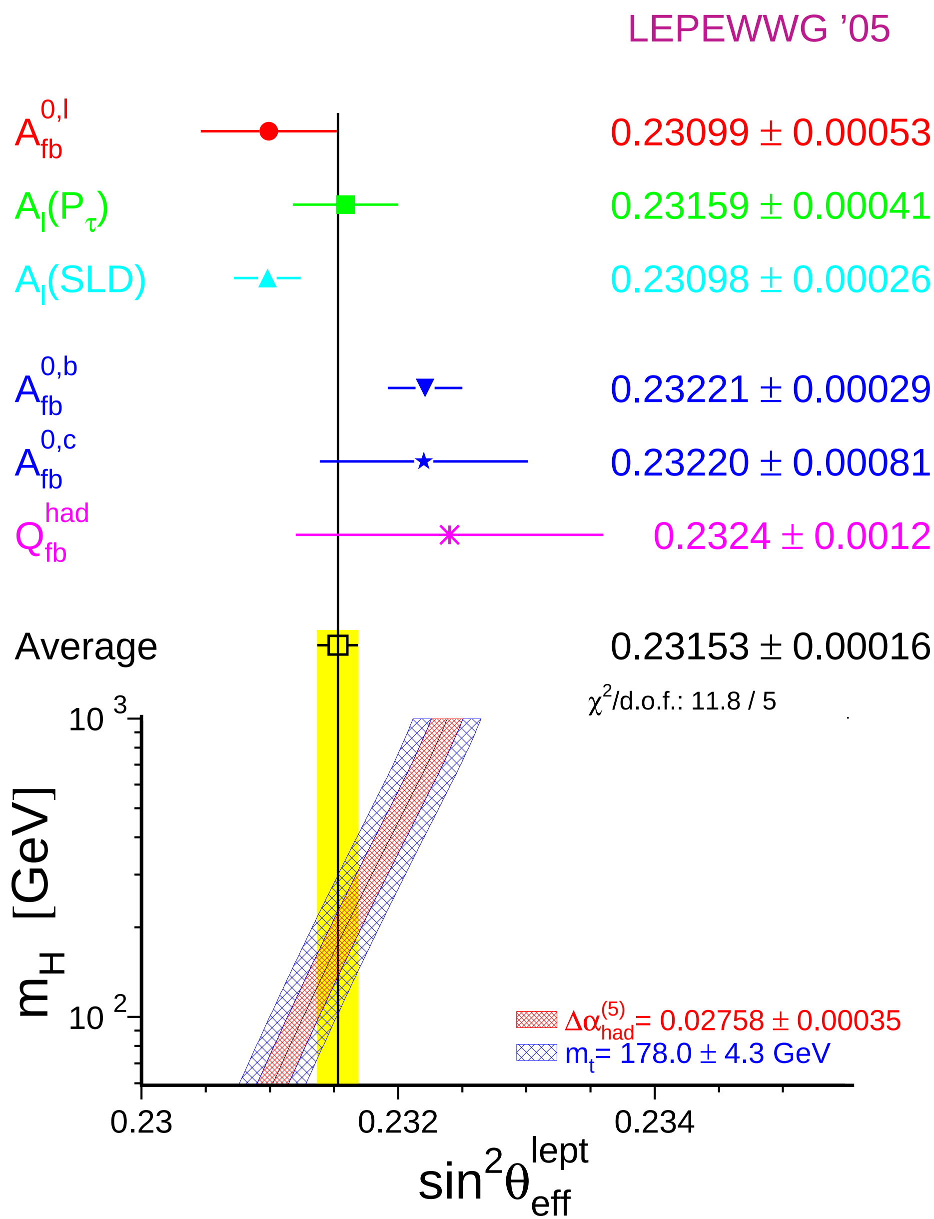
$$\sin^2\theta_{\rm eff}^{\rm lept} = \frac{1}{4}\left(1 - {\rm Re}\left\{\frac{g_{Vl}}{g_{Al}}\right\}\right)$$

Important features:

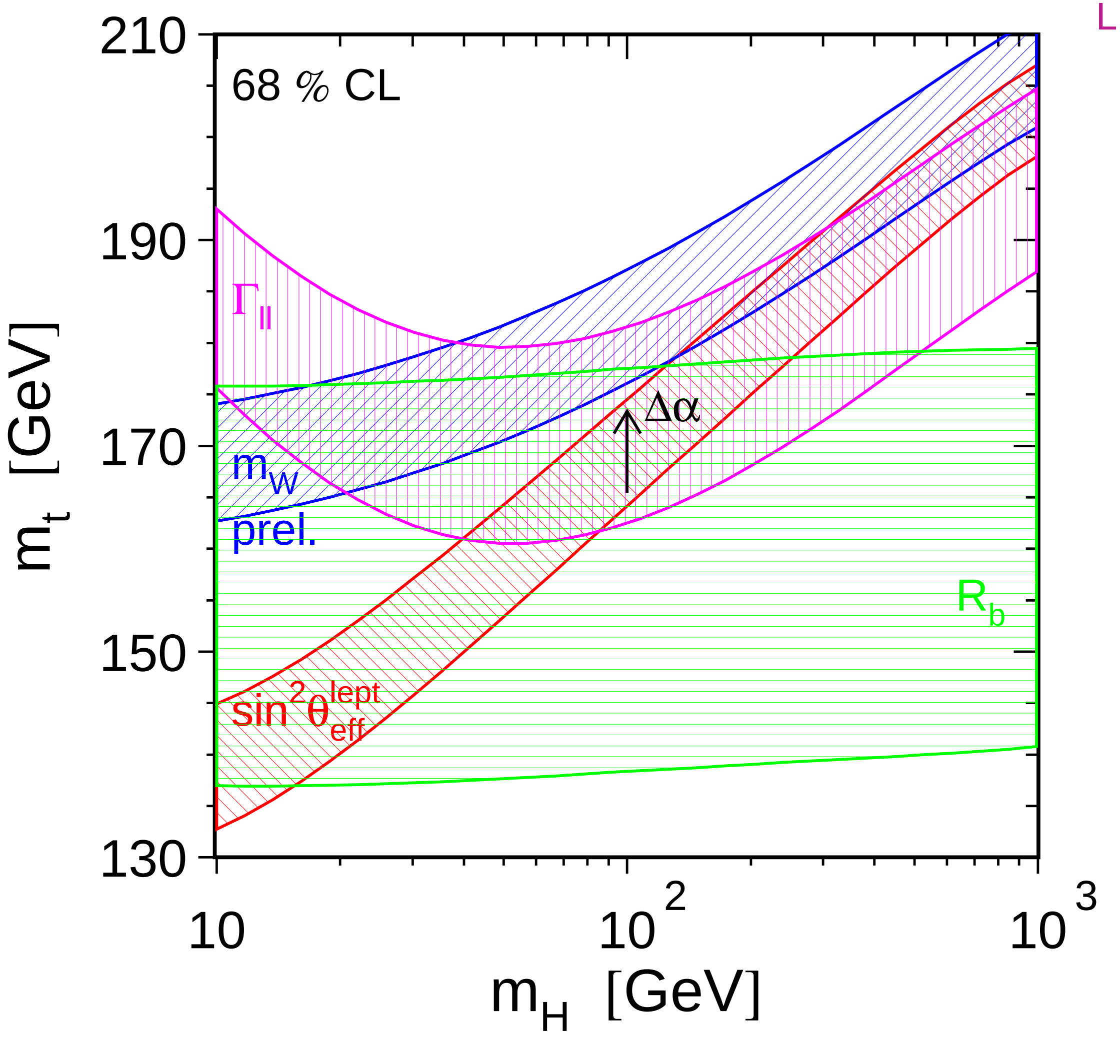
- high sensitivity to $M_{\rm H}$
- combination of very different observables
- $\sim 3\sigma$ difference between $A_{\rm FB}^{0,b}$(LEP) and $A_{\rm LR}^{0,l}$(SLD)

  with the initial-state pol. asymmetry

$$A_{\rm LR}^{0,l} = \frac{\sigma_{\rm L}^0 - \sigma_{\rm R}^0}{\sigma_{\rm L}^0 + \sigma_{\rm R}^0}\,\frac{1}{\langle|\mathcal{P}_{\rm e}|\rangle}$$

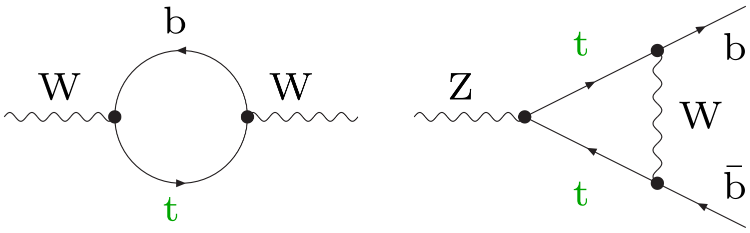


Observables most sensitive to m_t and M_H

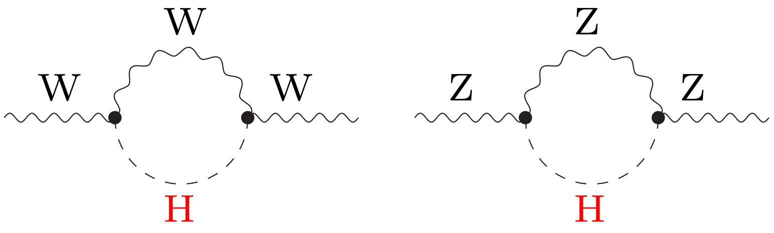


LEPEWWG '05

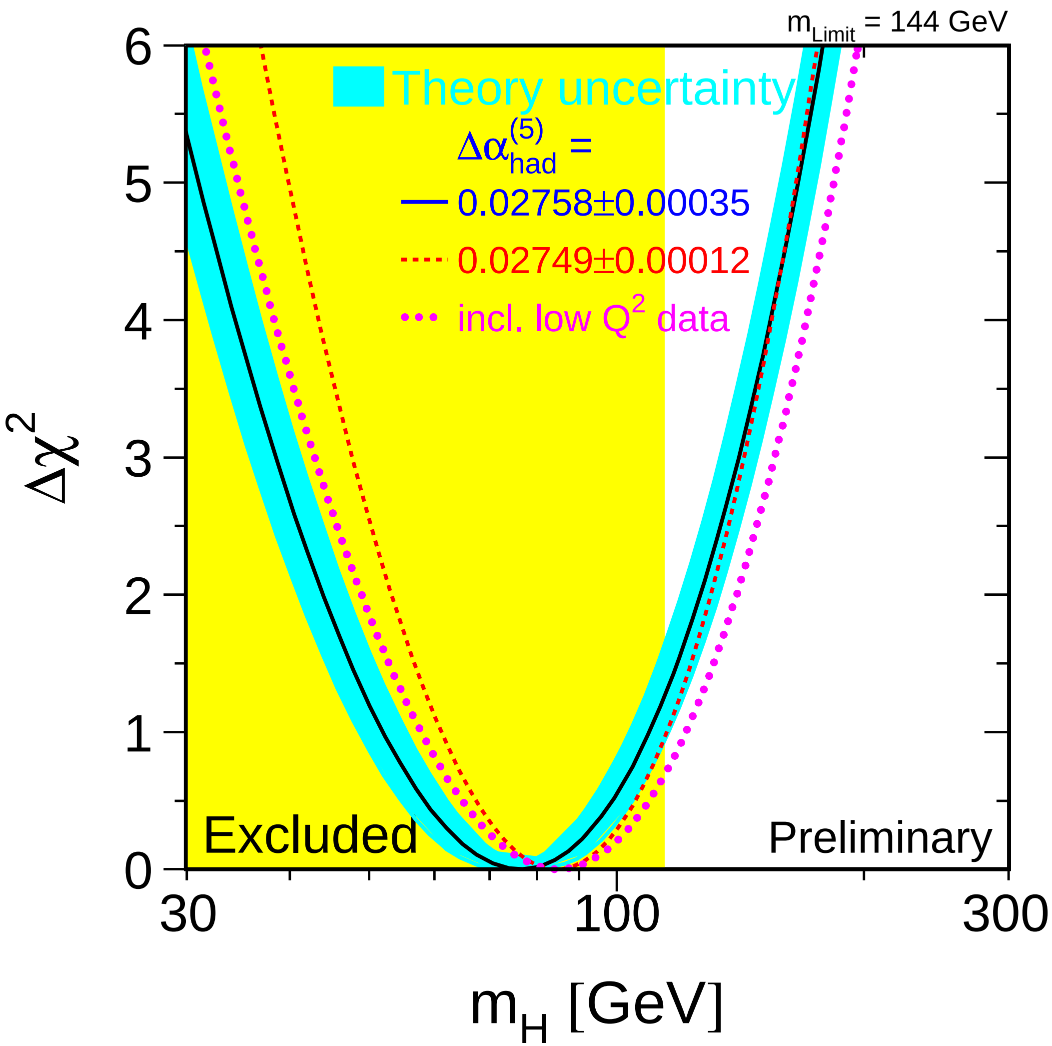
main sensitivity to m_t via



main sensitivity to M_H via



Bounds on M_{H} (95% C.L.)

