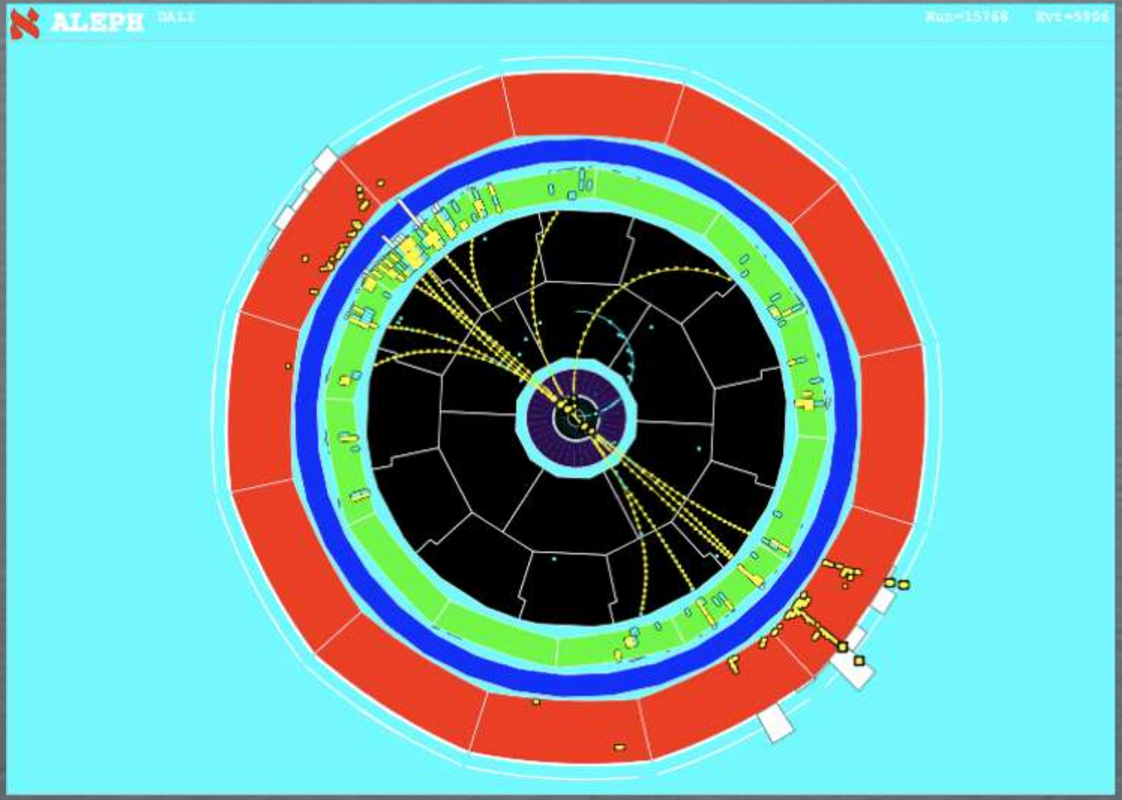
Coherence violation in jet observables



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In collaboration with J. Holguin and J. Forshaw

Vienna – 28 April 2026

Outline

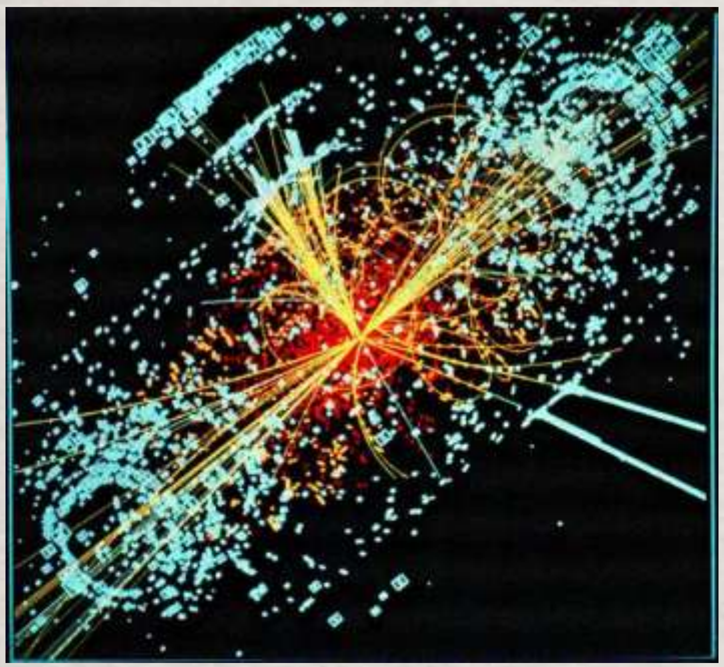
- Brief introduction of jet observables
- What is coherence of QCD radiation?
- Super-leading logarithms in gaps between jets
- Super-super-leading logarithms in one-jettiness

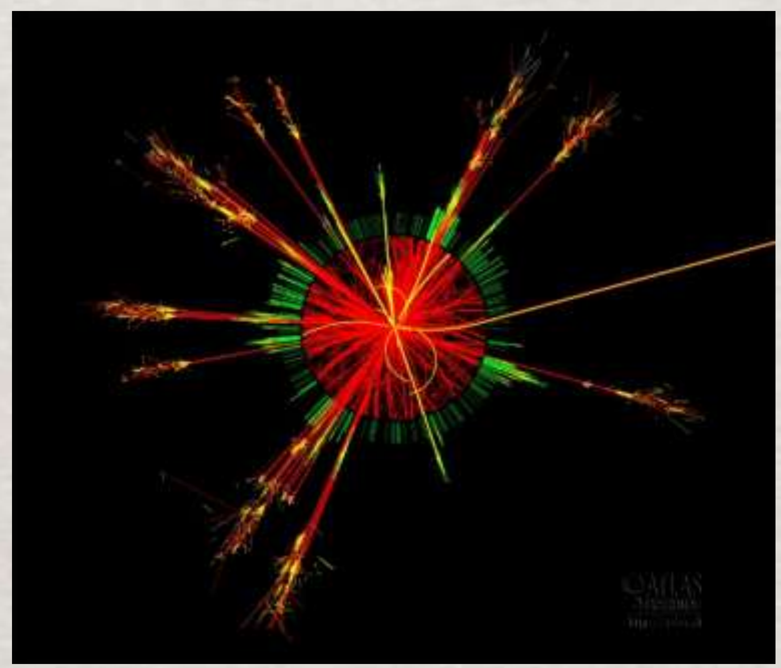
Introduction to jet observables

Hadronic jets are everywhere

Many interesting events at the LHC involve the presence of highly collimated bunches of hadrons, the jets

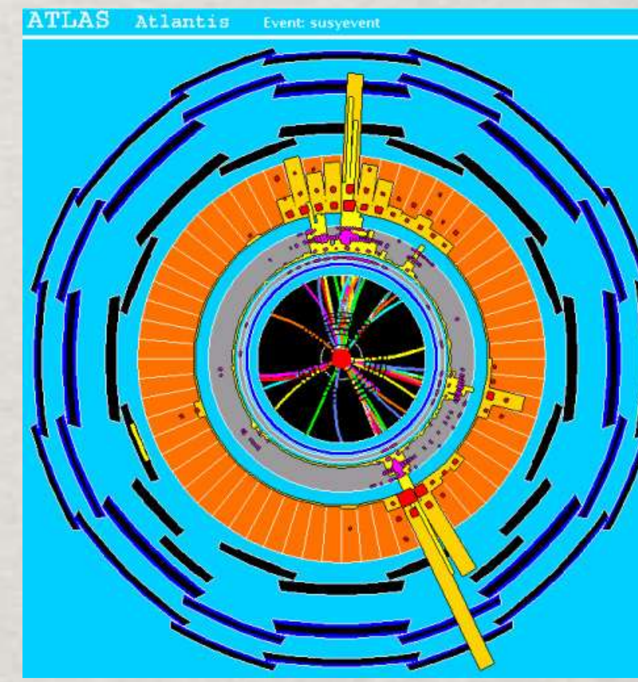
Higgs





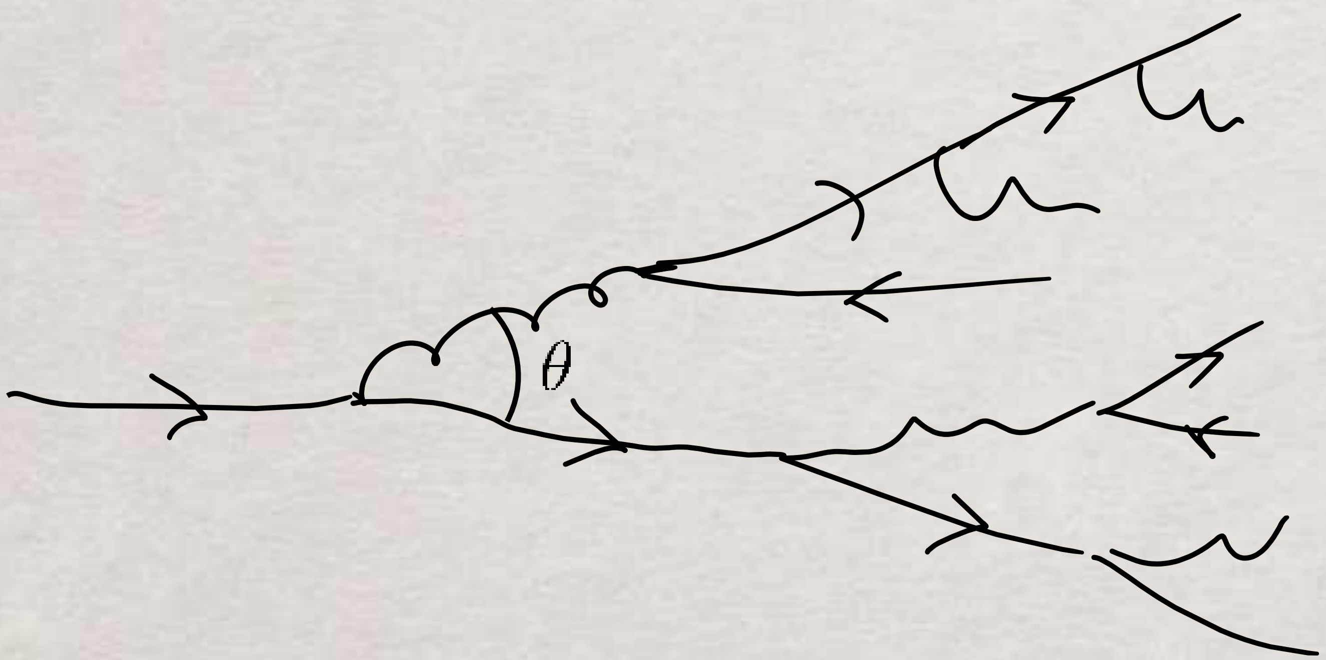
Black hole

Dark matter



Understanding jets ...

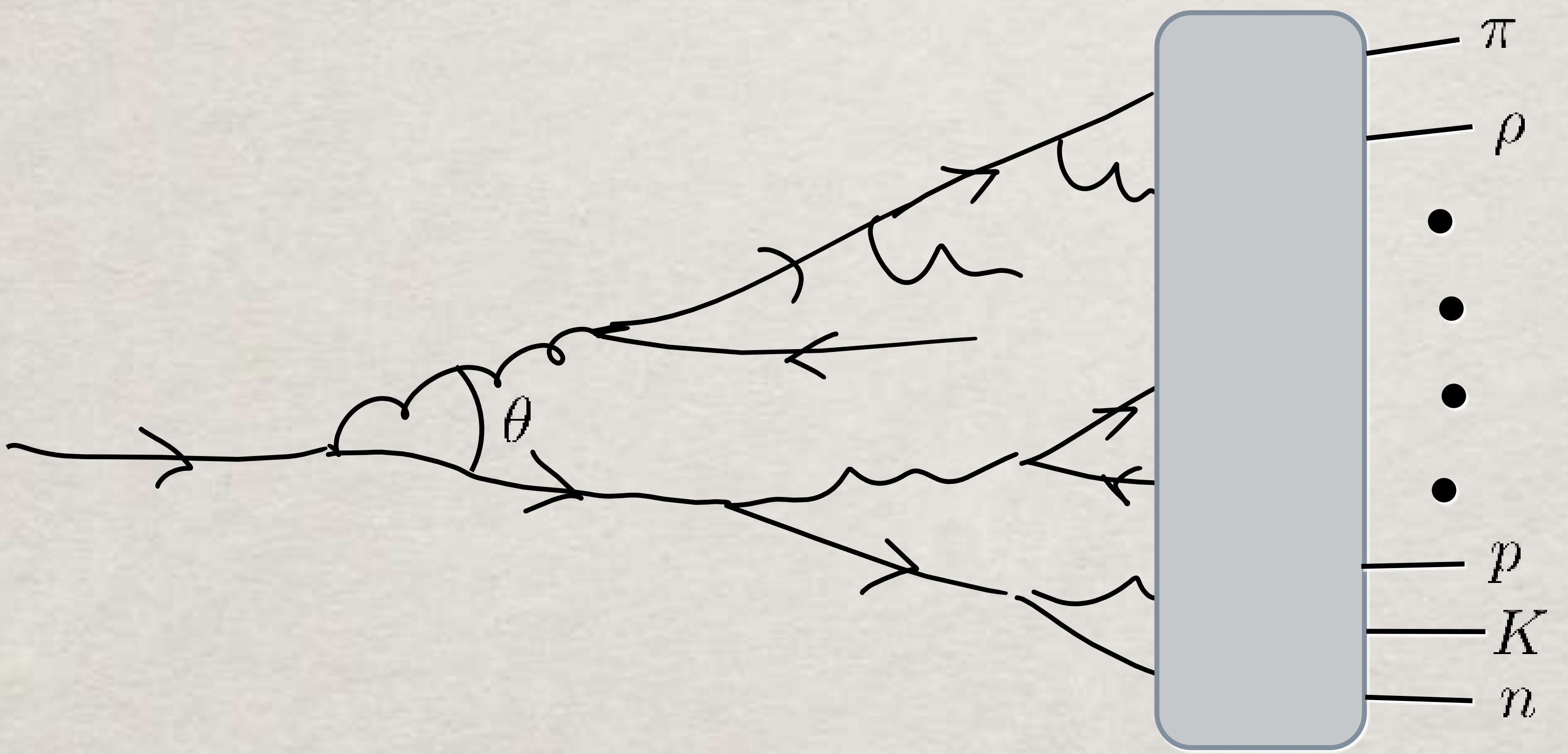
Jets start from energetic partons (quarks or gluons), which subsequently emit other partons, thus degrading their energy



$$dP \sim \frac{dE}{E}\frac{d\theta}{\theta}\alpha_s(\theta E)$$

Understanding jets ...

