# Towards a theory of dissipative Dark Matter

*Garance Lankester—Broche*[1,2,3]
*Assoc.-Prof. Josef Pradler*[1,2]

JCAP03(2026)034 [arXiv:2509.12317]
arXiv:26XX.XXXX

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Supported by:

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NLO-DM

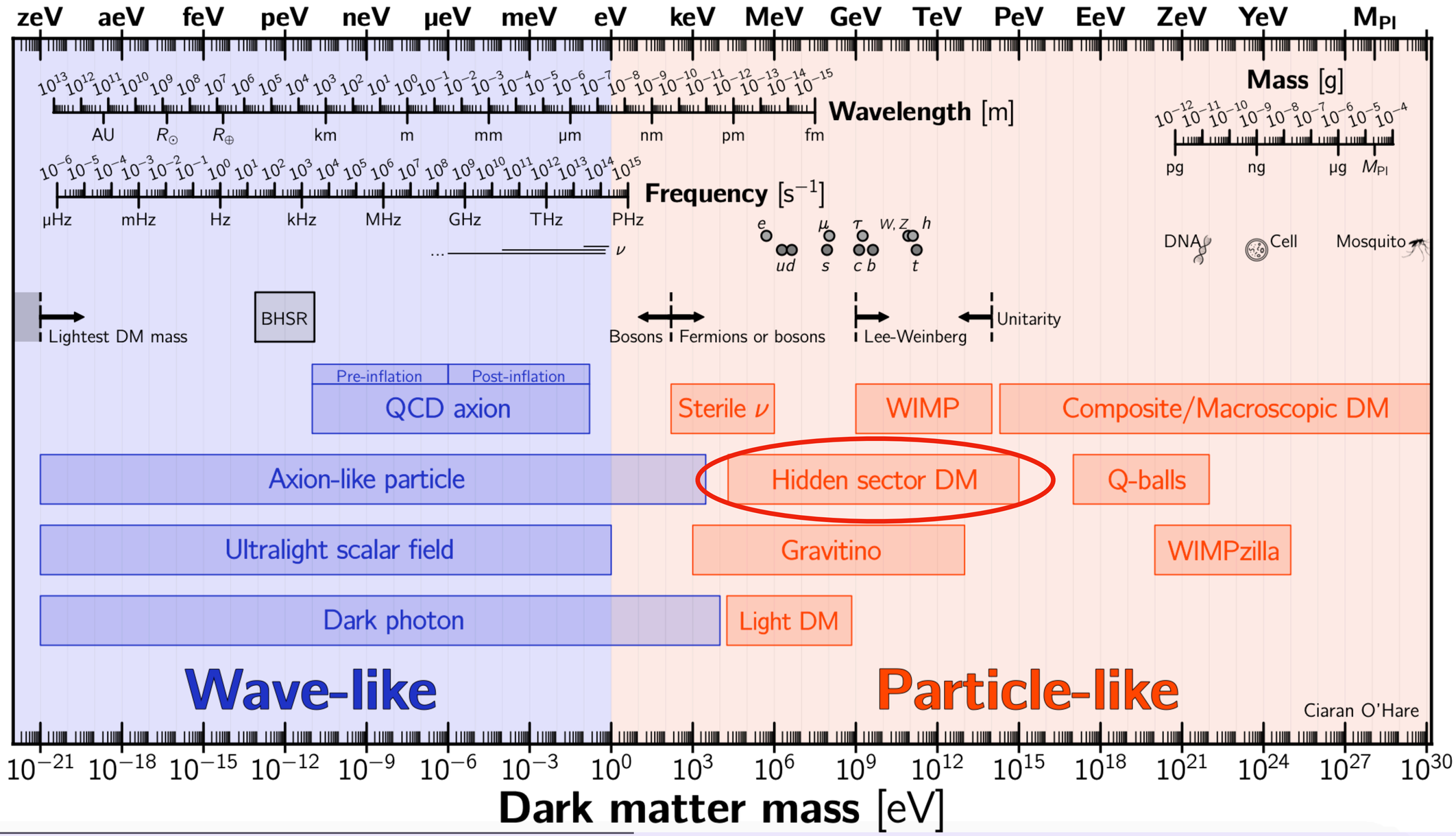
1. University of Vienna, Faculty of Physics 2.MBI - Marietta-Blau-Institut für Teilchenphysik der Österreichischen Akademie der Wissenschaften
3. University of Vienna, Vienna Doctoral School in Physics

# What is a good DM candidate?

Dark Matter is only observed through its gravitational fingerprints from **galactic scales** to the **CMB**

Gravitational observations do not select a preferred DM mass scale: huge landscape of candidates

# What is a good DM candidate ?

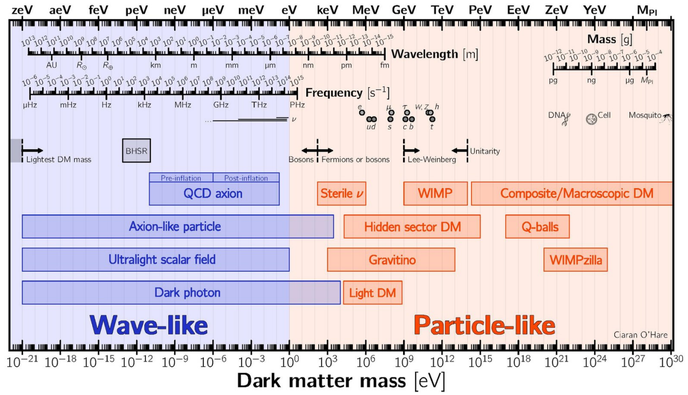


# What is a good DM candidate ?

Dark Matter is only observed through its gravitational fingerprints from **galactic scales** to the **CMB**

Huge landscape of DM candidates

→ This talk: Hidden Sector DM




Yet, a simple cosmological behavior during structure formation:

- Cold
- Very weakly coupled to visible matter
- Mostly collisionless

# Self-Interacting Dark Matter (SIDM)

$\Lambda CDM$

At large scales, is highly successful, while on galactic and sub-galactic scales, several persistent tensions remain

**Possible** small-scale deviations from collisionless DM



Review: Rev. Mod. Phys. **97**, 045004

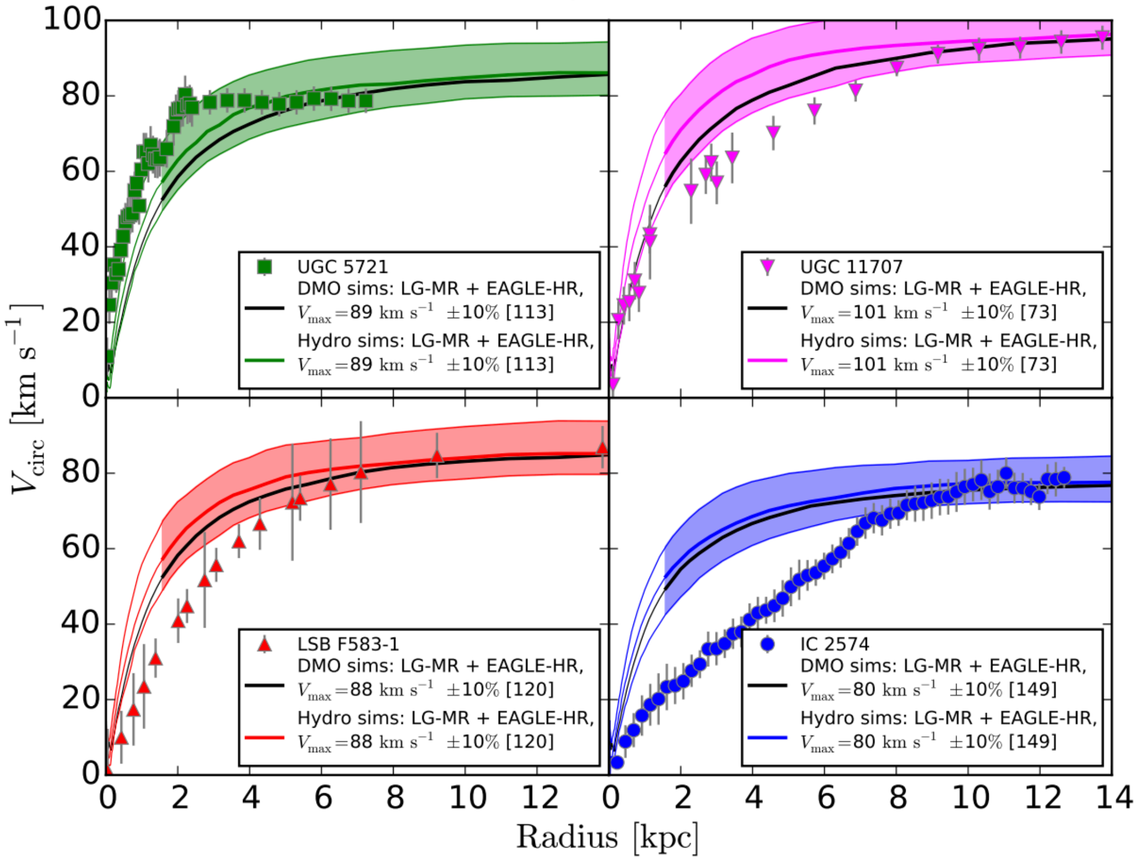
# Self-Interacting Dark Matter (SIDM)

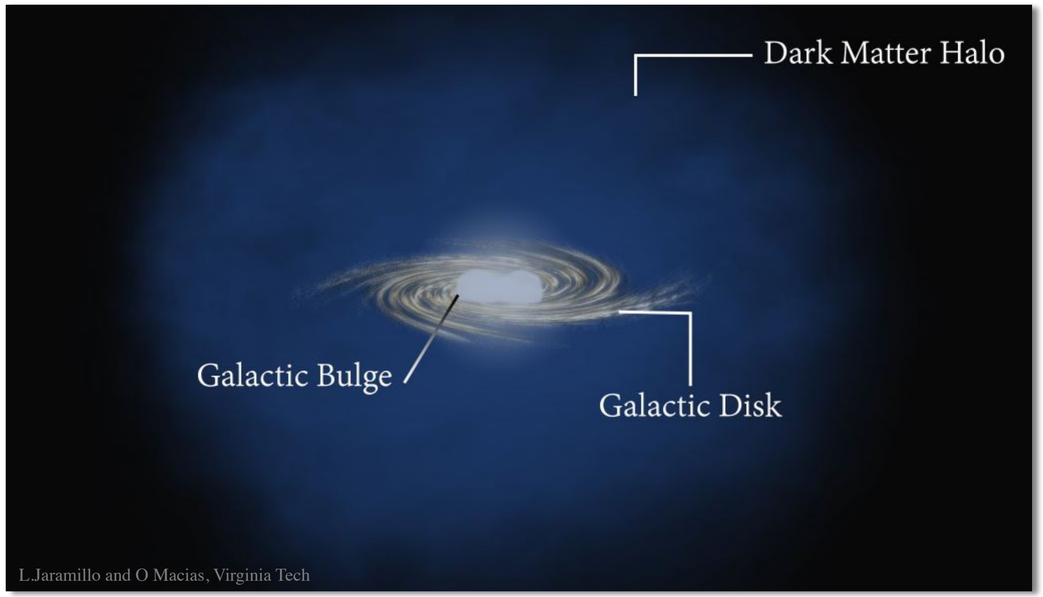
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Example diversity in rotation curves[2]






Review: Rev. Mod. Phys. **97**, 045004 [2] arXiv:1504.01437

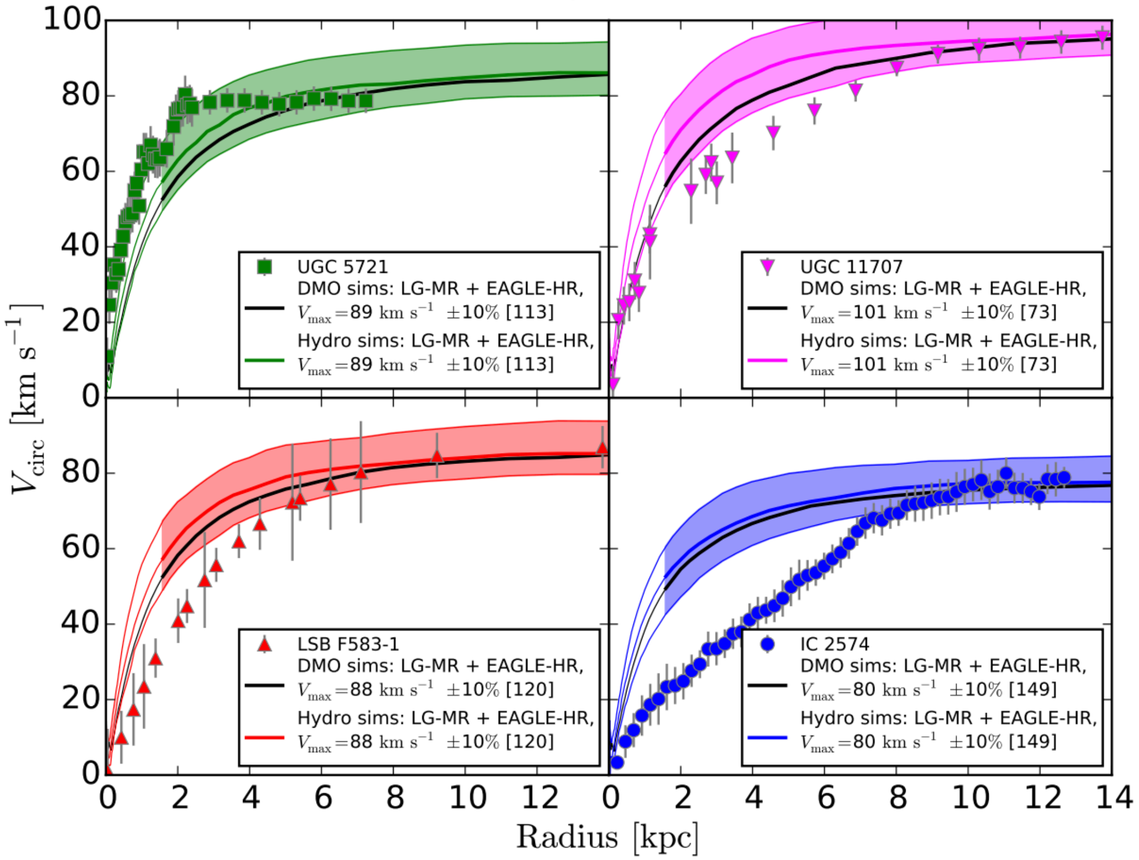
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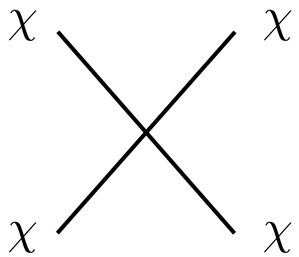
**Possible** small-scale deviations from collisionless DM

Example diversity in rotation curves[2]




**SIDM**[3]

Hidden sector:



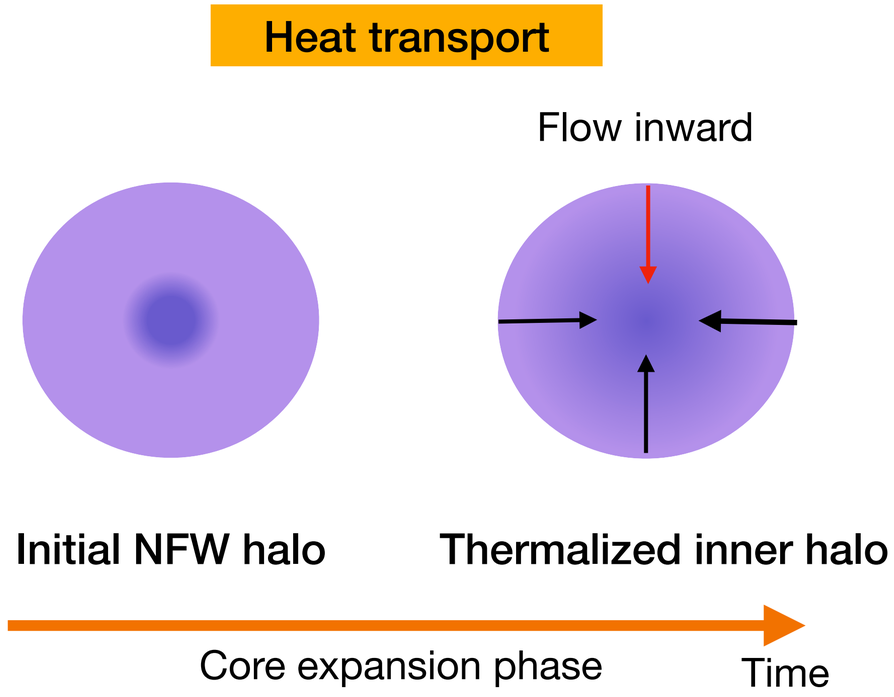

**Heat transfer through DM collisions**

Thermalization of the halo's inner region

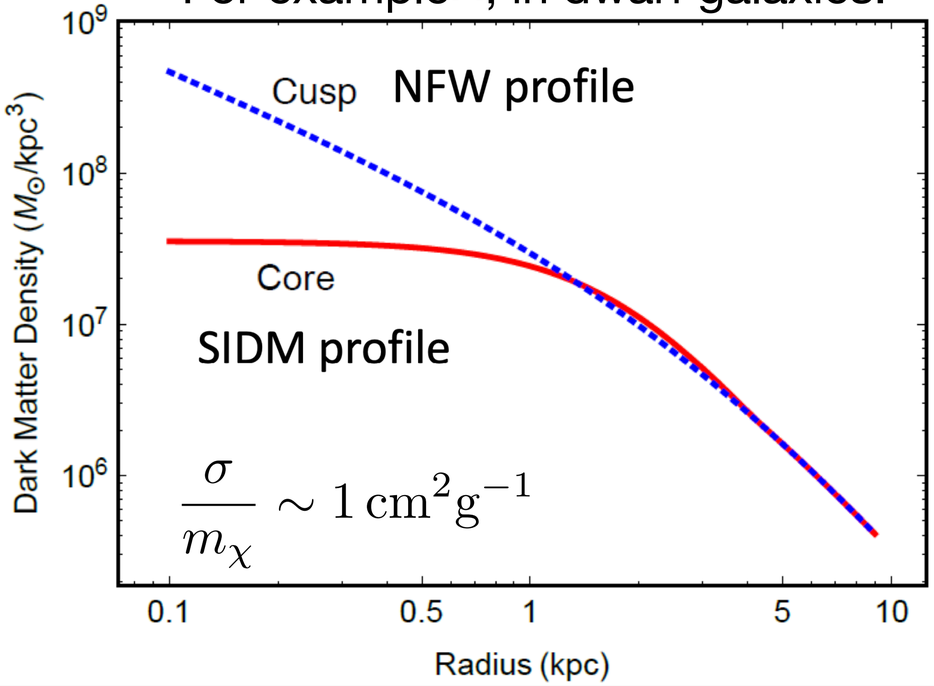


Review: Rev. Mod. Phys. **97**, 045004 [2] arXiv:1504.01437 [3] original paper: Phys.Rev.Lett. **84** (2000) 3760-3763

# Self-Interacting-Dark-Matter (SIDM)




For example[2], in dwarf galaxies:




Once thermalized[1]:

$$\rho_{\rm SIDM} \propto \rho_0 \exp\left(-\frac{\phi_{\rm tot}(r) - \phi_{\rm tot}(0)}{kT}\right)$$

$$\phi_{\rm tot} = \phi_{\rm DM} + \phi_{\rm baryon}$$



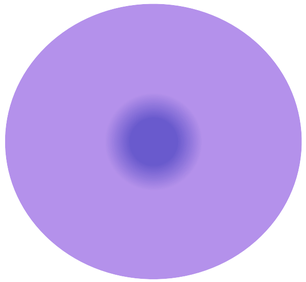
[1]. Review: Rev. Mod. Phys. **97**, 045004, [2] arXiv:1705.02358

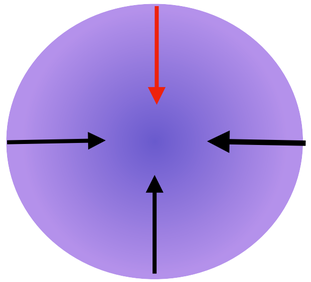
# Self-Interacting-Dark-Matter (SIDM)

Flow inward

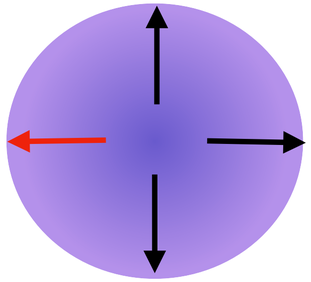
**Heat transport**

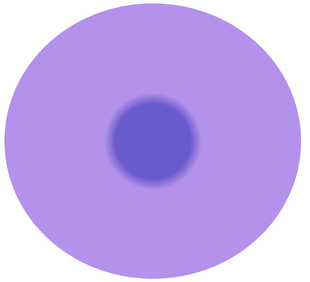
Flow outward





Self-gravitating system





**Initial NFW halo**

**Thermalized inner halo**

**Gravothermal collapse**

Core expansion

Core collapse

Time

Depending on the history, concentration, cross-section…

For example:

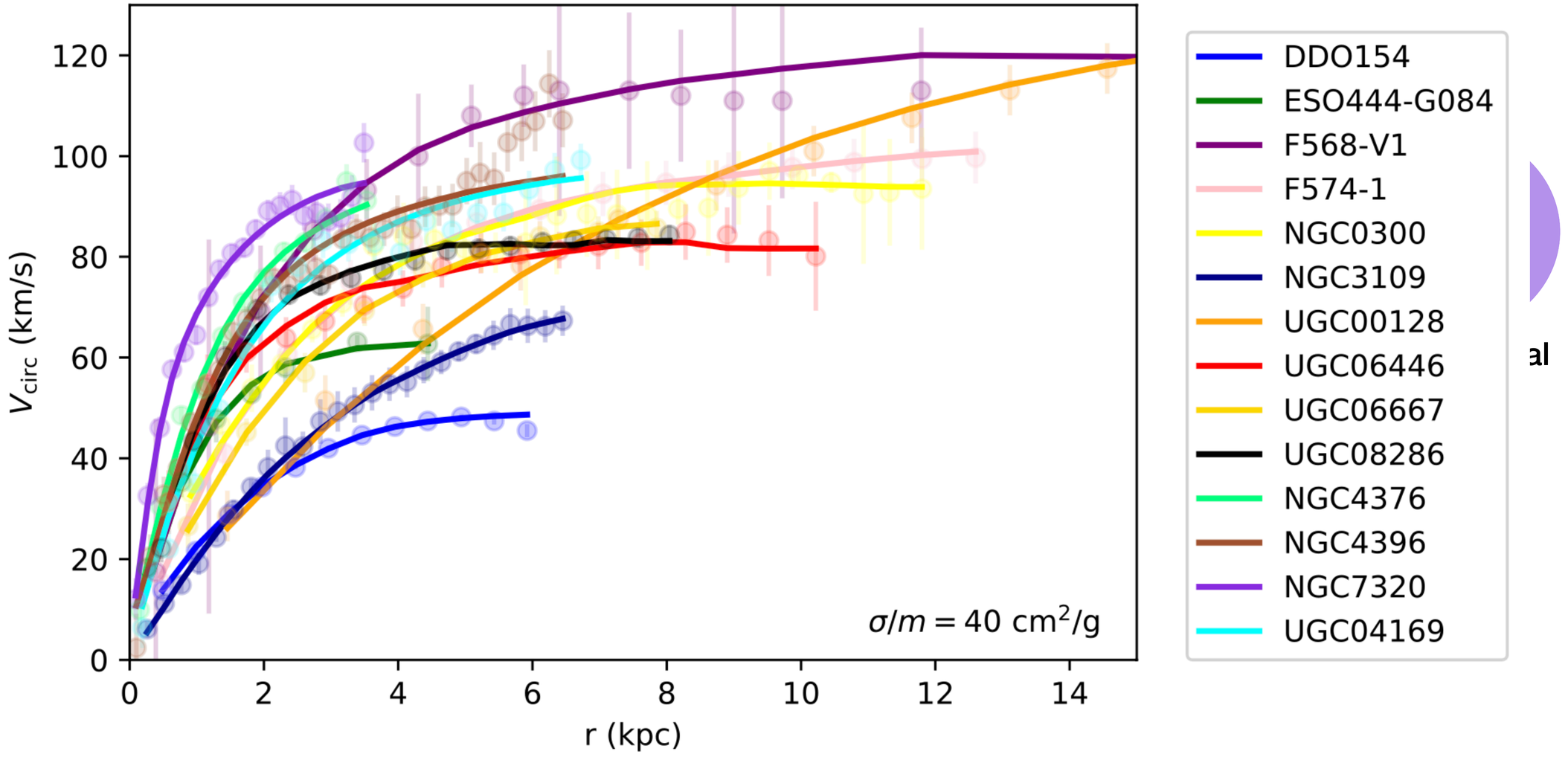
$$\frac{\sigma}{m_\chi} \sim \mathcal{O}(1)\,\mathrm{cm}^2/g : t_{\mathrm{collapse}} > t_{\mathrm{halo}}$$

$$\frac{\sigma}{m_\chi} \sim \mathcal{O}(50)\,\mathrm{cm}^2/g : t_{\mathrm{collapse}} \sim 10\,\mathrm{Gyr}$$