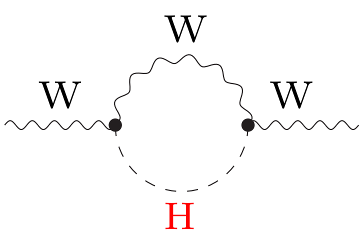


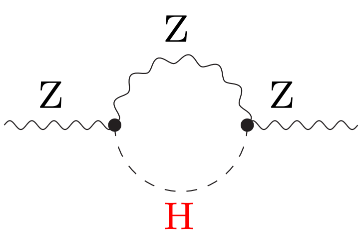
– $M_{\mathrm{H}} > 114.4\,\mathrm{GeV}$ (LEPHIGGS '02)

$e^+e^- \not\to \mathrm{ZH}$ at LEP2

– $M_{\mathrm{H}} < 144\,\mathrm{GeV}$ (LEPEWWG '07)

fit to precision data,
i.e. via quantum corrections





Sensitivity via "high-precision observables": m_{t}, M_{W}, $\sin^2\theta^{\mathrm{lept}}_{\mathrm{eff}}$, etc.

$\hookrightarrow$ precise measurement is possible at future ILC !

$\Rightarrow$ stronger bounds on M_{H}

4.3 W-boson physics at LEP2

W-pair production $e^+e^- \to WW \to 4f(+\gamma)$

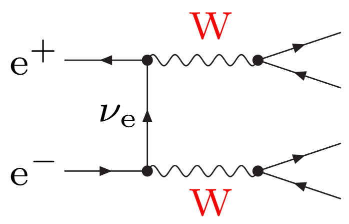


diagram dominates near W-pair threshold

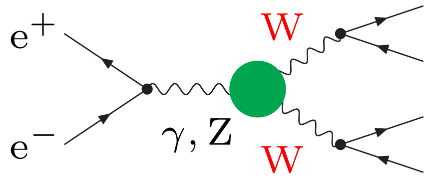


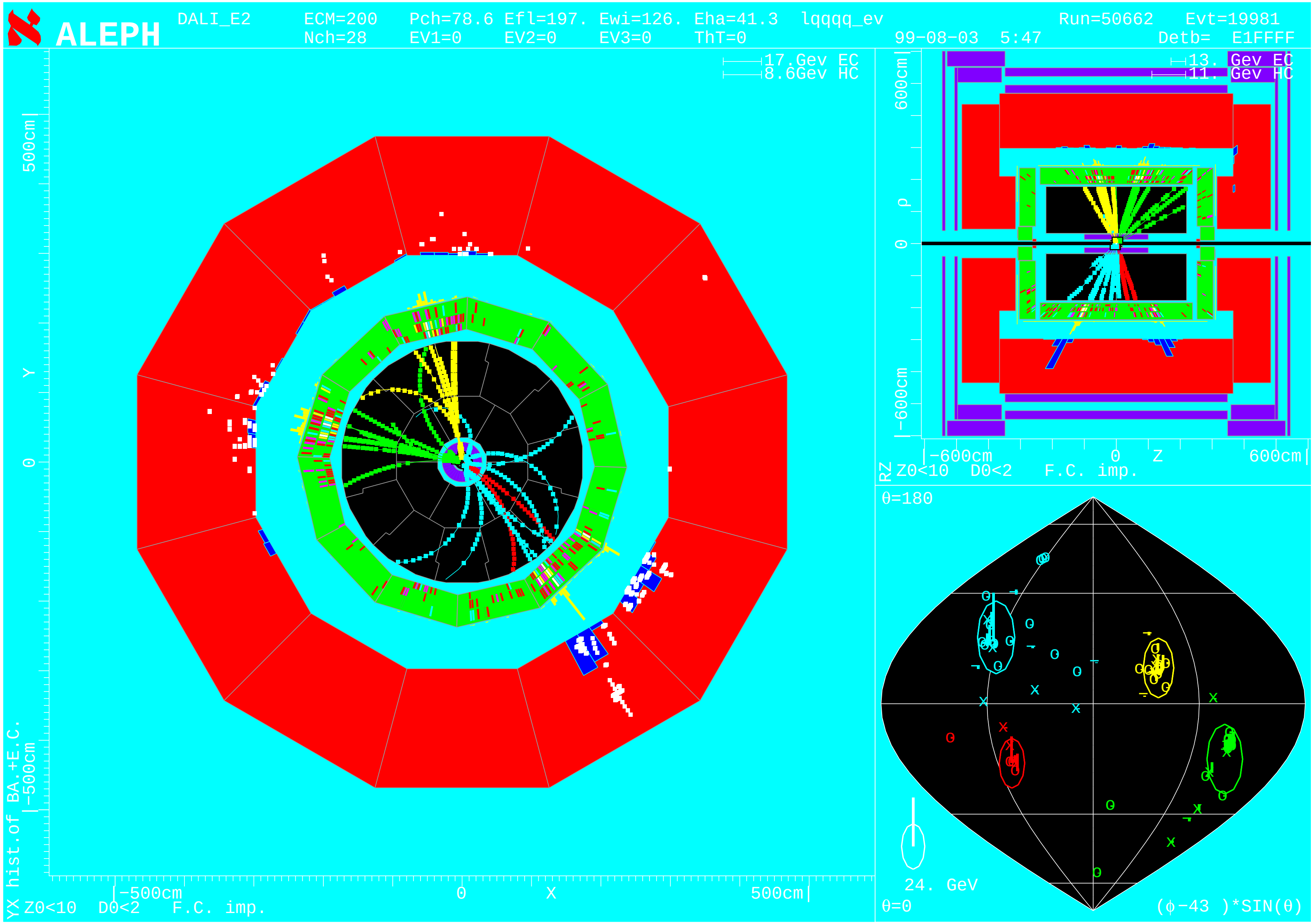
diagram contains γWW/ZWW couplings

Physics issues:

- test of non-abelian structure of triple gauge-boson couplings (TGCs)
  $\hookrightarrow$ constraint on non-standard $\gamma WW/ZWW$ couplings
- precision measurement of W-pair cross section
- precision measurement of W mass M_W
- first bounds on non-standard quartic gauge-boson couplings (QGCs)

$\Rightarrow$ Theoretical requirement:
precise understanding of $2 \to 4$ process (0.5% level for cross section)

A typical 4-jet event observed at ALEPH

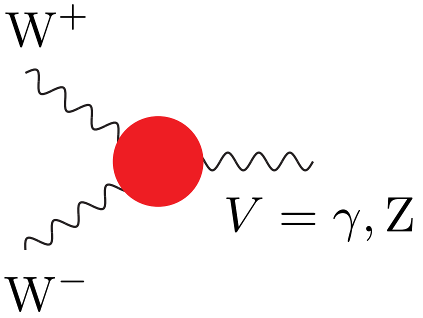


Made on 3-Aug-1999 14:42:48 by lancon with DALI_E2.
Filename: DC050662_019981_990803_1442.PS

(Non-)standard TGCs

Gaemers, Gounaris '79; Hagiwara, Hikasa, Peccei, Zeppenfeld '87; Bilenky, Kneur, Renard, Schildknecht '93; etc.

General parametrization (C- and P-conserving):



$$\mathcal{L}_{VWW} = -\mathrm{i}eg_{VWW}\Big\{ g_1^{\mathrm{V}}\left(W^+_{\mu\nu}W^{-,\mu}V^{\nu} - W^{-,\mu\nu}W^+_{\mu}V_{\nu}\right) + \kappa_{\mathrm{V}} W^+_{\mu}W^-_{\nu}V^{\mu\nu} + \frac{\lambda_{\mathrm{V}}}{M_{\mathrm{W}}^2} W^+_{\rho\mu}W^{-,\mu}_{\nu}V^{\nu\rho}\Big\}$$

Meaning for static W^+ bosons:

$$Q_{\mathrm{W}} = eg_1^{\gamma} = \text{electric charge } (= e \text{ by charge conservation})$$

$$\mu_{\mathrm{W}} = \frac{e}{2M_{\mathrm{W}}}\left(g_1^{\gamma} + \kappa_{\gamma} + \lambda_{\gamma}\right) = \text{magnetic dipole moment}$$

$$q_{\mathrm{W}} = -\frac{e}{M_{\mathrm{W}}^2}\left(\kappa_{\gamma} - \lambda_{\gamma}\right) = \text{electric quadrupole moment}$$

Standard Model values:

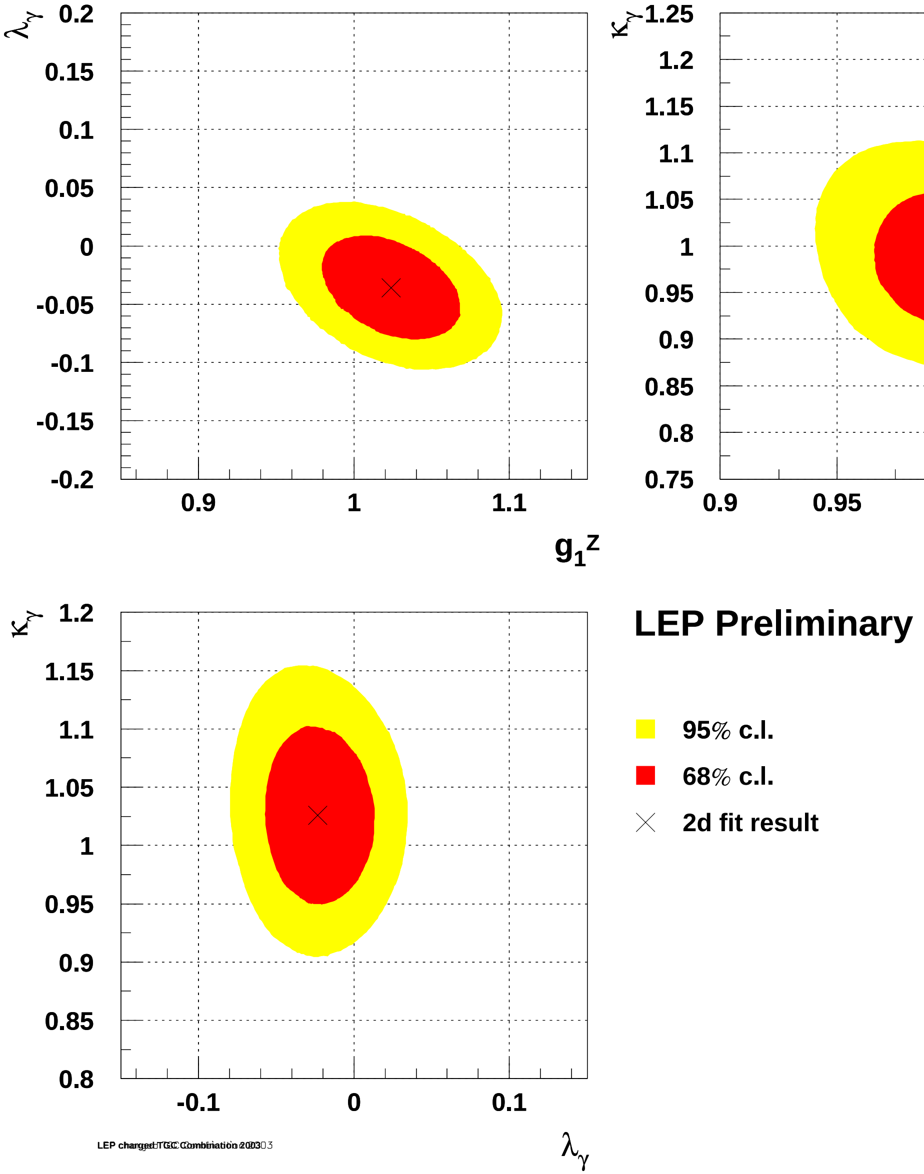
$$g_1^{\mathrm{V}} = \kappa_{\mathrm{V}} = 1, \qquad \lambda_{\mathrm{V}} = 0$$

Restriction to SU(2)×U(1)-symmetric dim-6 operators:

$$\kappa_{\mathrm{Z}} = g_1^{\mathrm{Z}} - (\kappa_{\gamma} - 1)\tan^2\theta_{\mathrm{W}}, \qquad \lambda_{\mathrm{Z}} = \lambda_{\gamma}$$