Jets start from energetic partons (quarks or gluons), which subsequently emit other partons, thus degrading their energy



At energies ~ 1GeV, hadronisation happens: fragmentation stops, and partons turn into hadrons with a mild reshuffling of their momenta

Final-state jet observables

- We consider a generic IRC safe final-state jet observable, a function $V(p_1, \ldots, p_n)$ of all final-state momenta $p_1, \ldots, p_n$

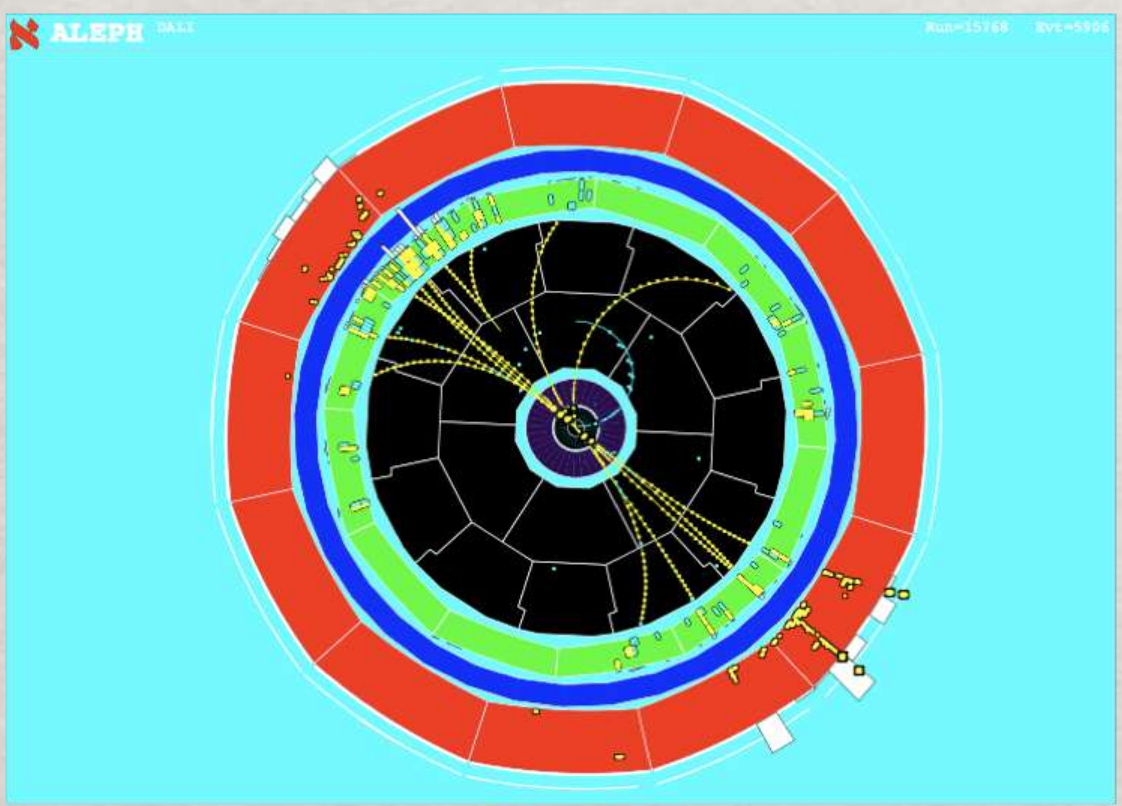
- Example: leading jet transverse momentum in Higgs production or thrust in $e^+e^- \to \mathrm{hadrons}$

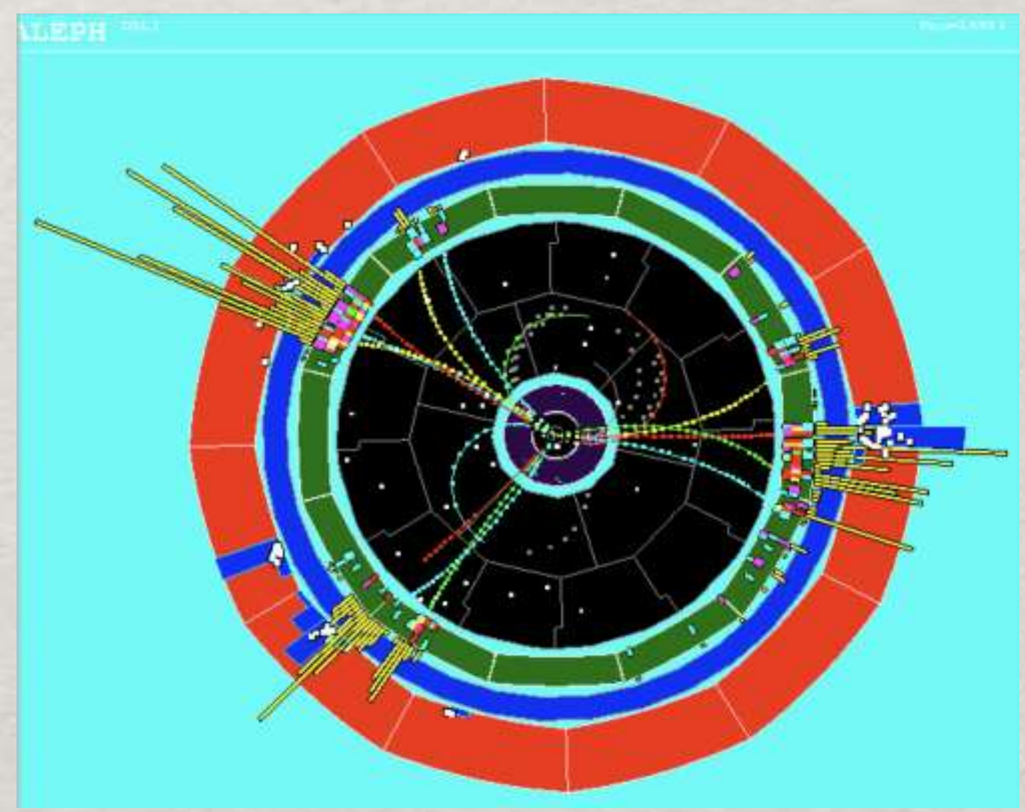
$$\frac{p_{\mathrm{t,max}}}{m_H} = \max_{j \in \mathrm{jets}} \frac{p_{t,j}}{m_H}$$

$$T \equiv \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}$$

Pencil-like events $T \to 1$

Planar events $T \to 2/3$

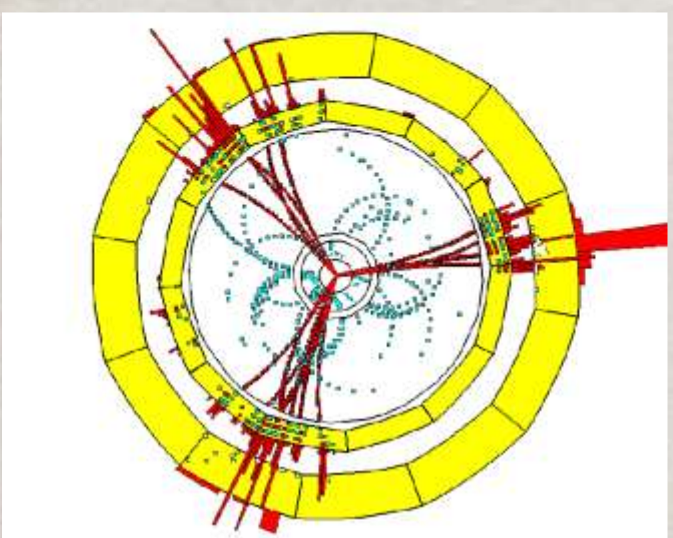


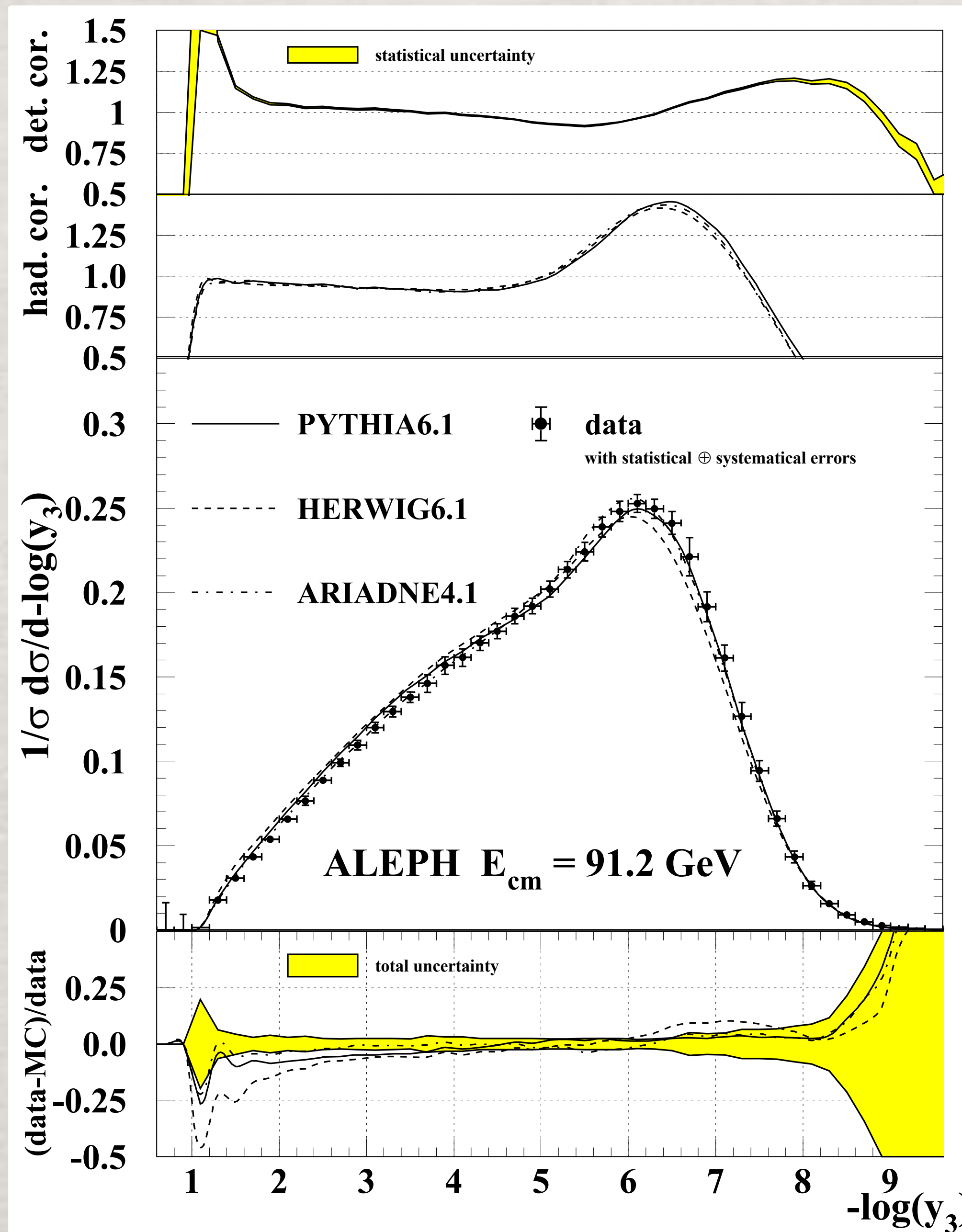


FROM SMALL TO LARGE SCALES

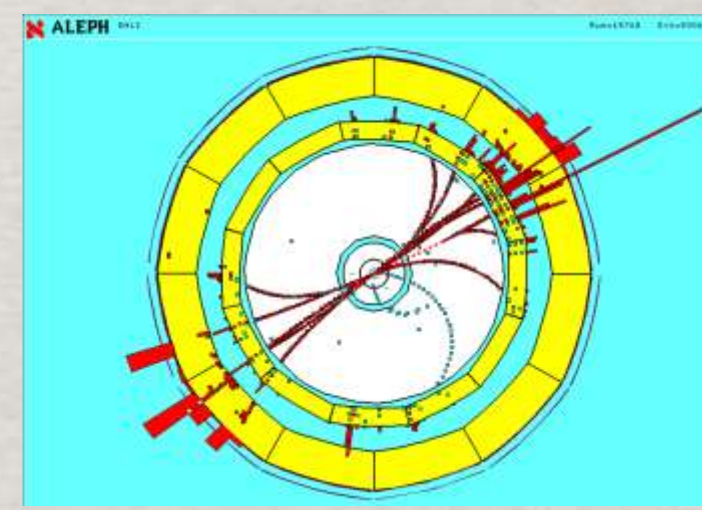
Jet observables probe a wide range of momentum scales

fixed order





resummation

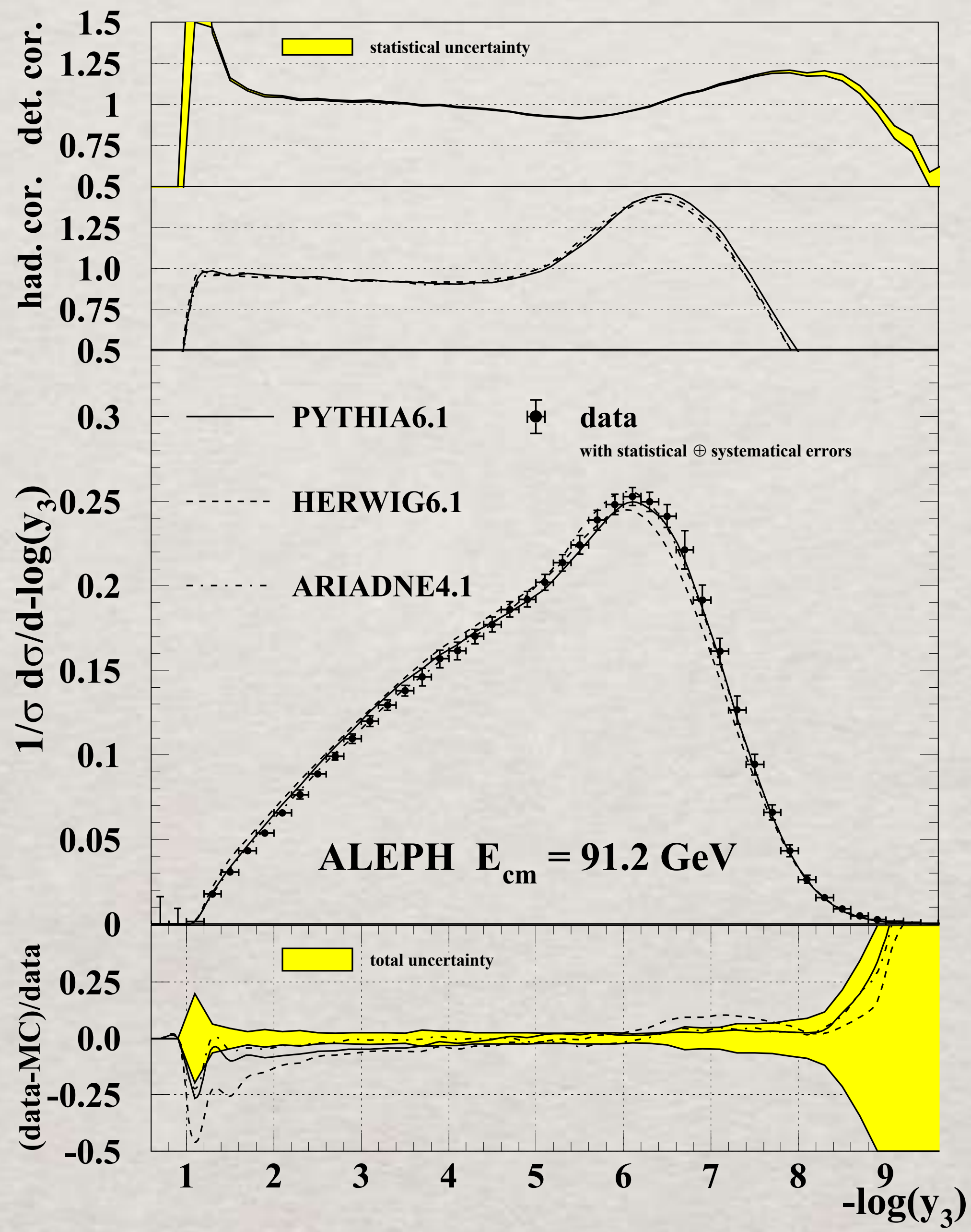


$$\sim \underbrace{\alpha_s}_{\text{LO}} + \underbrace{\alpha_s^2}_{\text{NLO}} + \underbrace{\alpha_s^3}_{\text{NNLO}} \ldots$$

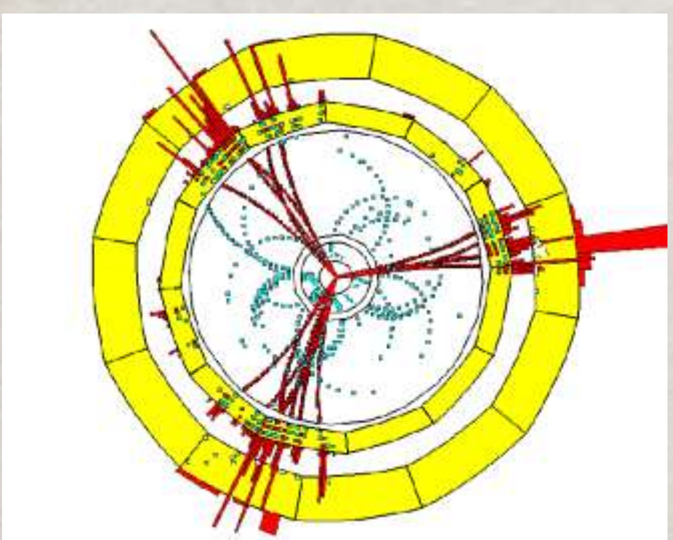
$$\sim \exp\left[\underbrace{L g_1(\alpha_s L)}_{\text{LL}} + \underbrace{g_2(\alpha_s L)}_{\text{NLL}} + \underbrace{\alpha_s g_3(\alpha_s L)}_{\text{NNLL}} + \ldots\right]$$