ArXiv:2407.15005

# Self-Interacting-Dark-Matter (SIDM)

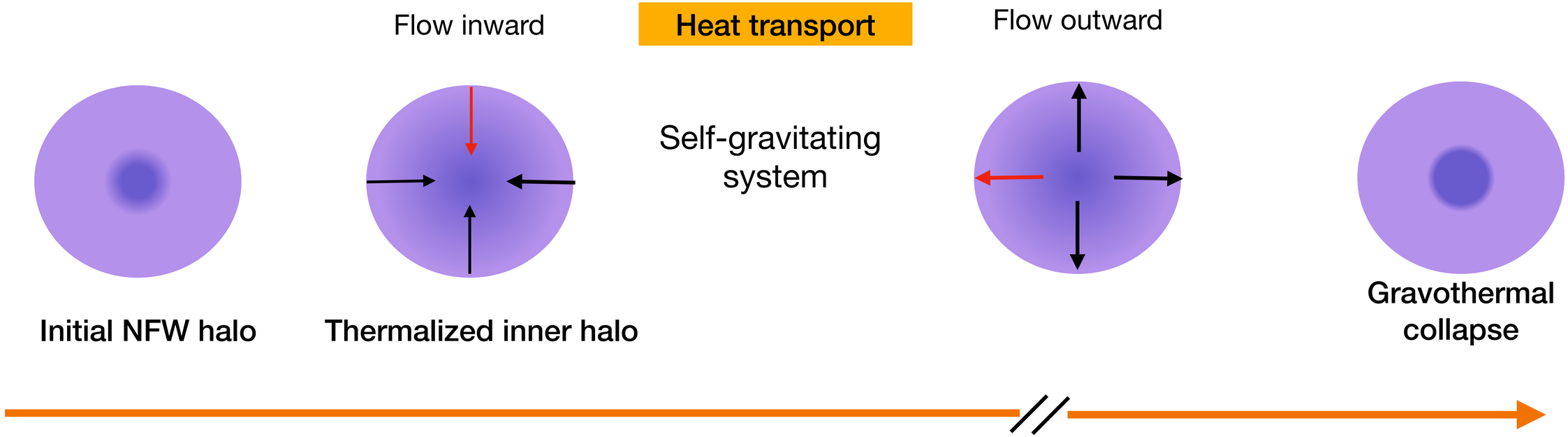



Core expansion phase // Core collapse phase Time

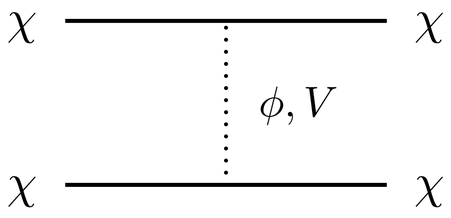


ArXiv:2407.15005

# Self-Interacting-Dark-Matter (SIDM)




Light mediator[1]




velocity-dependent cross section

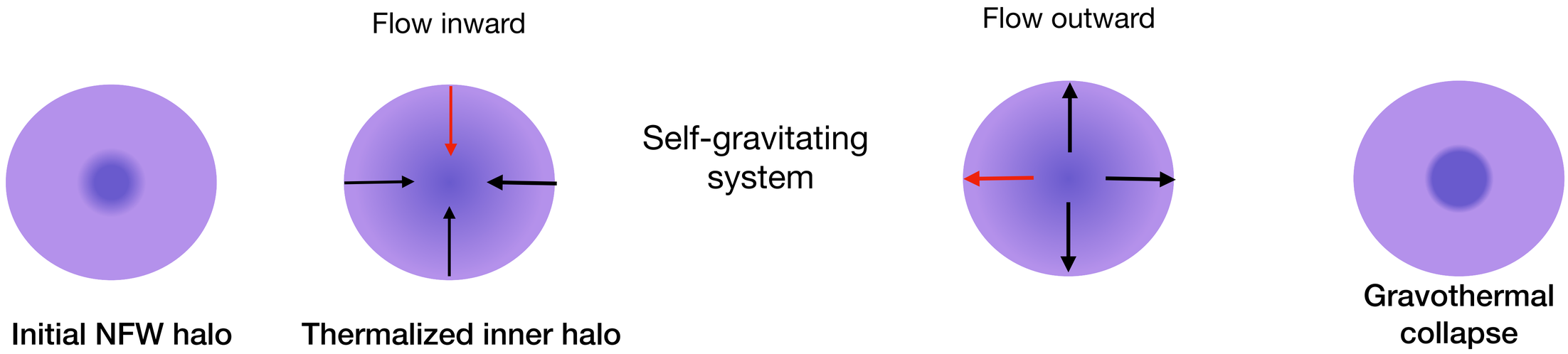
The constraints are scale dependent, each with their own phenomenology[2]:

Dwarf galaxy scales $v_i \sim 10$ km/s  Galaxy scales $v_i \sim 100$ km/s  Galaxy cluster scales $v_i \sim 1000$ km/s



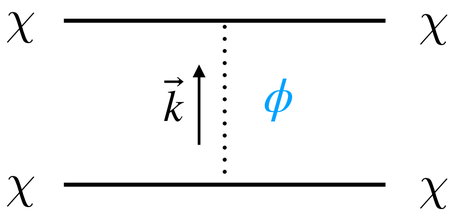
[1] arXiv:1302.3898 [2] Rev. Mod. Phys. 97, 045004 (2025)

# Self-Interacting-Dark-Matter (SIDM)



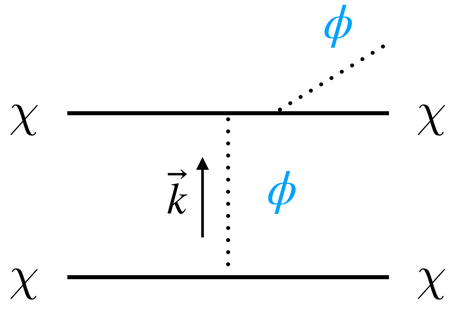

**dSIDM[1]**

**Heat transport**




Light particles

**Cooling effect**



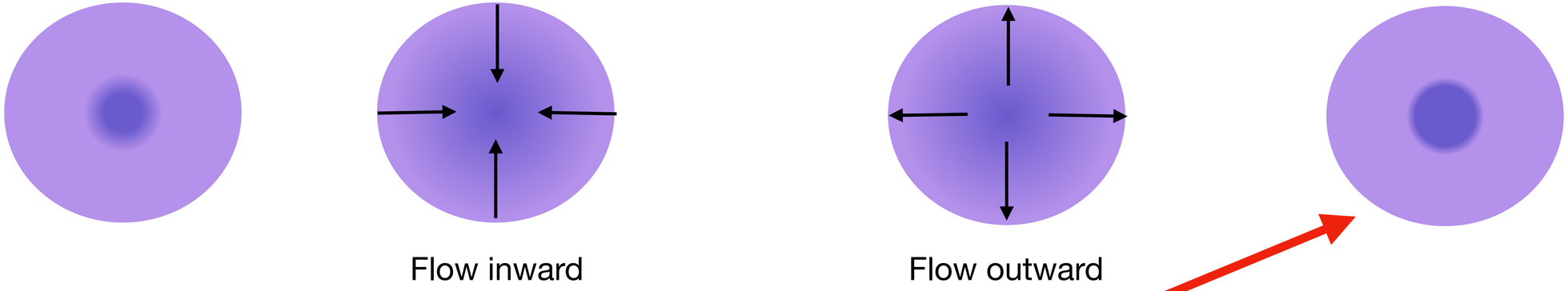



1] Examples of simple dissipative dark sectors: arXiv: 0810.5126 arXiv:1807.04750

# Dissipative SIDM (dSIDM)

**SIDM**

**Heat transport**




**dSIDM**

**Cooling**



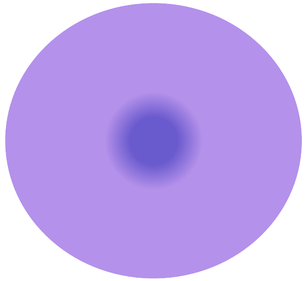


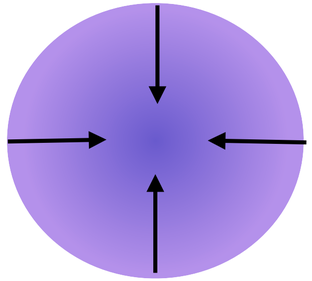
[1] ArXiv:1912.06757

# Dissipative SIDM (dSIDM)

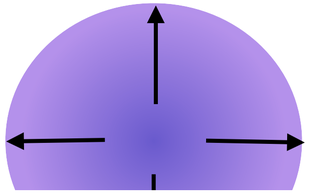
**SIDM**

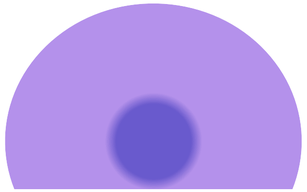
**Heat transport**





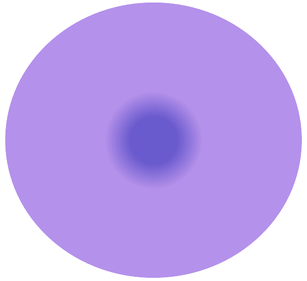
Flow inward

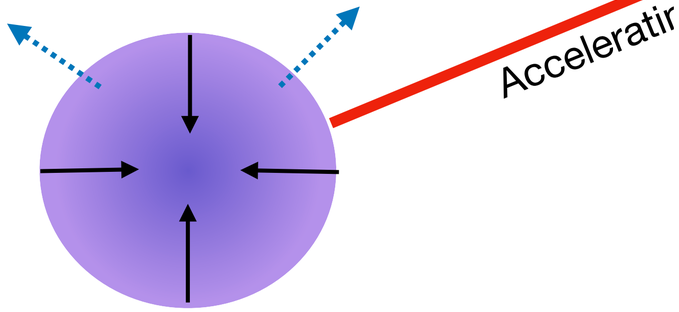


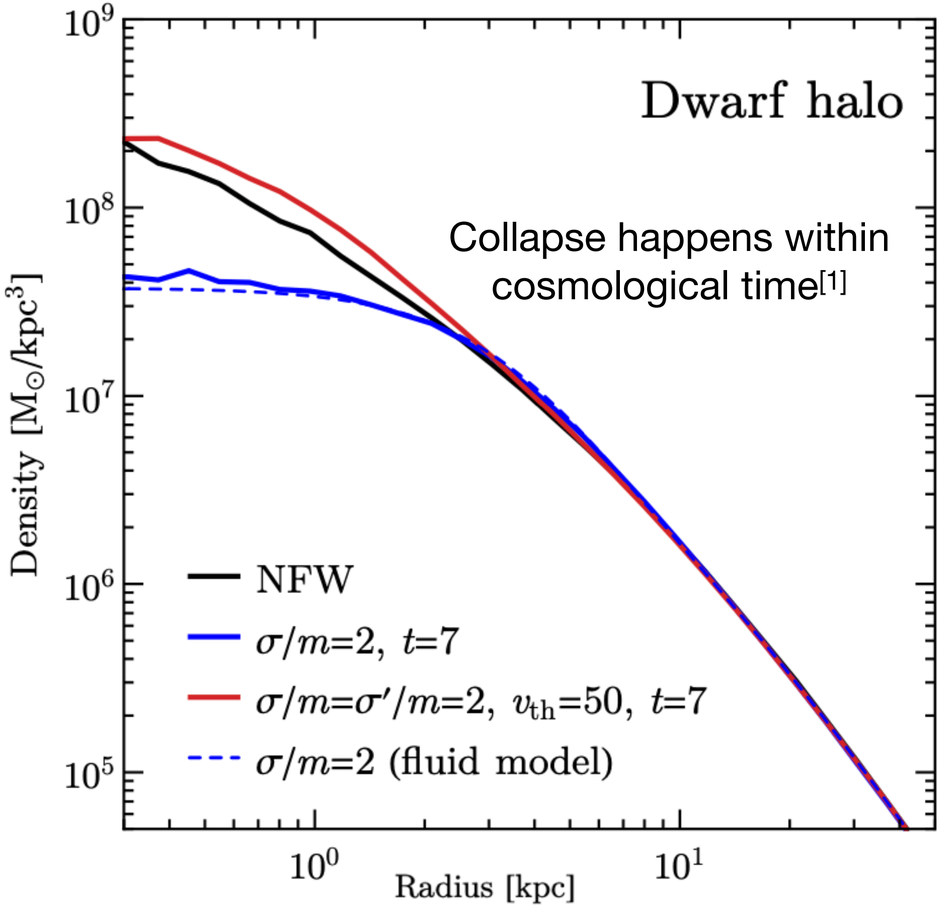


**dSIDM**

**Cooling**





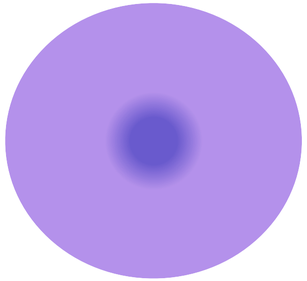



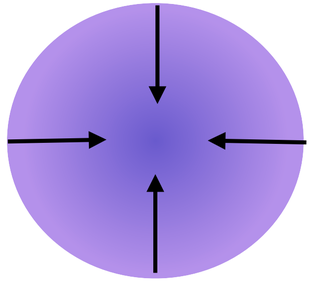

[1] ArXiv:1912.06757

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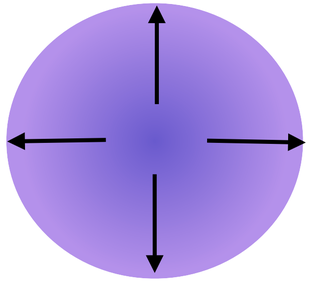
**SIDM**

Heat transport





Flow inward



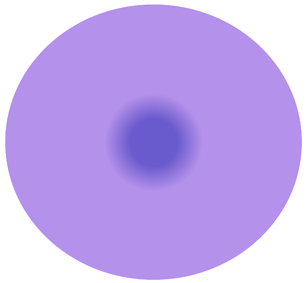
Flow outward

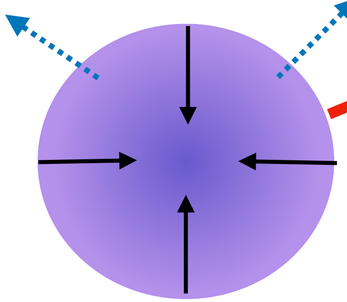


Accelerating gravothermal collapse time

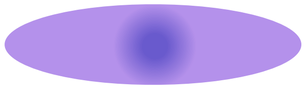
**dSIDM**

Cooling

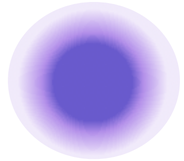




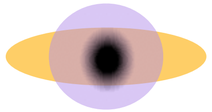
Large phenomenologic spectrum



Formation of dark disks[4]



Formation of compact substructure[3]



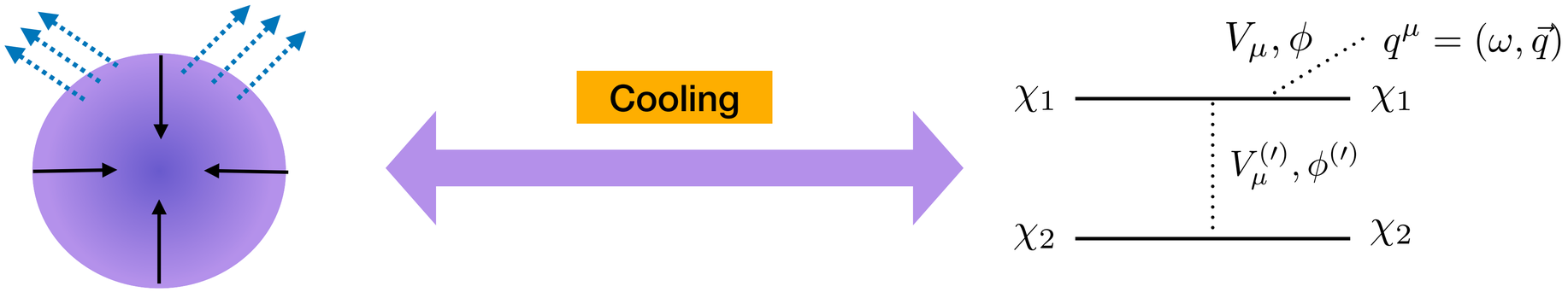
Evolution of supermassive black holes[2]



[1] ArXiv:1912.06757 [2] arXiv:2410.06252 [3] arXiv:1412.1839 [4] arXiv:1303.3271

# dSIDM: microphysics

JCAP03(2026)034
arXiv26XX.XXXXX




**This talk:** how the microscopic properties of the DM candidates shape the resulting cooling in minimal dissipative scenarios

# dSIDM Models

JCAP03(2026)034
arXiv26XX.XXXXX

DM candidates
- Fermion
- Scalar

$\chi_1\chi_2 \to \chi_1\chi_2 V \quad \tilde{\chi}_1\tilde{\chi}_2 \to \tilde{\chi}_1\tilde{\chi}_2\phi \quad \chi_1\chi_2 \to \chi_1\chi_2\phi$

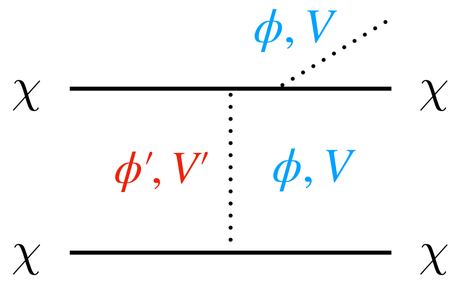
$S_1 S_2 \to S_1 S_2 V \quad \tilde{S}_1\tilde{S}_2 \to \tilde{S}_1\tilde{S}_2\phi \quad S_1 S_2 \to S_1 S_2\phi$

Emission/Mediation:
- Gauge vector $V$
- Scalar $\phi$

Including massive emission

Two mediation regimes:

Long-range Short-range



# dSIDM Models

JCAP03(2026)034
arXiv26XX.XXXXX

DM candidates
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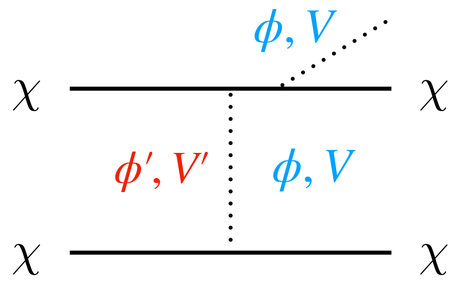
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Emission/Mediation:
- Gauge vector $V$
- Scalar $\phi$

Including massive emission

Two mediation regimes:

Long-range Short-range




$$\mathcal{L}^{V}_{\rm int} = g \sum_{j=1,2} \left\{ -Q_{\chi_j} \overline{\chi}_j \not{V} \chi_j + i Q_{S_j} V^\mu \left[ (\partial_\mu S_j^\dagger) S_j - S_j^\dagger \partial_\mu S_j \right] \right\} + g^2 \sum_{j=1,2} Q^2_{S_j} S_j^\dagger S_j V_\mu V^\mu$$

$$\mathcal{L}^{\phi}_{\rm int} = \sum_{j=1,2} \left\{ -y_j \overline{\chi}_j \chi_j \phi - \frac{1}{2}\tilde{y}_j \tilde{\chi}_j^T C \tilde{\chi}_j \phi - A_j |S_j|^2 \phi - \frac{1}{2}\tilde{A}_j \tilde{S}_j^2 \phi - \frac{1}{2}\lambda_j |S_j|^2 \phi^2 - \frac{1}{4}\tilde{\lambda}_j \tilde{S}_j^2 \phi^2 \right\}$$

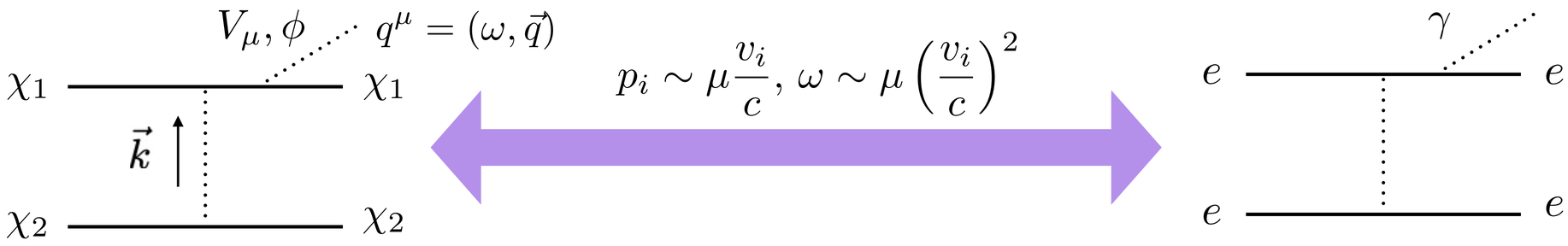
$$\mathcal{L}^{\rm self}_{\rm int} = -\frac{1}{3!}A_\phi \phi^3 - \frac{1}{4!}\lambda_\phi \phi^4 - \frac{1}{(1+3\delta_{S_1 S_2})}\lambda_S |S_1|^2 |S_2|^2 - \frac{1}{4(1+2\delta_{\tilde{S}_1\tilde{S}_2})!}\tilde{\lambda}_S \tilde{S}_1^2 \tilde{S}_2^2$$

+ primed couplings for heavy mediators

# Bremsstrahlung

JCAP03(2026)034
arXiv26XX.XXXXX

In DM halos: $v_i \lesssim 100 - 1000\,\mathrm{km.s}^{-1}$




Long range: $m_{\phi,V} \ll |\vec{k}|$

Short range: $m_{\phi',V'} \gg |\vec{k}|$

Non-relativistic scattering:

$$V(r) = \frac{a_1 a_2}{4\pi r}$$

$$V(r) = \frac{a_1' a_2' e^{-m_{\phi,V} r}}{4\pi r}$$

$$r = |x_1 - x_2|$$

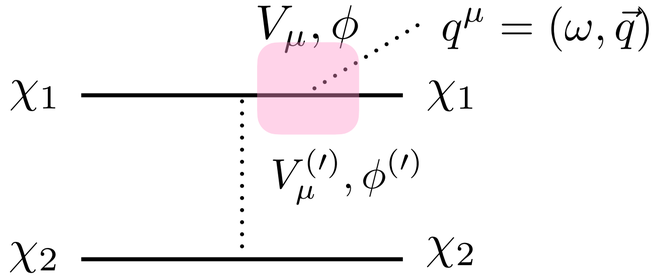
$$\mathcal{T}_{f(V,\phi),i} = -2\pi i \delta(E_i - E_f - \omega) \langle \psi_f | a(\vec{q}, \lambda) V_{\mathrm{int}} | \psi_i \rangle$$

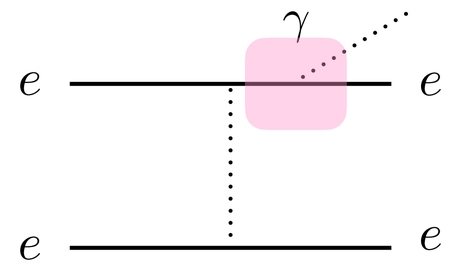


Textbook discussion: Lectures on Quantum Mechanics, Steven Weinberg

# Bremsstrahlung: Multipole expansion

JCAP03(2026)034
arXiv26XX.XXXXX




Long range

Short range

Non-relativistic scattering:

$$V(r) = \frac{a_1 a_2}{4\pi r}$$

$$V(r) = \frac{a_1' a_2' e^{-m_{\phi,V} r}}{4\pi r}$$

$$\mathcal{T}_{f(V,\phi),i} = -2\pi i \delta(E_i - E_f - \omega) \langle \psi_f | a(\vec{q}, \lambda) V_{\rm int} | \psi_i \rangle$$

Initial and final states:

$$H_0 \left| \varphi_{i(f)} \right\rangle = E_{i,f} \left| \varphi_{i(f)} \right\rangle$$

$$H_0 = H_{\rm kin} + V(r)$$

$$\propto a(\vec{q}, \lambda) \vec{A}(\vec{x}_n) \cdot \vec{p}_n \qquad \propto a(\vec{q}, \lambda) \phi(\vec{x}_n) \mathcal{O}_n(v_i)$$

Model-dependent:
- nature of the emitted particle
- DM candidates

Textbook discussion: Lectures on Quantum Mechanics, Steven Weinberg
V-emission: arXiv:2007.06592v3, $\phi$-emission: arXiv:2303.00778v2

# Bremsstrahlung: Multipole expansion

JCAP03(2026)034
arXiv26XX.XXXXX




$$\phi(\vec{x}_n),\ \vec{A}(\vec{x}_n)$$

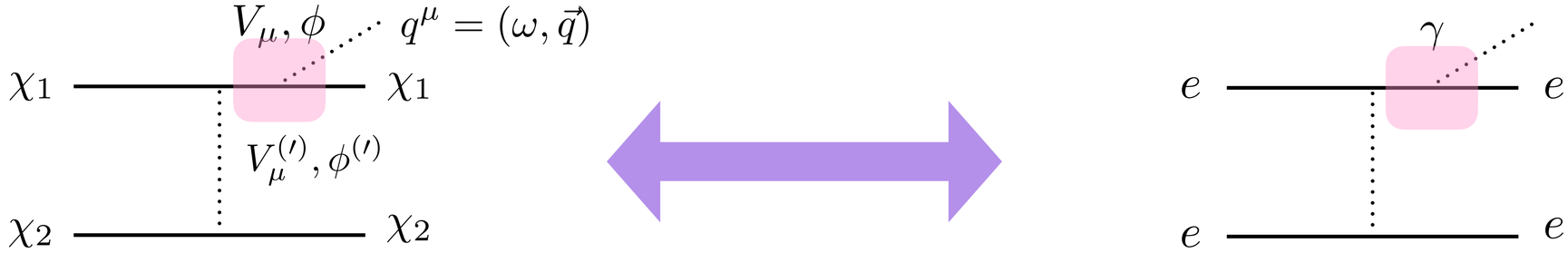
$$\mathcal{T}_{fi} \propto \frac{a_1}{\sqrt{2\omega}} \left\langle \varphi_{\bar{f}} \left| e^{-i\mu \vec{r}\cdot\vec{q}/m_1} \mathcal{O}_1 \right| \varphi_i \right\rangle + \frac{a_2}{\sqrt{2\omega}} \left\langle \varphi_{\bar{f}} \left| e^{i\mu \vec{r}\cdot\vec{q}/m_2} \mathcal{O}_2 \right| \varphi_i \right\rangle$$

$$\vec{r}\cdot\vec{q} \sim v_i \ll 1 \qquad e^{i\vec{q}\cdot\vec{r}} = 1 + i\vec{q}\cdot\vec{r} - \frac{1}{2}(\vec{q}\cdot\vec{r})^2 + \cdots$$