LEP2 constraints on charged TGCs

LEPEWWG '04



$$\Delta g_1^Z = -0.009^{+0.022}_{-0.021}$$

$$\Delta \kappa_\gamma = -0.016^{+0.042}_{-0.047}$$

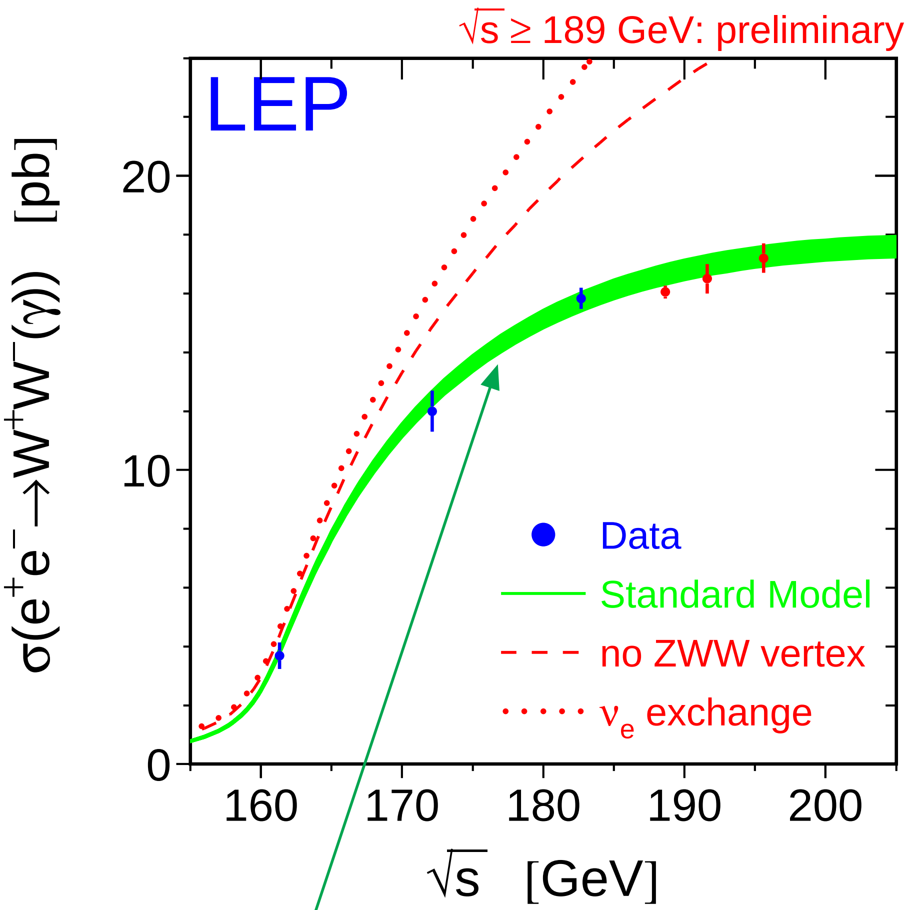
$$\lambda_\gamma = -0.016^{+0.021}_{-0.023}$$

Standard Model values verified
at the level of 2–4%

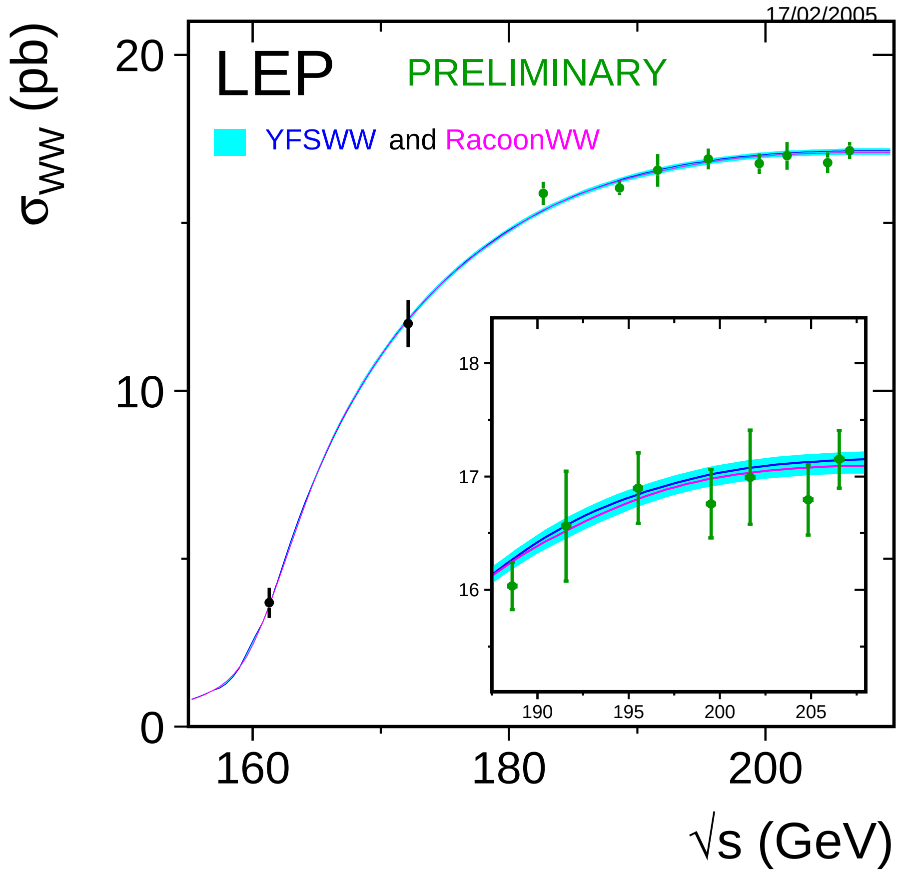
Note: TGC bounds $\sim$ $\mathcal{O}$(EW corrections)

Total WW cross section at LEP2

Status of 1999: (LEPEWWG ’99)



Final (?) result: (LEPEWWG ’05)

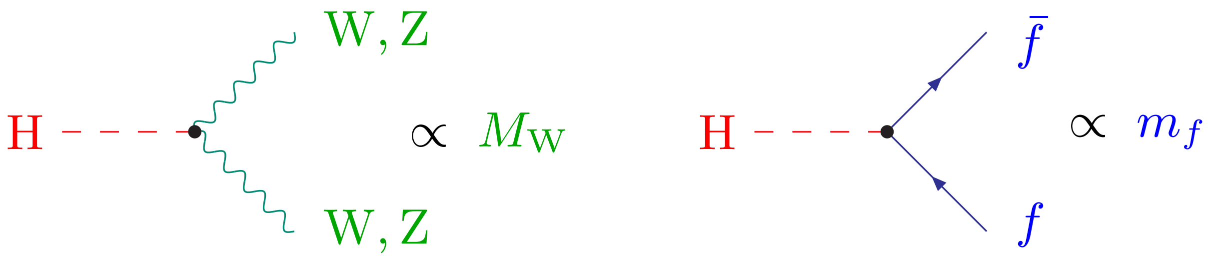


GENTLE (Bardin et al.)
only universal EW corrections
$\hookrightarrow$ theoretical uncertainty $\sim \pm 2\%$

YFSWW (Jadach et al.) / RacoonWW (Denner et al.)
non-universal corrections included
$\hookrightarrow$ th. uncertainty $\sim \pm 0.5\%$ for $\sqrt{s} > 170\,\mathrm{GeV}$

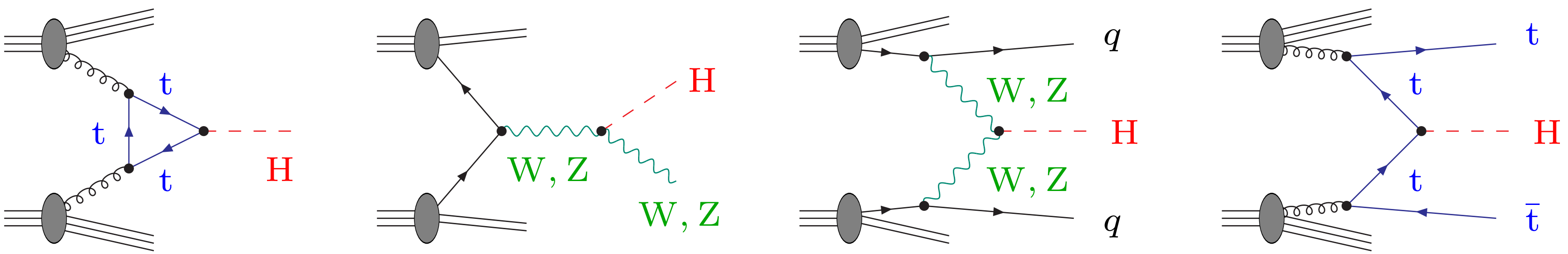
4.4 Higgs search at present and future colliders

Higgs bosons couple proportional to particle masses:

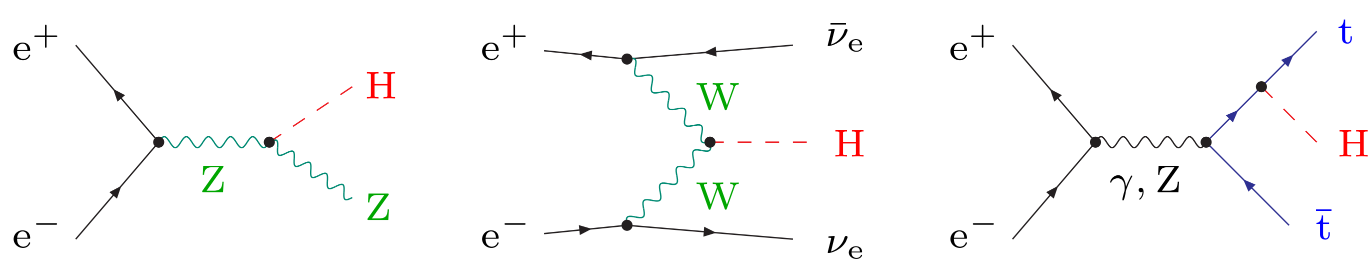


⇒ Higgs production mainly via coupling to W/Z bosons or top quarks

Processes at hadron colliders ($p\bar{p}/pp$):

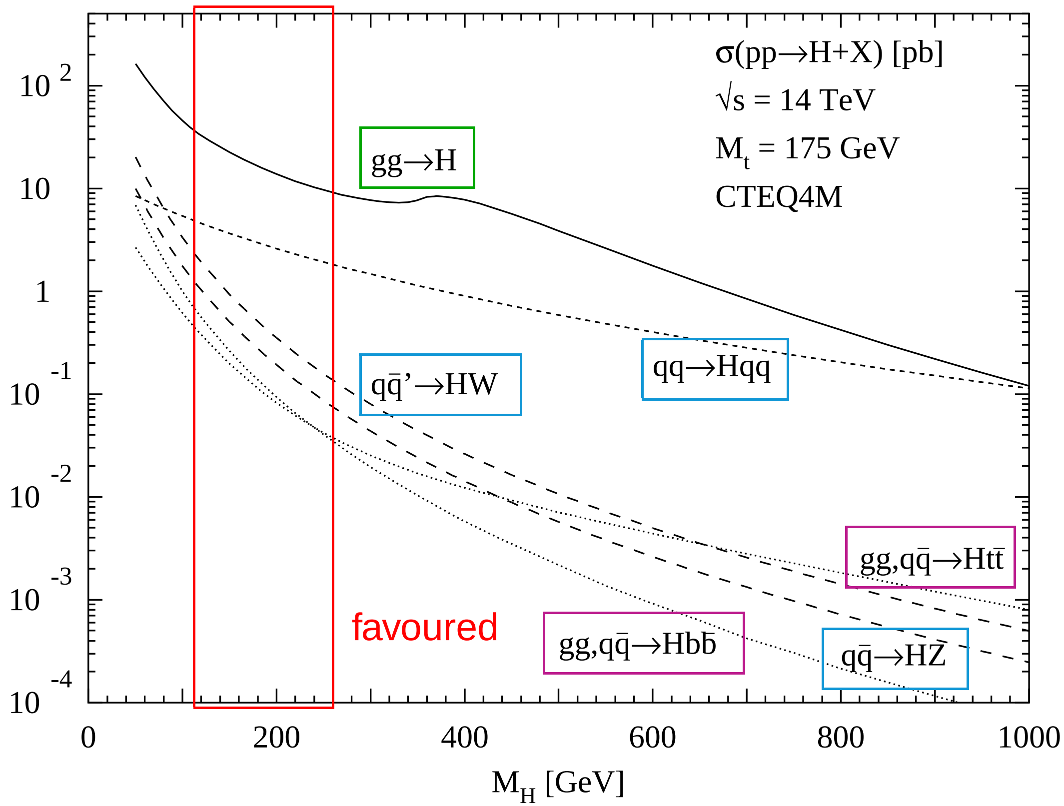


Processes at e^+e^- colliders:

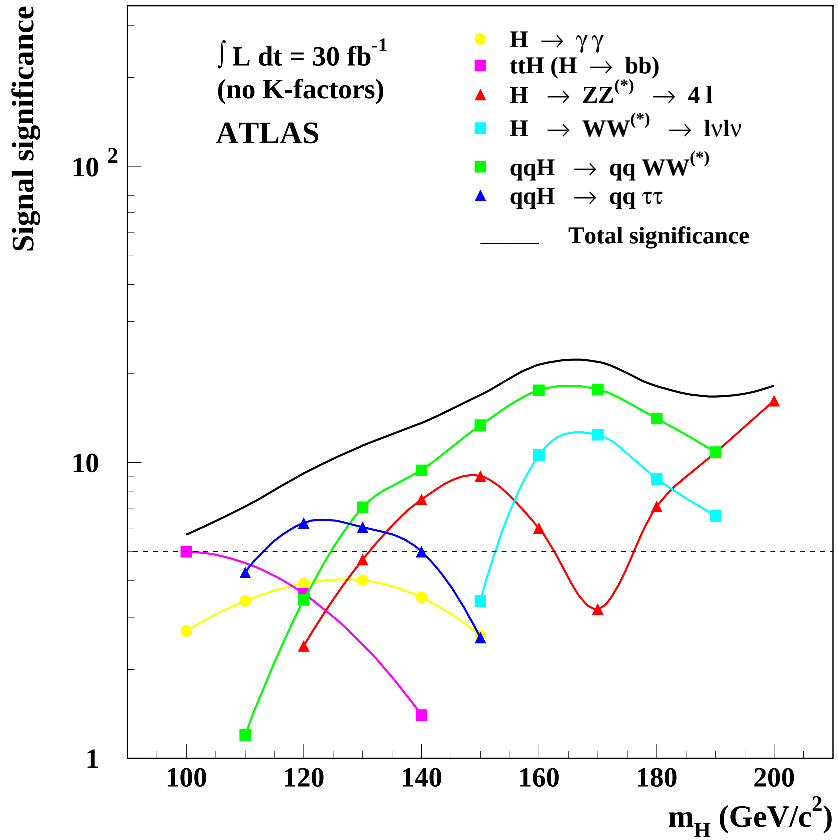


Cross sections and significance of the Higgs signal at the LHC

Spira et al. '98



ATLAS '03



Physics goals:

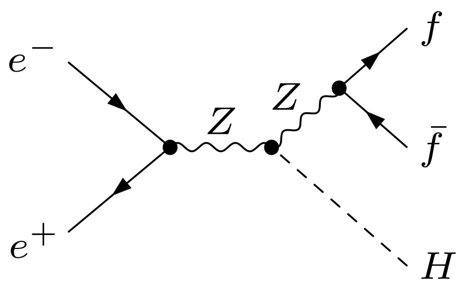
– Higgs discovery, M_{H} measurement, decay analyses

– ratios of couplings to W/Z bosons and quarks

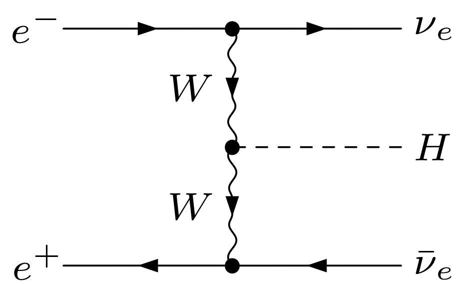
– extended Higgs sectors (MSSM: $\mathrm{h, H, A, H^{\pm}}$)

Higgs-boson production in e^+e^- annihilation

ZH production ("'Higgs-strahlung"')



WW fusion



WW fusion dominates
at high energies ($\sqrt{s} \gg M_{\rm H}$):

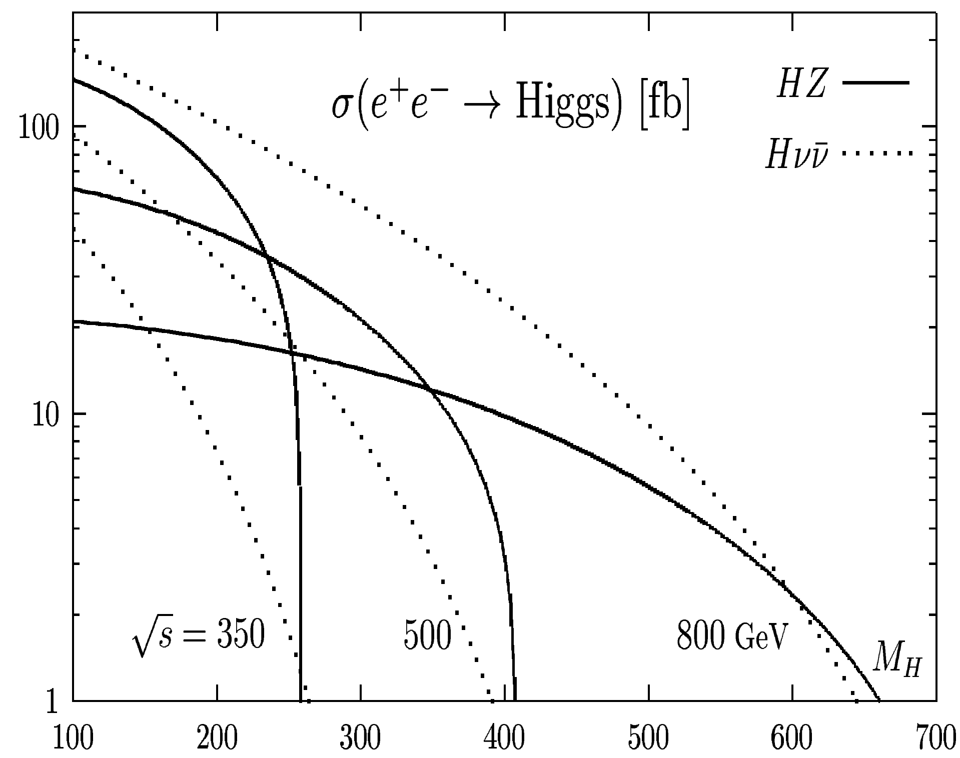
$\sigma_{\rm ZH} \;\sim\; \text{const} \,/\, s$

$\sigma_{\rm WW} \;\sim\; \text{const} \,\times\, \ln(s/M_{\rm W}^2)$

Physics issues:

– Higgs decay width
– quantum numbers (spin, P, CP)
– measurement of couplings
– extended Higgs sectors ?

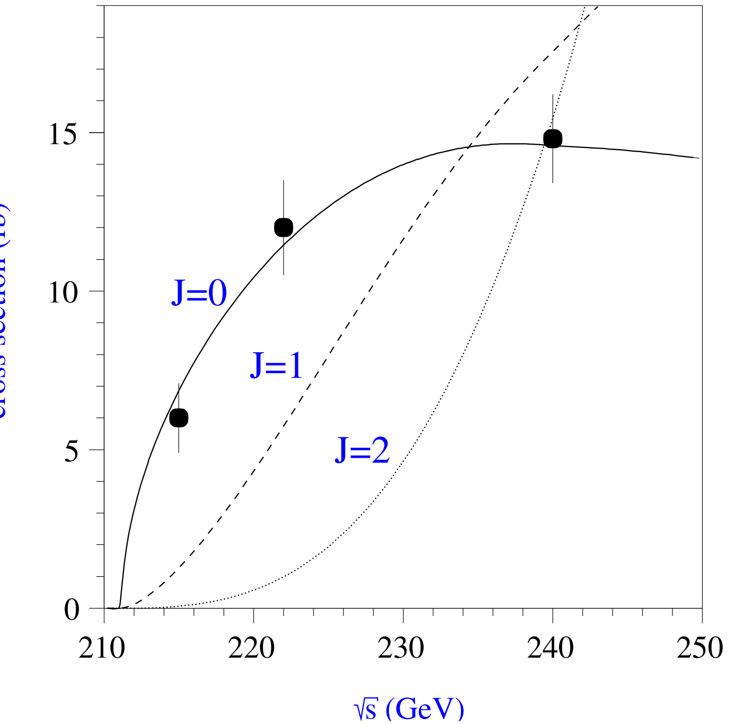
TESLA-TDR '01



Examples for Higgs studies at the ILC:

A qualitative study – spin:

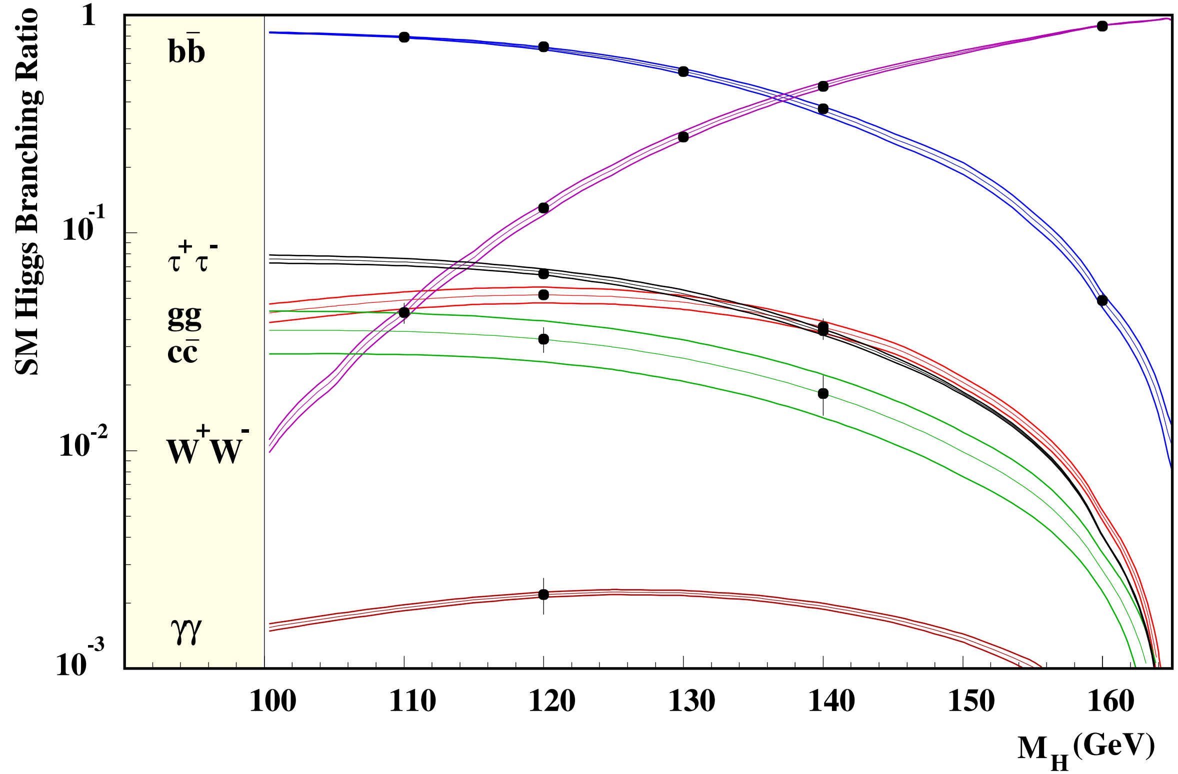
Miller et al. '01; TESLA-TDR '01



↪ spin J from rise of cross section

Precision BR measurements:

Battaglia '00; TESLA-TDR '01

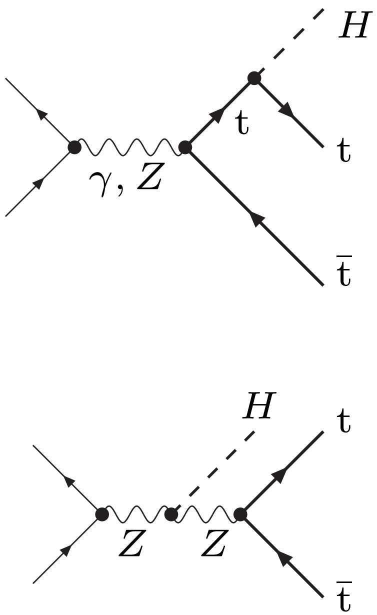


(assumed data versus theory error bands)

↪ precision test of Higgs mechanism, demarcation of SUSY Higgs bosons

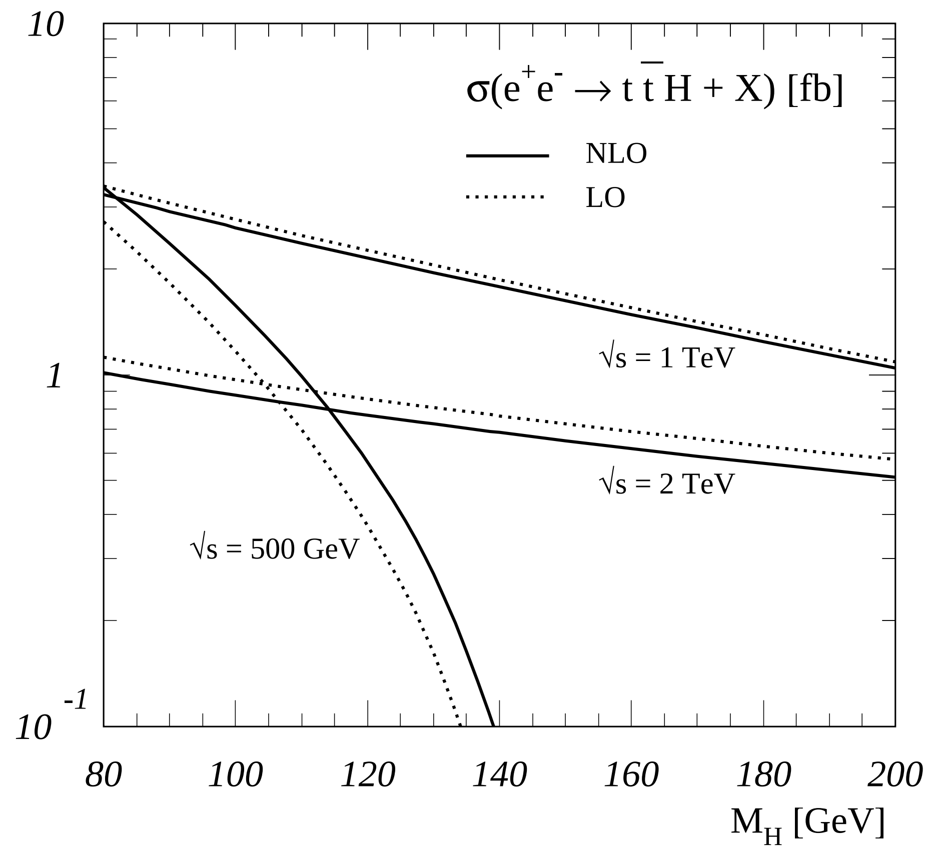
Channel for analyzing the top-Yukawa coupling:

Associated Higgs production: $e^+e^- \to t\bar{t}H$



QCD-corrected cross section:

Dittmaier, Krämer, Liao, Spira, Zerwas '98



expected accuracy:

$\Delta g_{\mathrm{ttH}}/g_{\mathrm{ttH}} \sim 5\%$

4.5 The role of precision at LHC and ILC

LHC: the discovery machine (Higgs & EWSB, SUSY, etc.?)