From small to large scales

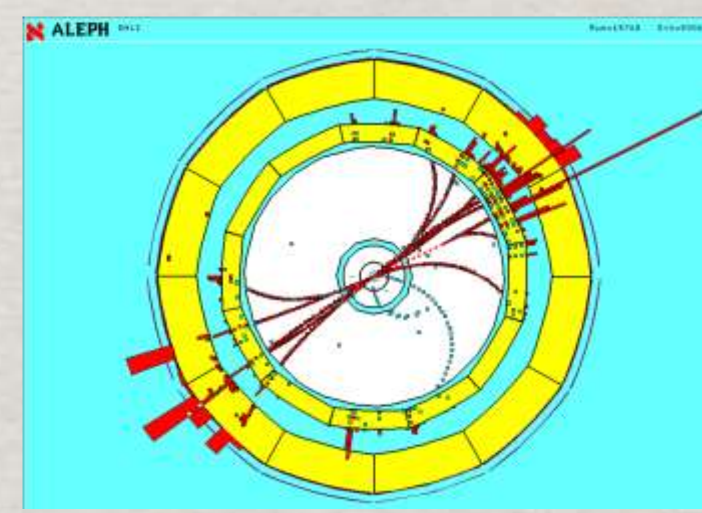
Jet observables probe a wide range of momentum scales



fixed order



resummation



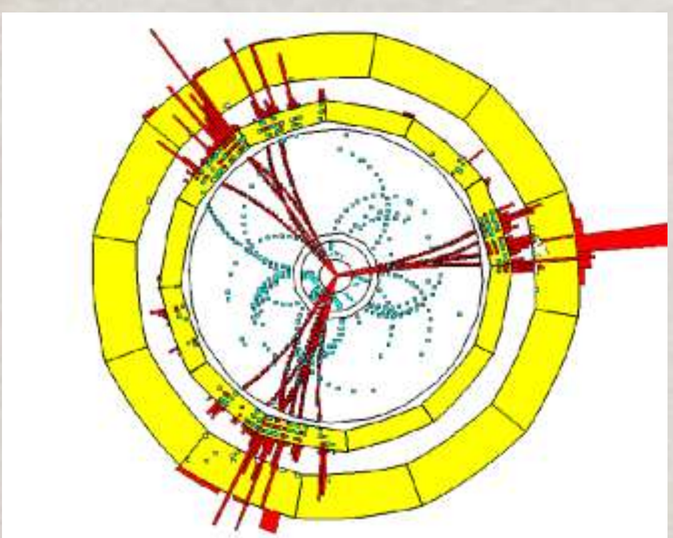
$$\sim \underbrace{\alpha_s}_{\text{LO}} + \underbrace{\alpha_s^2}_{\text{NLO}} + \underbrace{\alpha_s^3}_{\text{NNLO}} \ldots$$

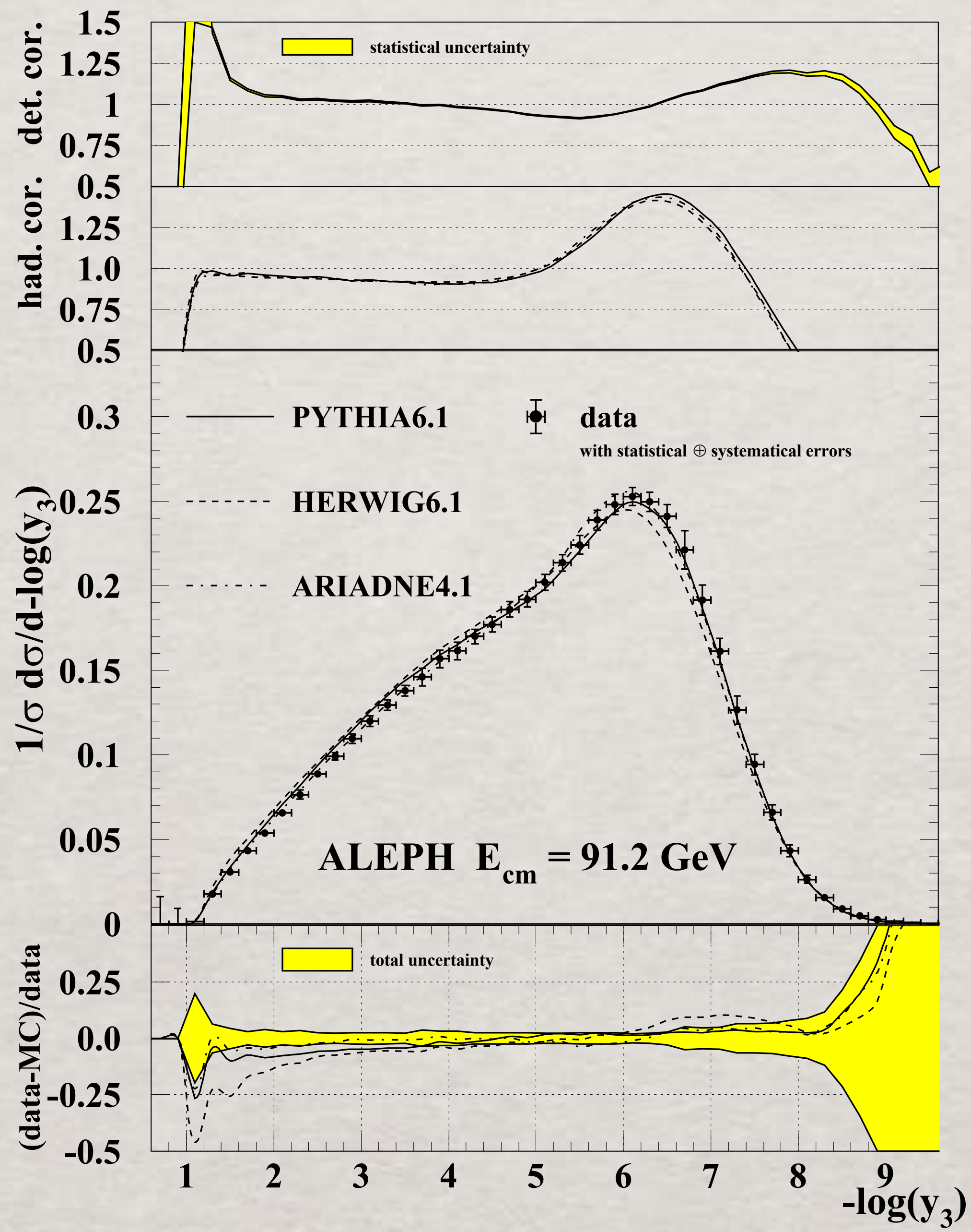
$$\underbrace{e^L}_{\text{LL}} \left[\underbrace{1}_{\text{NLL}} + \underbrace{\alpha_s}_{\text{NNLL}} + \underbrace{\alpha_s^2}_{\text{NNNLL}} + \ldots \right]$$

From small to large scales

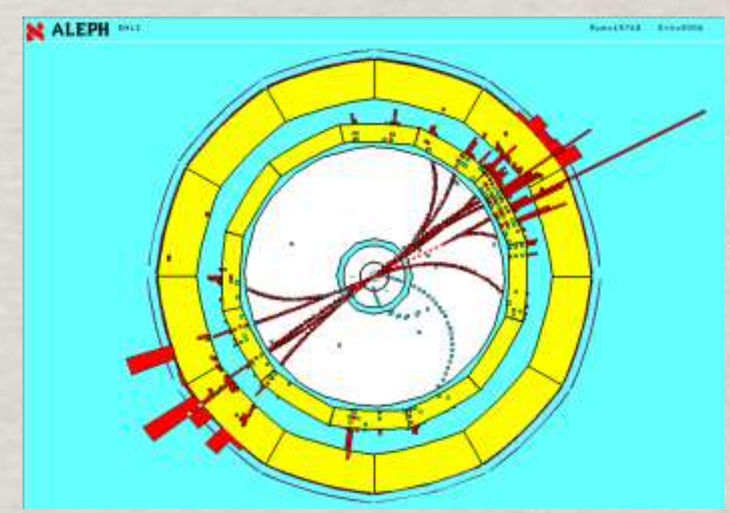
Jet observables probe a wide range of momentum scales

fixed order





resummation



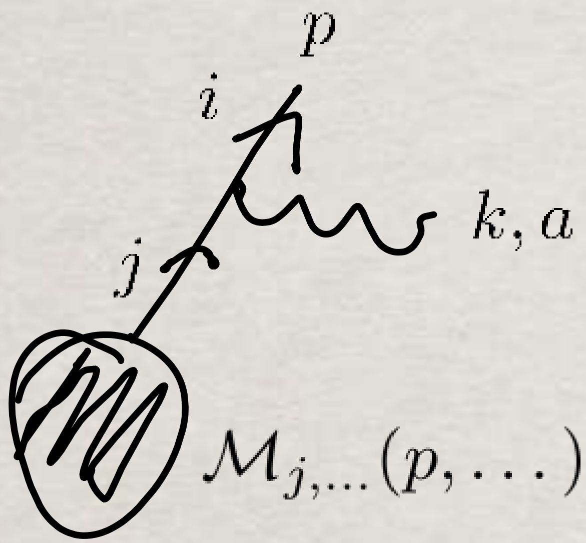
$$\sim \underbrace{\alpha_s}_{\text{LO}} + \underbrace{\alpha_s^2}_{\text{NLO}} + \underbrace{\alpha_s^3}_{\text{NNLO}} \ldots$$

$$\underbrace{e^L}_{\text{S LL}} \left[\underbrace{1}_{\cancel{\text{N}}\text{LL}} + \underbrace{\alpha_s}_{\cancel{\text{N}}\text{NLL}} + \underbrace{\alpha_s^2}_{\cancel{\text{N}}\text{NNLL}} + \ldots \right]$$

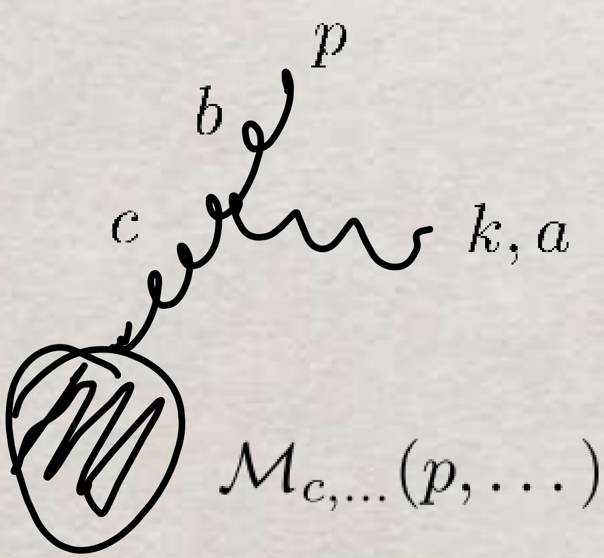
What is coherence?

Soft gluon emission

Soft gluon emission is known to factorise from hard amplitudes



$$t^a_{ij}\,\frac{p\cdot\varepsilon(k)}{p\cdot k}\,\mathcal{M}_{j,\dots}(p,\dots)$$

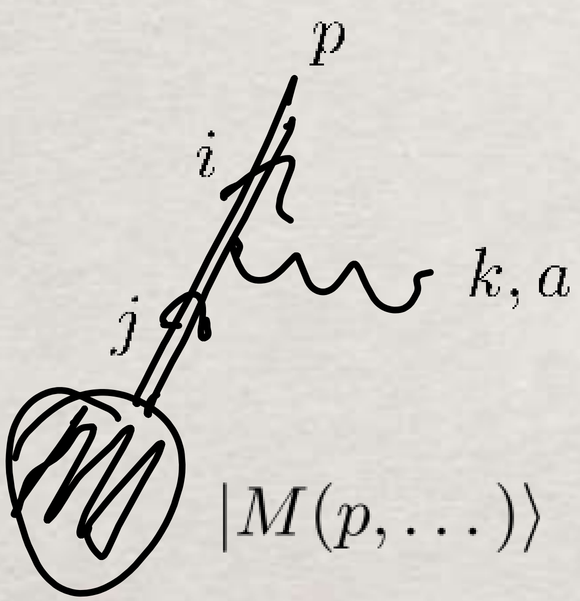


$$-if^{abc}\,\frac{p\cdot\varepsilon(k)}{p\cdot k}\,\mathcal{M}_{c,\dots}(p,\dots)$$

Colour matrices

It is useful to introduce a representation-independent notation

$$\langle i|\mathbf{T}^a|j\rangle = t^a_{ij} \qquad\qquad \langle b|\mathbf{T}^a|c\rangle = -if^{abc}$$



$$\mathbf{T}^a\,\frac{p\cdot\varepsilon(k)}{p\cdot k}|M(p,\ldots)\rangle$$

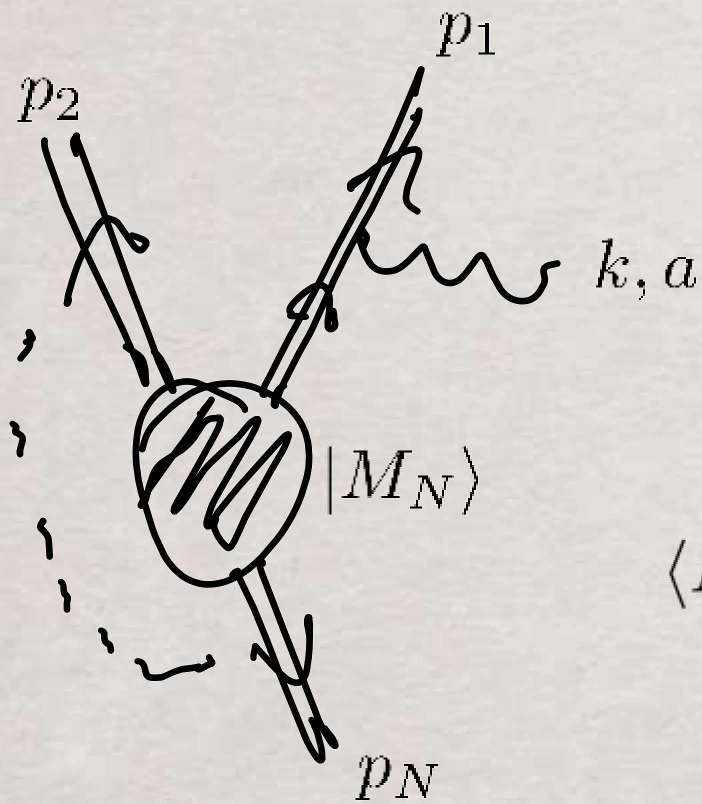
In this notation, Casimir operators can be understood as dot products

$$\sum_a \mathbf{T}^a\mathbf{T}^a = \mathbf{T}\cdot\mathbf{T} = \mathbf{T}^2 = C_R\mathbf{1}$$

$$C_R = \begin{cases} C_F = \dfrac{N_c^2-1}{2N_c} & \text{quark} \\ C_A = N_c & \text{gluon} \end{cases}$$

Soft amplitude squared

It is useful to introduce a representation-independent notation



$$\underbrace{|M(p_1,\ldots,p_N,k)\rangle}_{\equiv|M_{N+1}\rangle} = \sum_{i=1}^{N} \mathbf{T}_i \frac{p_i\cdot\varepsilon(k)}{p_i\cdot k} \underbrace{|M(p_1,\ldots,p_N)\rangle}_{\equiv|M_N\rangle}$$

$$\Downarrow$$

$$\langle M_{N+1}|M_{N+1}\rangle = -\sum_{i=1}^{N}\sum_{j\neq i}\frac{p_i\cdot p_j}{(p_i\cdot k)(p_j\cdot k)}\langle M_N|\mathbf{T}_i\cdot\mathbf{T}_j|M_N\rangle$$