Textbook discussion: Lectures on Quantum Mechanics, Steven Weinberg
V-emission: arXiv:2007.06592v3, $\phi$-emission: arXiv:2303.00778v2

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$$\mathcal{T}_{fi} = -i(2\pi)^4 \delta(E_i - E_f - \omega)\delta^{(3)}(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4 - \vec{q})\mathcal{M}$$

Dipole Quadrupole

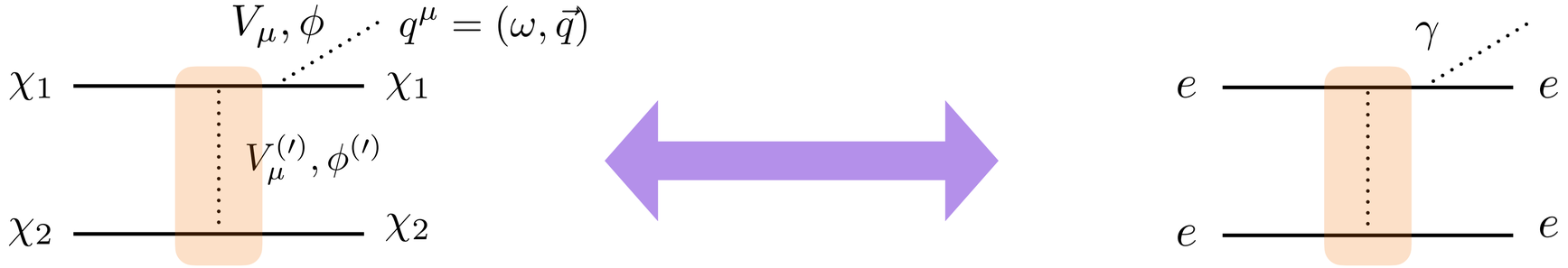
$$\mathcal{M} \simeq \left(\frac{a_1}{m_1} - \frac{a_2}{m_2}\right)\langle f|\mathcal{O}_{\mathcal{D}}|i\rangle + \left(\frac{a_1}{m_1^2} + \frac{a_2}{m_2^2}\right)\langle f|\mathcal{O}_{\mathcal{Q}}|i\rangle$$

Quadrupole emission amplitude is **suppressed** by $\vec{r}\cdot\vec{q} \sim v_i$ compared to dipole

Textbook discussion: Lectures on Quantum Mechanics, Steven Weinberg
V-emission: arXiv:2007.06592v3, $\phi$-emission: arXiv:2303.00778v2

# Bremsstrahlung

JCAP03(2026)034
arXiv26XX.XXXXX




Long range

Short range

Non-relativistic scattering:

$$V(r) = \frac{a_1 a_2}{4\pi r}$$

$$V(r) = \frac{a_1' a_2' e^{-m_{\phi,V} r}}{4\pi r}$$

$$\mathcal{T}_{f(V,\phi),i} = -2\pi i \delta(E_i - E_f - \omega) \langle \psi_f | a(\vec{q}, \lambda) V_{\rm int} | \psi_i \rangle$$

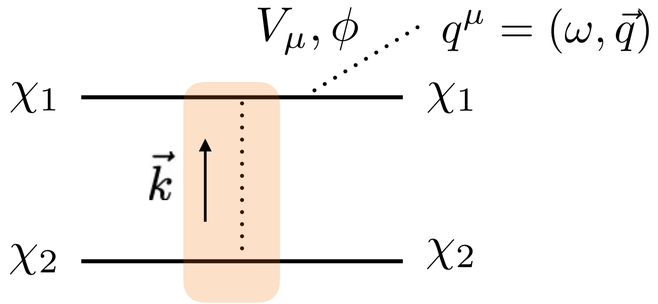
Initial and final states:

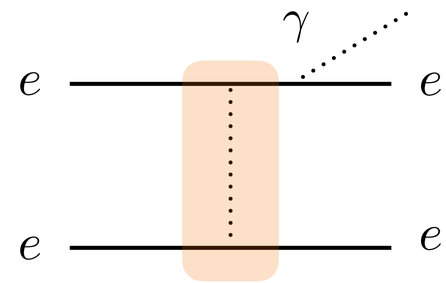
$$H_0 \left| \psi_{i(f)} \right\rangle = E_{i(f)} \left| \psi_{i(f)} \right\rangle$$

$$H_0 = H_{\rm kin} + V(r)$$

Textbook discussion: Lectures on Quantum Mechanics, Steven Weinberg
V-emission: arXiv:2007.06592v3, $\phi$-emission: arXiv:2303.00778v2

# Bremsstrahlung: Kinematic regimes




Long range: $m_{\phi,V} \ll |\vec{k}|$

Non-relativistic scattering:

$$V(r) = \frac{a_1 a_2}{4\pi r}$$

$$\mathcal{T}_{f(V,\phi),i} = -2\pi i \delta(E_i - E_f - \omega) \langle \psi_f | a(\vec{q}, \lambda) V_{\text{int}} | \psi_i \rangle$$

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$$H_0 \left| \psi_{i(f)} \right\rangle = E_{i(f)} \left| \psi_{i(f)} \right\rangle$$

Analytical solution

all orders in the Coulomb potential

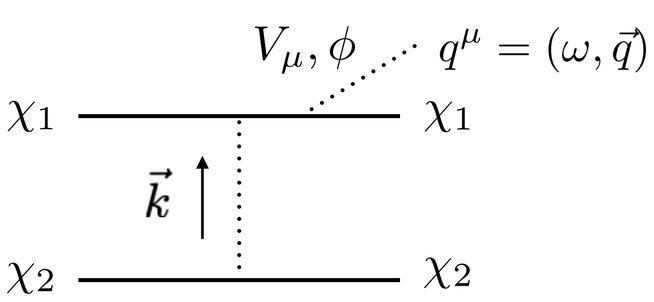
$$\chi^{(+)}_{\vec{p}_i}(\vec{r}) = \Gamma(1 - i\nu_i) e^{\frac{\pi}{2}\nu_i} e^{i\vec{p}_i \cdot \vec{r}} {}_1F_1\left(i\nu_i, 1; i\zeta_i^-\right)$$

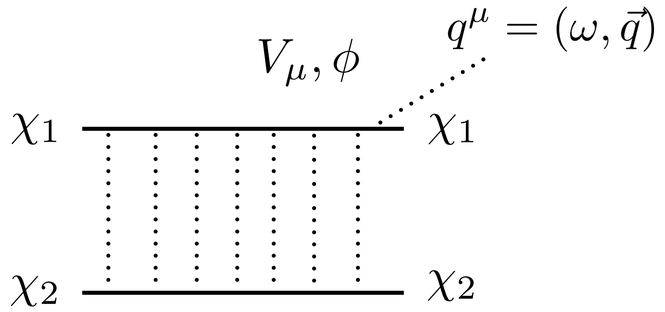
$$\chi^{(-)}_{\vec{p}_f}(\vec{r}) = \Gamma(1 + i\nu_f) e^{\frac{\pi}{2}\nu_f} e^{i\vec{p}_f \cdot \vec{r}} {}_1F_1\left(-i\nu_f, 1; -i\zeta_f^+\right)$$



Textbook discussion: Quantum Mechanics: Non-Relativistic Theory, Landau

# Bremsstrahlung: Kinematic regimes




Long range: $m_{\phi,V} \ll |\vec{k}|$

Non-relativistic scattering:

$$V(r) = \frac{a_1 a_2}{4\pi r}$$

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The different scattering regimes are characterized by the Sommerfeld parameters: $|\nu_{if}| = \dfrac{a_1 a_2}{4\pi v_{i,f}}$

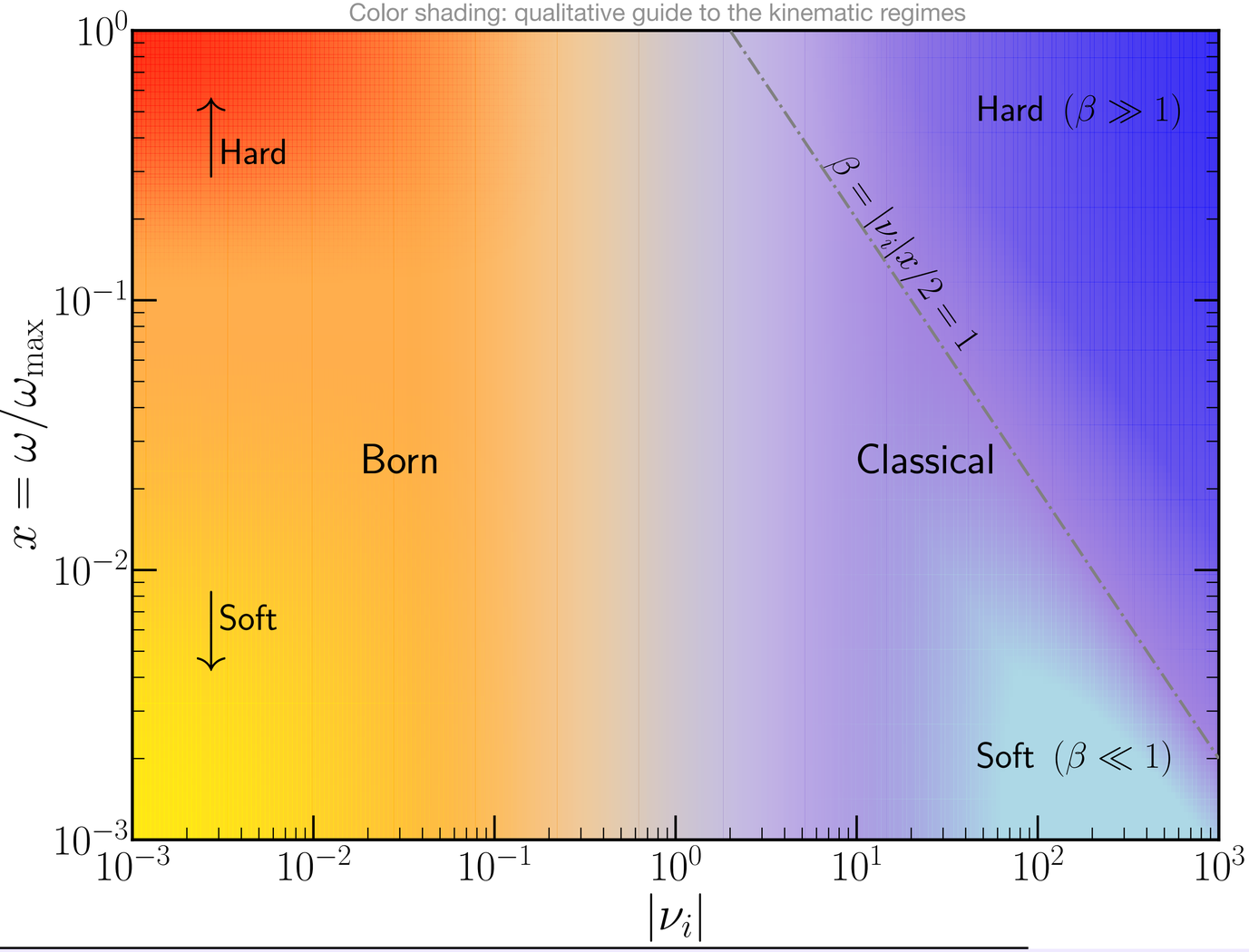


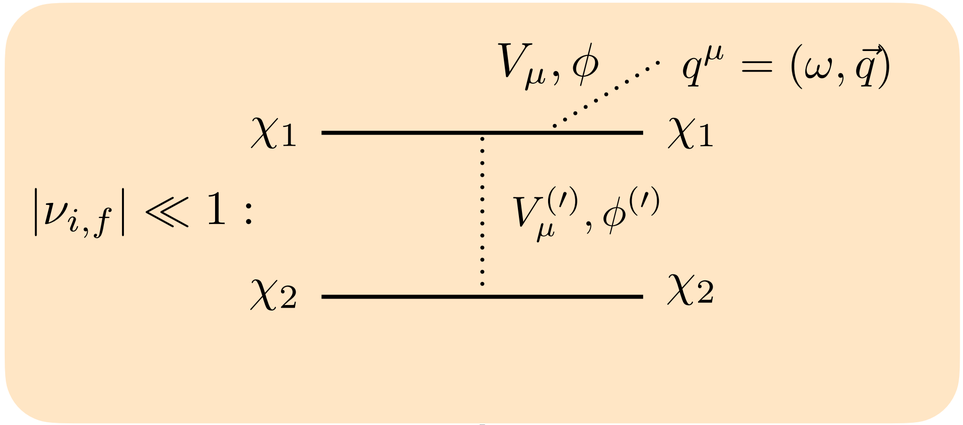
Textbook discussion: Quantum Mechanics: Non-Relativistic Theory, Landau

# Bremsstrahlung: Kinematic regimes

arXiv:26XX.XXXX

$$\mathcal{T}_{f(V,\phi),i} = -2\pi i \delta(E_i - E_f - \omega)\langle\psi_f|a(\vec{q},\lambda)V_{\rm int}|\psi_i\rangle$$




$$\psi_{i,f} = e^{i\vec{p}_{i,f}\cdot\vec{r}}$$ Born

Quantum Field Theory,
Tree level

$$\mathcal{M}_{2\to3}$$

Multipole expansion: $v_i \ll 1$

**Heavy mediator** $\dfrac{|g_1 g_2|\mu}{m_{\phi',V'}} \ll 1$

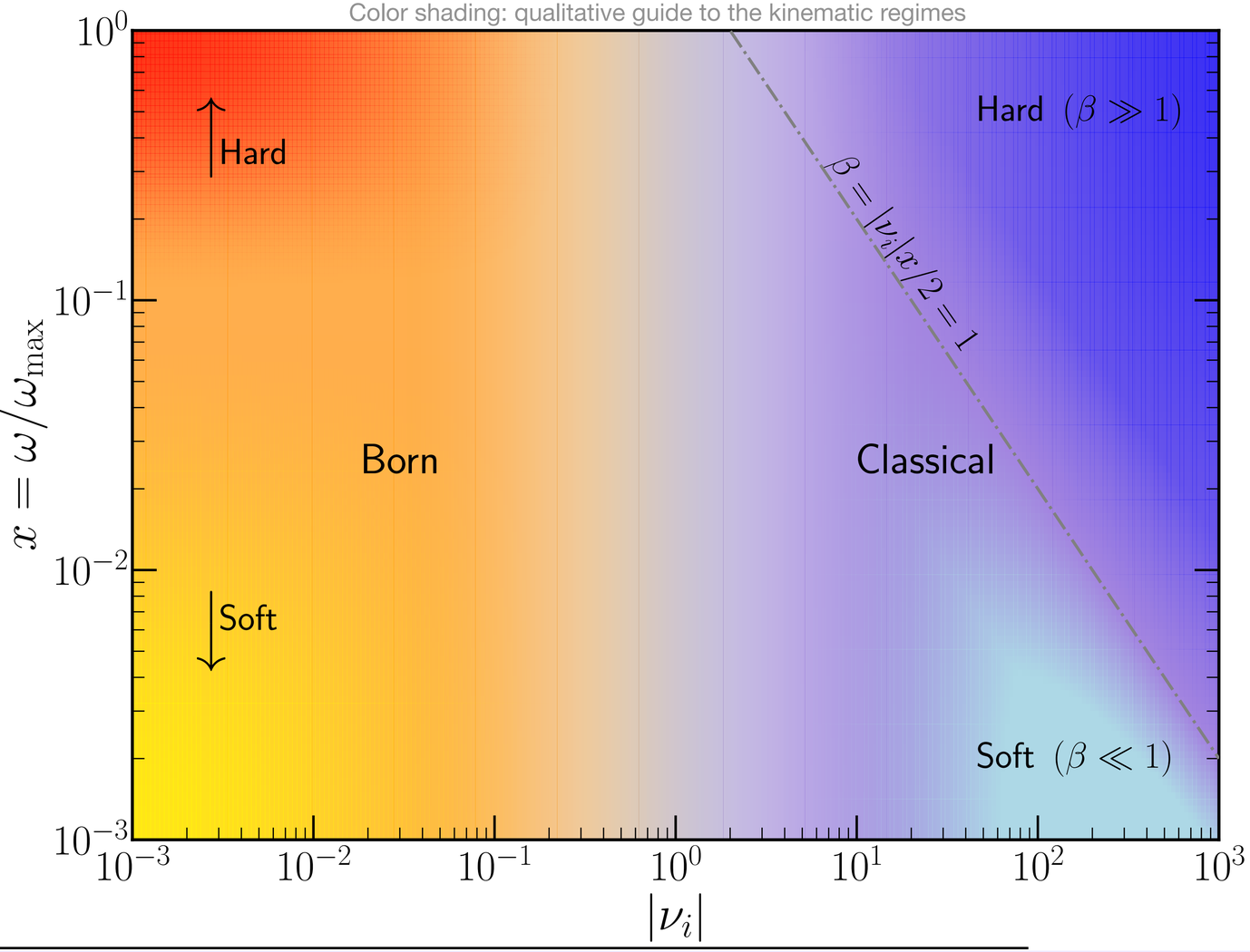


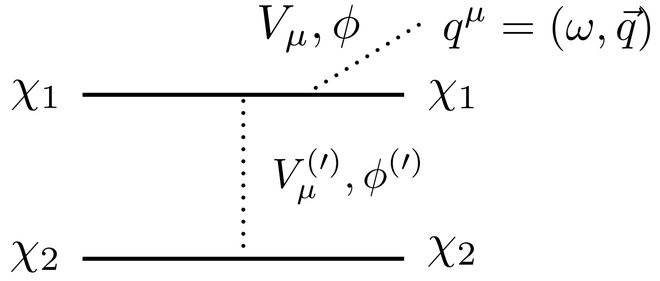
V-emission and Coulomb potential: arXiv:2007.06592

# Bremsstrahlung: Kinematic regimes

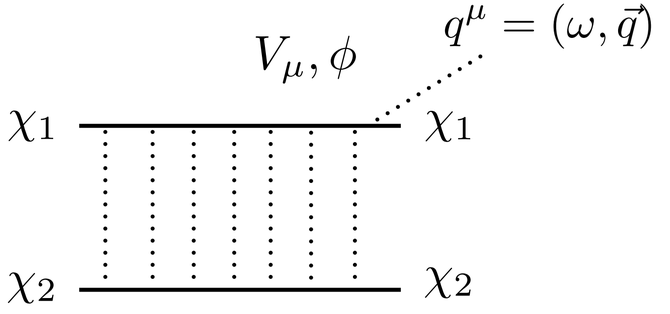
arXiv:26XX.XXXX

$$\mathcal{T}_{f(V,\phi),i} = -2\pi i \delta(E_i - E_f - \omega)\langle\psi_f|a(\vec{q},\lambda)V_{\rm int}|\psi_i\rangle$$




$|\nu_{i,f}| \sim 1$ : Intermediate regime, need the full Coulomb state



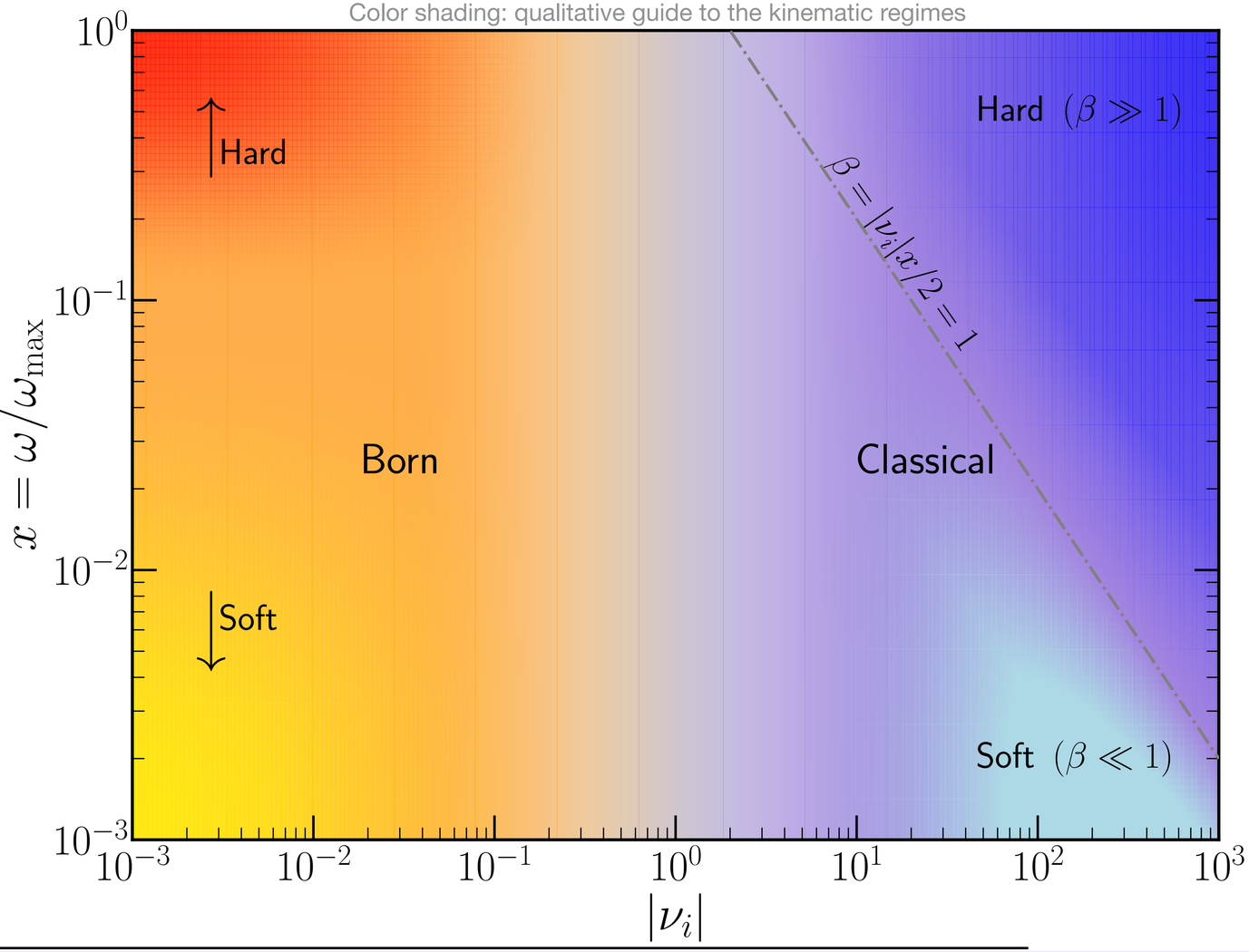



V-emission and Coulomb potential: arXiv:2007.06592

# Bremsstrahlung: Kinematic regimes

arXiv:26XX.XXXX

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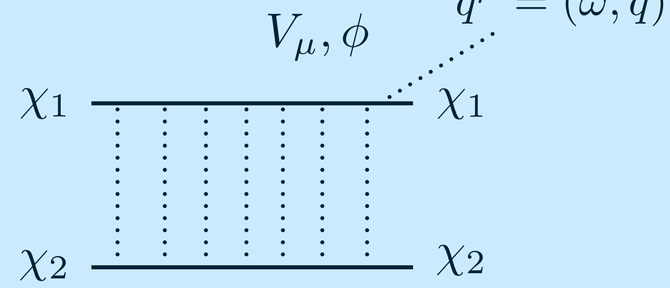

Particles moving along a classical path

$I$ ➡ Retarded potential: $\vec{A}, \phi$

Multipole expansion: long wavelength approx.