- QCD corrections (at least NLO) are substantial parts of predictions

  typical LO uncertainties $\sim$ several $10\%-100\%$

  corrections needed for signals and many background processes

- EW corrections also important for many observables

  (precision physics, searches at high scales, particle reconstruction, etc.)

ILC: the high-precision machine (precision $\to$ window to higher energy)

- old and new physics with high accuracy (typically $\delta\sigma/\sigma \lesssim 1\%$)

  $\hookrightarrow$ QCD and EW corrections required

- the ultimate precision at GigaZ/MegaW:

  precision increases by factor ~ 10 w.r.t. LEP/SLC

  EXP: $\Delta\sin^2\theta^{\mathrm{lept}}_{\mathrm{eff}} \sim 0.00001, \qquad \Delta M_{\mathrm{W}} \sim 7\,\mathrm{MeV}$

  TH: go from a few 10^2 to a few 10^4 (more complicated) diagrams

$\Rightarrow$ Precision calculations mandatory for LHC and ILC !

5 Quantum field theories and higher perturbative orders

5.1 General procedure

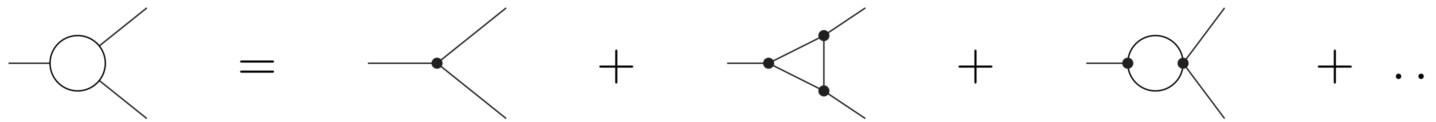
Formulate theory:	Lagrangian
	$\Downarrow$
	quantization $\longrightarrow$ gauge fixing, Faddeev–Popov ghosts
	$\Downarrow$
Perturbative evaluation:	Feynman rules
	$\Downarrow$
	Feynman graphs
	$\Downarrow$
	loop integrals $\longrightarrow$ technical problem: divergences (UV, IR)
	$\Downarrow$
	regularization $\longrightarrow$ divergences mathematically meaningful
	$\Downarrow$
Define input parameters:	renormalization $\longrightarrow$ eliminates UV divergences
	$\Downarrow$
Theoretical predictions:	calculation of observables (cross sections, decay widths, etc.)
	$\hookrightarrow$ IR divergences cancel for sufficiently inclusive quantities (e.g. inclusion of photon bremsstrahlung)

5.2 Green functions, transition amplitudes, and observables

"Amputated" Green functions $G_{\mathrm{amp}}^{\phi_1\ldots\phi_n}$:

calculated as sum of all connected Feynman diagrams with external n legs $\phi_1, \ldots, \phi_n$ with external propagators (and propagator corrections) omitted

$G_{\mathrm{amp}}^{\phi_1\phi_2\phi_3} =$



Transition amplitude $\mathcal{M}_{fi}$ for $|i\rangle \to |f\rangle$:

calculated from amputated Green functions $G_{\mathrm{amp}}^{\phi_1\ldots\phi_n}$ by "LSZ reduction":

- put external momenta to their mass shell, $p_i^2 = m_i^2$
- contract with wave functions of external particles (Dirac spinors, polarization vectors)

  Note: fields must be normalized: $R_{\phi_i} = 1$ (= residue of propagator pole), otherwise multiply by $\sqrt{R_{\phi_i}}$ for each external leg

Cross section for transition $|i\rangle \to |f\rangle$:

$$\sigma = \text{flux} \times \int \mathrm{dLIPS}\, |\mathcal{M}_{fi}|^2$$

“Vertex functions” $\Gamma^{\phi_1\cdots\phi_n}$ as irreducible building blocks:

- $\Gamma^{\phi_1\phi_2} \equiv -(G^{\phi_1\phi_2})^{-1} = -$(inverse propagator)

  example: scalar 2-point function

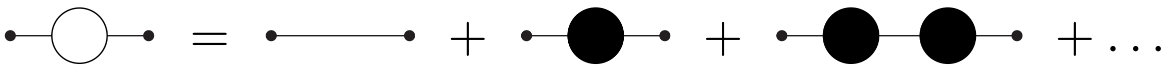
  $$\Gamma^{\phi\phi}(p) = \mathrm{i}(p^2 - m^2) + \mathrm{i}\Sigma(p^2),$$

  $\Sigma =$ self-energy = sum of 1PI graphs

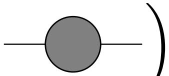
  

  1PI = 1-particle-irreducible
  (graph cannot be disconnected by cutting *one* line)

  $$G^{\phi\phi}(p) = \frac{\mathrm{i}}{p^2 - m^2} + \frac{\mathrm{i}}{p^2 - m^2}\,\mathrm{i}\Sigma(p^2)\,\frac{\mathrm{i}}{p^2 - m^2} + \ldots$$ (Dyson series)

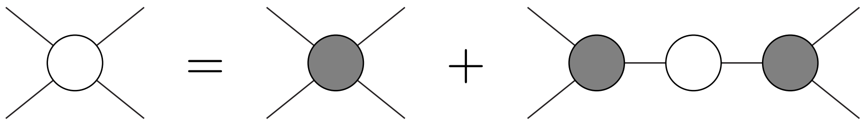
  

  $$= \frac{\mathrm{i}}{p^2 - m^2 + \Sigma(p^2)} = -\left(\Gamma^{\phi\phi}(p)\right)^{-1} = -\left(\quad\right)^{-1}$$

- $\Gamma^{\phi_1\cdots\phi_n} \equiv G^{\phi_1\cdots\phi_n}_{\mathrm{amp}}\Big|_{\text{only 1PI graphs}}$

  example:

  $+$ two permutations

  $G^{\phi\phi\phi\phi}_{\mathrm{amp}}$ $\qquad \Gamma^{\phi\phi\phi\phi}$ $\qquad \Gamma^{\phi\phi\phi}G^{\phi\phi}\Gamma^{\phi\phi\phi}$

5.3 Loop integrals and regularization

Regularization of divergences

Observation: loop integrals involve divergences

- UV divergences for $q \to \infty$, e.g.:

$$\int \mathrm{d}^4 q \, \frac{1}{(q^2 - m_0^2)(q^2 - m_1^2)} \sim \int \frac{\mathrm{d}q}{q} \text{ for } q \to \infty \qquad \to \text{logarithmic divergence}$$

- IR divergences for $q \to q_0$, e.g.:

$$\int \mathrm{d}^4 q \, \frac{1}{q^2(q^2 + 2qp_1)(q^2 + 2qp_2)} \sim \int \frac{\mathrm{d}q}{q} \text{ for } q \to 0 \qquad \to \text{logarithmic divergence}$$

"Regularization": extension of theory by free parameter δ such that

- integrals (and thus the theory) become finite, i.e. well defined
- original theory is obtained as limiting case $\delta \to \delta_0$

$\hookrightarrow$ fix input parameters x_i of regularized theory ($\delta \neq \delta_0$) by experiment

$\Rightarrow$ observables must have finite limit $\delta \to \delta_0$ as functions of x_i

(independent of regularization scheme)

Convenient regularization schemes:

- Dimensional regularization: switch to $D \neq 4$ space-time dimensions
  - ◇ regularizes UV (and IR) divergences, respects gauge invariance, easy use
  - ◇ prescription: (μ = arbitrary reference mass, drops out in observables)

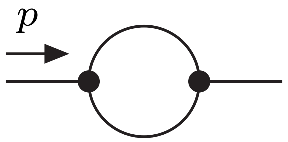
    $$\int \mathrm{d}^4 q \;\to\; (2\pi\mu)^{4-D} \int \mathrm{d}^D q$$ and D-dim. momenta, metric, Dirac algebra

    and analytic continuation to complex D !
  - ◇ divergences appear as poles $\frac{1}{4-D}$ in results

    $\hookrightarrow$ define $\Delta \equiv \frac{2}{4-D} - \gamma_{\mathrm{E}} + \ln(4\pi) = \frac{2}{4-D} +$ const.
- IR regularization by infinitesimal photon mass m_γ
  and (if relevant) by small fermion mass m_f
  - ◇ prescription: photon propagator pole $\frac{1}{q^2} \to \frac{1}{q^2 - m_\gamma^2}$
  - ◇ divergences appear as $\ln(m_\gamma)$ and $\ln(m_f)$ terms

Standard 1-loop integrals:

- 2-point integrals:

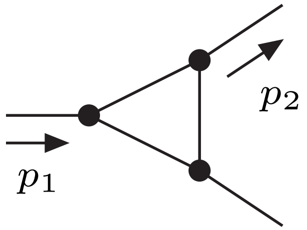


$$B_{0,\mu,\mu\nu,\dots}(p,m_0,m_1) \;=\; \frac{(2\pi\mu)^{4-D}}{i\pi^2}\int \mathrm{d}^D q\,\frac{1,q_\mu,q_\mu q_\nu,\dots}{(q^2-m_0^2+\mathrm{i}0)[(q+p)^2-m_1^2+\mathrm{i}0]}$$

scalar integral $B_0 =$ logarithmically UV divergent $= \Delta +$ finite,

vector integral $B_\mu = -\frac{1}{2}p_\mu\Delta +$ finite, etc.

- 3-point integrals:



$$C_{0,\mu,\mu\nu,\dots}(p_1,p_2,m_0,m_1,m_2)$$
$$= \frac{(2\pi\mu)^{4-D}}{i\pi^2}\int \mathrm{d}^D q\,\frac{1,q_\mu,q_\mu q_\nu,\dots}{(q^2-m_0^2+\mathrm{i}0)[(q+p_1)^2-m_1^2+\mathrm{i}0][(q+p_2)^2-m_2^2+\mathrm{i}0]}$$

C_0, $C_\mu =$ UV finite,

$C_{\mu\nu} =$ logarithmically UV divergent $= \frac{1}{4}g_{\mu\nu}\Delta +$ finite, etc.

- 4-point integrals: $D_{\dots}$ functions, etc.