The amplitude squared can be also written in terms of a colour-density matrix

$$\langle M_{N+1}|M_{N+1}\rangle = -\sum_{i=1}^{N}\sum_{j\neq i}\frac{p_i\cdot p_j}{(p_i\cdot k)(p_j\cdot k)}\mathrm{Tr}\left[(\mathbf{T}_i\cdot\mathbf{T}_j)\,\mathbf{H}_N\right]$$

$$\mathbf{H}_N \equiv |M_N\rangle\langle M_N| \qquad\qquad \mathrm{Tr}(\mathbf{H}_N) = \mathcal{M}_N^2$$

Colour conservation

Colour conservation can be used to further simplify the square amplitude

$$\langle M_{N+1}|M_{N+1}\rangle = -\sum_{i=1}^{N}\sum_{j\neq i}\frac{p_i\cdot p_j}{(p_i\cdot k)(p_j\cdot k)}\langle M_N|\mathbf{T}_i\cdot\mathbf{T}_j|M_N\rangle$$

- Reconstruction of Casimir once one sums over all legs

$$\mathbf{T}_1+\mathbf{T}_2+\cdots+\mathbf{T}_N=0 \implies \mathbf{T}_i=-\sum_{j\neq i}\mathbf{T}_j \implies -\sum_{j\neq i}\mathbf{T}_i\cdot\mathbf{T}_j=\mathbf{T}_i^2=C_i$$

- Simplification of colour algebra up to three hard emitters

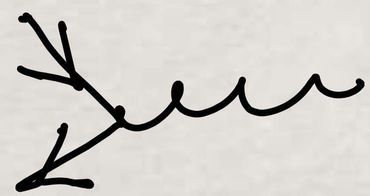
$$\mathbf{T}_1+\mathbf{T}_2+\mathbf{T}_3=0 \implies (\mathbf{T}_i+\mathbf{T}_j)^2=-\mathbf{T}_k^2=-C_k \implies -\mathbf{T}_i\cdot\mathbf{T}_j=\frac{C_i+C_j-C_k}{2}\mathbf{1}$$

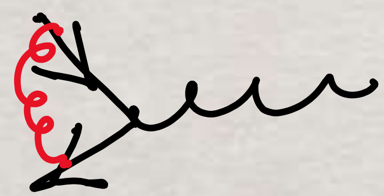
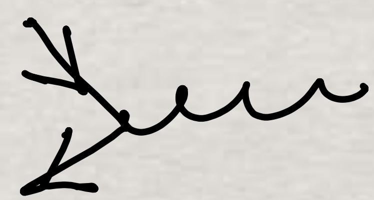
Colour conservation

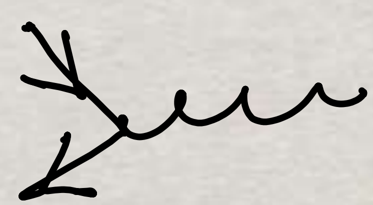
Colour conservation can be used to further simplify the square amplitude

$$\langle M_{N+1}|M_{N+1}\rangle = -\sum_{i=1}^{N}\sum_{j\neq i}\frac{p_i\cdot p_j}{(p_i\cdot k)(p_j\cdot k)}\langle M_N|\mathbf{T}_i\cdot\mathbf{T}_j|M_N\rangle$$

Compare with the explicit expression in terms of $SU(N_c)$ matrices

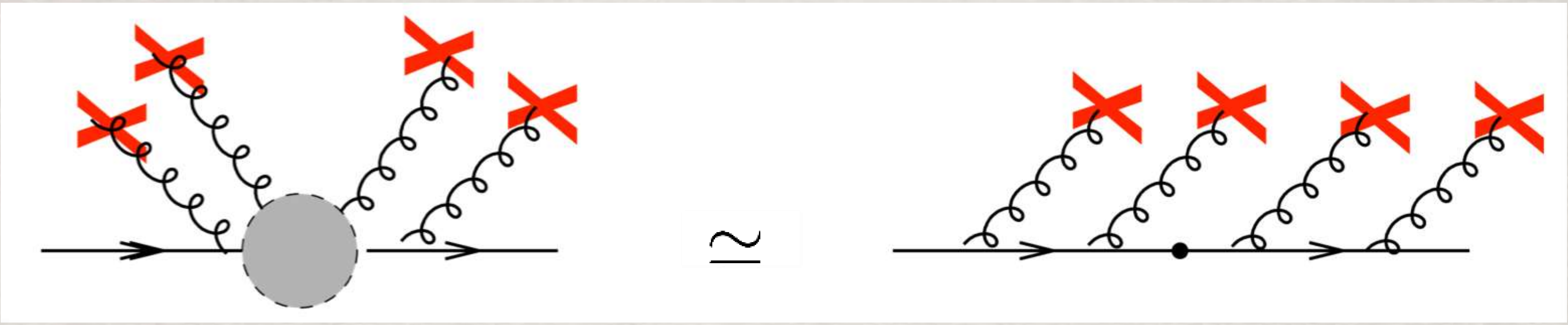
$|M_3\rangle \sim t^a$  $-\mathbf{T}_i\cdot\mathbf{T}_j = \dfrac{C_i+C_j-C_k}{2}\mathbf{1}$

$-(\mathbf{T}_1\cdot\mathbf{T}_2)|M_3\rangle$  $= -\dfrac{1}{2N_c}$  $-\dfrac{1}{2N_c} = \dfrac{C_F+C_F-C_A}{2}$

$-(\mathbf{T}_1\cdot\mathbf{T}_3)|M_3\rangle$  $= \dfrac{C_A}{2}$  $\dfrac{C_A}{2} = \dfrac{C_A+C_F-C_F}{2}$

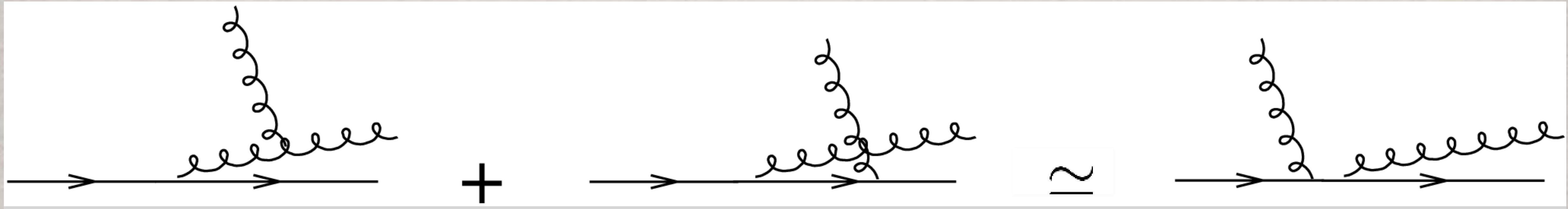
Coherence

Success of resummation of large logarithms relies heavily on the fact that soft emissions widely separated in angle are emitted independently off the hard legs



This property is a consequence of "coherence" of QCD radiation: gluons at large angles feel only the total colour charge of emitters

[Bassetto Ciafaloni Marchesini Phys. Rept. 100 (1983) 201]



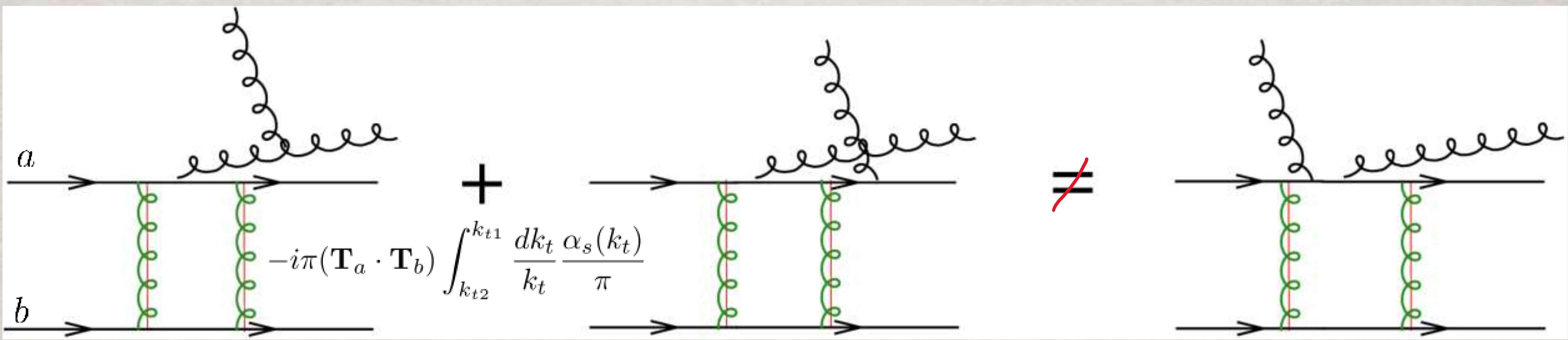
$$\mathbf{T}_p \frac{p\cdot\varepsilon(k_2)}{p\cdot k_2} + \mathbf{T}_1 \frac{k_1\cdot\varepsilon(k_2)}{k_1\cdot k_2} \simeq (\mathbf{T}_p+\mathbf{T}_1)\frac{p\cdot\varepsilon(k_2)}{p\cdot k_2}$$

Coherence violating logarithms

In hadron collisions, coherence is violated in the presence of Coulomb gluons, which can transfer colour from one incoming parton to the other

$$\boldsymbol{Sp}^{(1)}(p_1,p_2;\widetilde{P};p_3,\ldots,p_n) = \boldsymbol{Sp}_H^{(1)}(p_1,p_2;\widetilde{P}) + \boldsymbol{I}_C(p_1,p_2;p_3,\ldots,p_n)\ \boldsymbol{Sp}^{(0)}(p_1,p_2;\widetilde{P})\ , \qquad (4.2)$$

[Catani De Florian Rodrigo 1112.4405]



Coulomb-gluon exchanges (subleading in N_c)

- can endanger the factorisation of collinear singularities into parton distribution functions $\Rightarrow$ factorisation proofs needed

[Becher Hager Jaskievicz Neubert Schwienbacher 2408.10308]

- even in the presence of collinear factorisation, they give rise to coherence-violating logarithms (CVLs) that are not captured by “traditional” resummation techniques

[Forshaw Kyrieleis Seymour hep-ph/0604094]

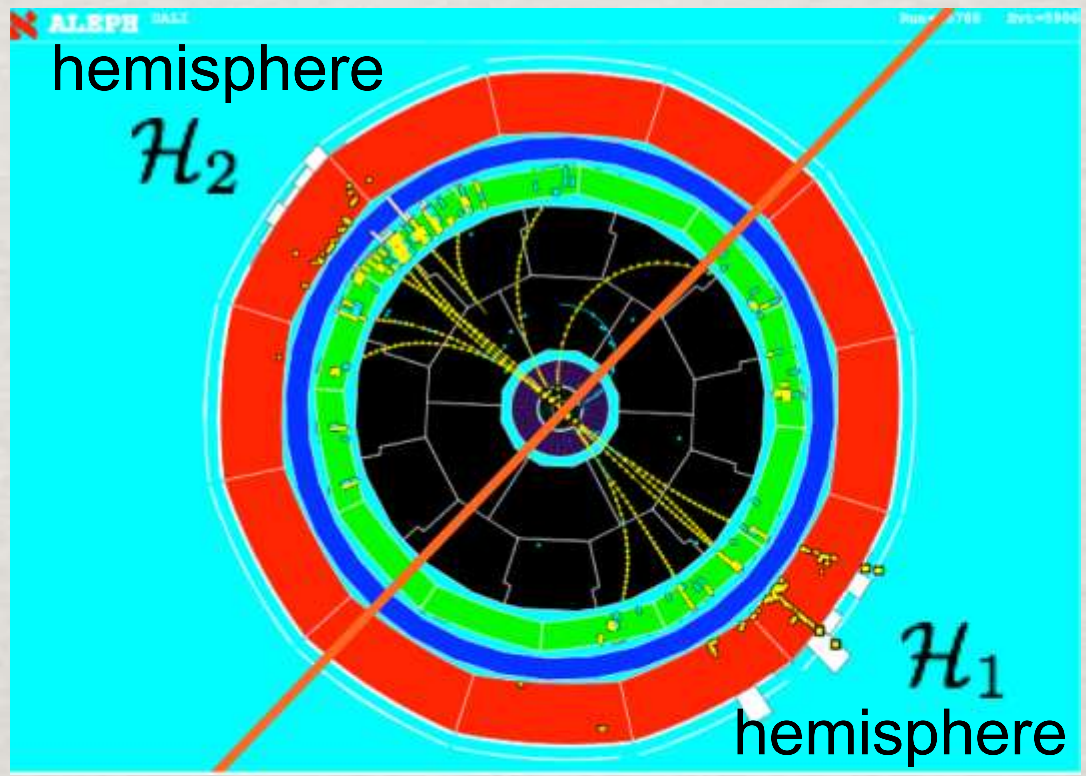
Super-leading logarithms

Global vs non-global

Global observables are those whose measurement is sensitive to emissions everywhere in the phase space

global

$$\rho_H = \max\left(\frac{M_1^2}{Q^2}, \frac{M_2^2}{Q^2}\right)$$



non-global

$$\rho_L = \min\left(\frac{M_1^2}{Q^2}, \frac{M_2^2}{Q^2}\right)$$

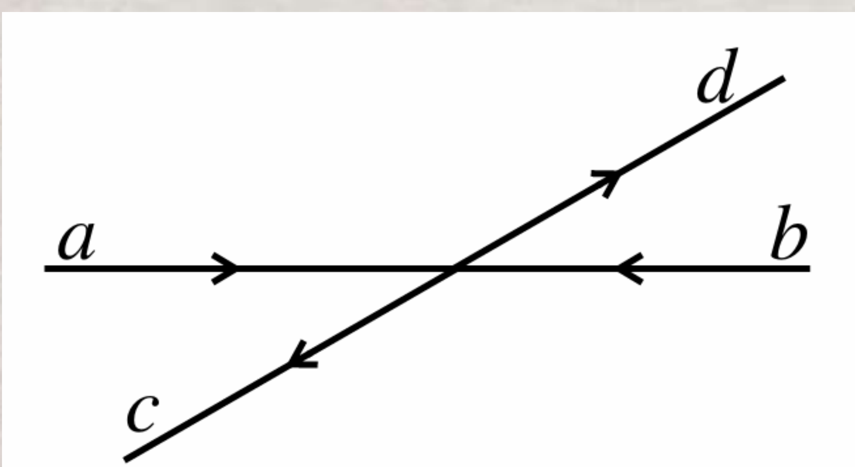
Non-global observables are most common in hadron collisions, where the definition of jet observables often imposes restrictions on hadron phase space (e.g. only measure hadrons inside the central tracker region)