Quasi-Classical

$|\nu_{i,f}| \gg 1$ :






V-emission and Coulomb potential: arXiv:2007.06592

# Classical radiation



For particles moving along a classical path, observables are obtained from:

$$\Sigma(v_i)|^{\text{tot}}_{\text{class.}} = \int_0^{\infty} d\rho\, 2\pi\rho \int_{-\infty}^{+\infty} dt'\, I$$

$$\vec{A} = \frac{1}{4\pi R_0} \int \vec{j}(t' + \vec{x}\cdot\hat{q})\, d^3x$$

Far from the source: radiated field acts as an outgoing plane wave

$$\phi = \frac{1}{4\pi R_0} \int n_s(t' + \vec{x}\cdot\hat{q})\, d^3x$$

$v_i \ll c$ — Long wavelength approximation

Multipole expansion

$$\vec{A} \simeq \frac{1}{4\pi R_0}\mu\left(\frac{g_1}{m_1} - \frac{g_2}{m_2}\right)\partial_{t'}\vec{r} + \frac{1}{8\pi R_0}\mu^2\left(\frac{g_1}{m_1^2} + \frac{g_2}{m_2^2}\right)\partial_{t'}^2\left[\vec{r}(\vec{r}\cdot\hat{q})\right] + \cdots$$

$$\phi \simeq \frac{y_1 + y_2}{4\pi R_0} + \frac{1}{4\pi R_0}\mu\left(\frac{y_1}{m_1} - \frac{y_2}{m_2}\right)\partial_{t'}(\hat{q}\cdot\vec{r}) + \frac{1}{8\pi R_0}\mu^2\left(\frac{y_1}{m_1^2} + \frac{y_2}{m_2^2}\right)\left[\partial_{t'}^2(\hat{q}\cdot\vec{r})^2 + \eta|\dot{\vec{r}}|^2\right] + \cdots$$

$$dI = \begin{matrix} |\partial_{t'}\vec{A}\times\hat{q}|^2 \\ |\partial_{t'}\phi|^2 \end{matrix} \Bigg\} \times R_0^2 d\Omega_{\hat{q}}$$

# Born

arXiv:2509.12317

DM candidates
- Fermion
- Scalar

$$\chi_1\chi_2 \to \chi_1\chi_2 V \quad \tilde{\chi}_1\tilde{\chi}_2 \to \tilde{\chi}_1\tilde{\chi}_2\phi \quad \chi_1\chi_2 \to \chi_1\chi_2\phi$$

$$S_1 S_2 \to S_1 S_2 V \quad \tilde{S}_1\tilde{S}_2 \to \tilde{S}_1\tilde{S}_2\phi \quad S_1 S_2 \to S_1 S_2\phi$$

Emission/Mediation:
- Gauge vector $V$
- Scalar $\phi$

Including massive emission

**Heavy mediator**

$$\frac{|g_1 g_2|\mu}{m_{\phi',V'}} \ll 1$$

**Light mediator**

$$\frac{|g_1 g_2|}{v_{i,f}} \ll 1$$

$$\psi_{i,f} = e^{i\vec{p}_{i,f}\cdot\vec{r}}$$

Tree level

Quantum Field Theory

# Born

arXiv:2509.12317

DM candidates
- Fermion
- Scalar

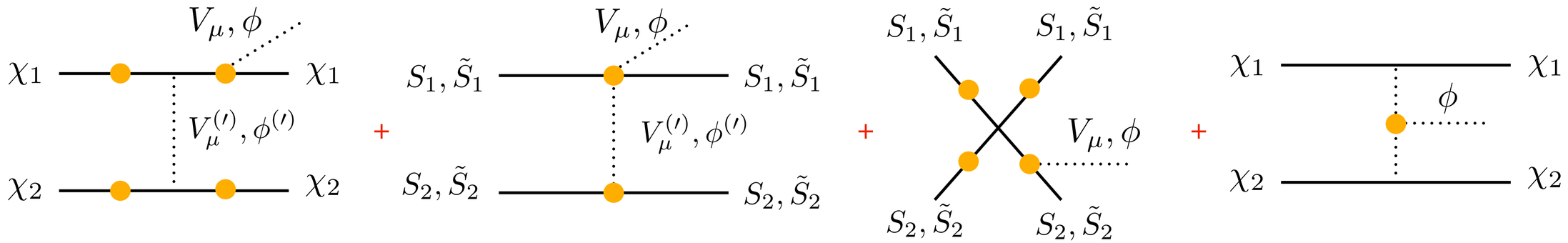
$\chi_1\chi_2 \to \chi_1\chi_2 V \quad \tilde{\chi}_1\tilde{\chi}_2 \to \tilde{\chi}_1\tilde{\chi}_2\phi \quad \chi_1\chi_2 \to \chi_1\chi_2\phi$

$S_1 S_2 \to S_1 S_2 V \quad \tilde{S}_1\tilde{S}_2 \to \tilde{S}_1\tilde{S}_2\phi \quad S_1 S_2 \to S_1 S_2\phi$

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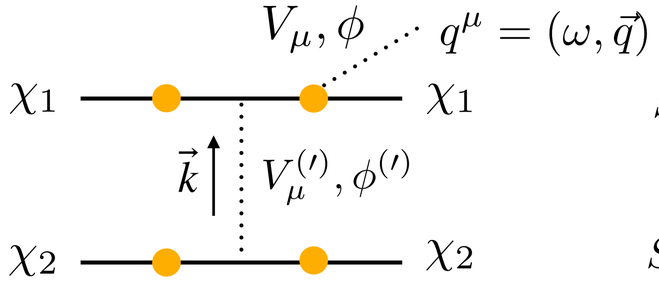
$\mathcal{M} =$

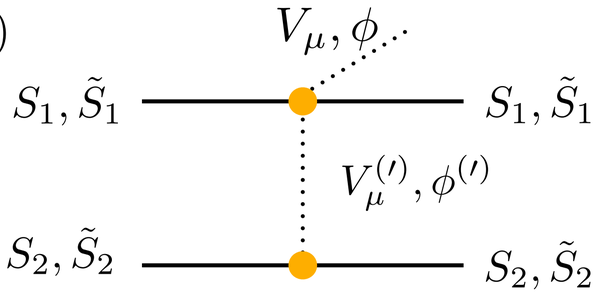


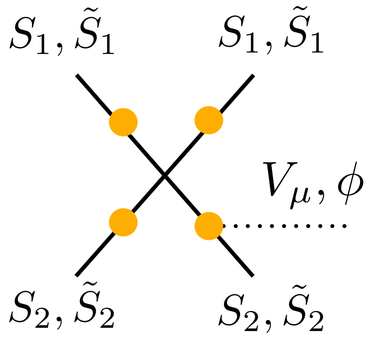

Finite but large set of dissipative channels to evaluate

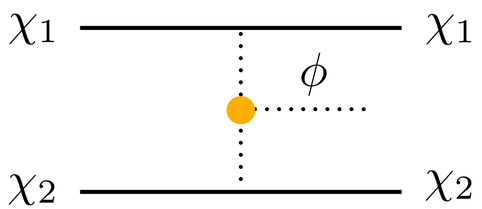
# Non-relativistic regime

arXiv:2509.12317




In DM halos: $v_i \lesssim 100 - 1000\,\mathrm{km.s}^{-1}$

**Light mediator:** $m_\phi \ll |\vec{k}|$

$$p_i \sim \mu \frac{v_i}{c}, \omega \sim \mu \left(\frac{v_i}{c}\right)^2$$

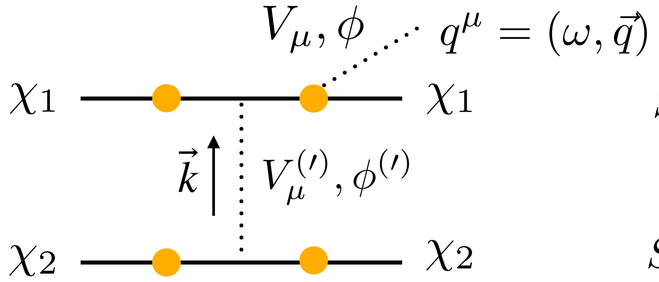
Separation of scales in the CM

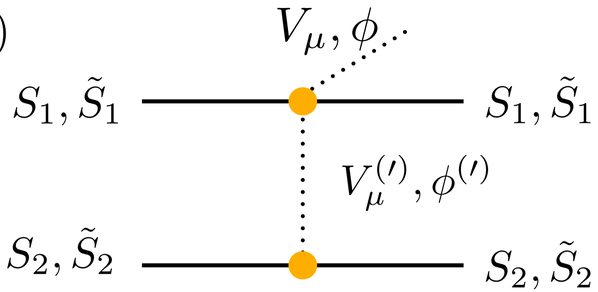
**Heavy mediator:** $m_{\phi'} \gg |\vec{k}|$

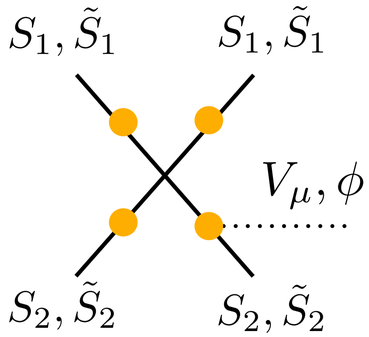
$$\frac{\mu v_i}{m_{\phi', V'} c} \ll 1$$

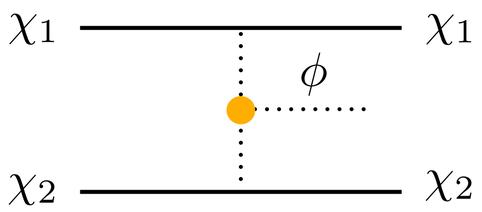
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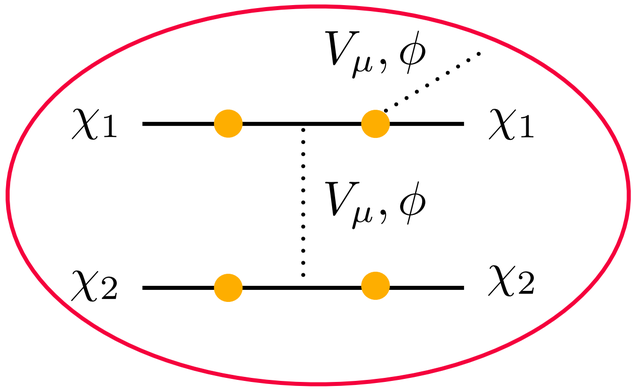
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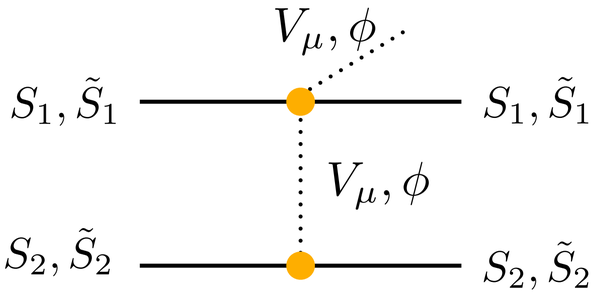
$$\mathcal{M}_{2\to3} = \mathcal{M}_{\mathcal{O}\left(\frac{c}{v_i}\right)^3} + \mathcal{M}_{\mathcal{O}\left(\frac{c}{v_i}\right)^2} + \ldots$$

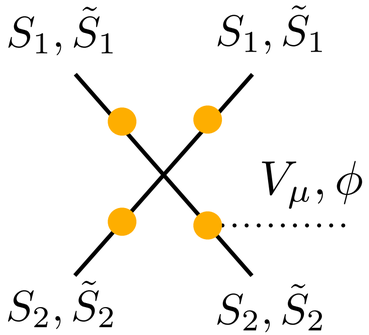
**Compact analytical expression**

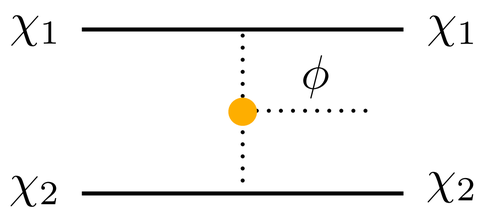
# Amplitudes: long-range mediation

arXiv:2509.12317



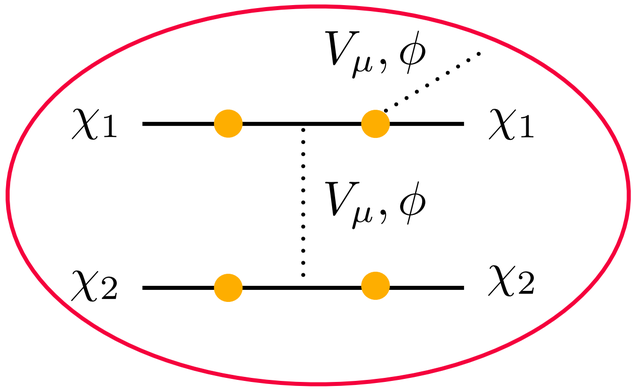



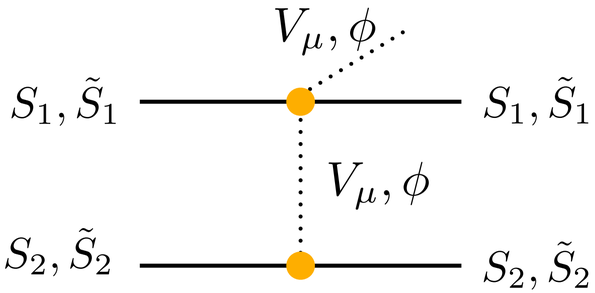



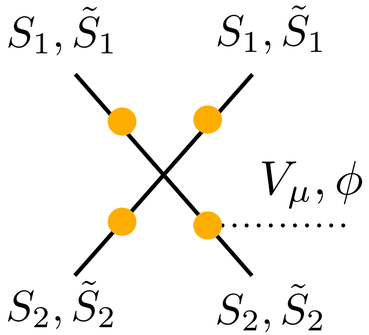

$$\mathcal{M}_{2\to3} = \mathcal{M}_{\mathcal{O}\left(\frac{c}{v_i}\right)^3} + \mathcal{M}_{\mathcal{O}\left(\frac{c}{v_i}\right)^2} + \ldots$$

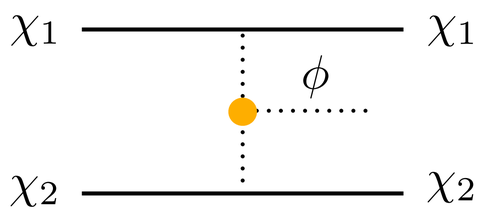
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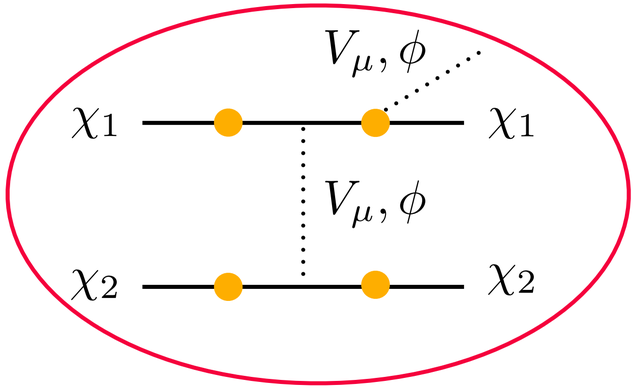
Dipole emission

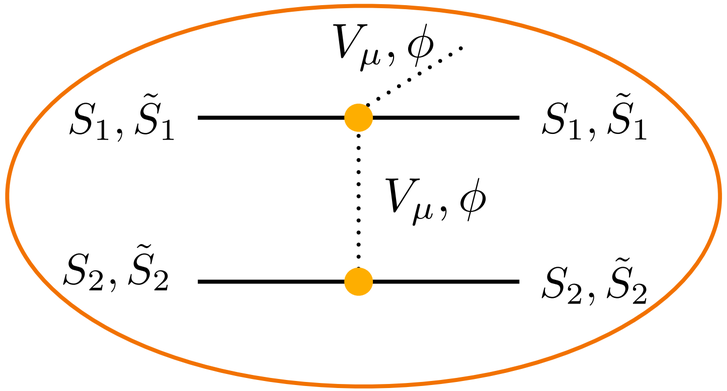
$$\frac{1}{d_1 d_2}\langle|\mathcal{M}_{\mathcal{D}}|^2\rangle_{\hat{q}} = a_1^2 a_2^2 \left(\frac{a_1}{m_1} - \frac{a_2}{m_2}\right)^2 \frac{m_1^2 m_2^2}{\mu^2} \frac{16 f_{\phi,V}}{3\omega^2 |\vec{v}_i - \vec{v}_f|^2} \qquad a_j \equiv g_j, y_j \text{ or } \frac{A_j}{2m_j}$$

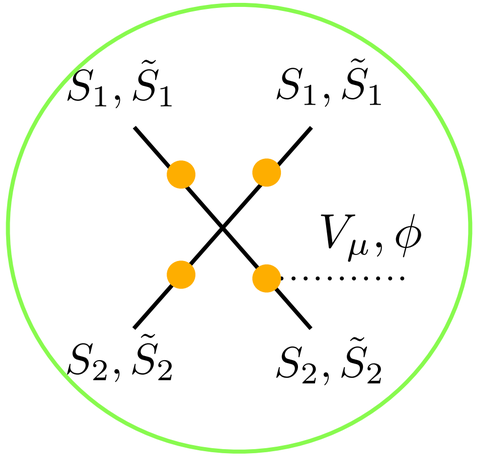
Details about the nature of the emission/DM candidate are velocity suppressed at dipole order

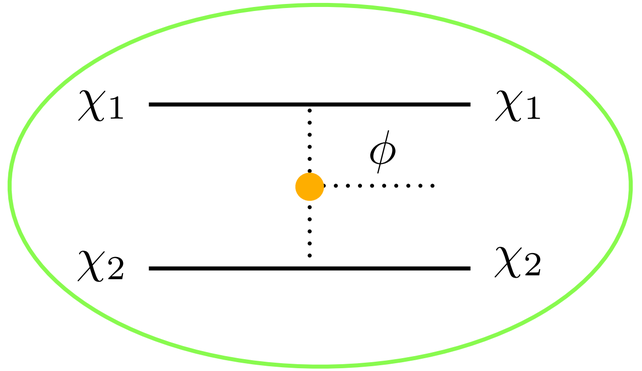
# Amplitudes: long-range mediation

arXiv:2509.12317



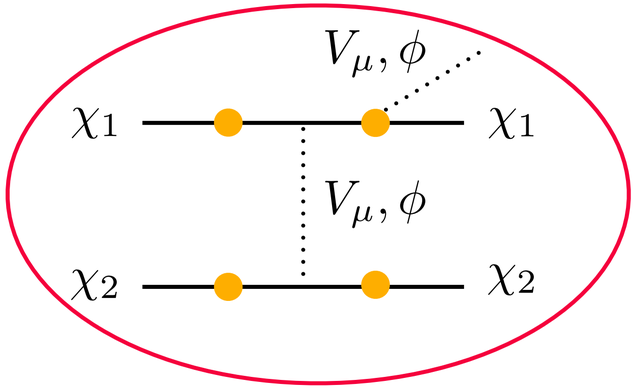

$$\mathcal{M}_{2\to 3} = \mathcal{M}_{\mathcal{O}\left(\frac{c}{v_i}\right)^3} + \mathcal{M}_{\mathcal{O}\left(\frac{c}{v_i}\right)^2} + \dots$$

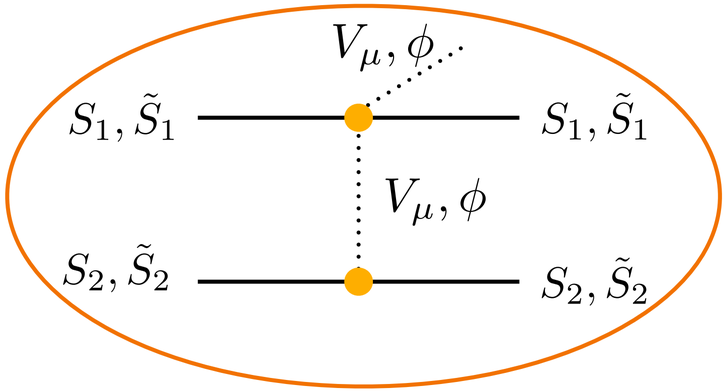
Quadrupole emission $\quad \frac{1}{d_1 d_2}\langle|\mathcal{M}_{\mathcal{Q}}|^2\rangle_{\hat{q}} = a_1^2 a_2^2 \left(\frac{a_1}{m_1^2} + \frac{a_2}{m_2^2}\right)^2 \mathcal{G}(\alpha, \beta, v_{i,f}, \omega)$

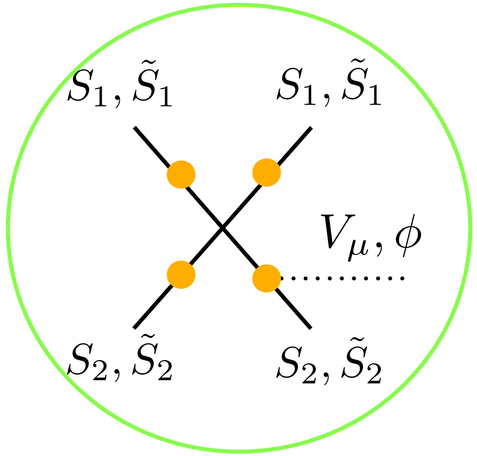
Model dependence arises

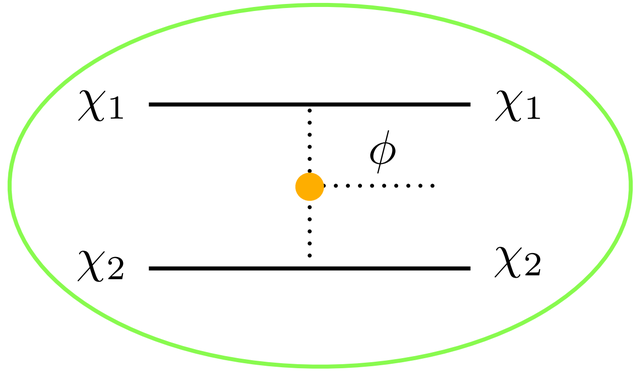
# Amplitudes: long-range mediation

arXiv:2509.12317




$$\mathcal{M}_{2\to3} = \mathcal{M}_{\mathcal{O}\left(\frac{c}{v_i}\right)^3} + \mathcal{M}_{\mathcal{O}\left(\frac{c}{v_i}\right)^2} + \dots$$

Quadrupole emission $\quad \frac{1}{d_1 d_2}\langle|\mathcal{M}_{\mathcal{Q}}|^2\rangle_{\hat{q}} = a_1^2 a_2^2 \left(\frac{a_1}{m_1^2} + \frac{a_2}{m_2^2}\right)^2 \mathcal{G}(\alpha, \beta, v_{i,f}, \omega)$

Model dependence arises

Spin effects suppressed

$\chi_1\chi_2 \to \chi_1\chi_2 V$

$S_1 S_2 \to S_1 S_2 V$

$\tilde{S}_1\tilde{S}_2 \to \tilde{S}_1\tilde{S}_2\phi$

$S_1 S_2 \to S_1 S_2 \phi$

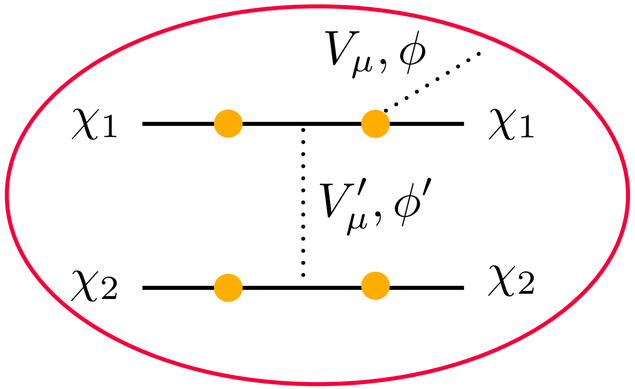
$\chi_1\chi_2 \to \chi_1\chi_2\phi$

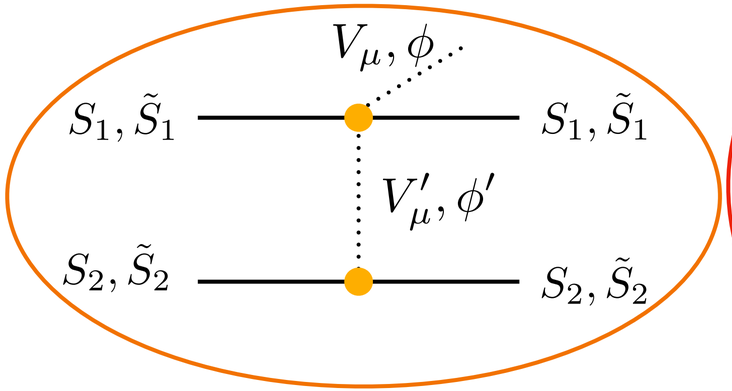
$\tilde{\chi}_1\tilde{\chi}_2 \to \tilde{\chi}_1\tilde{\chi}_2\phi$

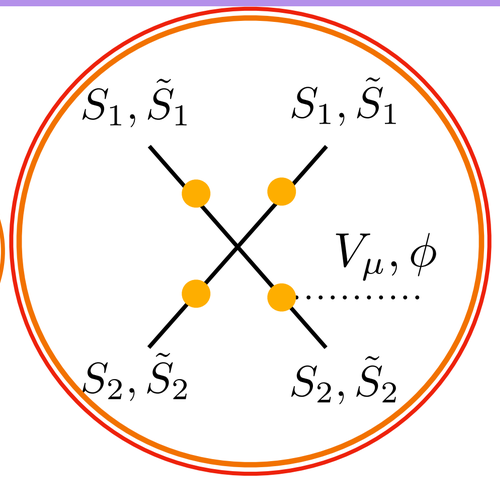
Annihilation channels suppressed

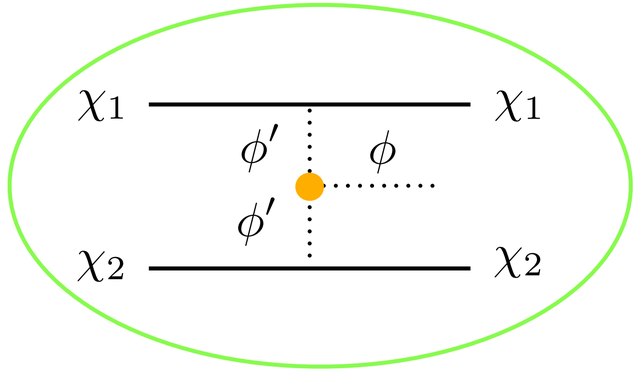
# Amplitudes: short-range mediation

arXiv:2509.12317




$$\mathcal{M}_{2\to3} = \mathcal{M}_{\mathcal{O}\left(\frac{c}{v_i}\right)} + \mathcal{M}_{\mathcal{O}\left(\frac{c}{v_i}\right)^0} + \cdots$$

Dipole emission

Quadrupole emission

$$\frac{1}{d_1 d_2}\langle|\mathcal{M}_{\mathcal{D}}|^2\rangle_{\hat{q}} = \left(a'_1 a'_2 \pm C\lambda_S \frac{m^2_{\phi',V'}}{m_1 m_2}\right)^2 \left(\frac{a_1}{m_1} - \frac{a_2}{m_2}\right)^2 \mathcal{F}'(v_{i,f}, \omega, m_{\phi',V'})$$

= Model independent

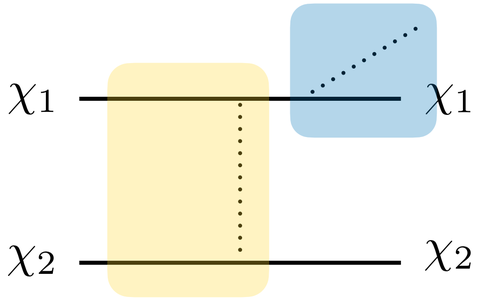
$$\frac{1}{d_1 d_2}\langle|\mathcal{M}_{\mathcal{Q}}|^2\rangle_{\hat{q}} = \left(a'_1 a'_2 \pm C\lambda_S \frac{m^2_{\phi',V'}}{m_1 m_2}\right)^2 \left(\frac{a_1}{m_1^2} + \frac{a_2}{m_2^2}\right)^2 \mathcal{G}'(\alpha', \beta', v_{i,f}, \omega, m_{\phi',V'})$$