Features of one-loop integrals:

- sign of infinitesimally small imaginary part $\mathrm{i}0$ in mass terms reflects causality
- general results for 1-loop integrals known
  (complicated but straightforward calculation)
  - ◇ momentum integrals can be carried out after "Feynman parametrization"
    $\hookrightarrow$ $(n-1)$-dimensional integrals for n-point functions
  - ◇ B functions $\rightarrow$ can be expressed in terms of log's
  - ◇ C, D, etc. $\rightarrow$ involve dilogarithms $\mathrm{Li}_2(x) = -\int_0^x \frac{\mathrm{d}t}{t} \ln(1-t)$
- tensor integrals can be decomposed into Lorentz covariants:

$$B^\mu = p^\mu B_1, \qquad B^{\mu\nu} = g^{\mu\nu} B_{00} + p^\mu p^\nu B_{11},$$

$$C^\mu = p_1^\mu C_1 + p_2^\mu C_2, \quad C^{\mu\nu} = p_1^\mu p_1^\nu C_{11} + p_2^\mu p_2^\nu C_{22} + (p_1^\mu p_2^\nu + p_1^\nu p_2^\mu) + g^{\mu\nu} C_{00}, \quad \text{etc.}$$

$\hookrightarrow$ tensor coefficients B_1, B_{ij}, C_i, etc. can be obtained as
linear combinations of scalar integrals B_0, C_0, etc.
(e.g. by "Passarino–Veltman reduction")

5.4 Renormalization

Propagators and 2-point functions:

Structure of one-loop self-energies (scalar case as example):

$$\Sigma(p^2) \;=\; C_1\, p^2\, \Delta \;+\; C_2\, \Delta \;+\; \Sigma_{\mathrm{finite}}(p^2) \;=\; \text{UV divergent}$$

Behaviour of propagator near pole for free propagation:

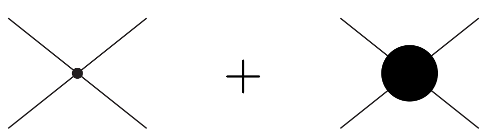
$$G^{\phi\phi}(p^2) \;=\; \frac{\mathrm{i}}{p^2 - m^2 + \Sigma(p^2)} \;\underset{p^2 \to m^2}{\sim}\; \frac{1}{1+\Sigma'(m^2)}\,\frac{\mathrm{i}}{p^2 - m^2 + \Sigma(m^2)}$$

$\hookrightarrow$ higher-order corrections change location and residue of propagator pole

Interaction vertices:

Example: scalar 4-point interaction $\mathcal{L}_{\phi^4} = \lambda\phi^4/4!$

$$\Gamma^{\phi\phi\phi\phi}(p_1,p_2,p_3) = \quad \mathrm{i}\lambda \quad + \quad \mathrm{i}\Lambda\,(p_1,p_2,p_3)$$



momentum-dependent one-loop correction:

$$\Lambda\,(p_1,p_2,p_3) \;=\; C_3\Delta \;+\; \Lambda_{\mathrm{finite}}\,(p_1,p_2,p_3) \;=\; \text{UV divergent}$$

$\hookrightarrow$ higher-order corrections change coupling strengths

Structure of UV divergences:

- Renormalizable field theories:

  UV divergences in vertex functions have analytical form of elementary vertex structures (directly related to $\mathcal{L}$)

  $\hookrightarrow$ idea: absorb divergences in free parameters

  $\Rightarrow$ Reparametrization of theory (=renormalization)

  Different types of renormalizable theories:

  ◇ theories with unrelated couplings of non-negative mass dimensions

  $\hookrightarrow$ renormalizability proven by power counting and "BPHZ procedure"

  ◇ gauge theories (couplings unified by gauge invariance)

  $\hookrightarrow$ renormalizability non-trivial consequence of gauge symmetry 't Hooft '71

- Non-renormalizable field theories:

  e.g. theories with couplings of negative mass dimensions (cf. Fermi model)

  operators of higher and higher mass dimensions needed to absorb UV divergences

  $\hookrightarrow$ infinitely many free parameters, much less predictive power

Practical procedure for renormalization:

consider original (“bare”) parameters and fields as preliminary
(denoted with subscripts “0” in the following)

$\hookrightarrow$ switch to new “renormalized” parameters and fields that obey certain conditions

Propagators and 2-point functions:

- mass renormalization: $m_0^2 = m^2 + \delta m^2$,
  $m^2 \stackrel{!}{=}$ location of propagator pole = “physical mass” $\quad \to \delta m^2 = \Sigma(m^2)$

- wave-function ren.: rescale fields $\phi_0 = \sqrt{Z_\phi}\phi$, $\quad G^{\phi\phi} = Z_\phi^{-1} G^{\phi_0\phi_0}$
  fix $Z_\phi = 1 + \delta Z_\phi$ such that residue of $G^{\phi\phi}$ at $p^2 = m^2$ equals 1
  $\hookrightarrow \delta Z_\phi = -\Sigma'(m^2)$

$\Rightarrow$ Renormalized propagator $G^{\phi\phi}$ is UV finite:

$$G^{\phi\phi}(p^2) = \frac{\mathrm{i}}{p^2 - m^2 + \Sigma_{\mathrm{ren}}(p^2)},$$

$$\begin{aligned}\Sigma_{\mathrm{ren}}(p^2) &= \Sigma(p^2) - \Sigma(m^2) + (p^2 - m^2)\Sigma'(m^2) \ = \ \text{ren. self-energy} \\ &= \Sigma_{\mathrm{finite}}(p^2) - \Sigma_{\mathrm{finite}}(m^2) + (p^2 - m^2)\Sigma'_{\mathrm{finite}}(m^2) \ = \ \text{UV finite}\end{aligned}$$

Vertex functions for interactions:

- coupling renormalization: $\lambda_0 \;=\; \lambda + \delta\lambda$

  fix $\delta\lambda$ such that λ assumes a measured value for special kinematics $p_i^{\rm exp}$

  note: $\Gamma^{\phi\phi\phi\phi} \;=\; Z_\phi^2 \Gamma^{\phi_0\phi_0\phi_0\phi_0}$

  $\hookrightarrow \;\; \delta\lambda = -2\delta Z_\phi \lambda - \Lambda(p_1^{\rm exp}, p_2^{\rm exp}, p_3^{\rm exp})$

$\Rightarrow$ Renormalized vertex function is UV finite:

$$\Gamma^{\phi\phi\phi\phi}\,(p_1, p_2, p_3) = {\rm i}\lambda + {\rm i}\Lambda_{\rm ren}\,(p_1, p_2, p_3)\,,$$

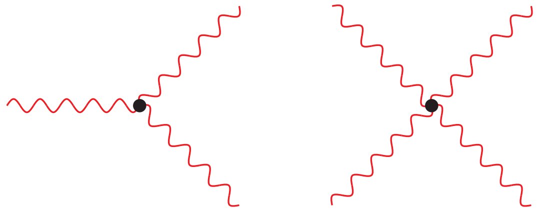
$$\Lambda_{\rm ren}\,(p_1, p_2, p_3) = \Lambda_{\rm finite}\,(p_1, p_2, p_3) \;-\; \Lambda_{\rm finite}(p_1^{\rm exp}, p_2^{\rm exp}, p_3^{\rm exp}) \;=\; \text{UV finite}$$

6 Electroweak Standard Model — radiative corrections

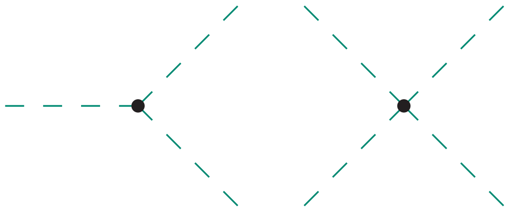
6.1 Loop corrections

Recapitulation of elementary SM couplings (vertices)

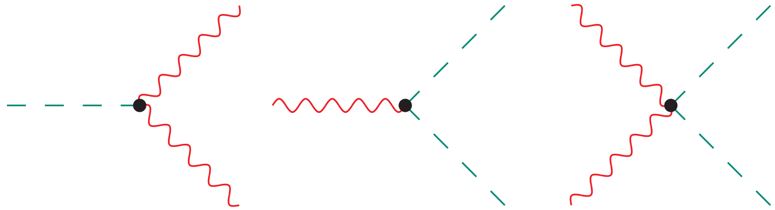
gauge-boson self-couplings:



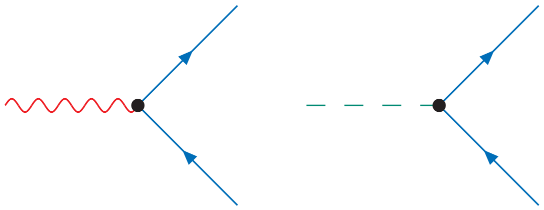
Higgs self-couplings:



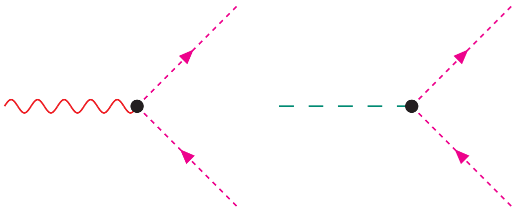
gauge-boson–Higgs couplings:



fermion couplings:



Faddeev–Popov couplings:

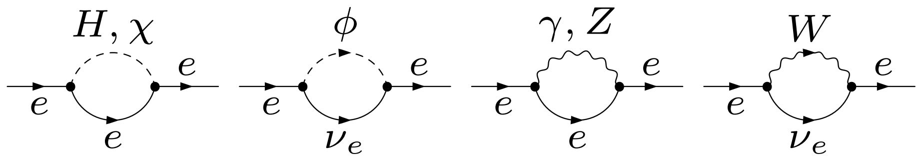


$\Rightarrow$ Large variety of loop diagrams !

Examples for 2-point functions at one loop: ('t Hooft–Feynman gauge)

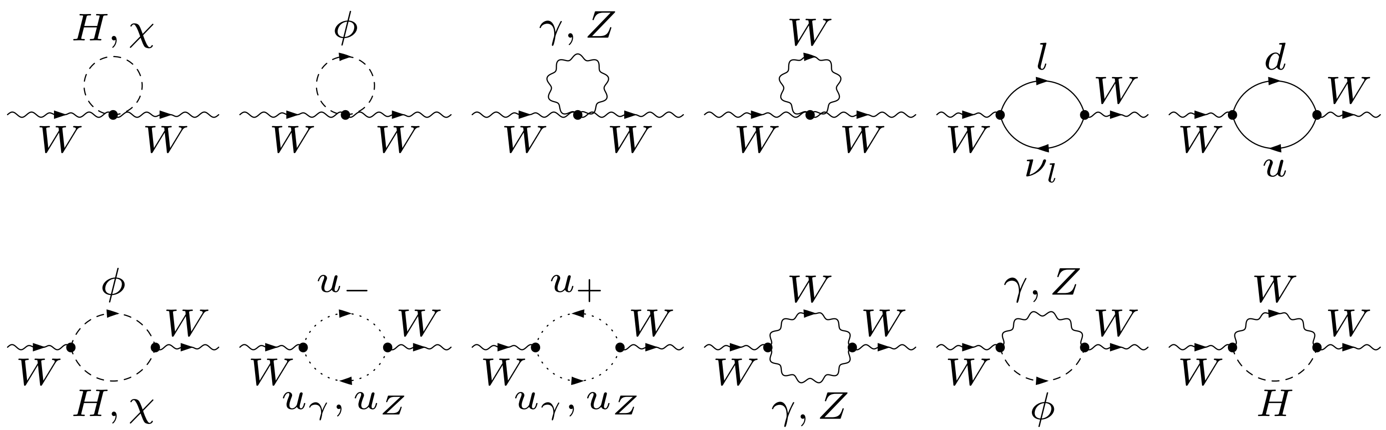
Electron self-energy:

$$\Gamma^{e\bar{e}}(p) \;=\; \mathrm{i}(\not{p} - m_\mathrm{e}) + \mathrm{i}\not{p}\omega_+\Sigma^\mathrm{e}_\mathrm{R}(p^2) + \mathrm{i}\not{p}\omega_-\Sigma^\mathrm{e}_\mathrm{L}(p^2) + \mathrm{i}m_\mathrm{e}\Sigma^\mathrm{e}_\mathrm{S}(p^2)$$



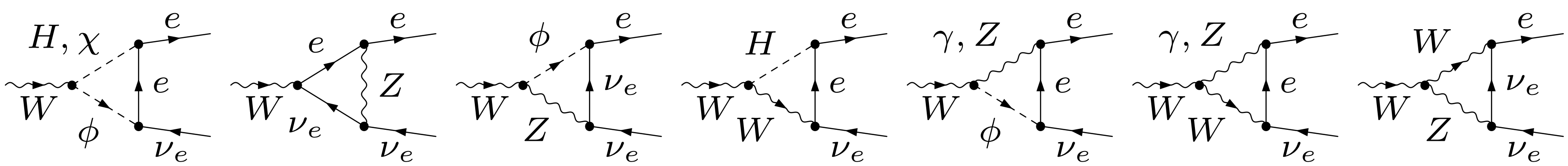
W-boson self-energy:

$$\Gamma^{\mathrm{W}^-\mathrm{W}^+}_{\mu\nu}(k) \;=\; -\mathrm{i}g_{\mu\nu}(k^2 - M_\mathrm{W}^2) - \mathrm{i}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\Sigma^\mathrm{W}_\mathrm{T}(k^2) - \mathrm{i}\frac{k_\mu k_\nu}{k^2}\Sigma^\mathrm{W}_\mathrm{L}(k^2)$$