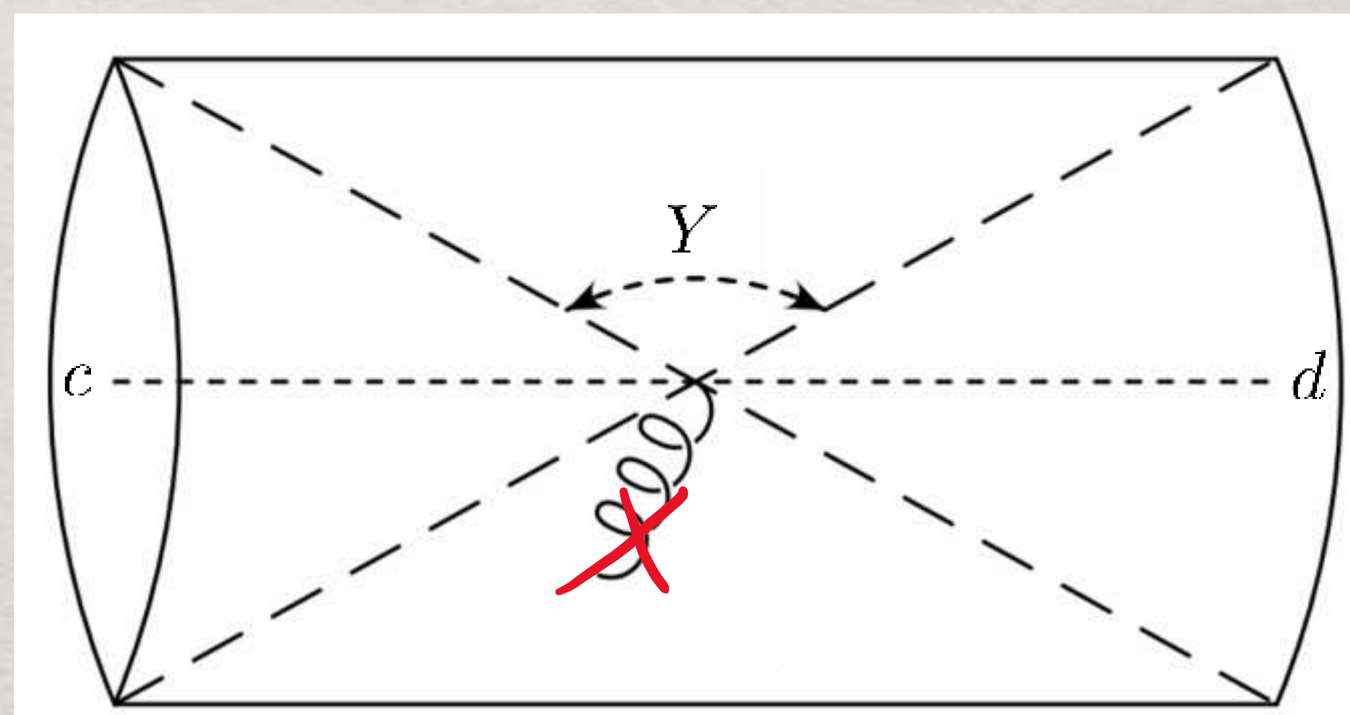
Rapidity gap between jets

Prototype of non-global observable: cross section for two jets with large transverse momentum $\sim Q$ and no jets with transverse momentum larger than $Q_0 \ll Q$ in a rapidity gap between the jets of width Y

(gap) survival probability

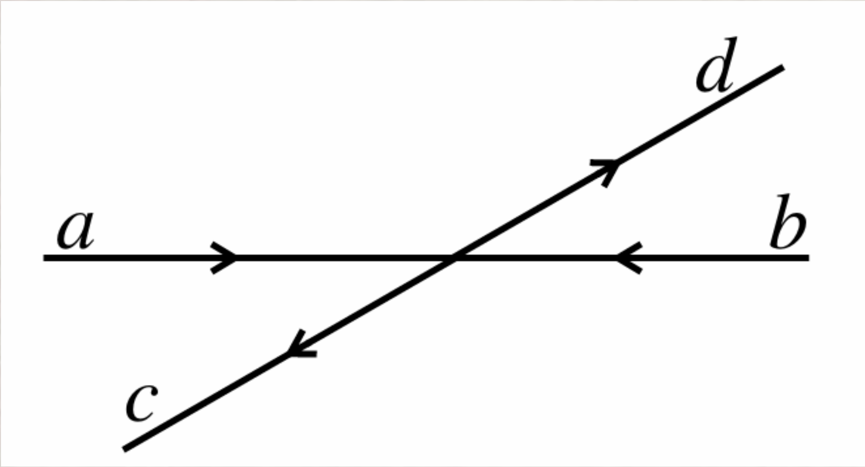
$$\frac{d\sigma}{dY} \approx f_a(Q) \otimes f_b(Q) \otimes \frac{d\hat{\sigma}_{ab}}{dY} \times \Sigma(Q_0) \qquad \Sigma(Q_0) \approx \mathrm{Prob}\left[\max_{i\in\mathrm{out}} k_{Ti} < Q_0\right]$$

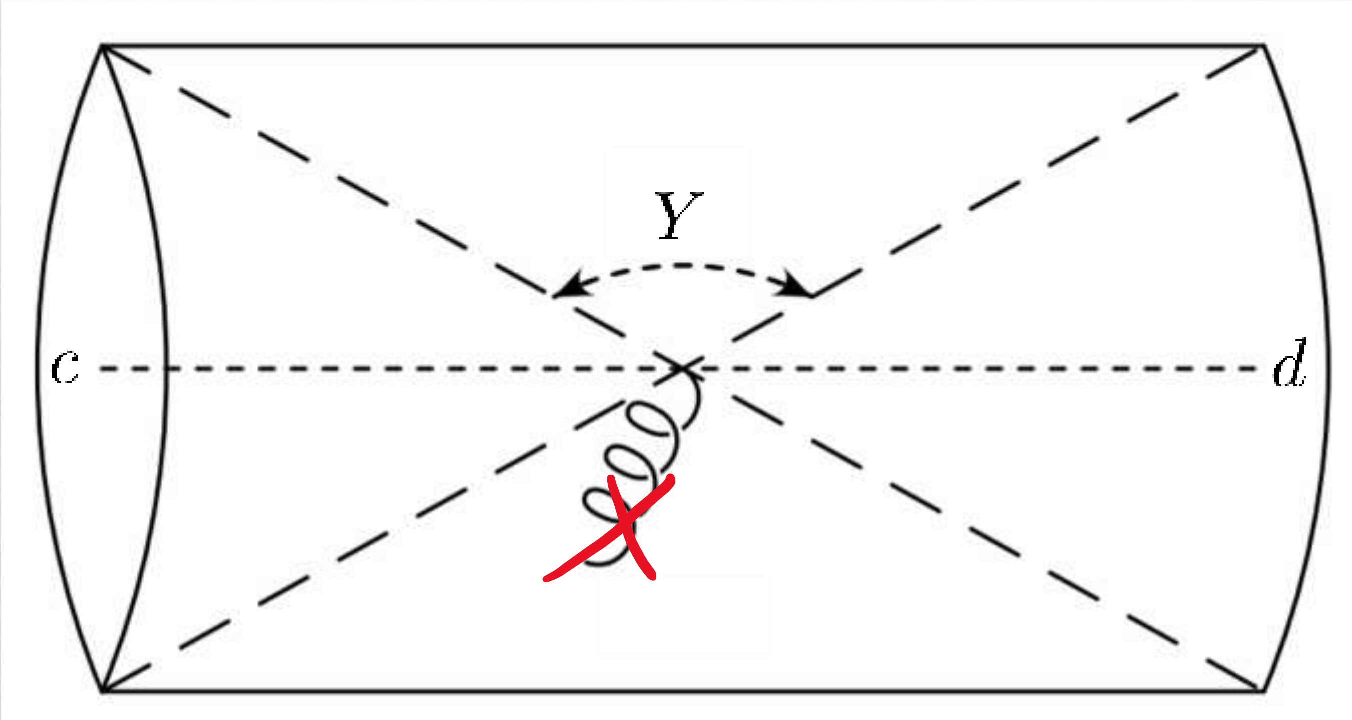




COMPUTING THE SURVIVAL PROBABILITY

With one soft gluon only, the survival probability is one minus the probability that that gluon is inside the gap and has transverse momentum larger than Q_0





$$\Sigma(Q_0) \approx 1 - 4C_F \frac{\alpha_s}{\pi} \int_{Q_0}^{Q} \frac{dk_T}{k_T} \int_{-Y/2}^{Y/2} dy = 1 - 4C_F \frac{\alpha_s}{\pi} Y \ln \frac{Q}{Q_0}$$

The first-ever calculation by Oderda and Sterman corresponds to the exponentiation of one-gluon result [Oderda Sterman hep-ph/9806530]

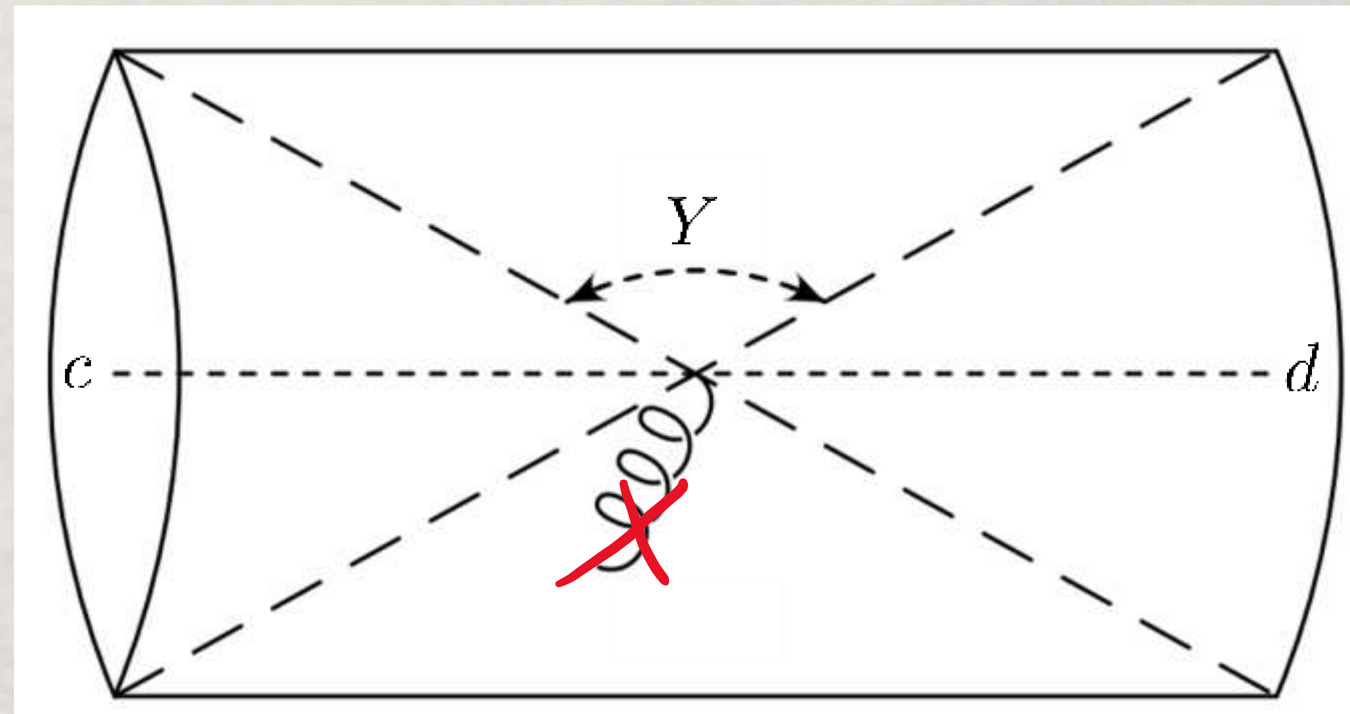
$$\Sigma^{\mathrm{OS}}(Q, Q_0) \approx \exp\left[-4C_F \frac{\alpha_s}{\pi} Y L\right] \qquad L \equiv \ln(Q/Q_0)$$

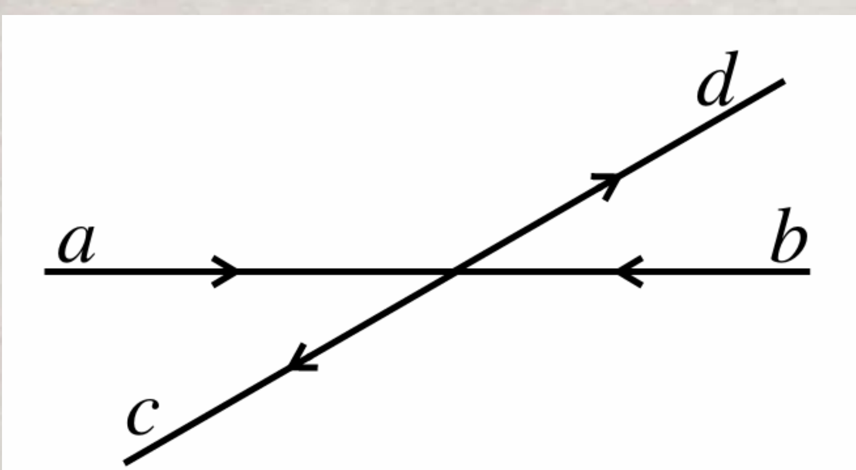
Computing the survival probability

The Oderda-Sterman formula considers vetoes all gluons emitted from the hard legs only

$$\Sigma^{\mathrm{OS}}(Q_0) = \mathrm{Tr}\Big(\mathbf{V}_{Q_0,Q}\mathbf{H}\,\mathbf{V}^{\dagger}_{Q_0,Q}\Big)$$

[Forshaw Holguin 2109.03665]





$$\mathbf{V}_{Q_0,Q} \approx \exp\left[-\frac{\alpha_s}{\pi}\ln\frac{Q}{Q_0}\left(Y\mathbf{T}_t^2 + i\pi\mathbf{T}_s^2\right)\right]$$

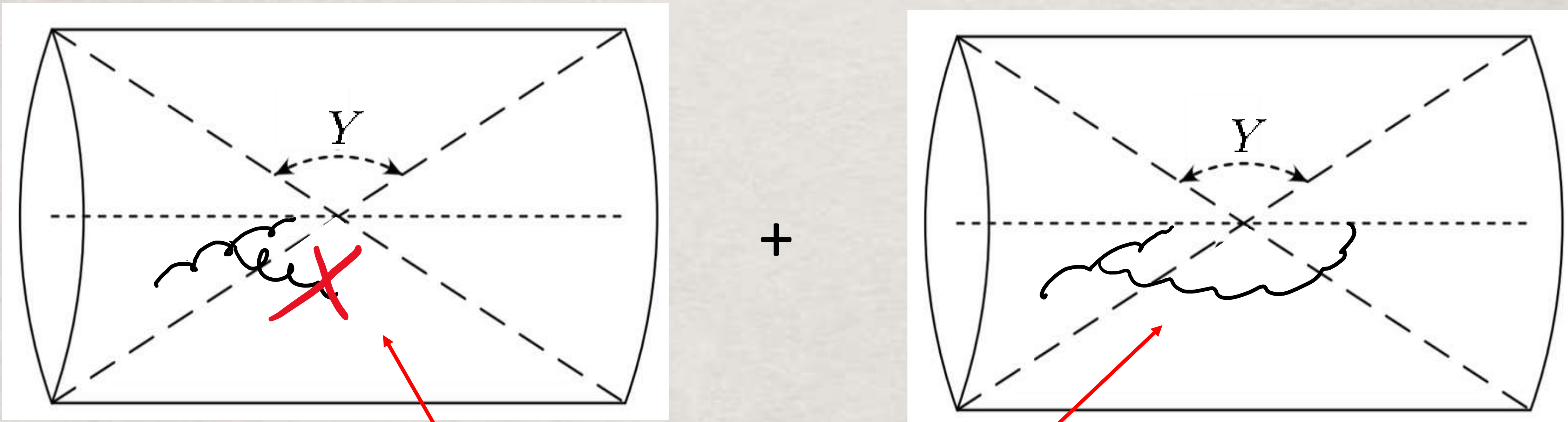
$$\mathbf{T}_t^2 = (\mathbf{T}_a + \mathbf{T}_c)^2 = (\mathbf{T}_b + \mathbf{T}_d)^2$$

$$\mathbf{T}_s^2 = (\mathbf{T}_a + \mathbf{T}_b)^2 = (\mathbf{T}_c + \mathbf{T}_d)^2$$

$$[\mathbf{T}_t^2, \mathbf{T}_s^2] \neq 0$$

Non-global logarithms

Additional logarithmic contributions arise when harder emissions in the jets coherently emit a softer gluon inside the gap [Dasgupta Salam hep-ph/0104277]



$$N_c^2\alpha_s^2\left[\int_0^{Q_0}\frac{dk_{T2}}{k_{T2}}\int_{k_{T2}}^{Q}\frac{dk_{T1}}{k_{T1}}-\int_0^{Q_0}\frac{dk_{T2}}{k_{T2}}\int_{k_{T2}}^{Q}\frac{dk_{T1}}{k_{T1}}\right]\sim -N_c^2\alpha_s^2L^2$$

- Such contributions, universally known as “non-global” logarithms, are typical of observables sensitive to emissions in a portion of the phase space
- Non-global logarithms have been resummed at NLL in the large-N_c limit