= Model dependent

# Soft emission

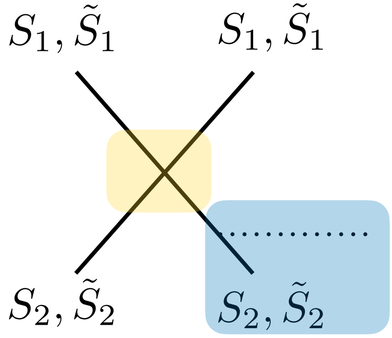
arXiv:2509.12317

NR = separation of scales $\omega \sim \mathcal{O}(\mu v_i^2)$




Soft factorization of the amplitudes $\omega \to 0$ and massless emission :

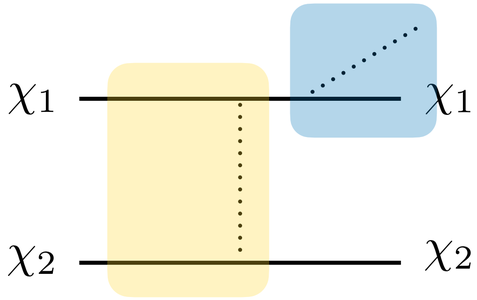
$$\mathcal{M}_{2\to 3} = \mathcal{M}_{2\to 2} \times \mathcal{E}_{\text{soft}}$$



# Soft emission

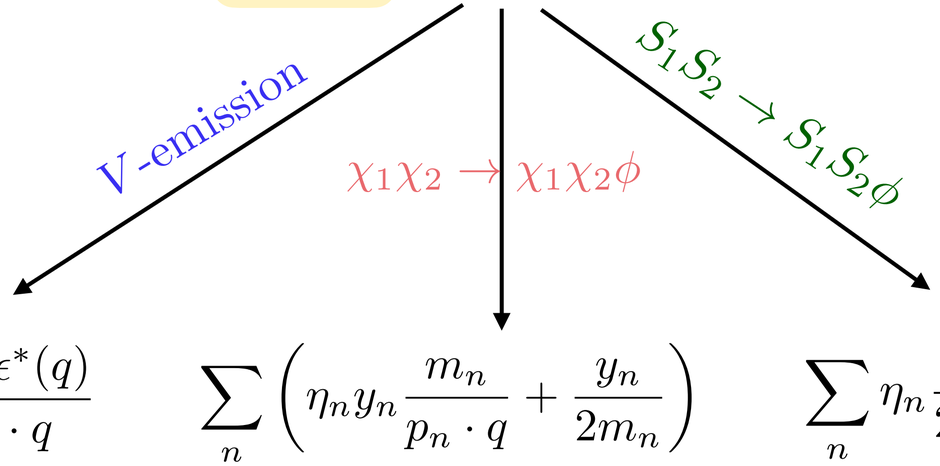
arXiv:2509.12317

NR = separation of scales $\omega \sim \mathcal{O}(\mu v_i^2)$

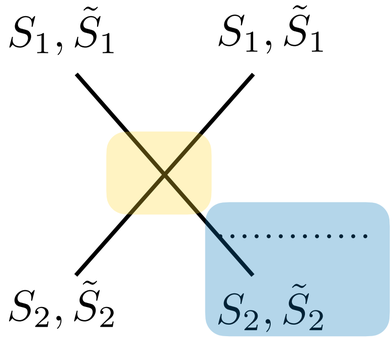



Soft factorization of the amplitudes $\omega \to 0$ and massless emission :

$$\mathcal{M}_{2\to 3} = \mathcal{M}_{2\to 2} \times \mathcal{E}_{\text{soft}}$$




$$\sum_n \eta_n g_n \frac{p_n \cdot \epsilon^*(q)}{p_n \cdot q} \qquad \sum_n \left(\eta_n y_n \frac{m_n}{p_n \cdot q} + \frac{y_n}{2m_n}\right) \qquad \sum_n \eta_n \frac{A_n}{2p_n \cdot q}$$




The soft energy differential cross-section is derived in the limit $|\vec{v}_i| \to |\vec{v}_f|$

# Soft emission

arXiv:2509.12317

NR = separation of scales $\omega \sim \mathcal{O}(\mu v_i^2)$

Soft factorization of the amplitudes $\omega \to 0$ :

$$\mathcal{M}_{2\to 3} = \mathcal{M}_{2\to 2} \times \mathcal{E}_{\rm soft}$$

Dipole emission

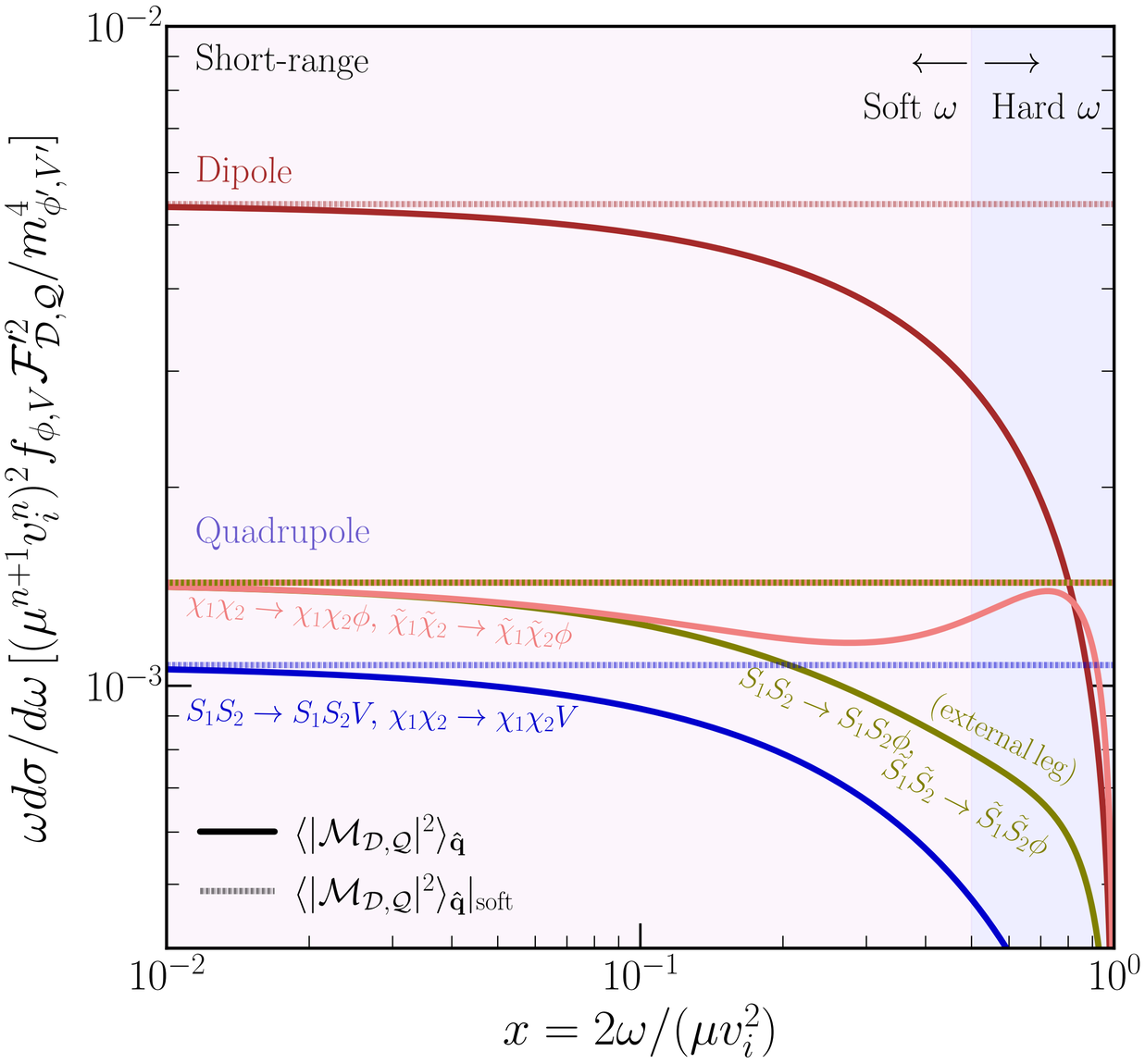
$$\langle|\mathcal{M}_{\mathcal{D}}|^2\rangle_{\hat{\mathbf{q}}}\Big|_{\rm soft} = \langle|\mathcal{M}_{\mathcal{D}}|^2\rangle_{\hat{\mathbf{q}}}$$

Quadrupole emission

$$\langle|\mathcal{M}_{\mathcal{Q}}|^2\rangle_{\hat{\mathbf{q}}}\Big|_{\rm soft} = \langle|\mathcal{M}_{\mathcal{Q}}|^2\rangle_{\hat{\mathbf{q}}}$$

1) Full 2-3 kinematics

$$\vec{p}_1 + \vec{p}_2 = \vec{p}_3 + \vec{p}_4 + \vec{q}$$



# Soft emission

arXiv:2509.12317

NR = separation of scales $\omega \sim \mathcal{O}(\mu v_i^2)$

Soft factorization of the amplitudes $\omega \to 0$ :

$$\mathcal{M}_{2\to 3} = \mathcal{M}_{2\to 2} \times \mathcal{E}_{\text{soft}}$$

Dipole emission

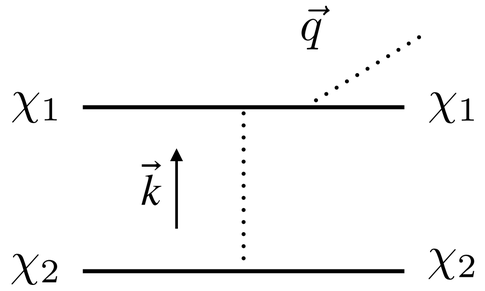
$$\langle |\mathcal{M}_{\mathcal{D}}|^2 \rangle_{\hat{\mathbf{q}}}\Big|_{\text{soft}} = \langle |\mathcal{M}_{\mathcal{D}}|^2 \rangle_{\hat{\mathbf{q}}}$$

Quadrupole emission

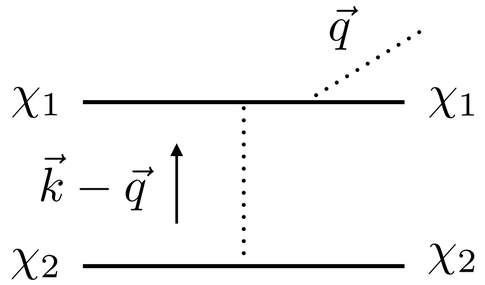
$$\langle |\mathcal{M}_{\mathcal{Q}}|^2 \rangle_{\hat{\mathbf{q}}}\Big|_{\text{soft}} = \langle |\mathcal{M}_{\mathcal{Q}}|^2 \rangle_{\hat{\mathbf{q}}}$$

1) Full 2-3 kinematics
2) Corrections on the exchange momenta

$$\mathcal{E}_{\text{soft}}|_{\text{corrected}} = \sum_n \frac{|\vec{k}_{\text{elastic}}|^2}{k_n^2} \mathcal{E}_{n,\text{soft}}$$




Vs



# Soft emission

arXiv:2509.12317

NR = separation of scales $\omega \sim \mathcal{O}(\mu v_i^2)$

Soft factorization of the amplitudes $\omega \to 0$ :

$$\mathcal{M}_{2\to 3} = \mathcal{M}_{2\to 2} \times \mathcal{E}_{\rm soft}$$

Dipole emission

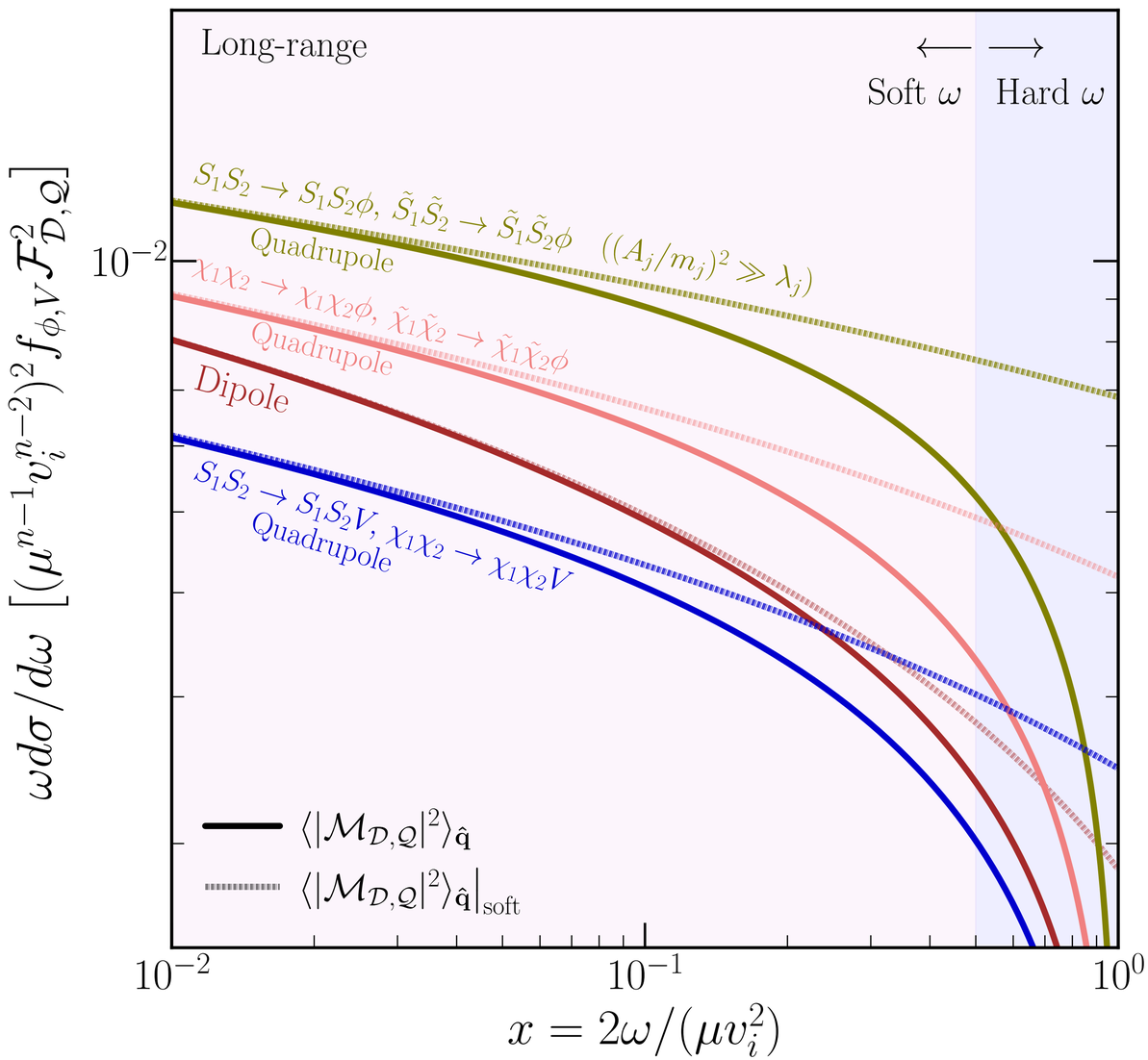
$$\langle|\mathcal{M}_{\mathcal{D}}|^2\rangle_{\hat{\mathbf{q}}}\Big|_{\rm soft} = \langle|\mathcal{M}_{\mathcal{D}}|^2\rangle_{\hat{\mathbf{q}}}$$

Quadrupole emission

$$\langle|\mathcal{M}_{\mathcal{Q}}|^2\rangle_{\hat{\mathbf{q}}}\Big|_{\rm soft} = \langle|\mathcal{M}_{\mathcal{Q}}|^2\rangle_{\hat{\mathbf{q}}}$$

1) Full 2-3 kinematics
2) Corrections on the exchange momenta

$$\mathcal{E}_{\rm soft}|_{\rm corrected} = \sum_n \frac{|\vec{k}_{\rm elastic}|^2}{k_n^2}\mathcal{E}_{n,\rm soft}$$



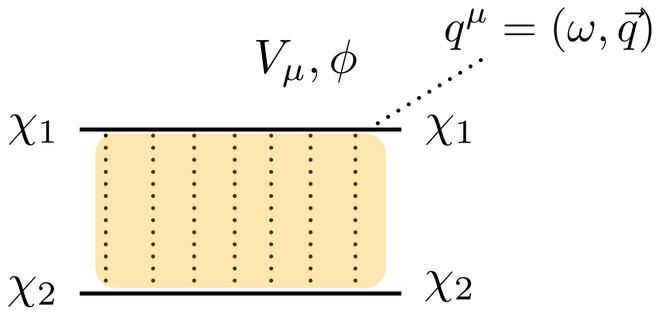

Additional corrections are necessary to recover the correct low-energy behavior at **quadrupole** order.

# Beyond Born

arXiv:26XX.XXXX

$$\mathcal{T}_{f(V,\phi),i} = -2\pi i\delta(E_i - E_f - \omega)\langle\psi_f|a(\vec{q},\lambda)V_{\rm int}|\psi_i\rangle$$

$$|\nu_{if}| = \frac{a_1 a_2}{4\pi v_{i,f}}$$



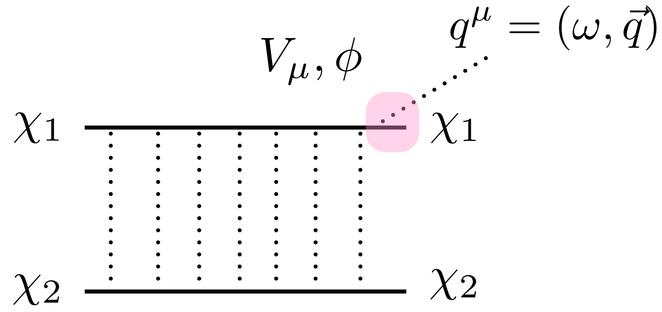

Initial and final states are known:

$$\chi^{(+)}_{\vec{p}_i}(\vec{r}) = \Gamma(1 - i\nu_i)e^{\frac{\pi}{2}\nu_i}e^{i\vec{p}_i\cdot\vec{r}}{}_1F_1\left(i\nu_i, 1; i\zeta_i^-\right),\ \chi^{(-)}_{\vec{p}_f}(\vec{r}) = \Gamma(1 + i\nu_f)e^{\frac{\pi}{2}\nu_f}e^{i\vec{p}_f\cdot\vec{r}}{}_1F_1\left(-i\nu_f, 1; -i\zeta_f^+\right)$$

# Beyond Born: emission operators

arXiv:26XX.XXXX

$$\mathcal{T}_{f(V,\phi),i} = -2\pi i\delta(E_i - E_f - \omega)\langle\psi_f|a(\vec{q},\lambda)V_{\rm int}|\psi_i\rangle$$




Initial and final states are known:

$$\chi^{(+)}_{\vec{p}_i}(\vec{r}) = \Gamma(1-i\nu_i)e^{\frac{\pi}{2}\nu_i}e^{i\vec{p}_i\cdot\vec{r}}{}_1F_1\left(i\nu_i,1;i\zeta_i^-\right),\ \chi^{(-)}_{\vec{p}_f}(\vec{r}) = \Gamma(1+i\nu_f)e^{\frac{\pi}{2}\nu_f}e^{i\vec{p}_f\cdot\vec{r}}{}_1F_1\left(-i\nu_f,1;-i\zeta_f^+\right)$$

Emission operators are obtained from:

$$H = \underbrace{m\beta + \vec{\alpha}\cdot\vec{p} + y\beta\phi}_{\chi_1\chi_2\to\chi_1\chi_2\phi} \Big| \underbrace{-\frac{1}{2m}\left[\nabla^2 - A\phi\right](\sigma_3 + i\sigma_2) + m\sigma_3}_{S_1S_2\to S_1S_2\phi} \Big| \underbrace{\vec{\alpha}\cdot(\vec{p} - g\vec{A}) + gA_0 + \beta m}_{\chi_1\chi_2\to\chi_1\chi_2 V} \Big| \underbrace{\frac{1}{2m}\left[\vec{p} - g\vec{V}\right]^2(\sigma_3 + i\sigma_2) + m\sigma_3 + gV_0}_{S_1S_2\to S_1S_2 V}$$

Foldy-Wouthuysen-Tani transformations

Hard scale: $m$

Vs

$|\vec{p}_i| \sim \mu v_i$

$|\vec{k}| \sim \mu v_i$

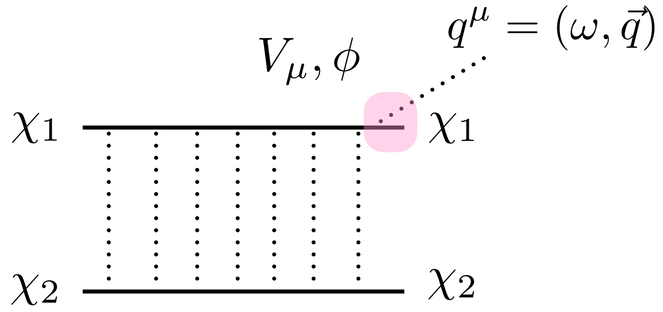
$\omega \sim \mu v_i^2$

→

# Beyond Born: emission operators

arXiv:26XX.XXXX

$$\mathcal{T}_{f(V,\phi),i} = -2\pi i\delta(E_i - E_f - \omega)\langle\psi_f|a(\vec{q},\lambda)V_{\rm int}|\psi_i\rangle$$




Initial and final states are known:

$$\chi^{(+)}_{\vec{p}_i}(\vec{r}) = \Gamma(1-i\nu_i)e^{\frac{\pi}{2}\nu_i}e^{i\vec{p}_i\cdot\vec{r}}{}_1F_1\left(i\nu_i,1;i\zeta_i^-\right),\ \chi^{(-)}_{\vec{p}_f}(\vec{r}) = \Gamma(1+i\nu_f)e^{\frac{\pi}{2}\nu_f}e^{i\vec{p}_f\cdot\vec{r}}{}_1F_1\left(-i\nu_f,1;-i\zeta_f^+\right)$$

Emission operators are obtained from:

$$H = \underbrace{m\beta + \vec{\alpha}\cdot\vec{p} + y\beta\phi}_{\chi_1\chi_2\to\chi_1\chi_2\phi} \Big| \underbrace{-\frac{1}{2m}\left[\nabla^2 - A\phi\right](\sigma_3 + i\sigma_2) + m\sigma_3}_{S_1S_2\to S_1S_2\phi} \Big| \underbrace{\vec{\alpha}\cdot(\vec{p} - g\vec{A}) + gA_0 + \beta m}_{\chi_1\chi_2\to\chi_1\chi_2 V} \Big| \underbrace{\frac{1}{2m}\left[\vec{p} - g\vec{V}\right]^2(\sigma_3 + i\sigma_2) + m\sigma_3 + gV_0}_{S_1S_2\to S_1S_2 V}$$

Foldy-Wouthuysen-Tani transformations

Hard scale: $m$

Vs

$|\vec{p}_i| \sim \mu v_i$

$|\vec{k}| \sim \mu v_i$

$\omega \sim \mu v_i^2$

$$\rightarrow H = \begin{cases} \beta m + \hat{O} + \hat{E} \\ \sigma_3 m + \hat{O} + \hat{E} \end{cases} \xrightarrow{H' = -ie^{i\hat{S}}\partial_t e^{-i\hat{S}} + e^{i\hat{S}}He^{-i\hat{S}}} H' = \begin{pmatrix} H^+ & 0 \\ 0 & H^- \end{pmatrix} \rightarrow$$

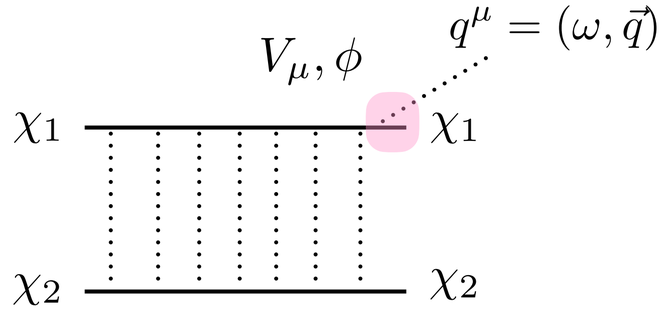
Order by order in $\sim \mathcal{O}(\frac{1}{m})$ :

One-particle Hamiltonian description including relativistic corrections

# Beyond Born: emission operators

arXiv:26XX.XXXX

$$\mathcal{T}_{f(V,\phi),i} = -2\pi i\delta(E_i - E_f - \omega)\langle\psi_f|a(\vec{q},\lambda)V_{\rm int}|\psi_i\rangle$$




Initial and final states are known:

$$\chi^{(+)}_{\vec{p}_i}(\vec{r}) = \Gamma(1-i\nu_i)e^{\frac{\pi}{2}\nu_i}e^{i\vec{p}_i\cdot\vec{r}}{}_1F_1\left(i\nu_i,1;i\zeta_i^-\right),\ \chi^{(-)}_{\vec{p}_f}(\vec{r}) = \Gamma(1+i\nu_f)e^{\frac{\pi}{2}\nu_f}e^{i\vec{p}_f\cdot\vec{r}}{}_1F_1\left(-i\nu_f,1;-i\zeta_f^+\right)$$

Emission operators are obtained from:

$$H = \underbrace{m\beta + \vec{\alpha}\cdot\vec{p} + y\beta\phi}_{\chi_1\chi_2\to\chi_1\chi_2\phi} \Big| \underbrace{-\frac{1}{2m}\left[\nabla^2 - A\phi\right](\sigma_3 + i\sigma_2) + m\sigma_3}_{S_1S_2\to S_1S_2\phi} \Big| \underbrace{\vec{\alpha}\cdot(\vec{p} - g\vec{A}) + gA_0 + \beta m}_{\chi_1\chi_2\to\chi_1\chi_2V} \Big| \underbrace{\frac{1}{2m}\left[\vec{p} - g\vec{V}\right]^2(\sigma_3 + i\sigma_2) + m\sigma_3 + gV_0}_{S_1S_2\to S_1S_2V}$$