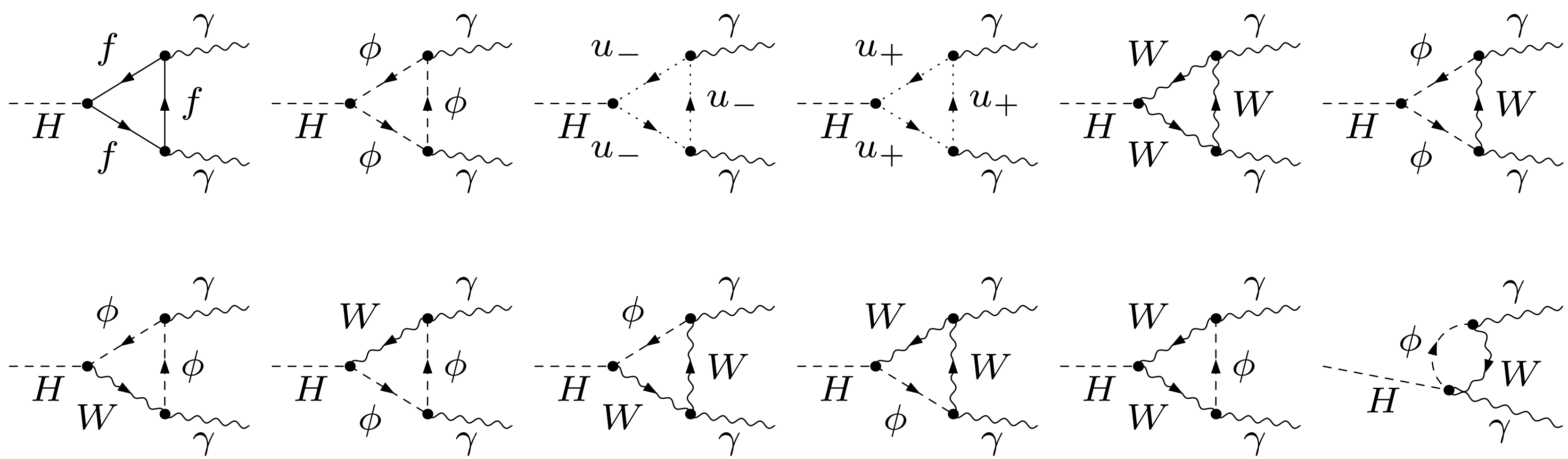


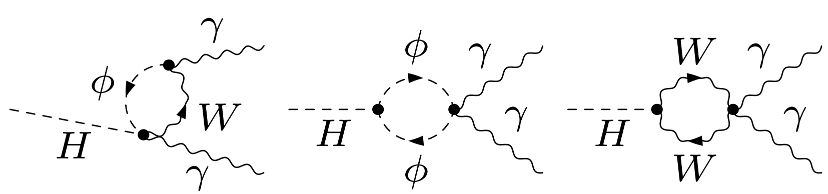
Examples for 3-point functions at one loop:

$\mathrm{We}\nu_\mathrm{e}$ vertex correction:



$\mathrm{H}\gamma\gamma$ vertex (loop induced):





$+$ diagrams (if different) with photons interchanged

6.2 Renormalization

Bare input parameters: $e_0, M_{\mathrm{W},0}, M_{\mathrm{Z},0}, M_{\mathrm{H},0}, m_{f,0}, V_{ij,0}$

Renormalization transformation:

- Parameter renormalization:

$$e_0 = (1 + \delta Z_e)e,$$

$$M_{\mathrm{W},0}^2 = M_\mathrm{W}^2 + \delta M_\mathrm{W}^2, \quad M_{\mathrm{Z},0}^2 = M_\mathrm{Z}^2 + \delta M_\mathrm{Z}^2, \quad M_{\mathrm{H},0}^2 = M_\mathrm{H}^2 + \delta M_\mathrm{H}^2,$$

$$m_{f,0} = m_f + \delta m_f, \quad V_{ij,0} = V_{ij} + \delta V_{ij}, \quad \text{(both } V_{ij,0}, V_{ij} \text{ unitary)}$$

Note: renormalization of $c_\mathrm{W}, s_\mathrm{W}$ fixed by on-shell condition $c_\mathrm{W} = \dfrac{M_\mathrm{W}}{M_\mathrm{Z}}$
(s_W is *not* a free parameter if M_W, M_Z are used as input parameters)

- Field renormalization

$$W_0^\pm = \sqrt{Z_W}\, W^\pm, \quad \begin{pmatrix} Z_0 \\ A_0 \end{pmatrix} = \begin{pmatrix} \sqrt{Z_{ZZ}} & \sqrt{Z_{ZA}} \\ \sqrt{Z_{AZ}} & \sqrt{Z_{AA}} \end{pmatrix} \begin{pmatrix} Z \\ A \end{pmatrix}, \quad H_0 = \sqrt{Z_H}\, H,$$

$$\psi_{f,0}^\mathrm{L} = \sqrt{Z_{ff'}^\mathrm{L}}\, \psi_{f'}^\mathrm{L}, \quad \psi_{f,0}^\mathrm{R} = \sqrt{Z_{ff'}^\mathrm{R}}\, \psi_{f'}^\mathrm{R}$$

Note: matrix renormalization necessary to account for loop-induced mixing

Renormalization conditions:

- Mass renormalization:

  on-shell definition: mass^2 is location of pole in propagator

  $\hookrightarrow \; \delta M_\mathrm{W}^2 = \mathrm{Re}\{\Sigma_\mathrm{T}^W(M_\mathrm{W}^2)\}$, similar expressions for $\delta M_\mathrm{Z}^2, \delta M_\mathrm{H}^2, \delta m_f$

  Note: ◇ location of pole is complex for unstable particles

  $\hookrightarrow$ subtlety in all-orders definition, but not relevant at one loop

  (gauge-invariant definition: mass^2 as real part of pole location)

  ◇ other definitions of quark masses often more appropriate

  (running masses, masses in effective field theories)

- Field renormalization: (bosons and leptons)

  ◇ residues of propagators (diagonal, transverse parts) normalized to 1

  $\hookrightarrow \; \delta Z_W = -\mathrm{Re}\{\Sigma_\mathrm{T}^{W\,\prime}(M_\mathrm{W}^2)\}$,

  similar expressions for $\delta Z_{AA}, \delta Z_{ZZ}, \delta Z_H, \delta Z_{ff}^{\mathrm{L/R}}$

  ◇ suppression of mixing propagators on particle poles

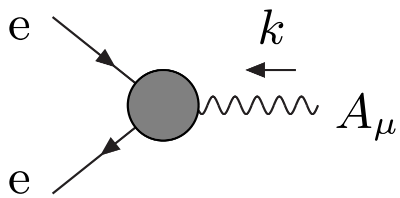
  $\hookrightarrow$ fixes non-diagonal constants $\delta Z_{AZ}, \delta Z_{ZA}, \delta Z_{ff'}^{\mathrm{L/R}} \;\; (f \neq f')$

  Note: problems for unstables particles beyond one loop

  (field-renormalization constants become complex)

Renormalization conditions: (continued)

- Charge renormalization: define e in Thomson limit



$$\xrightarrow{k\to 0} \quad \mathrm{i}e\gamma_\mu \quad \text{for on-shell electrons}$$

$\Rightarrow$ $e =$ elementary charge of classical electrodynamics

fine-structure constant $\alpha(0) = \dfrac{e^2}{4\pi} = 1/137.03599976$

Gauge invariance relates δZ_e to photon wave-function renormalization:

$$\delta Z_e \;=\; -\frac{1}{2}\delta Z_{AA} - \frac{s_{\mathrm{W}}}{2c_{\mathrm{W}}}\delta Z_{ZA}$$

- Quark-field and CKM-matrix renormalization $\to$ fixes $\delta Z^{\mathrm{L/R}}_{qq'}, \delta V_{ij}$

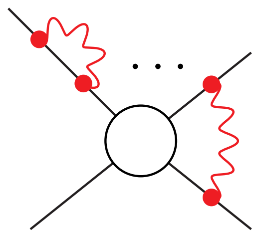
  rotation to mass eigenstates;
  CKM part requires a careful (non-trivial) investigation
  of mixing self-energies, mass eigenstates, LSZ reduction, etc.

General result: all renormalization constants can be obtained from self-energies.

6.3 IR divergences and photon bremsstrahlung

Consider processes with charged external particles, e.g., $\mathrm{e}^+\mathrm{e}^- \to \mu^+\mu^-$

- Virtual corrections: loop diagrams

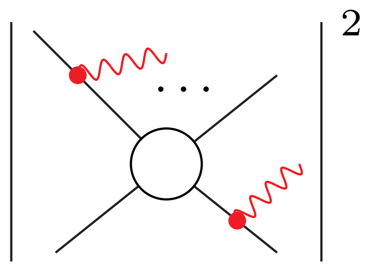


IR divergences from soft virtual photons ($q \to 0$)

$$\int \frac{\mathrm{d}^4 q \ldots}{(q^2 - m_\gamma^2)(2qp_1)(2qp_2)} \to C \ln(m_\gamma)$$

- "Real" corrections: photon bremsstrahlung

$$\int \frac{\mathrm{d}^3 \mathbf{q}}{2q_0}$$



IR divergences from soft real photons ($\mathbf{q} \to 0$)

$$\int \frac{\mathrm{d}^3 \mathbf{q} \ldots}{\sqrt{\mathbf{q}^2 + m_\gamma^2}(2qp_1)(2qp_2)} \to -C \ln(m_\gamma)$$

Bloch–Nordsieck theorem:

IR divergences of virtual and real corrections cancel in the sum

$\hookrightarrow$ virtual and soft-photonic corrections cannot be discussed separately

$\leftrightarrow$ related to limited experimental resolution of soft photons

$\Rightarrow$ Cross-section predictions necessarily depend on treatment of photon emission (energy and angular cuts)

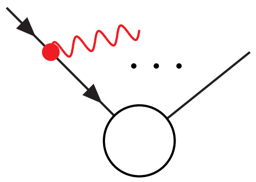
Separation of soft and hard photons:

Why? cancellation of $\ln(m_\gamma)$ terms delicate in practice, but terms are universal

- soft photons, $m_\gamma < E_\gamma < \Delta E \ll Q$ = typical scale of the process
  $\hookrightarrow$ correction is universal factor δ_{soft} to Born cross section
  relatively simple analytical expression with explicit $C\ln(\Delta E/m_\gamma)$ terms
- hard photons, $E_\gamma > \Delta E$
  $\hookrightarrow$ Monte Carlo integration of full radiative process, but with $m_\gamma = 0$
  $-C\ln(\Delta E)$ terms emerge numerically

$\ln(\Delta E)$ contributions cancel numerically in sum for small ΔE up to $\mathcal{O}(\Delta E/E)$

Calculation of soft-photon factor:



$$= \quad A(p-q)\,\frac{\mathrm{i}(\not{p}-\not{q}+m_f)}{(p-q)^2-m_f^2}(\mathrm{i}Q_f e)\not{\varepsilon}^*_\gamma u_f(p)$$

$$\underset{q\to 0}{\sim} \quad -Q_f e\,\frac{\varepsilon^*_\gamma p}{qp}A(p)u_f(p) \;=\; -Q_f e\,\frac{\varepsilon^*_\gamma p}{qp}\mathcal{M}_{\mathrm{Born}}$$

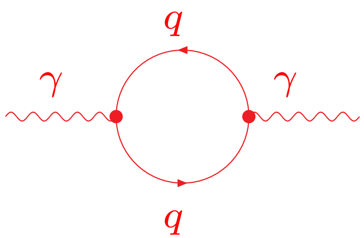
"Eikonal factorization" holds for all charged particles (spin $0, \frac{1}{2}, 1$)

$$\Rightarrow \quad \delta_{\mathrm{soft}} = -\frac{\alpha}{2\pi^2}\int_{m_\gamma<q_0<\Delta E}\frac{\mathrm{d}^3\mathbf{q}}{2q_0}\sum_{i,j}\frac{(\pm Q_i)(\pm Q_j)(p_i p_j)}{(qp_i)(qp_j)}$$

(i = particle with charge Q_i incoming(+) or outgoing ($-$))

6.4 The universal radiative corrections $\Delta\alpha$ and $\Delta\rho$

Running electromagnetic coupling $\alpha(s)$:



becomes sensitive to unphysical quark masses m_q
for $|s|$ in GeV range and below (non-perturbative regime)
$\hookrightarrow$ charge-renormalization constant δZ_e sensitive to m_q

Solution: fit hadronic part of $\Delta\alpha(s) = -\mathrm{Re}\{\Sigma^{AA}_{\mathrm{T,ren}}(s)/s\}$ and thus of δZ_e

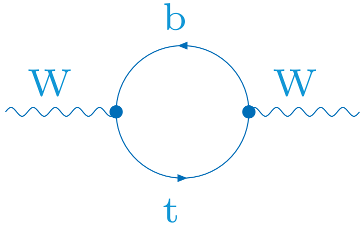
via dispersion relations to $R(s) = \dfrac{\sigma(\mathrm{e^+e^-} \to \text{hadrons})}{\sigma(\mathrm{e^+e^-} \to \mu^+\mu^-)}$ Jegerlehner et al.