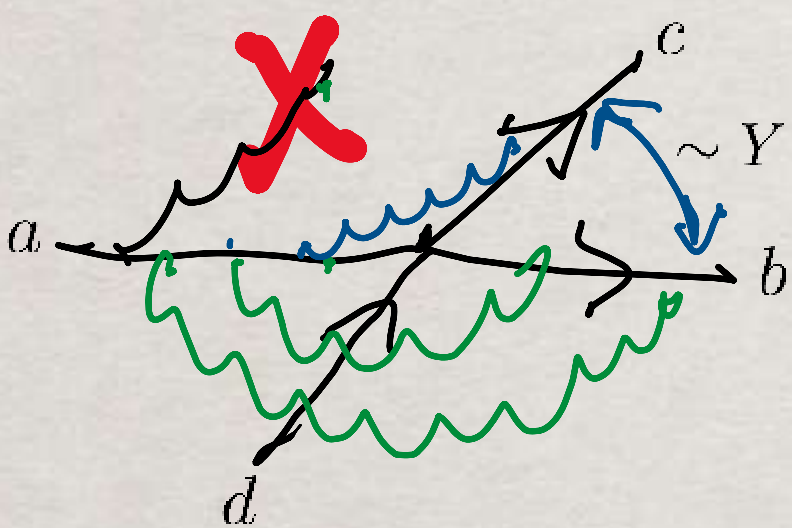
[AB Dreyer Monni 2104.06416 , 2111.02413]

[Becher Rau Xu 2112.02108]

Super-leading logarithms

Beyond the large-N_c limit, real emissions outside the gap can change the colour space of virtual corrections

[Forshaw Kyrieleis Seymour 0808.1269]



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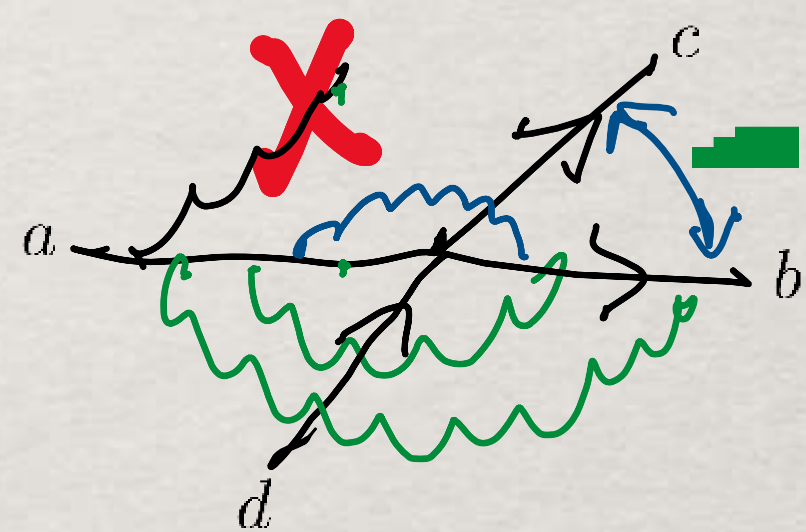
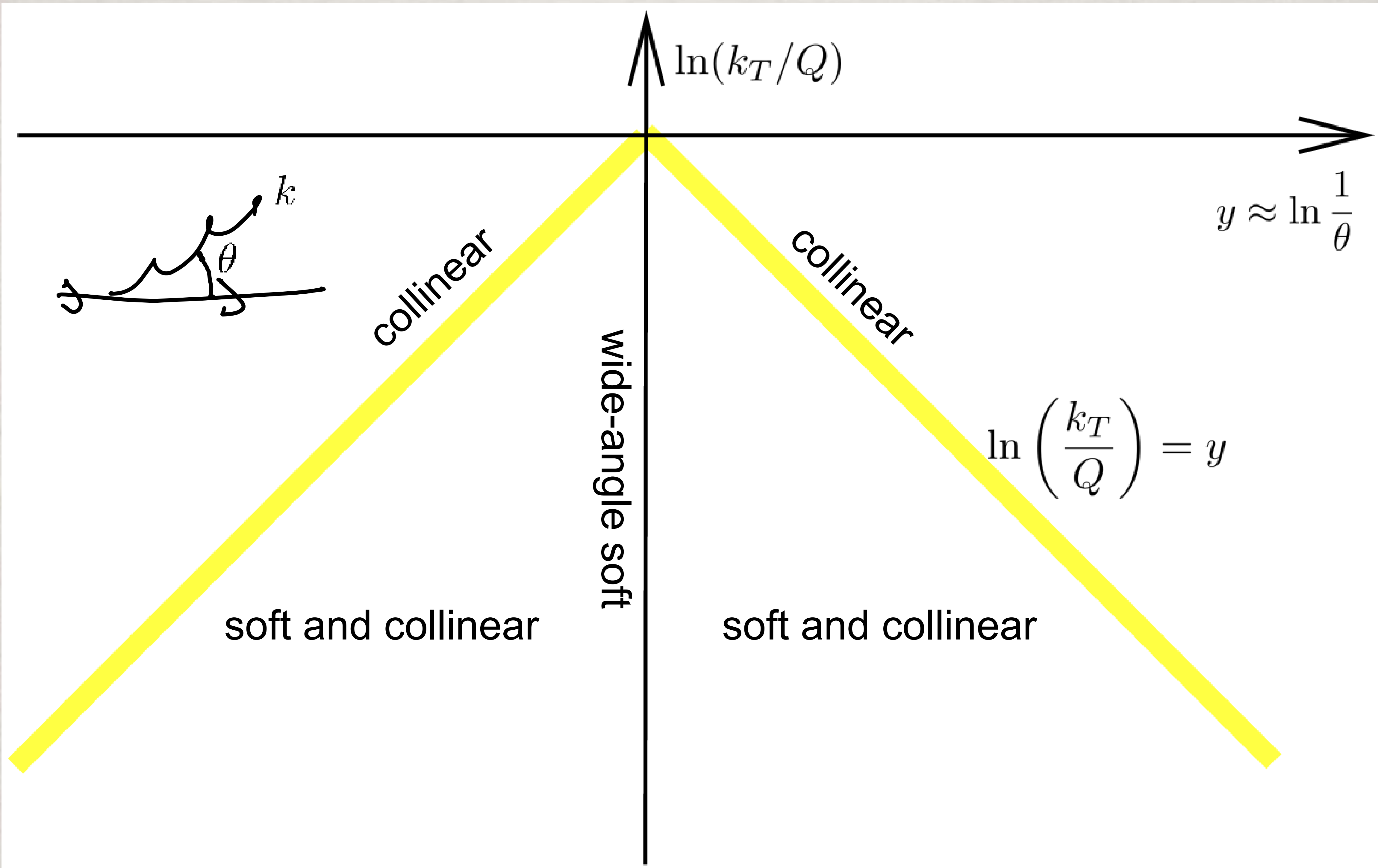


$$\Sigma^{\mathrm{CVLs}}(Q_0) \approx \frac{\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{dk_T}{k_T}\int_0^{\ln(Q/k_T)} dy\, \Theta(y-Y) \times$$
$$\times \Big[\mathrm{Tr}\Big(\mathbf{V}_{Q_0,k_T}\,\mathbf{T}_a\,\mathbf{V}_{k_T,Q}\,\mathbf{H}\,\mathbf{V}^\dagger_{k_T,Q}\,\mathbf{T}^\dagger_a\,\mathbf{V}^\dagger_{Q_0,k_T}\Big) - C_F\,\mathrm{Tr}\Big(\mathbf{V}_{Q_0,Q}\mathbf{H}\,\mathbf{V}^\dagger_{Q_0,Q}\Big)\Big]$$
$$\sim N_c^2\left(\frac{\alpha_s}{\pi}\right)^4 Y(-i\pi)^2\ln^5\left(\frac{Q}{Q_0}\right) \sim \alpha_s^4 L^5$$

Such contributions are one log higher that the leading logarithms $(\alpha_s \ln(Q/Q_0))^n$, hence the name super-leading logarithms

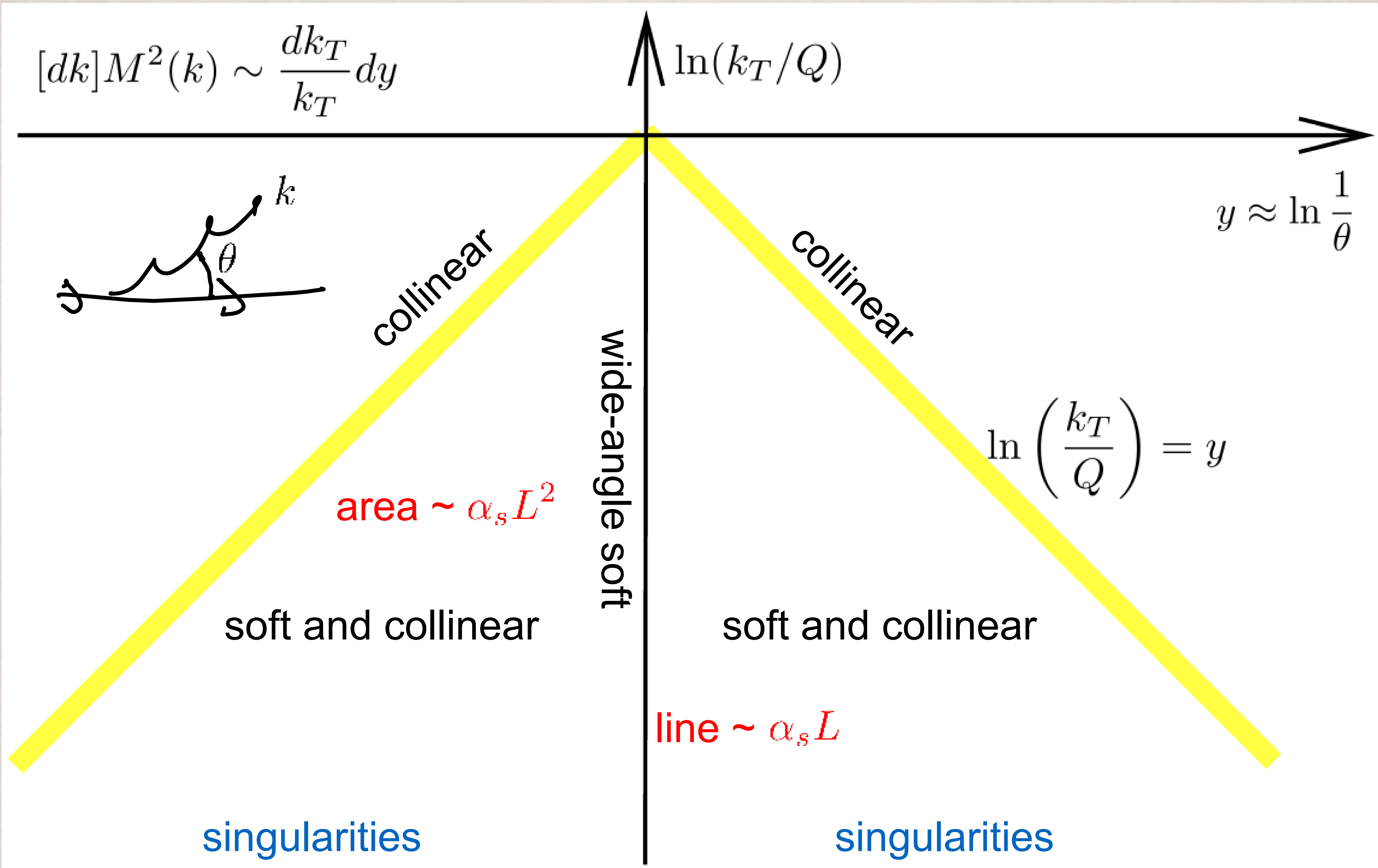
SLL IN THE LUND PLANE

The Lund plane is an effective way to picture soft/collinear emissions



SLL IN THE LUND PLANE

The Lund plane is an effective way to picture soft/collinear emissions



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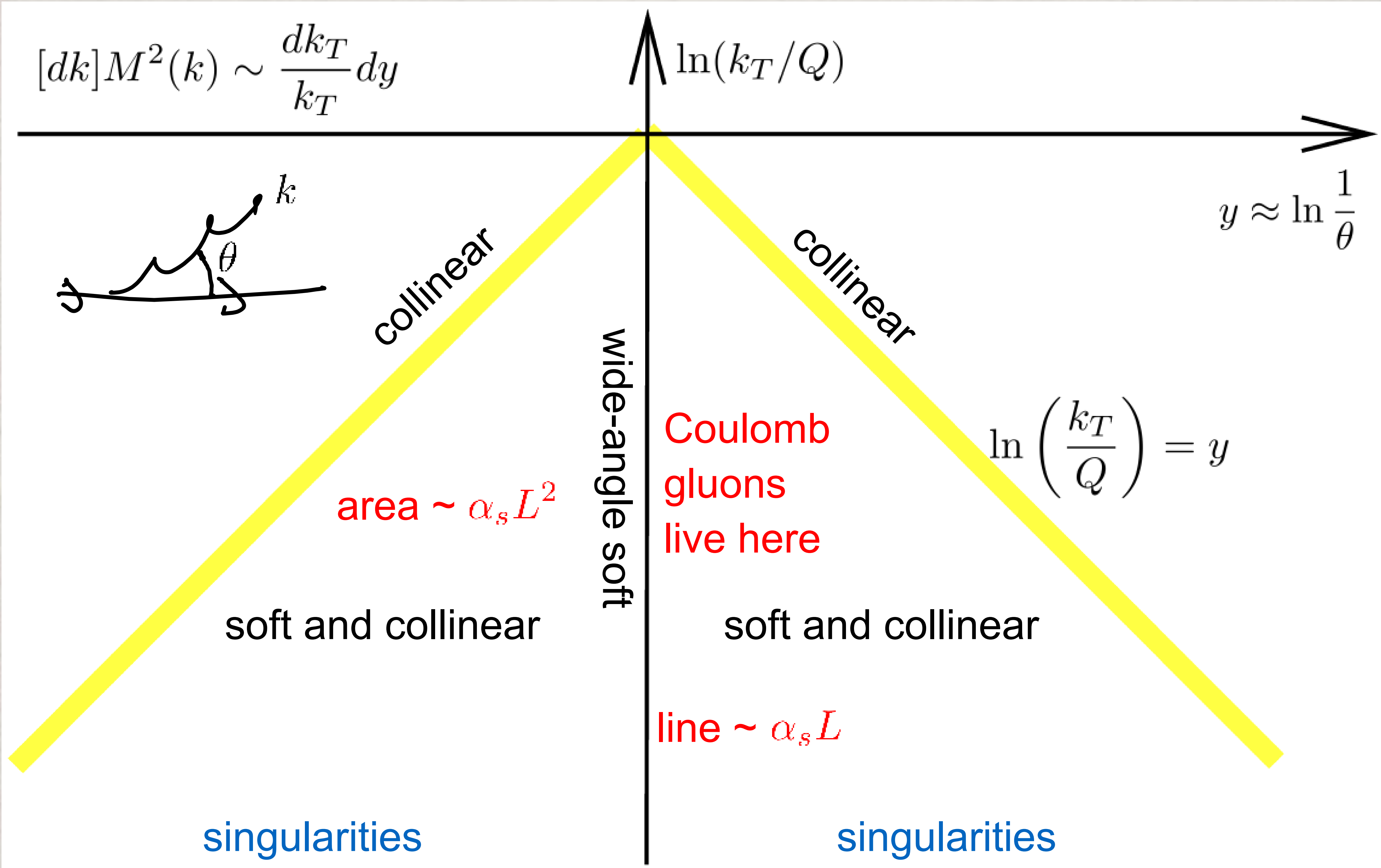
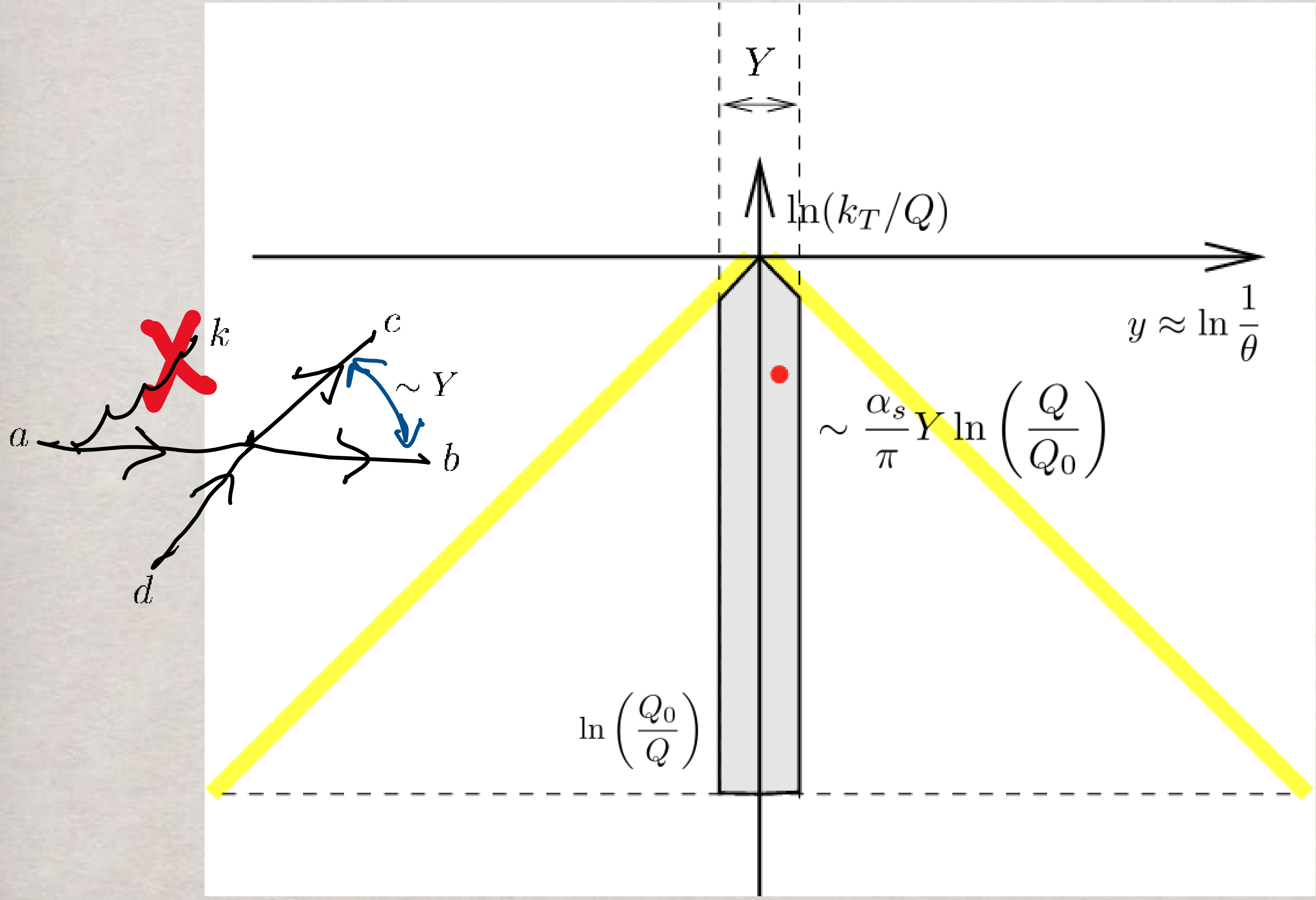