Foldy-Wouthuysen-Tani transformations

One-particle Hamiltonian description including relativistic corrections

$$H_{\rm FWT} = H_\chi + H_\phi + V_{\rm int}$$

$$V_{\rm int} = \sum_n y_n\phi(\vec{x}_n)\left(1 - \frac{\vec{p}_n^2}{2m_n^2}\right) \Big| \sum_n \frac{A_n}{2m_n}\phi(\vec{x}_n)\left(1 - \frac{\vec{p}_n^2}{2m_n^2} - \frac{1}{m_n}V(r)\right) \Big| -\sum_n g_n\left(\frac{\vec{A}^{(L)}(\vec{x}_n)\cdot\vec{p}_n}{m_n} - A_0^{(L)}(\vec{x}_n)\right) \Big| -\sum_n g_n\frac{\vec{A}^{(T)}(\vec{x}_n)\cdot\vec{p}_n}{m_n}$$

# Beyond Born: emission amplitudes

arXiv:26XX.XXXX

$$\mathcal{T}_{f(V,\phi),i} = -2\pi i\delta(E_i - E_f - \omega)\langle\psi_f|a(\vec{q},\lambda)V_{\rm int}|\psi_i\rangle$$

$$V_{\rm int} = \sum_n y_n\phi(\vec{x}_n)\left(1-\frac{\vec{p}_n^{\,2}}{2m_n^2}\right) \Big| \sum_n \frac{A_n}{2m_n}\phi(\vec{x}_n)\left(1-\frac{\vec{p}_n^{\,2}}{2m_n^2}-\frac{1}{m_n}V(r)\right) \Big| -\sum_n g_n\left(\frac{\vec{A}^{(L)}(\vec{x}_n)\cdot\vec{p}_n}{m_n} - A_0^{(L)}(\vec{x}_n)\right) \Big| -\sum_n g_n\frac{\vec{A}^{(T)}(\vec{x}_n)\cdot\vec{p}_n}{m_n}$$

$v_i$ $v_i$

$$\phi(\vec{x}_n),\ \vec{A}(\vec{x}_n)$$

$$\vec{r}\cdot\vec{q} \sim v_i \ll 1 \quad \Big\downarrow \quad e^{i\vec{q}\cdot\vec{r}} = 1 + i\vec{q}\cdot\vec{r} - \frac{1}{2}(\vec{q}\cdot\vec{r})^2 + \cdots$$

$v_i$

$$\mathcal{T}_{fi} = -i(2\pi)^4\delta(E_i - E_f - \omega)\delta^{(3)}(\vec{p}_1+\vec{p}_2-\vec{p}_3-\vec{p}_4-\vec{q})\mathcal{M}$$

---

Dipole emission: $D^a_{fi} = \int d^3r\ \chi^*_{\vec{p}_f}(\vec{r})p^a\chi_{\vec{p}_i}(\vec{r})$

$$\mathcal{M}_{\mathcal{D}} = \begin{cases} -\dfrac{1}{\sqrt{2\omega}}\left(\dfrac{a_1}{m_1}-\dfrac{a_2}{m_2}\right)D^a_{fi}\,e^{*a} & V-\text{emission} \\ \dfrac{1}{\omega\sqrt{2\omega}}\left(\dfrac{a_1}{m_1}-\dfrac{a_2}{m_2}\right)D^a_{fi}\,q^a & \phi-\text{emission} \end{cases}$$

# Beyond Born: emission amplitudes

arXiv:26XX.XXXX

$$\mathcal{T}_{f(V,\phi),i} = -2\pi i\delta(E_i - E_f - \omega)\langle\psi_f|a(\vec{q},\lambda)V_{\rm int}|\psi_i\rangle$$

$$V_{\rm int} = \sum_n y_n\phi(\vec{x}_n)\left(1 - \frac{\vec{p}_n^{\,2}}{2m_n^2}\right) \Big| \sum_n \frac{A_n}{2m_n}\phi(\vec{x}_n)\left(1 - \frac{\vec{p}_n^{\,2}}{2m_n^2} - \frac{1}{m_n}V(r)\right) \Big| -\sum_n g_n\left(\frac{\vec{A}^{(L)}(\vec{x}_n)\cdot\vec{p}_n}{m_n} - A_0^{(L)}(\vec{x}_n)\right) \Big| -\sum_n g_n\frac{\vec{A}^{(T)}(\vec{x}_n)\cdot\vec{p}_n}{m_n}$$

$v_i^2$ $v_i^2$

$$\phi(\vec{x}_n),\ \vec{A}(\vec{x}_n)$$

$$\vec{r}\cdot\vec{q} \sim v_i \ll 1 \quad \Big\downarrow \quad e^{i\vec{q}\cdot\vec{r}} = 1 + i\vec{q}\cdot\vec{r} - \frac{1}{2}(\vec{q}\cdot\vec{r})^2 + \cdots \quad (v_i^2)$$

$$\mathcal{T}_{fi} = -i(2\pi)^4\delta(E_i - E_f - \omega)\delta^{(3)}(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4 - \vec{q})\mathcal{M}$$

---

Quadrupole emission:

$$Q^{ab} = \int d^3r\ \chi^*_{\vec{p}_f}(r)r^a\partial^b\chi_{\vec{p}_i} \qquad \mathcal{K} = \int d^3r\,\chi^*_{\vec{p}_f}(\vec{r})p^2\chi_{\vec{p}_i}(\vec{r})$$

$\chi_1\chi_2 \to \chi_1\chi_2 V \quad S_1S_2 \to S_1S_2V$

$$\mathcal{M}_{\mathcal{Q}} = \frac{\mu}{\sqrt{2\omega}}\left(\frac{g_1}{m_1^2} + \frac{g_2}{m_2^2}\right)q^a e^{*b}Q^{ab}$$

$\chi_1\chi_2 \to \chi_1\chi_2\phi \quad S_1S_2 \to S_1S_2\phi$

$$\mathcal{M}_{\mathcal{Q}} = -\frac{\mu}{\sqrt{2\omega}}\left(\frac{a_1}{m_1^2} + \frac{a_2}{m_2^2}\right)\left[\frac{1}{\omega}q^a q^b Q^{ab} + \frac{\eta}{2\mu}\mathcal{K}\right]$$

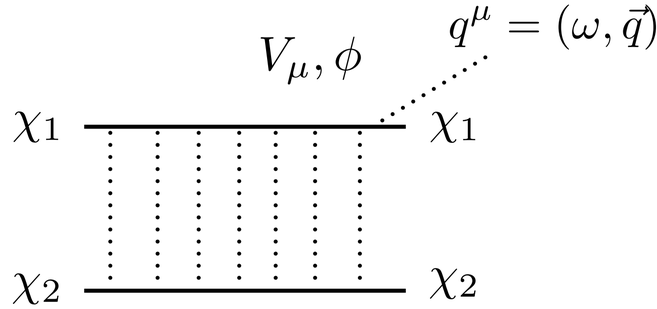
# Beyond Born: emission amplitudes

arXiv:26XX.XXXX

Initial and final states known:

$$\chi^{(+)}_{\vec{p}_i}(\vec{r}) = \Gamma(1-i\nu_i)e^{\frac{\pi}{2}\nu_i}e^{i\vec{p}_i\cdot\vec{r}}{}_1F_1\left(i\nu_i,1;i\zeta_i^-\right),$$

$$\chi^{(-)}_{\vec{p}_f}(\vec{r}) = \Gamma(1+i\nu_f)e^{\frac{\pi}{2}\nu_f}e^{i\vec{p}_f\cdot\vec{r}}{}_1F_1\left(-i\nu_f,1;-i\zeta_f^+\right)$$




Dipole emission:

$$\mathcal{M}_{\mathcal{D}} = \begin{cases} -\dfrac{1}{\sqrt{2\omega}}\left(\dfrac{a_1}{m_1}-\dfrac{a_2}{m_2}\right)D^a_{fi}\,e^{*a} & V-\text{emission} \\ \dfrac{1}{\omega\sqrt{2\omega}}\left(\dfrac{a_1}{m_1}-\dfrac{a_2}{m_2}\right)D^a_{fi}\,q^a & \phi-\text{emission} \end{cases}$$

Analytical solutions at dipole and quadrupole orders

Quadrupole emission:

$$\mathcal{M}_{\mathcal{Q}} = \begin{cases} \dfrac{\mu}{\sqrt{2\omega}}\left(\dfrac{g_1}{m_1^2}+\dfrac{g_2}{m_2^2}\right)q^a e^{*b}Q^{ab} & \chi_1\chi_2\to\chi_1\chi_2V \\ & S_1S_2\to S_1S_2V \\ -\dfrac{\mu}{\sqrt{2\omega}}\left(\dfrac{a_1}{m_1^2}+\dfrac{a_2}{m_2^2}\right)\left[\dfrac{1}{\omega}q^aq^bQ^{ab}+\dfrac{\eta}{2\mu}\mathcal{K}\right] & \chi_1\chi_2\to\chi_1\chi_2\phi \\ & S_1S_2\to S_1S_2\phi \end{cases}$$

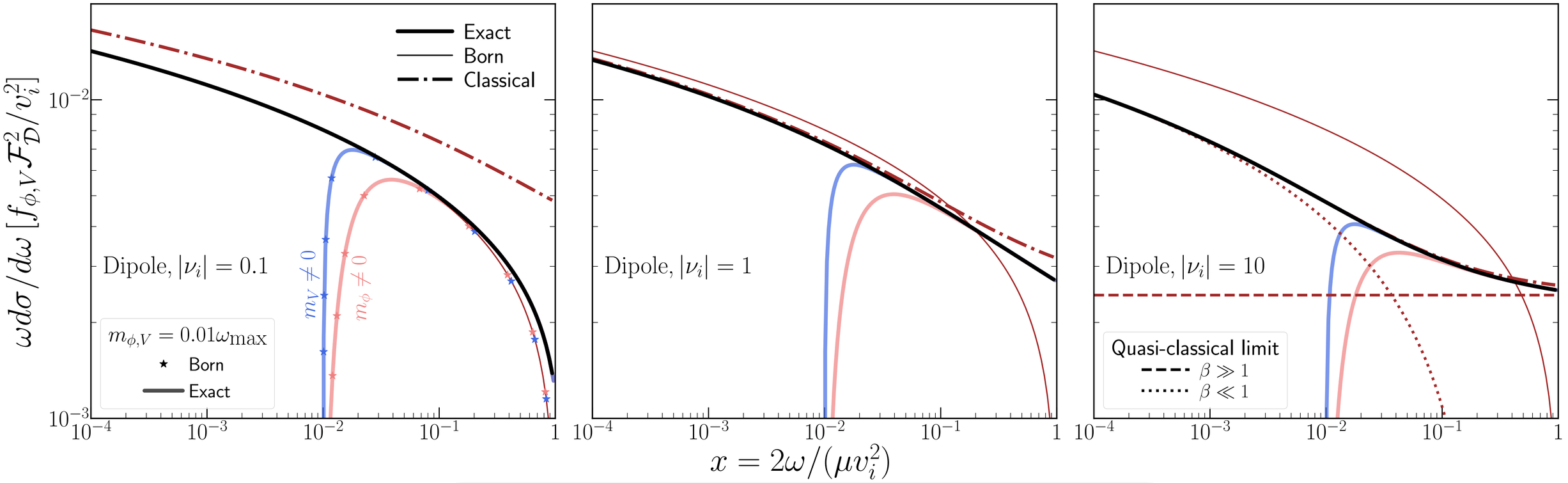
Full result $\propto S_iS_f \times$ [hypergeometric functions]

$$S_i = e^{-2\pi\nu_i}|\chi^{(+)}_{\vec{p}_i}(0)|^2,\ S_f = |\chi^{(-)}_{\vec{p}_f}(0)|^2$$

# Beyond Born: energy-differential cross section

arXiv:26XX.XXXX

Attractive Interaction




$$\omega\frac{d\sigma}{d\omega}\bigg|_{\text{Exact}} = \frac{f_{\phi,V}}{24\pi^3 v_i^2}\mathcal{F}_{\mathcal{D}}^2 S_i S_f \frac{z_{\min}}{4\nu_i\nu_f}\frac{d|{}_2F_1(i\nu_f, i\nu_i, 1; z_{\min})|^2}{dz_{\min}}$$

$|\nu_i| \ll 1$

$$\omega\frac{d\sigma}{d\omega}\bigg|_{\text{Born}} = \mathcal{F}_{\mathcal{D}}^2\frac{f_{\phi,V}}{24\pi^3 v_i^2}\ln\left(\frac{1+\sqrt{1-x}}{1-\sqrt{1-x}}\right)$$

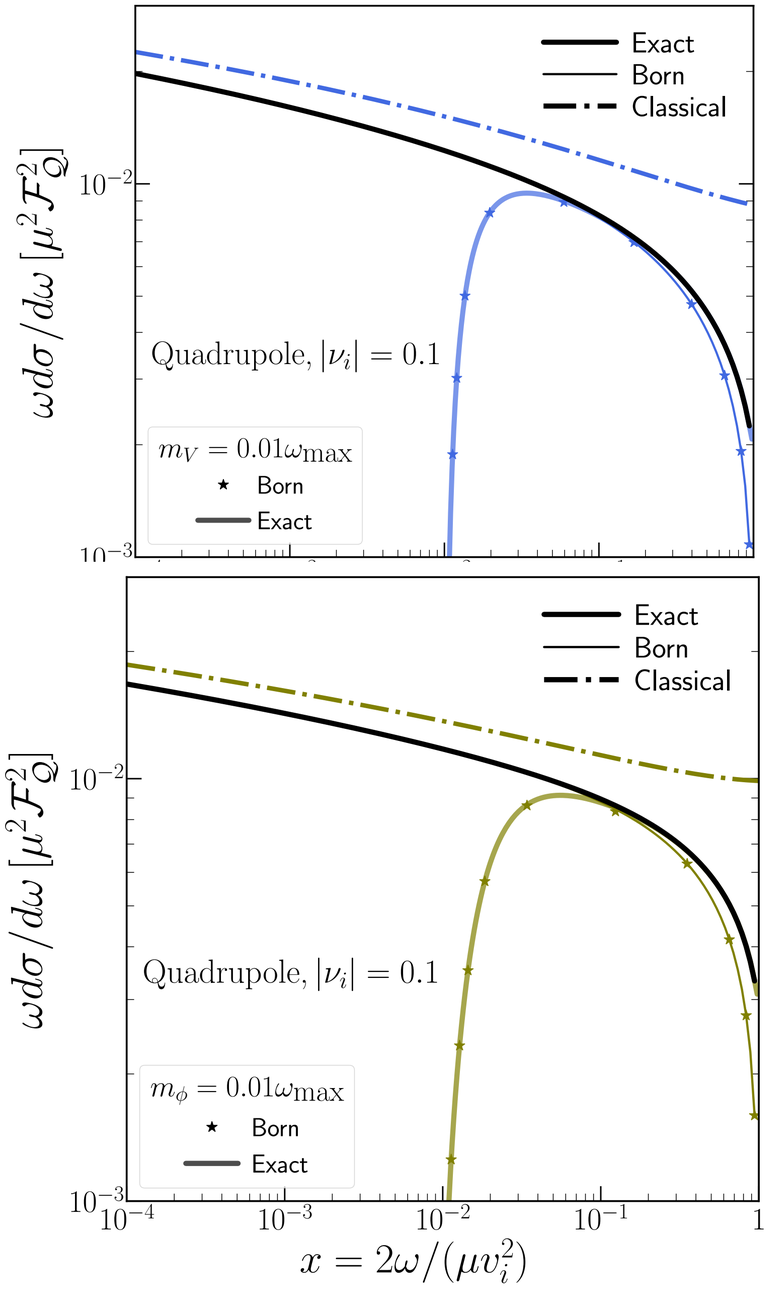
Independent calculations

$|\nu_i| \gg 1$

$$\omega\frac{d\sigma}{d\omega}\bigg|_{\text{classical}} = \mathcal{F}_{\mathcal{D}}^2\frac{f_{\phi,V}}{96\pi v_i^2}\beta|H_{i\beta}^{(1)}(i\beta)|H_{i\beta}^{(1)\prime}(i\beta)$$

# Beyond Born: energy-differential cross section

arXiv:26XX.XXXX




$\chi_1\chi_2 \to \chi_1\chi_2 V,\ S_1S_2 \to S_1S_2V$

$S_1S_2 \to S_1S_2\phi,\ \tilde{S}_1\tilde{S}_2 \to \tilde{S}_1\tilde{S}_2\phi$

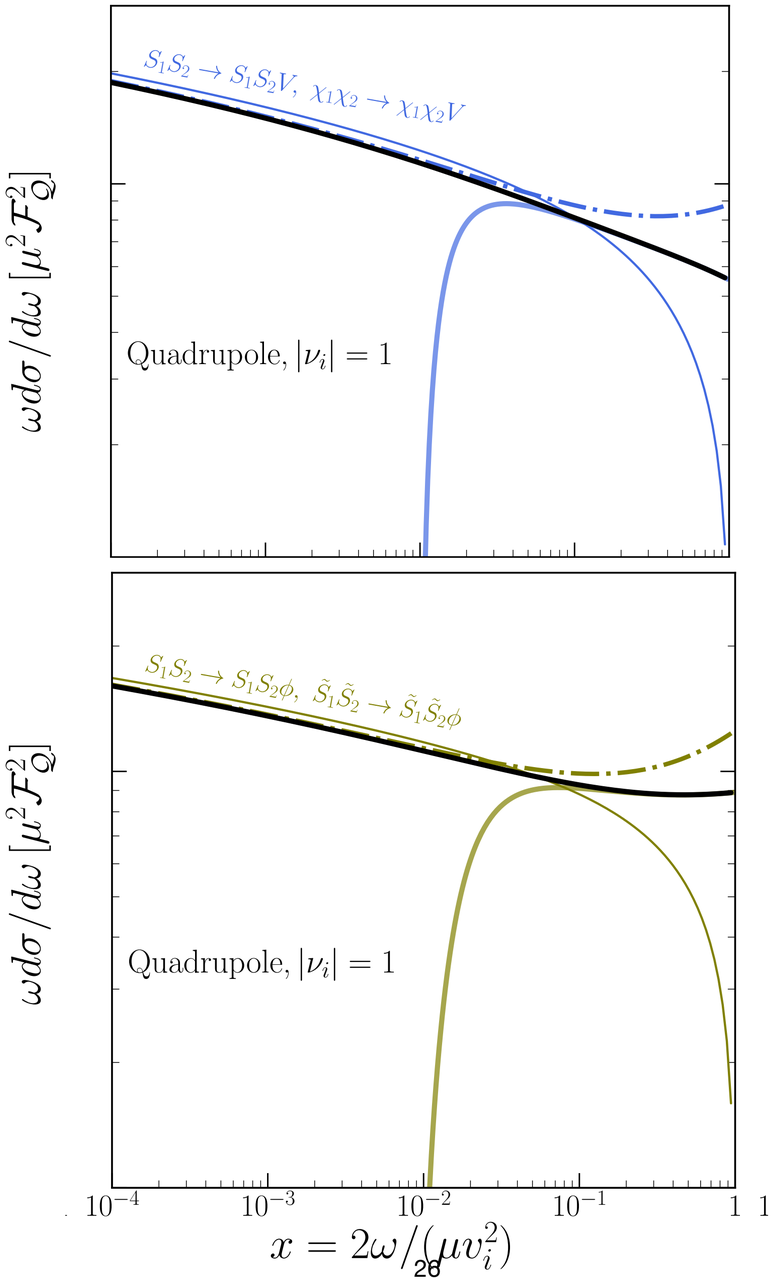
Relativistic corrections take a different form in each treatment:

- Born: momentum-dependent terms
- Exact QM: subleading emission operators
- Classical: corrected charge density:

$$\phi_{\mathcal{Q}} = \frac{1}{8\pi R_0}\mu^2\left(\frac{y_1}{m_1^2} + \frac{y_2}{m_2^2}\right)\left[\partial_{t'}^2(\hat{q}\cdot\vec{r})^2 + \eta|\dot{\vec{r}}|^2\right]$$

# Beyond Born: energy-differential cross section

arXiv:26XX.XXXX




Relativistic corrections take a different form in each treatment:

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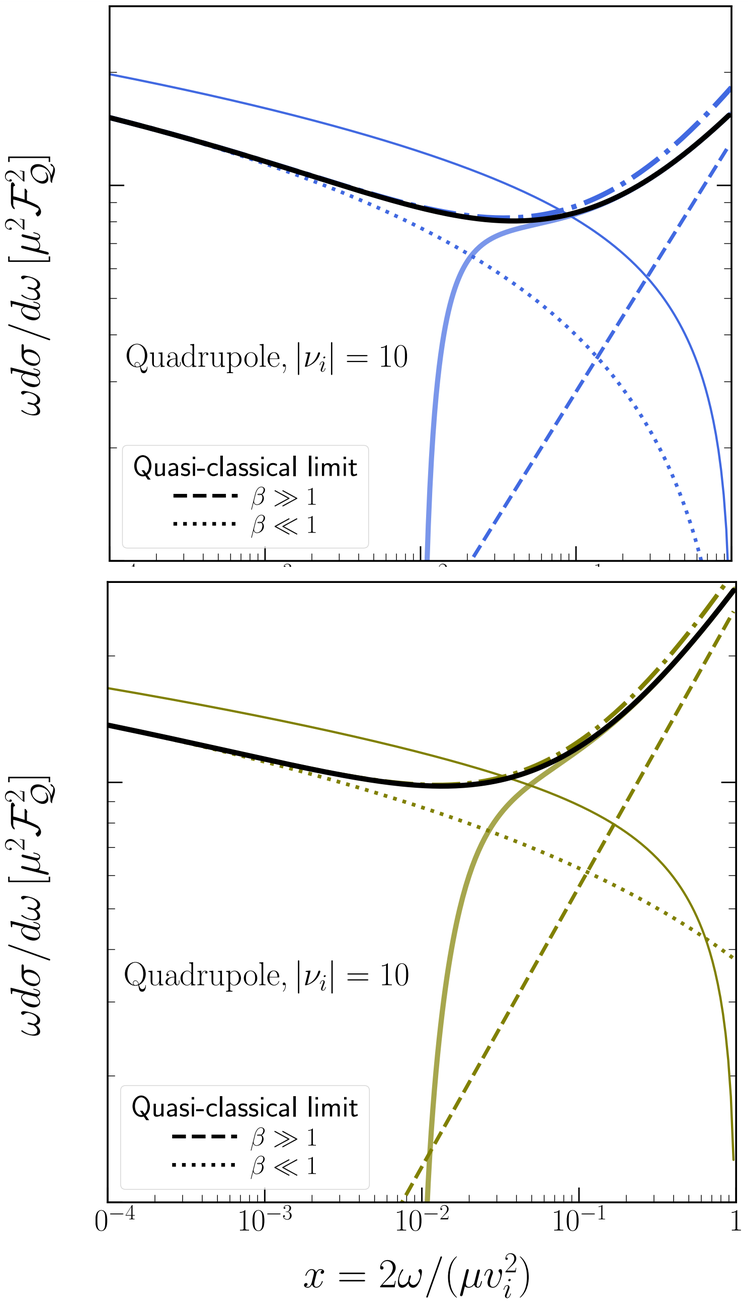
# Beyond Born: energy-differential cross section

arXiv:26XX.XXXX

Relativistic corrections take a different form in each treatment:

- Born: momentum-dependent terms
- Exact QM: subleading emission operators
- Classical: corrected charge density

Yet the full quantum result smoothly connects to both independently derived limits