$\Rightarrow$ Running elmg. coupling: $\alpha(s) = \dfrac{\alpha(0)}{1 - \Delta\alpha_{\mathrm{ferm}\neq\mathrm{top}}(s)}$

Leading correction to the ρ-parameter:

mass differences in fermion doublets break custodial SU(2) symmetry

$\hookrightarrow$ large effects from bottom–top loops in W self-energy Veltman ’77



$$\Delta\rho_{\mathrm{top}} \sim \frac{\Sigma^{ZZ}_{\mathrm{T}}(0)}{M_{\mathrm{Z}}^2} - \frac{\Sigma^{WW}_{\mathrm{T}}(0)}{M_{\mathrm{W}}^2} \sim \frac{3G_\mu m_{\mathrm{t}}^2}{8\sqrt{2}\pi^2}$$

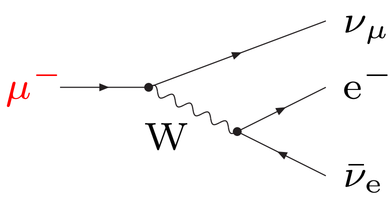
Precision calculation of M_W via μ decay

$\hookrightarrow$ M_W as function of $\alpha(0)$, G_μ, M_Z and the quantity Δr

$$M_\mathrm{W}^2\left(1-\frac{M_\mathrm{W}^2}{M_\mathrm{Z}^2}\right) = \frac{\pi\alpha(0)}{\sqrt{2}G_\mu}(1+\Delta r)$$

Δr comprises quantum corrections to μ decay
(beyond electromagnetic corrections in Fermi model)

Lowest order:

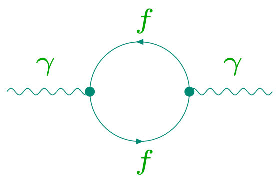


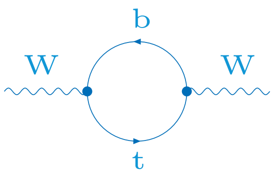
$\mathcal{O}(\alpha)$ corrections:

Sirlin '80, Marciano, Sirlin '80

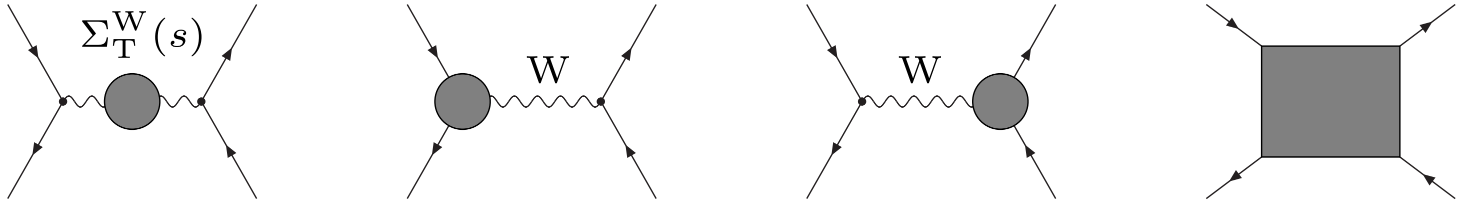
$$\Delta r_{1-\mathrm{loop}} = \Delta\alpha(M_\mathrm{Z}^2) - \frac{c_\mathrm{W}^2}{s_\mathrm{W}^2}\Delta\rho_\mathrm{top} + \Delta r_\mathrm{rem}(M_\mathrm{H})$$

$\sim 6\%$	$\sim 3\%$	$\sim 1\%$
$\alpha\ln(m_f/M_\mathrm{Z})$	$G_\mu m_\mathrm{t}^2$	$\alpha\ln(M_\mathrm{H}/M_\mathrm{Z})$



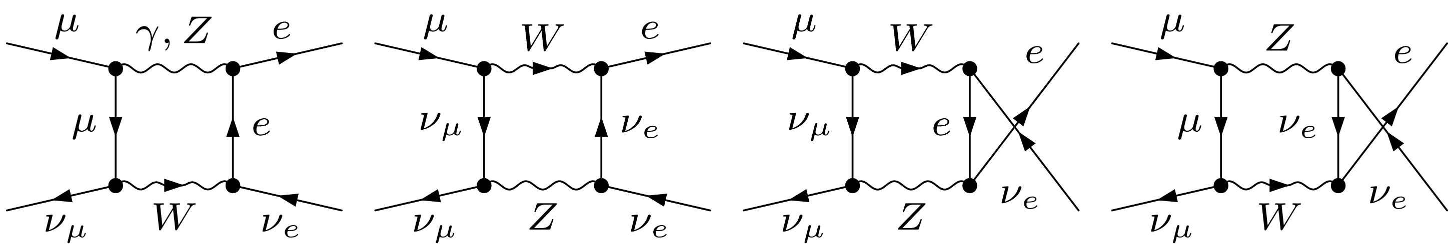


Virtual correction – 1-loop diagrams:

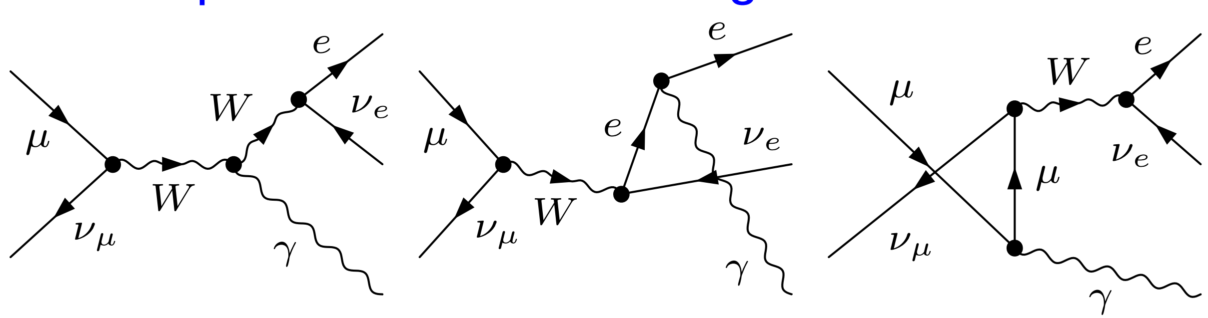


W self-energy — $Wl\nu_l$ vertex correction — box diagrams

e.g.:

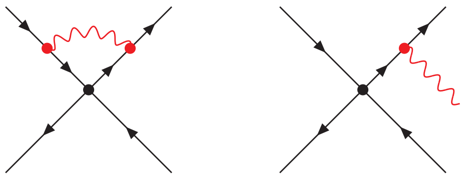


Real correction – 1-photon bremsstrahlung:



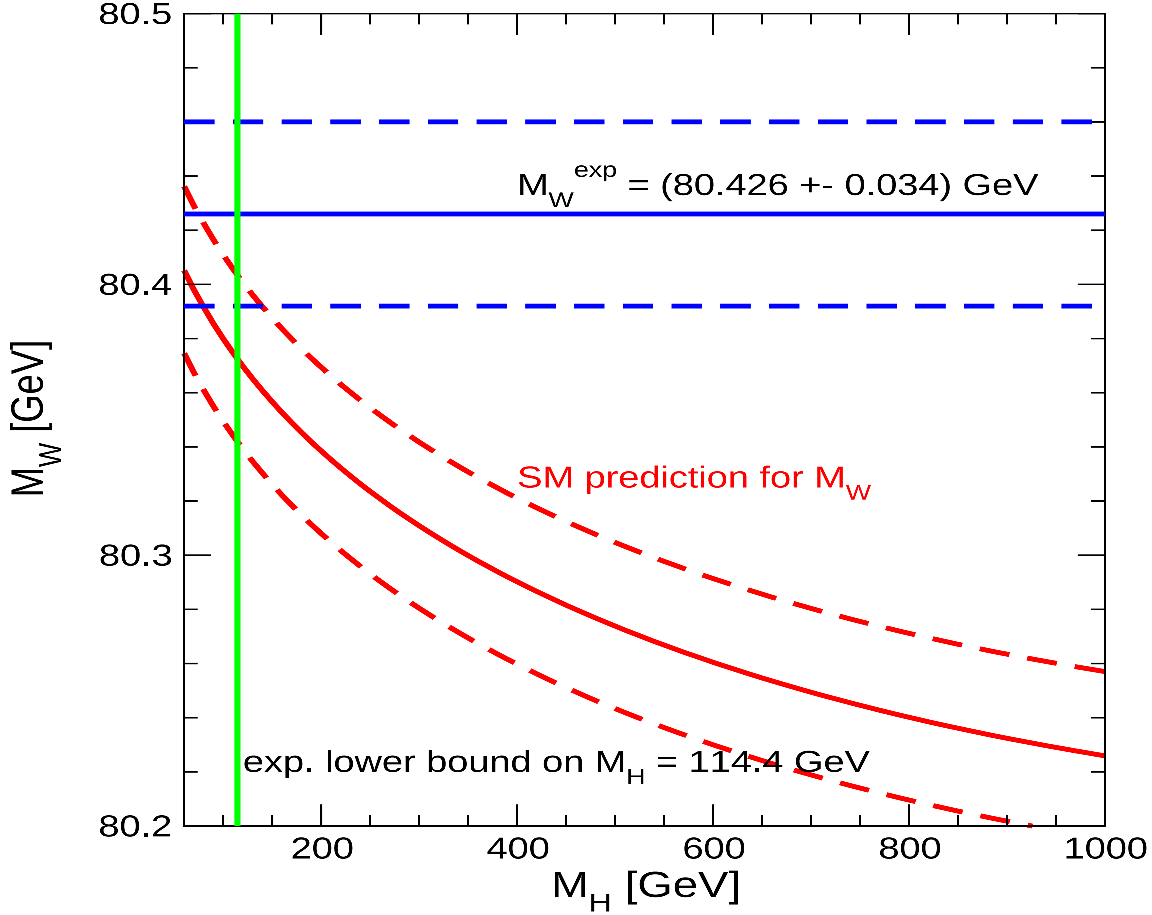
Consistent use of G_μ:

Photonic QED corrections are treated in the Fermi model and subtracted from Δr



etc.

State-of-the-art prediction of M_{W} from muon decay:



Hollik et al. '03

Theoretical uncertainty:

status '00: $\Delta M_{\mathrm{W}} \sim 6\,\mathrm{MeV}$

status '06: $\Delta M_{\mathrm{W}} \sim 4\,\mathrm{MeV}$

Experimental error:

status '06: $\Delta M_{\mathrm{W}} \sim 29\,\mathrm{MeV}$

ILC(?): $\Delta M_{\mathrm{W}} \sim 7\,\mathrm{MeV}$

Prediction includes:

- full electroweak corrections of $\mathcal{O}(\alpha)$ (1-loop level)
- full electroweak corrections of $\mathcal{O}(\alpha^2)$ (2-loop level)
  (v.Ritbergen,Stuart '98; Seidensticker,Steinhauser '99;
  Freitas,Hollik,Walter,Weiglein '00-'02; Awramik,Czakon '02/'03; Onishchenko,Veretin '02)
- various improvements by universal corrections to ρ-parameter

Literature

- Textbooks:
  - Böhm/Denner/Joos: "Gauge Theories of the Strong and Electroweak Interaction"
  - Cheng/Li: "Gauge Theory of Elementary Particle Physics"
  - Collins: "Renormalization"
  - Ellis/Stirling/Webber: "QCD and Collider Physics"
  - Itzykson/Zuber: "Quantum Field Theory"
  - Peskin/Schroeder: "An Introduction to Quantum Field Theory"
  - Weinberg: "The Quantum Theory of Fields, Vol. 1: Foundations";
    "The Quantum Theory of Fields, Vol. 2: Modern Applications"
- Some reviews on dedicated topics:
  - Z-boson production at LEP1/SLC:
    "Z Physics at LEP1", eds. G. Altarelli, R. Kleiss and C. Verzegnassi (CERN 89-08), Vol. 1;
    "Reports of the Working Group on Precision Calculations for the Z Resonance", eds. D. Bardin, W. Hollik and G. Passarino (CERN 95-03);
    D.Y. Bardin, M. Grünewald and G. Passarino, hep-ph/9902452.
  - W-pair production at LEP2:
    W. Beenakker *et al.*, in "Physics at LEP2", eds. G. Altarelli, T. Sjöstrand and F. Zwirner (CERN 96-01, Geneva, 1996), Vol. 1, p. 79 [hep-ph/9602351];
    M. W. Grünewald *et al.*, in "Reports of the Working Groups on Precision Calculations for LEP2 Physics", eds. S. Jadach, G. Passarino and R. Pittau (CERN 2000-009), p. 1 [hep-ph/0005309].
  - SM Higgs physics:
    A. Djouadi, hep-ph/0503172 and references therein

Literature (continued)

- Experimental results widely taken from:
  - LEPEWWG: http://lepewwg.web.cern.ch/LEPEWWG/
  - LEPHiggs: http://lephiggs.web.cern.ch/LEPHIGGS/www/Welcome.html
- (Incomplete) list of articles on techniques for radiative corrections:
  - one-loop integrals:
    G. 't Hooft and M. Veltman, Nucl. Phys. B 153 (1979) 365;
    G. Passarino and M. Veltman, Nucl. Phys. B 160 (1979) 151;
    W. Beenakker and A. Denner, Nucl. Phys. B 338 (1990) 349;
    A. Denner, U. Nierste and R. Scharf, Nucl. Phys. B 367 (1991) 637;
    A. Denner and S. Dittmaier, Nucl. Phys. B 734 (2006) 62 and references therein
  - renormalization of the electroweak SM:
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    M. Böhm, W. Hollik and H. Spiesberger, Fortsch. Phys. 34 (1986) 687;
    W. F. Hollik, Fortsch. Phys. 38 (1990) 165;
    A. Denner, Fortsch. Phys. 41 (1993) 307;
    A. Denner, S. Dittmaier and G. Weiglein, Nucl. Phys. B 440 (1995) 95
  - IR structure of photon radiation:
    D. R. Yennie, S. C. Frautschi and H. Suura, Annals Phys. 13 (1961) 379.