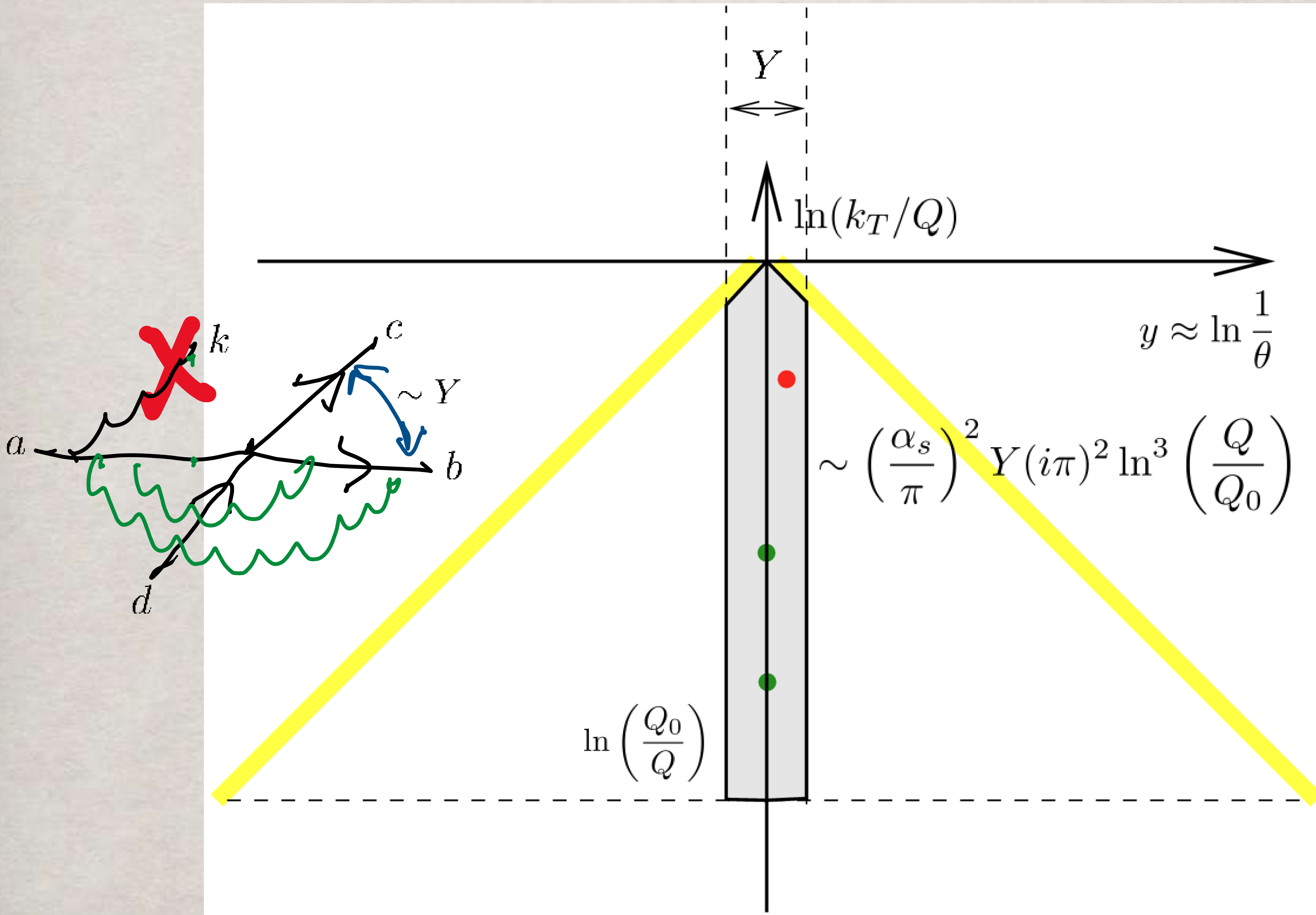
SLL IN THE LUND PLANE

Only virtual corrections survive inside the gap



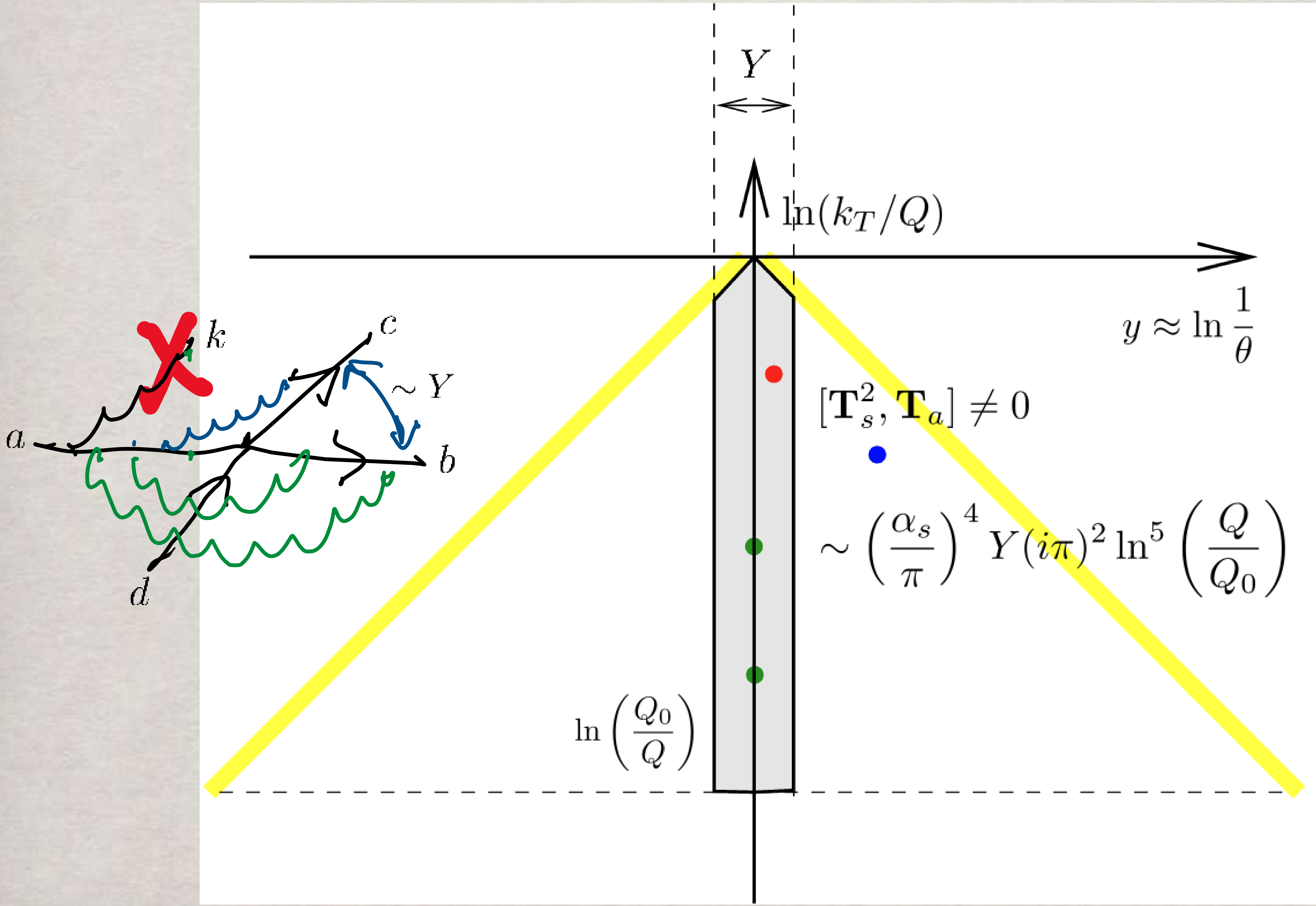
SLL in the Lund plane

Only virtual corrections survive inside the gap



SLL IN THE LUND PLANE

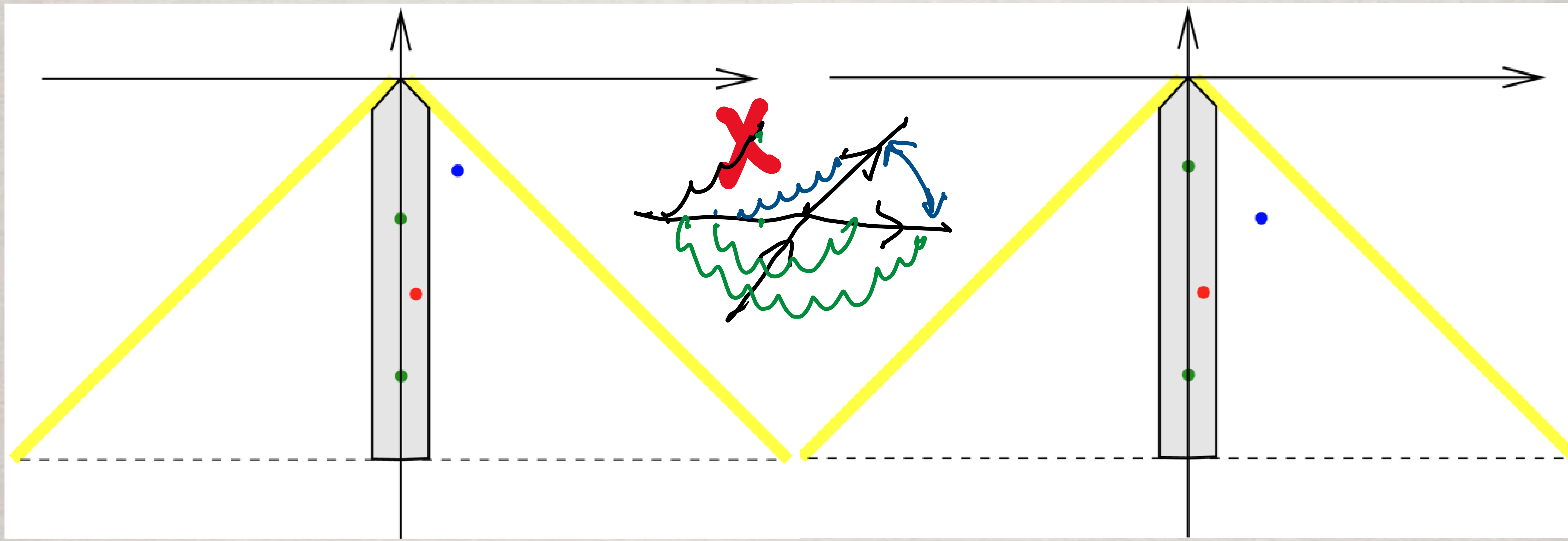
An additional collinear gluon outside the gap can be "colour trapped"



SLL AT FOURTH ORDER

hardest gluon outside the gap

second-hardest gluon outside the gap



$$\Sigma^{\text{CVLs}}(Q_0) \sim A \left(\frac{\alpha_s}{\pi}\right)^4 Y (i\pi)^2 \ln^5\left(\frac{Q}{Q_0}\right)$$

[Forshaw Kyrieleis Seymour 0808.1269]

$$A = \mathrm{Tr}\left([\mathbf{T}_s^2, [\mathbf{T}_s^2, \mathbf{T}_t^2]]\left(\mathbf{T}_a^\dagger \mathbf{H} \mathbf{T}_a - \mathbf{T}_a^2 \mathbf{H}\right)\right)$$

vanishes for less than 3 Born partons

$$A = \mathrm{Tr}\left([\mathbf{T}_s^2, \mathbf{T}_t^2]\left(\mathbf{T}_a^\dagger [\mathbf{T}_s^2, \mathbf{H}] \mathbf{T}_a - \mathbf{T}_a^2 [\mathbf{T}_s^2, \mathbf{H}]\right)\right)$$

vanishes for less than 4 Born partons

Recovering collinear factorisation

Below the scale Q_0 there is no constraint on real emissions

[Forshaw Holguin 2109.03665]

$$\frac{\alpha_s}{\pi}\int_{\mu_F}^{Q_0}\frac{dk_T}{k_T}\int_0^{\ln(Q/k_T)} dy\ f_a(x_a,\mu_F)f_b(x_b,\mu_F)\times$$

$$\times\left[\mathrm{Tr}\Big(\mathbf{V}_{\mu_F,k_T}\,\mathbf{T}_a\,\mathbf{V}_{k_T,Q_0}\,\mathbf{H}(Q_0,Q)\,\mathbf{V}^\dagger_{k_T,Q_0}\,\mathbf{T}^\dagger_a\,\mathbf{V}^\dagger_{\mu_F,k_T}\Big)-C_F\mathrm{Tr}\Big(\mathbf{V}_{\mu_F,Q_0}\mathbf{H}(Q_0,Q)\mathbf{V}^\dagger_{\mu_F,Q_0}\Big)\right]$$

$$\Downarrow$$

We replace the soft matrix element with the full splitting function

$$\frac{\alpha_s}{\pi}\int_{\mu_F}^{Q_0}\frac{dk_T}{k_T}\int_0^{1-k_T/Q} dz\left(\frac{1+z^2}{1-z}\right)f_b(x_b,\mu_F)\times$$

$$\times\left[\mathrm{Tr}\Big(\mathbf{V}_{\mu_F,k_T}\,\mathbf{T}_a\,\mathbf{V}_{k_T,Q_0}\,\mathbf{H}(Q_0,Q)\,\mathbf{V}^\dagger_{k_T,Q_0}\,\mathbf{T}^\dagger_a\,\mathbf{V}^\dagger_{\mu_F,k_T}\Big)\frac{1}{z}f_a\left(\frac{x_a}{z},\mu_F\right)\Theta(z-x_a)\right.$$

$$\left.-C_F\mathrm{Tr}\Big(\mathbf{V}_{\mu_F,Q_0}\mathbf{H}(Q_0,Q)\mathbf{V}^\dagger_{\mu_F,Q_0}\Big)f(x_a,\mu_F)\right]$$