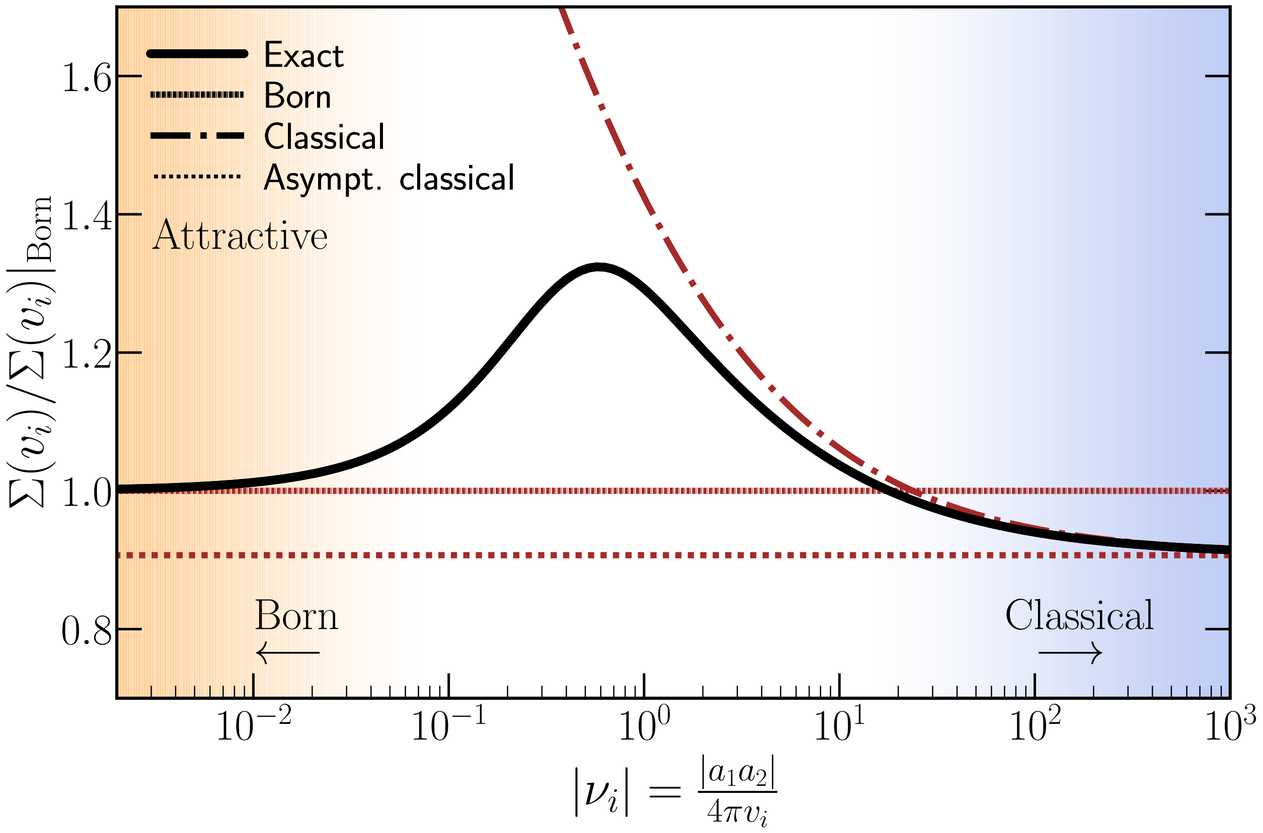


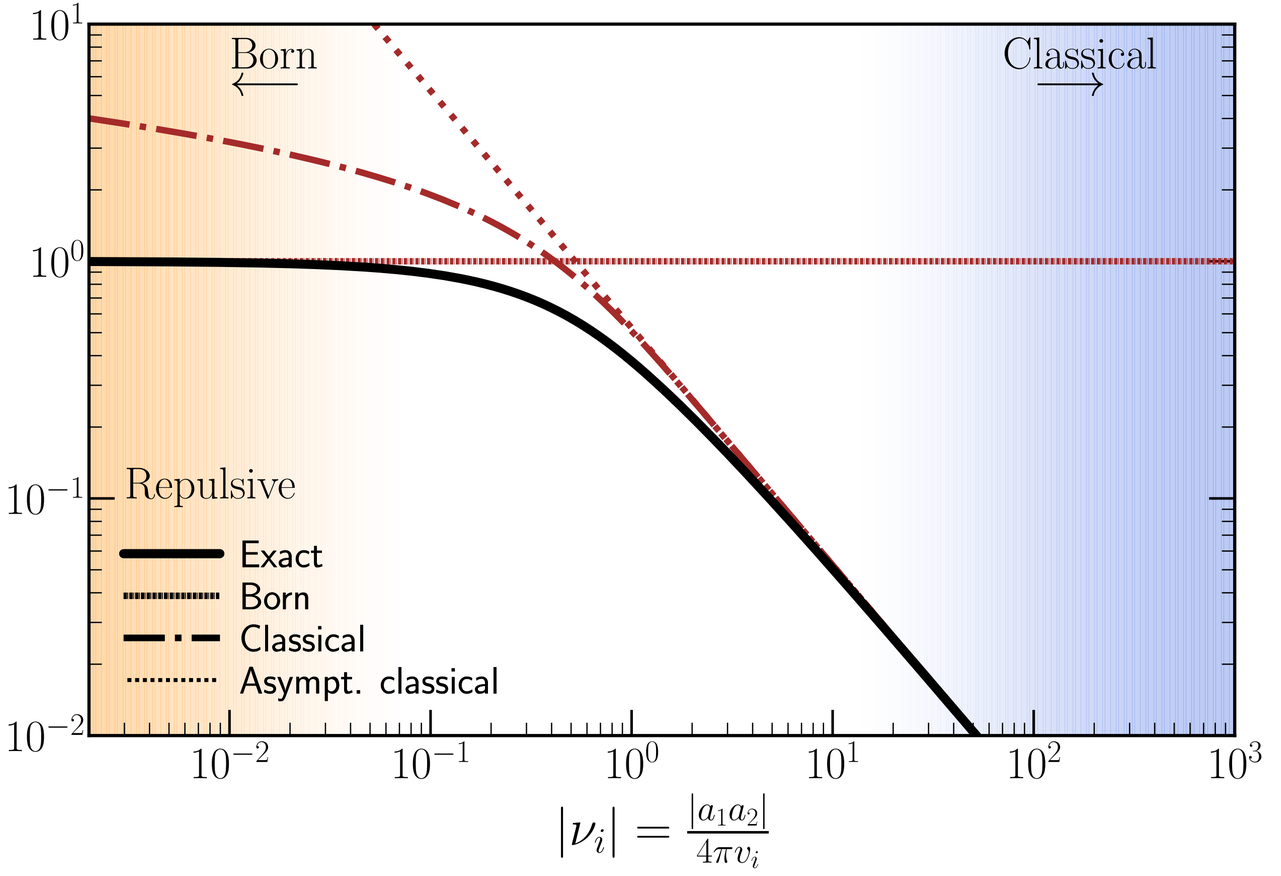
# Beyond Born: effective energy loss rate

arXiv:26XX.XXXX

$$\Sigma(v_i) = \int_{m_{\phi,V}}^{\mu v_i^2/2} d\omega\, \omega \frac{d\sigma}{d\omega}$$

Dipole emission



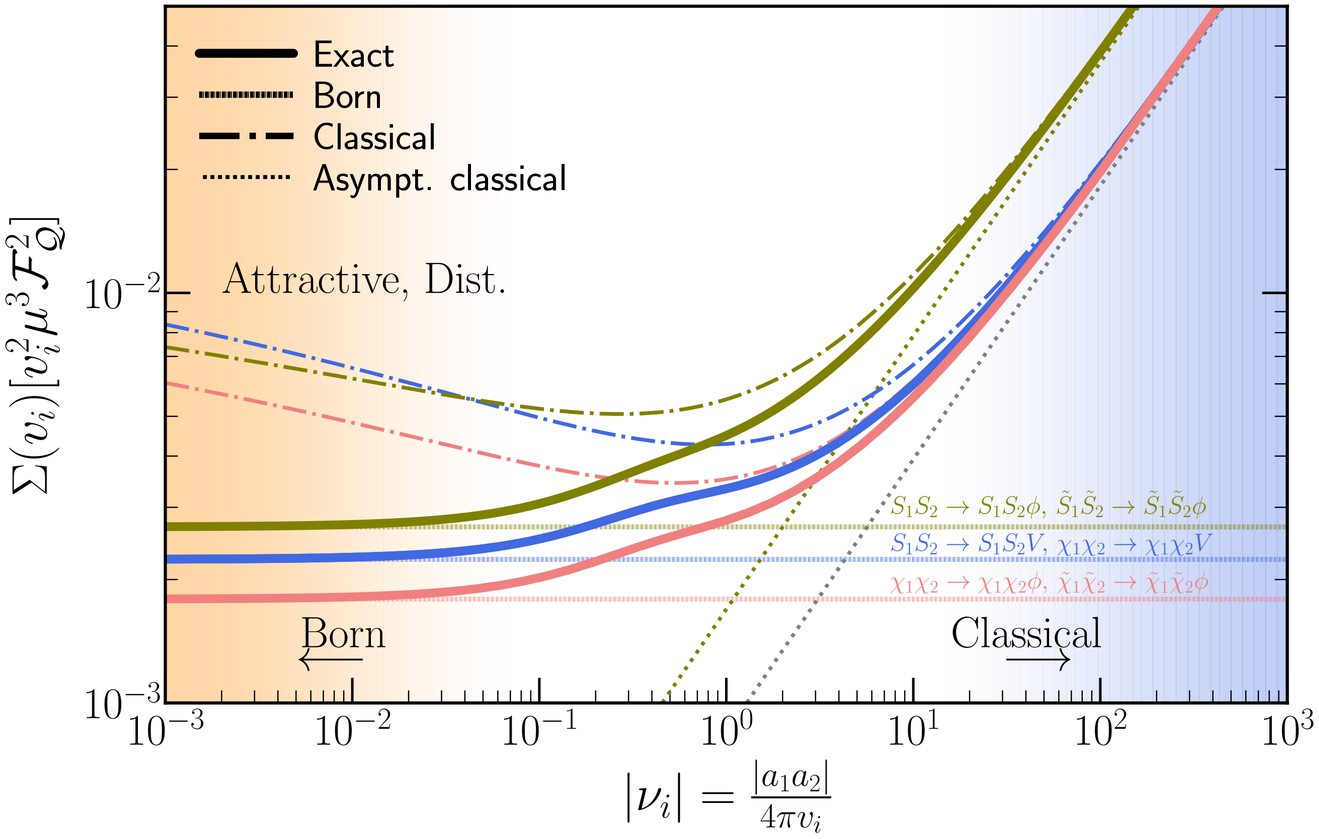



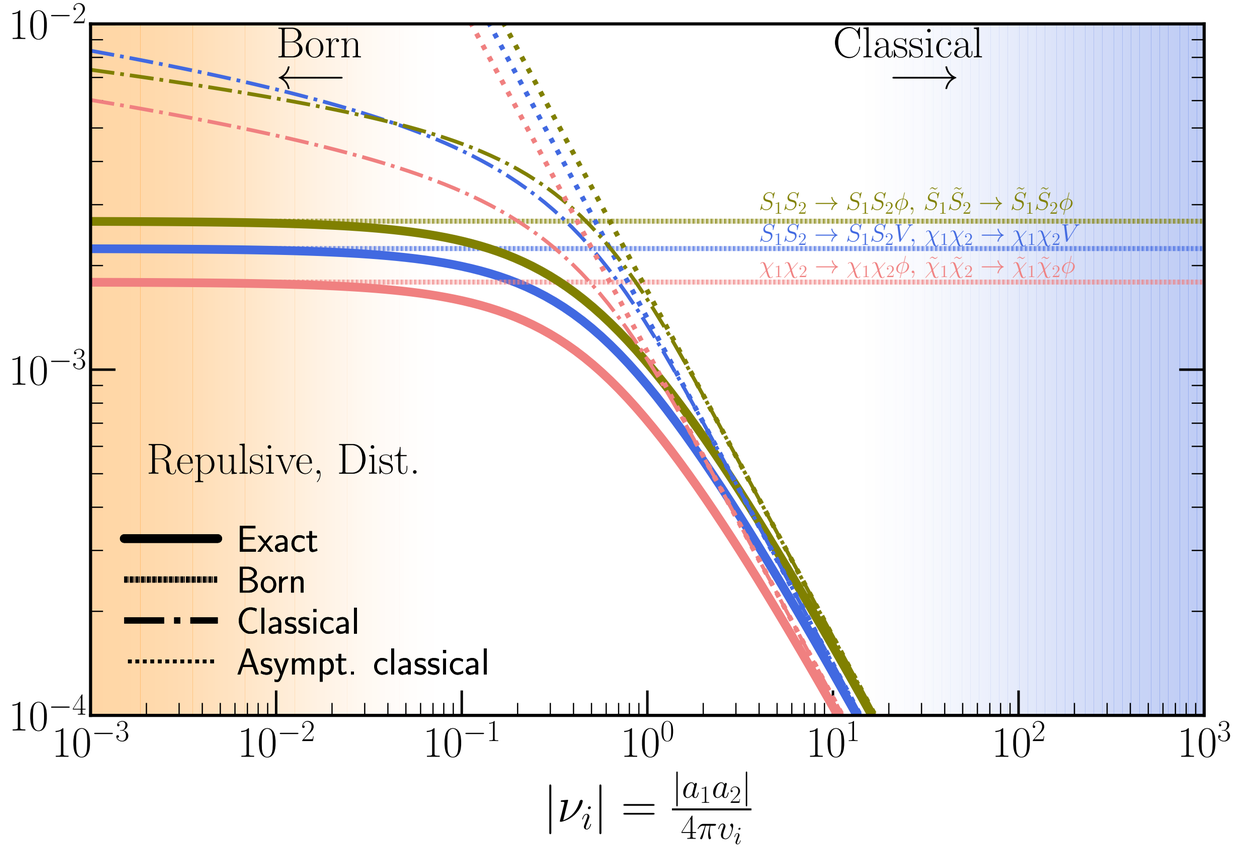
# Beyond Born: effective energy loss rate

arXiv:26XX.XXXX

$$\Sigma(v_i) = \int_{m_{\phi,V}}^{\mu v_i^2/2} d\omega\, \omega \frac{d\sigma}{d\omega}$$

Quadrupole emission: distinguishable particle scattering



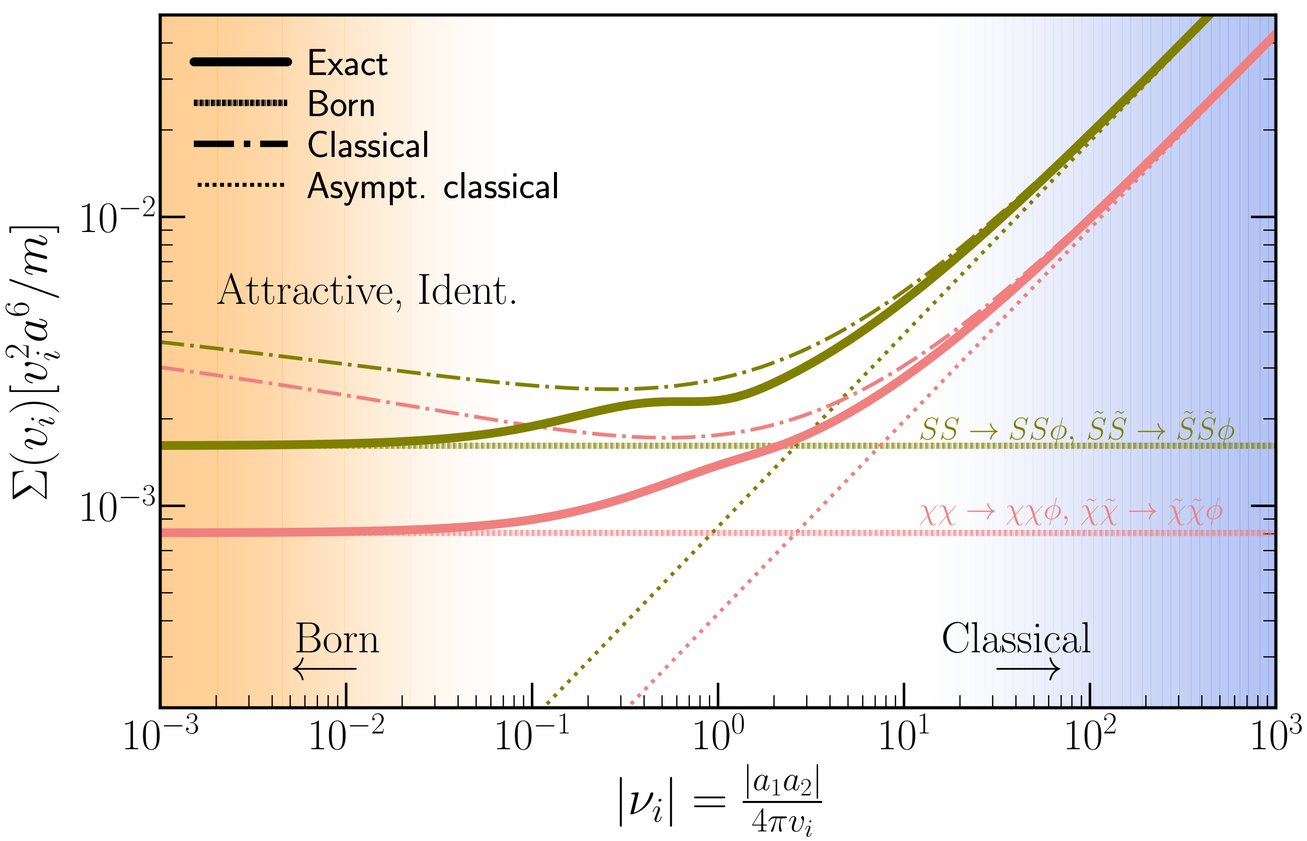



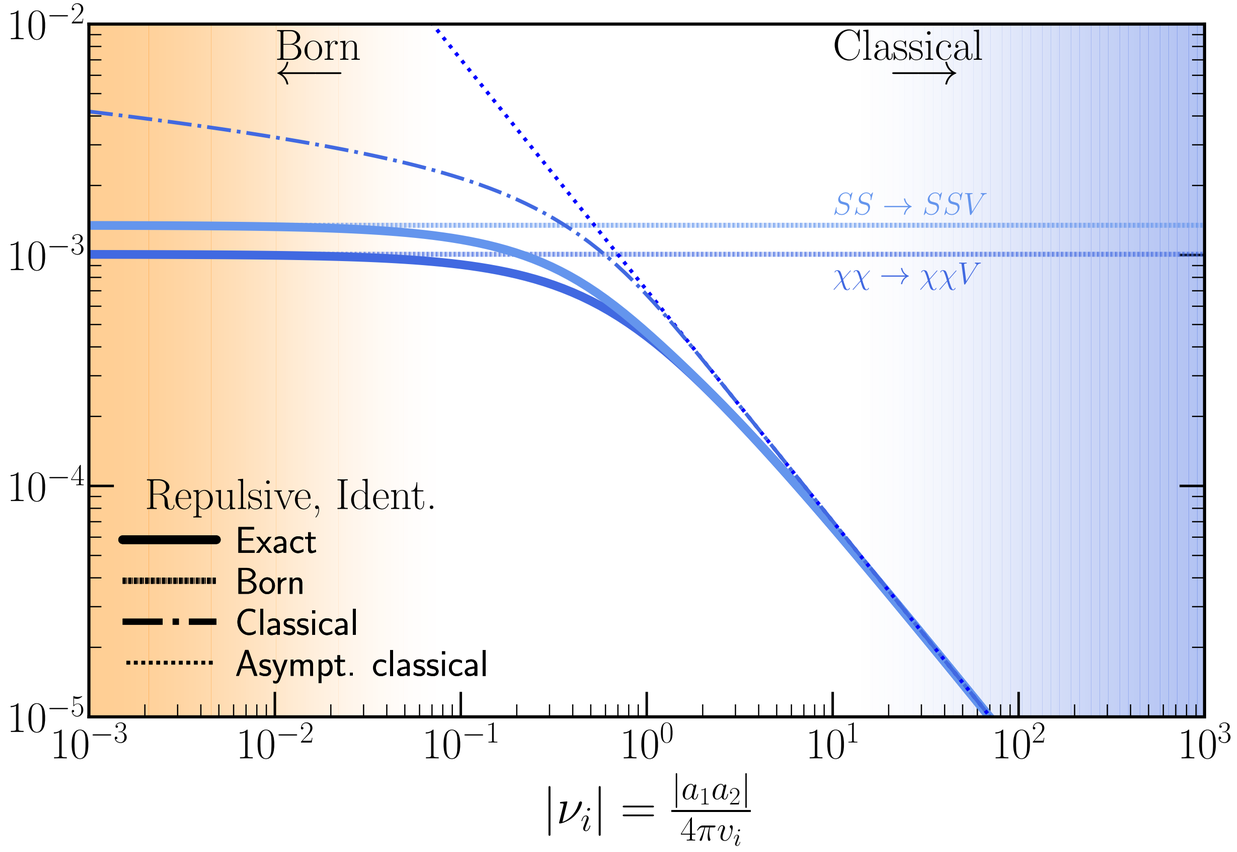
# Beyond Born: effective energy loss rate

arXiv:26XX.XXXX

$$\Sigma(v_i) = \int_{m_{\phi,V}}^{\mu v_i^2/2} d\omega\, \omega \frac{d\sigma}{d\omega}$$

Quadrupole emission: Identical particle scattering



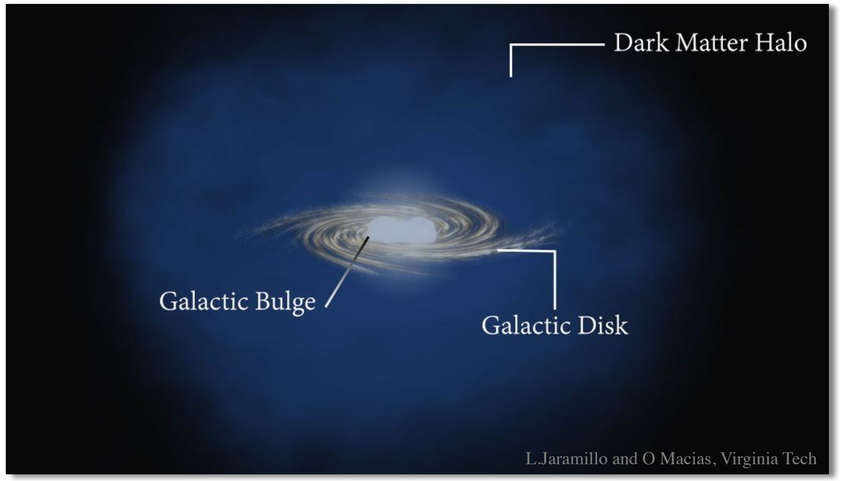



# Impact on structure formation

arXiv:2509.12317

In halos, DM can be treated as a non-degenerate Maxwellian DM gas

$$\frac{1}{2}\mu\langle v_i^2\rangle = \frac{3}{2}k_B T$$




$$\frac{\dot{\epsilon}}{n_1 n_2} \propto \frac{\mu^{3/2}}{T^{3/2}} \int_{v_{\rm min}}^{\infty} dv_i\, f(v_i) \int_{m_{\phi,V}}^{\omega_{\rm max}} d\omega\, \omega \frac{d\sigma}{d\omega}$$

→ Depends on the microphysics

**The energy per unit of time lost by the system**

$$\dot{\epsilon} = \dot{\epsilon}_{1-1} + \dot{\epsilon}_{2-2} + \dot{\epsilon}_{1-2}$$

# Energy loss rate

arXiv:2509.12317

## Light mediator

$$\dot{\epsilon} = \frac{n_1 n_2 f_{\phi,V}}{6\pi^{7/2}\sqrt{2}} \left[ \mathcal{F}_{\mathcal{D}}^2 (\mu T)^{1/2} + \frac{24}{45} \mathcal{F}_{\mathcal{Q}}^2 (\mu T)^{3/2} (4\alpha + \beta) \right]$$

$\chi_1 = \chi_2$

$$\dot{\epsilon} \propto g^6 \frac{n_\chi^2 T^{3/2}}{m_\chi^{5/2}}$$

## Heavy mediator

$$\dot{\epsilon} = \frac{32\sqrt{2} n_1 n_2 f_{\phi,V}}{15\pi^{7/2}} \left[ \mathcal{F}_{\mathcal{D}}'^2 (\mu T)^{5/2} + \frac{8}{35} \mathcal{F}_{\mathcal{Q}}'^2 (\mu T)^{7/2} (8\alpha + \beta) \right]$$

$\chi_1 = \chi_2$

$$\dot{\epsilon} \propto g^2 g'^4 \frac{n_\chi^2 T^{7/2}}{m_\chi^{1/2} m_{\phi',V'}^4}$$

$$\frac{\dot{\epsilon}_{\text{Short}}}{\dot{\epsilon}_{\text{Long}}} \sim \frac{(\mu T)^2}{m_{\phi',V'}^4}$$

The long-range processes **dominate** the dissipative dynamics.

# Energy loss rate

arXiv:2509.12317

## Light mediator

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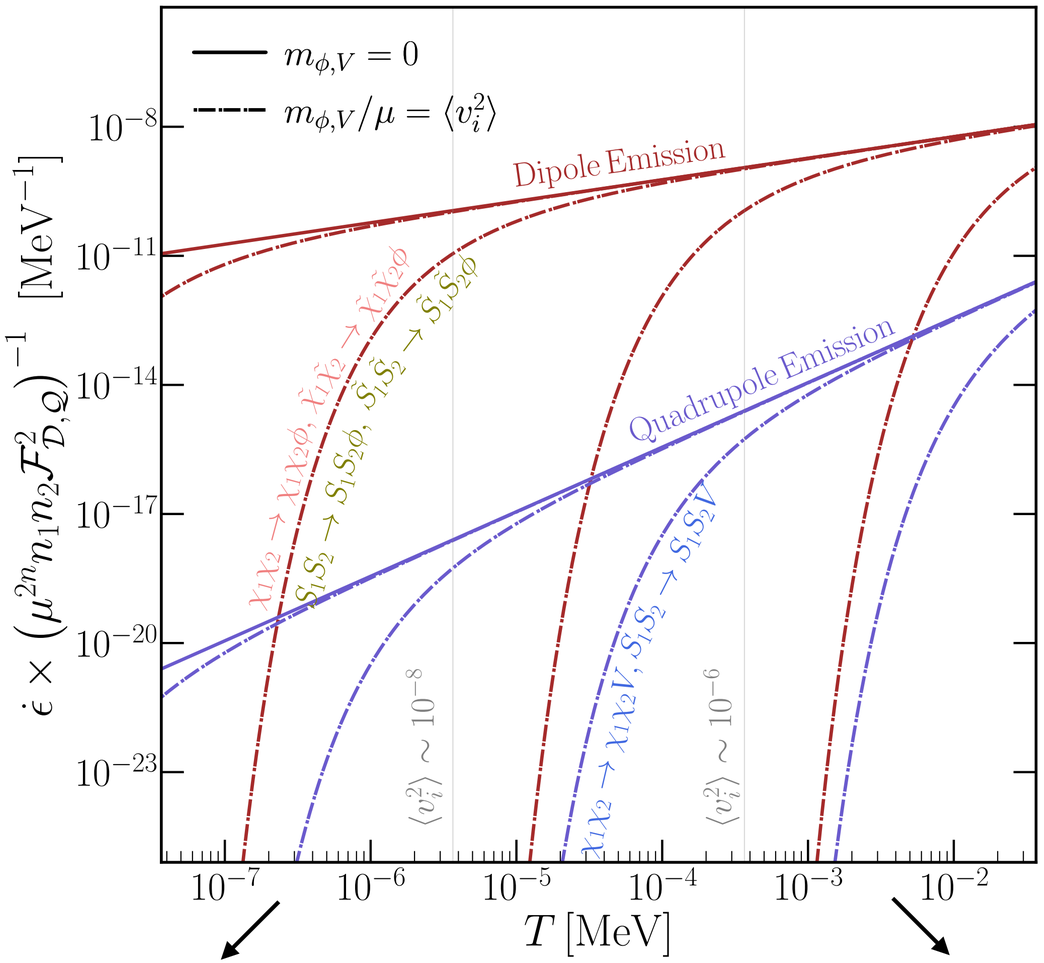
The long-range processes **dominate** the dissipative dynamics.

At each multipole order and for each mediation regime, the differences between dSIDM scenarios remain **mild**: typically within **one to two orders of magnitude** for comparable couplings and masses.

# Structure formation impact

arXiv:2509.12317

## Light mediator

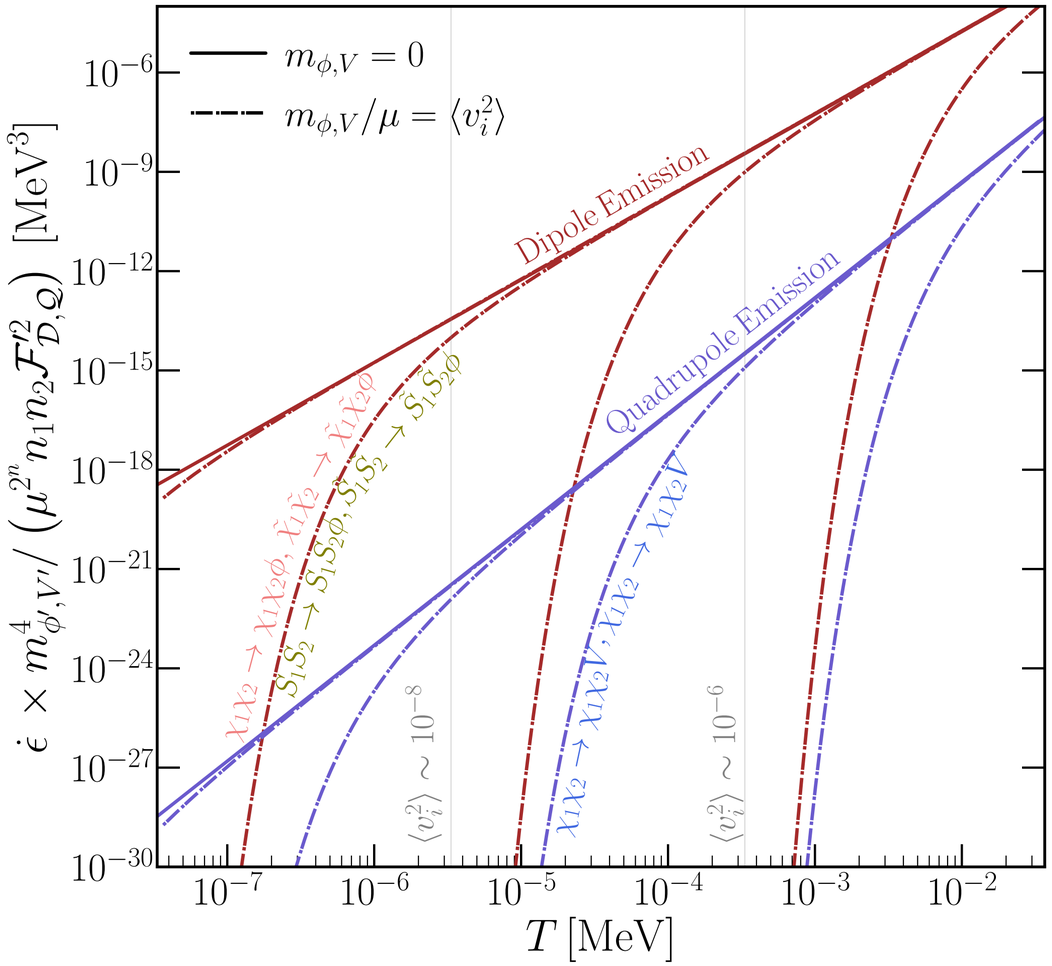



$$\frac{\dot{\epsilon}_{\rm Short}}{\dot{\epsilon}_{\rm Long}} \sim \frac{(\mu T)^2}{m^4_{\phi',V'}}$$

Dwarf galaxy

Galaxy Cluster

## Heavy mediator




Massive emissions naturally modulate the dissipative effects.

# Conclusion

DM could be self-interacting on small cosmic scales → Beyond elastic SIDM: **dissipation**

In this talk, a set of dSIDM models was investigated:

- Using a perturbative QFT and an exact QM treatment relevant in **all** the kinematic regimes, including classical emission

The different kinematic regimes highlight different aspects of the scattering dynamics:

- Born: identical vs distinguishable particle scattering
- Classical: attractive vs repulsive interaction
- Intermediate regime: non-trivial dependence on the Sommerfeld parameters

Reconstructed the dipole and quadrupole pieces of the emission amplitude:

- Dipole radiation is universal, while quadrupole emission probes model-dependent relativistic corrections

**Application:** Constrain the parameter space of the SIDM model and the formation of small-scale structures

# Nordsieck Integral

$$\mathcal{I}_N(\lambda,\vec{k},\vec{p}_i,\vec{p}_f)=\lim_{\lambda\to 0}\int d^3r\,\frac{1}{|\vec{r}|}e^{i\vec{k}\cdot\vec{r}-\lambda|\vec{r}|}\,{}_1F_1\left(i\nu_i,1;i\zeta_i^-\right)\,{}_1F_1\left(i\nu_f,1;i\zeta_f^+\right)$$

Closed-form solutions

$$\mathcal{I}_N(\lambda,\vec{k},\vec{p}_i,\vec{p}_f)=\frac{2\pi}{\eta_1}e^{-\pi\nu_i}\left(\frac{\eta_1}{\eta_3}\right)^{i\nu_i}\left(\frac{\eta_3+\eta_4}{\eta_3}\right)^{-i\nu_f}\,{}_2F_1\left(1-i\nu_i,i\nu_f;1;\frac{\eta_1\eta_4-\eta_2\eta_3}{\eta_1(\eta_3+\eta_4)}\right)$$

$$\eta_1=\frac{|\vec{k}|^2+\lambda^2}{2},\quad \eta_2=\vec{p}_f\cdot\vec{k}-i\lambda|\vec{p}_f|,$$

$$\eta_3=\vec{p}_i\cdot\vec{k}+i\lambda|\vec{p}_i|-\eta_1,\quad \eta_4=|\vec{p}_i||\vec{p}_f|+\vec{p}_i\cdot\vec{p}_f-\eta_2.$$

---

The emission amplitudes are obtained from:

$$D^a_{fi}=B_iB_f\lim_{\lambda\to 0}\left[i|\vec{p}_i|\nabla^a_{\vec{p}_i}-p_i^a\partial_\lambda\right]\mathcal{I}_N(\lambda,\vec{k},\vec{p}_i,\vec{p}_f)$$

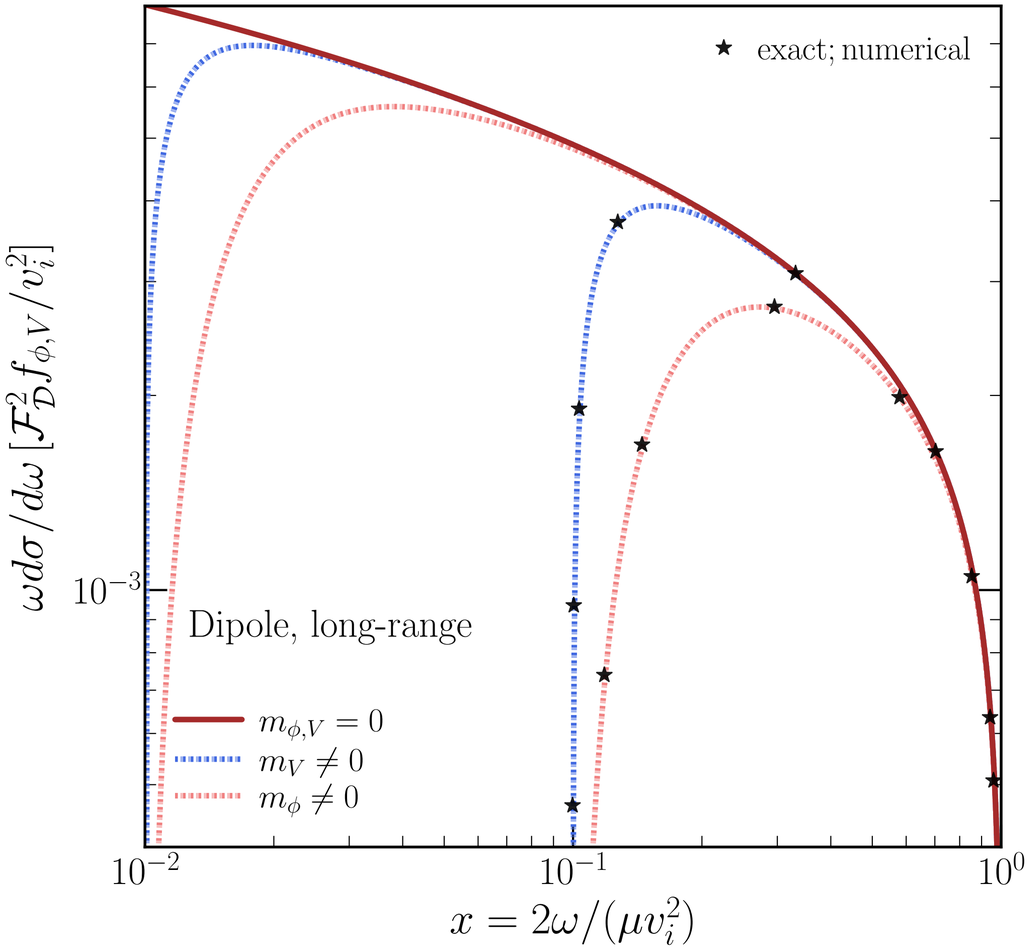
$$Q^{ab}=B_iB_f\lim_{\lambda\to 0}\partial_{k^a}\left[ip_i\partial_{p_i^b}-p_i^b\partial_\lambda\right]\mathcal{I}_N\left(\lambda,\vec{k},\vec{p}_i,\vec{p}_f\right)$$

$$\mathcal{K}=B_iB_f\lim_{\lambda\to 0}\frac{2\mu a_1a_2}{4\pi}\mathcal{I}_N\left(\lambda,\vec{k},\vec{p}_i,\vec{p}_f\right)$$

# Born: energy differential cross-section

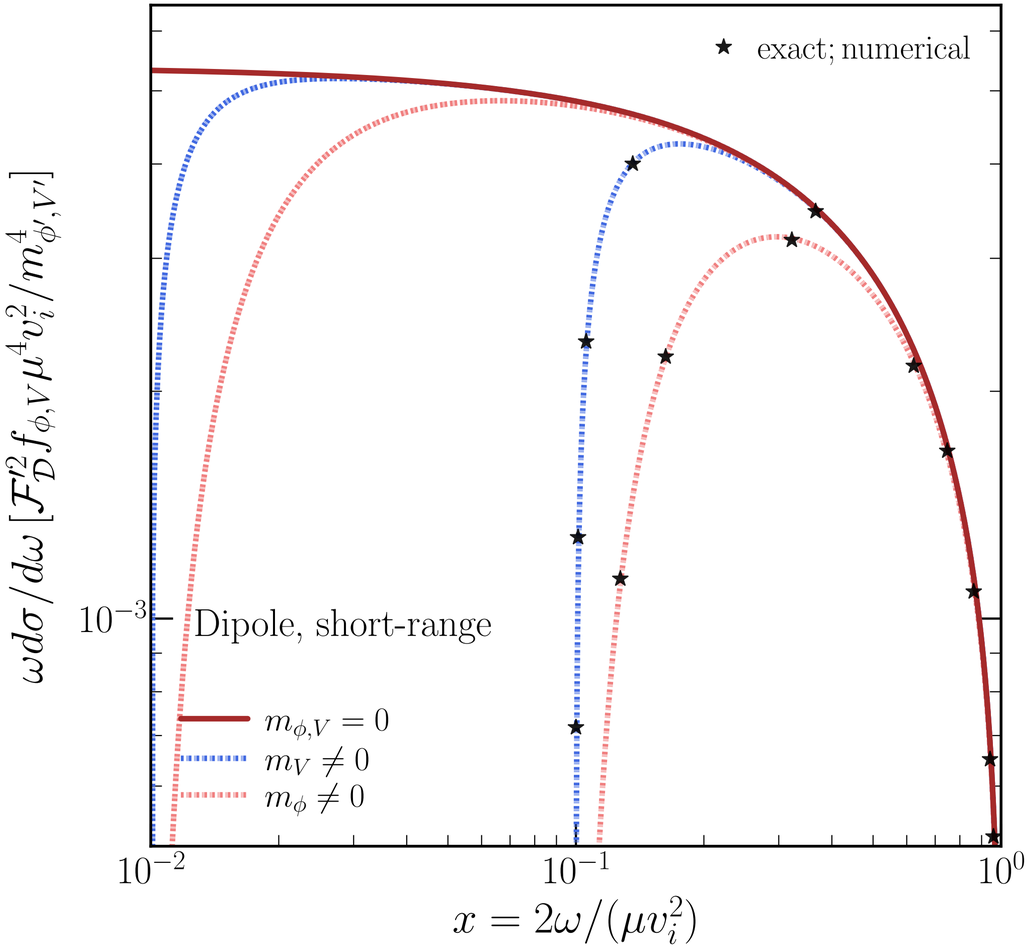
arXiv:2509.12317

## Light mediator




$$\omega\frac{d\sigma}{d\omega} = f_{\phi,V}\mathcal{F}_{\mathcal{D}}^2\frac{1}{24\pi^3 v_i^2}\ln\left(\frac{1+\sqrt{1-x}}{1-\sqrt{1-x}}\right)$$

## Heavy mediator



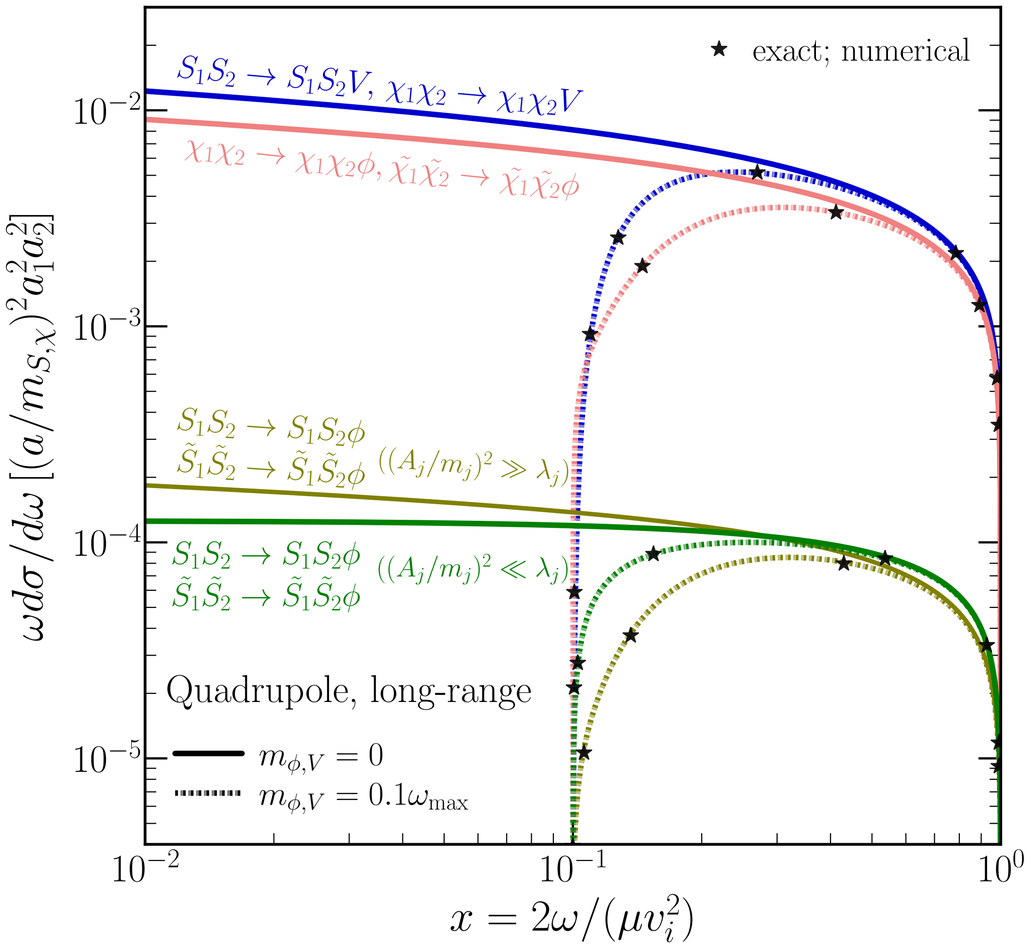

$$\omega\frac{d\sigma}{d\omega} = f_{\phi,V}\mathcal{F}_{\mathcal{D}}'^2\frac{\mu^4 v_i^2\sqrt{1-x}\,(2-x)}{12\pi^3 m_{\phi',V'}^4}$$

# Born: energy differential cross-section

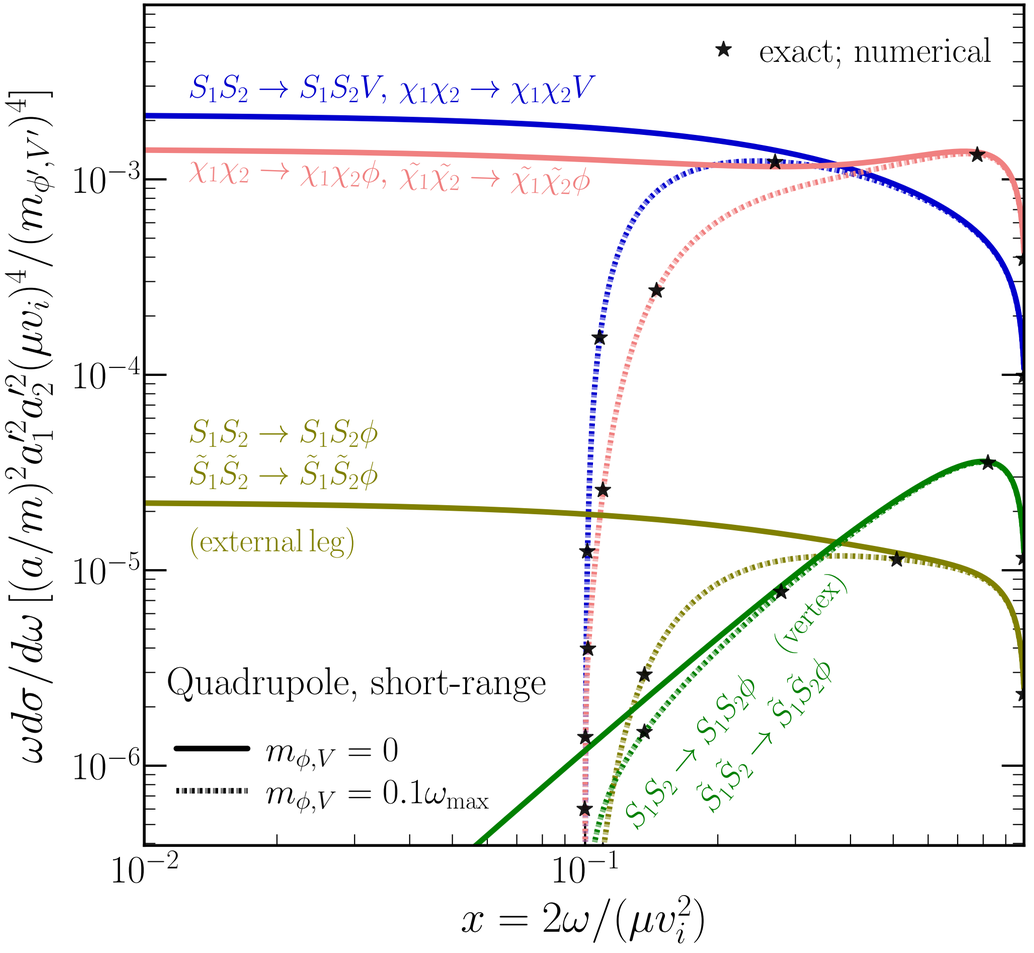
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Distinguishable particle scattering

**Light mediator**



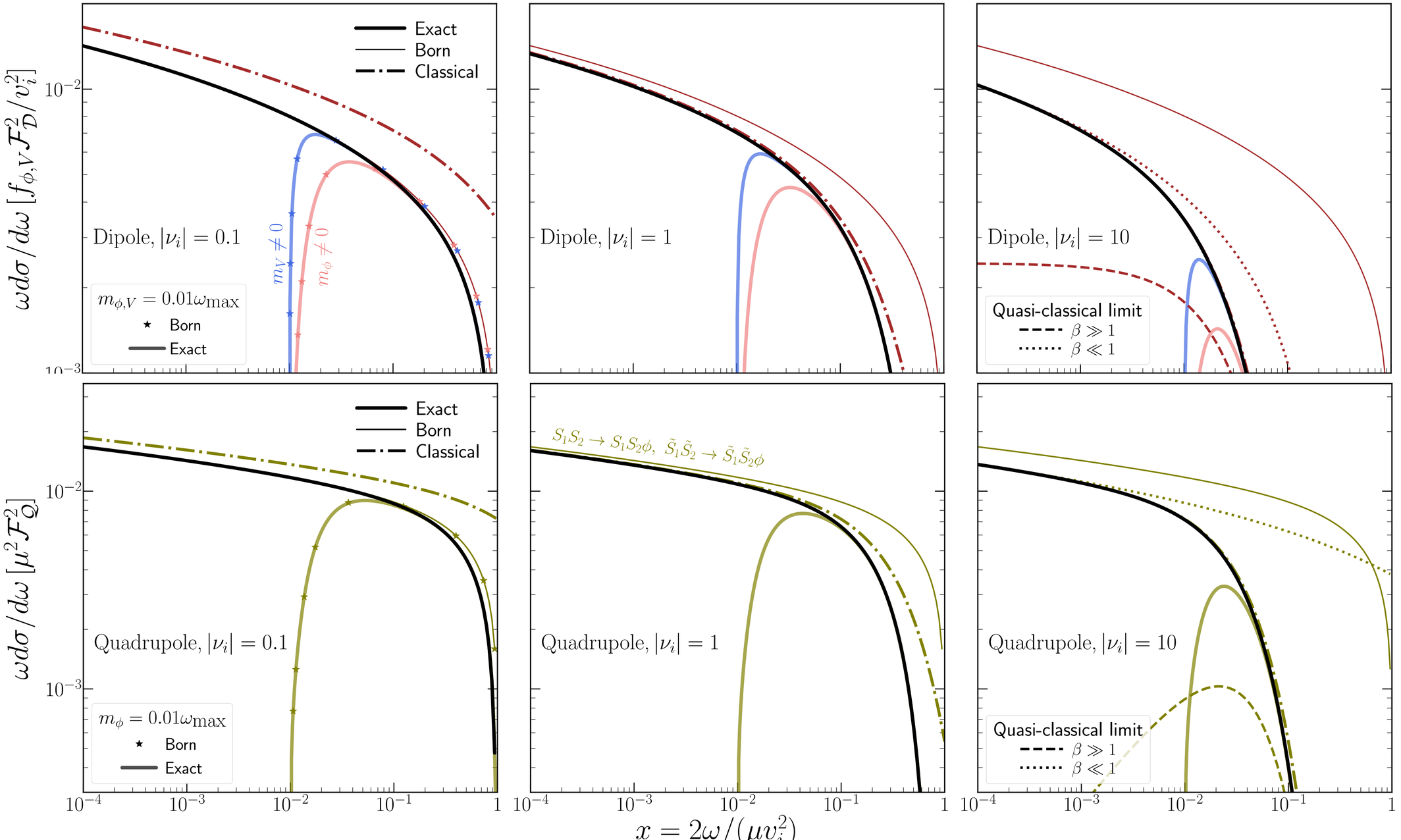

**Heavy mediator**




$$\omega\frac{d\sigma}{d\omega} = f_{\phi,V}\mathcal{F}_{\mathcal{Q}}^2\frac{\mu^2}{60\pi^3}\left[(\beta-\alpha)\sqrt{1-x}+\alpha(2-x)\ln\left(\frac{1+\sqrt{1-x}}{1-\sqrt{1-x}}\right)\right]$$

$$\omega\frac{d\sigma}{d\omega} = f_{\phi,V}\mathcal{F}_{\mathcal{Q}}'^2\frac{\mu^6 v_i^4\sqrt{1-x}}{180\pi^3 m_{\phi',V'}^4}\left[8\alpha'(1-x)+3x^2(\alpha'+\beta')\right]$$

# Energy differential cross-section: repulsive



# Amplitudes: massive emission

arXiv:2509.12317

$$\mathcal{M}_{2\to3} = \mathcal{M}_{\mathcal{O}\left(\frac{c}{v_i}\right)^3} + \mathcal{M}_{\mathcal{O}\left(\frac{c}{v_i}\right)^2} + \dots$$

Dipole emission

Quadrupole emission

---

**Massive emission:** the emitted particle carries a mass $m_{\phi,V} \lesssim \mu v_i^2$

Stuckelberg-like field

$$\varepsilon_\mu^{(L)} = \frac{1}{m_V}\left(|\vec{q}|,\, \omega\frac{\vec{q}}{|\vec{q}|}\right)$$

| | | |
|---|---|---|
| $\chi_1\chi_2 \to \chi_1\chi_2 V_L$ | $\tilde{S}_1\tilde{S}_2 \to \tilde{S}_1\tilde{S}_2\phi$ | $\chi_1\chi_2 \to \chi_1\chi_2\phi$ |
| $S_1S_2 \to S_1S_2V_L$ | $S_1S_2 \to S_1S_2\phi$ | $\tilde{\chi}_1\tilde{\chi}_2 \to \tilde{\chi}_1\tilde{\chi}_2\phi$ |

Conserved current + Ward identity: common kinematic mass-dependent correction at **dipole** and **quadrupole** orders

$$\varepsilon_\nu^{(L)}\mathcal{M}^\nu_{\mathcal{D},V} = \frac{m_V}{|\mathbf{q}|}\mathcal{M}_{\mathcal{D},\phi} \qquad \varepsilon_\nu^{(L)}\mathcal{M}^\nu = -\frac{m_V|\mathbf{q}|}{m_\phi^2}\Delta\mathcal{M}_Q(m_\phi)$$

→ Not the case for the transverse modes of V-emission