Recovering collinear factorisation

$$\frac{\alpha_s}{\pi}\int_{\mu_F}^{Q_0}\frac{dk_T}{k_T}\int_0^{1-k_T/Q}dz\left(\frac{1+z^2}{1-z}\right)f_b(x_b,\mu_F)\times$$
$$\times\Bigg[\mathrm{Tr}\Big(\mathbf{V}_{\mu_F,k_T}\,\mathbf{T}_a\,\mathbf{V}_{k_T,Q_0}\,\mathbf{H}(Q_0,Q)\,\mathbf{V}^\dagger_{k_T,Q_0}\,\mathbf{T}_a^\dagger\,\mathbf{V}^\dagger_{\mu_F,k_T}\Big)\frac{1}{z}f_a\left(\frac{x_a}{z},\mu_F\right)\Theta(z-x_a)$$
$$-C_F\mathrm{Tr}\Big(\mathbf{V}_{\mu_F,Q_0}\mathbf{H}(Q_0,Q)\mathbf{V}^\dagger_{\mu_F,Q_0}\Big)f(x_a,\mu_F)\Bigg]$$

Virtual correction contain only Coulomb phases

$$\mathbf{V}_{Q_1,Q_2}\approx \mathrm{Pexp}\left\{-\frac{\alpha_s}{\pi}\ln\left(\frac{Q_2}{Q_1}\right)\left[\mathbf{T}_t^2\underbrace{Y}_{=0}+i\pi\mathbf{T}_s^2\right]\right\}\implies \mathbf{V}_{Q_1,Q_2}\mathbf{V}^\dagger_{Q_1,Q_2}=\mathbf{1}$$

Together with the lowest order contribution, this gives

$$\underbrace{\mathrm{Tr}(\mathbf{H}(Q_0,Q))}_{=\Sigma(Q_0)}f_b(x_b,\mu_F)\underbrace{\left[f(x_a,\mu_F)+\frac{\alpha_s}{\pi}\int_{\mu_F}^{Q_0}\frac{dk_T}{k_T}\int_{x_a}^1\frac{dz}{z}C_F\left(\frac{1+z^2}{1-z}\right)_+f\left(\frac{x_a}{z},\mu_F\right)\right]}_{\approx f(x_a,Q_0)}$$

Unresolved real emissions reconstruct the pdfs at the veto scale Q_0

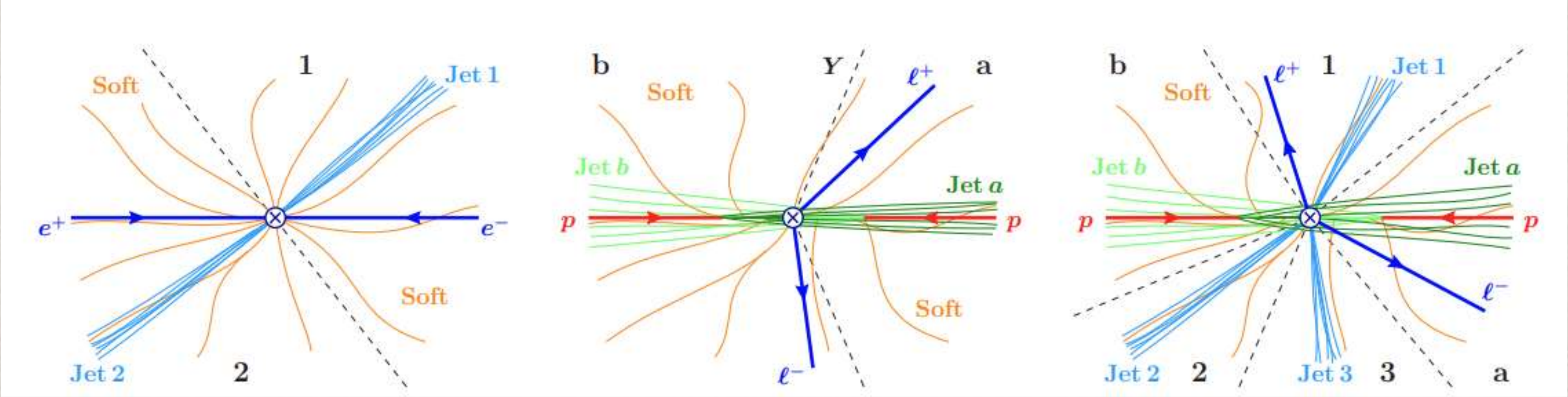
[Becher Hager Jaskievicz Neubert Schwienbacher 2408.10308]

One-jettiness

The beauty of N-jettiness

- Jettiness is a class of observables introduced to characterise the properties of jets [Stewart Tackmann Waalewjin 1004.2489]

- The N-jettiness τ_N is defined in such a way that, for $\tau_N \to 0$, an event contains only N highly collimated jets



- N-jettiness is recursively IRC safe $\Longrightarrow$ with coherence, this gives access to N^3LL accuracy in the N-jet region [Alioli et al 2312.06496]

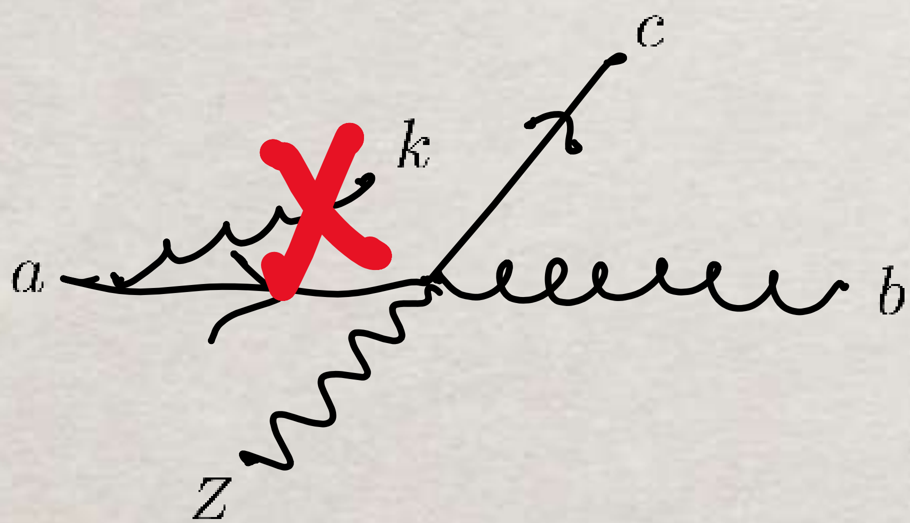
- Due to both its calculability and simplicity, N-jettiness is also used for
  - Isolation of exclusive N-jet events for BSM searches [Lindert et al 1705.04664]
  - Resolution variable for phase-space slicing in NNLO calculations [Boughezal Isgrò Petriello 1802.00456]

One-jettiness

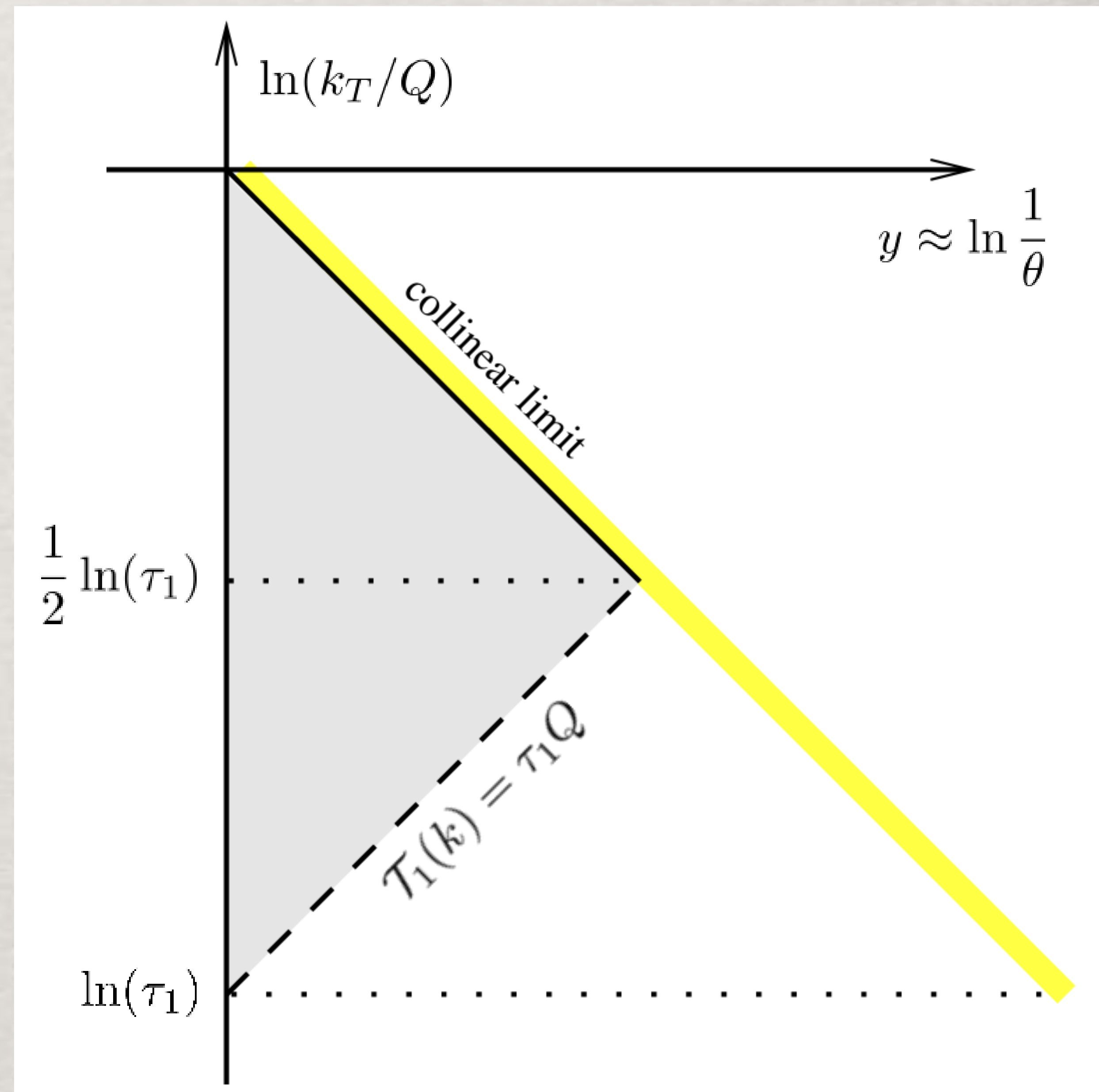
In Z+jet event, we define one-jettiness as

$$\mathcal{T}_1 \equiv \sum_i \min\{n_a \cdot k_i, n_b \cdot k_i, n_c \cdot k_i\}$$

$$\Sigma(\tau_1) = \mathrm{Prob}\left(\mathcal{T}_1 < \tau_1 Q\right)$$



$$\Sigma(\tau_1) \sim \exp\left(-(2C_F + C_A)\frac{\alpha_s}{2\pi}L^2 + \dots\right)$$

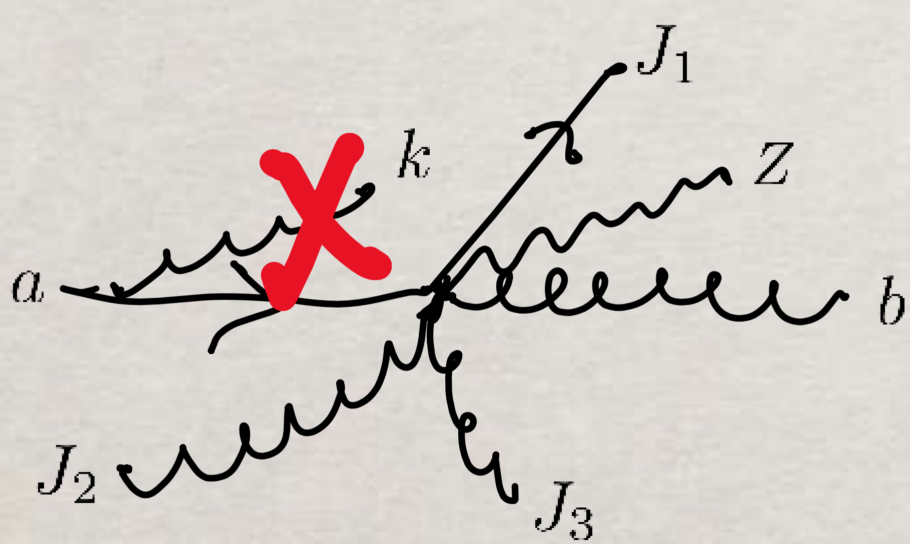


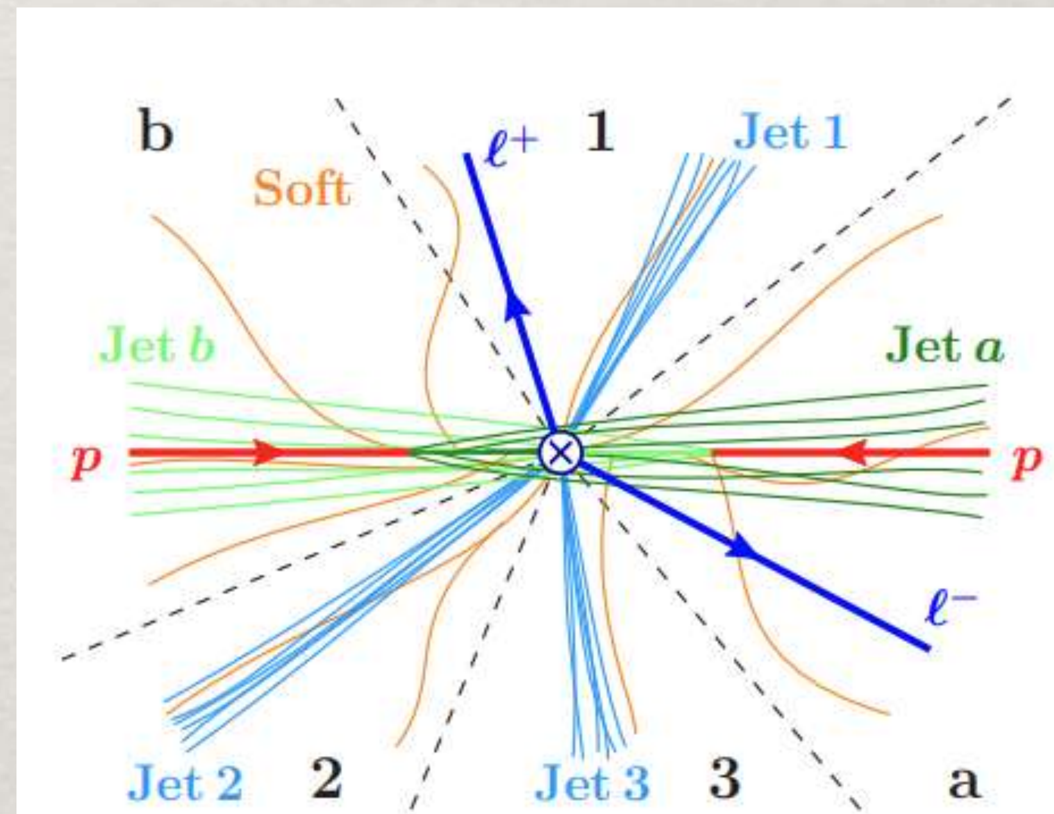
Factorisation theorem

In general, one can define N-jettiness as

$$\mathcal{T}_N \equiv \sum_i \min\left\{ \frac{p_a \cdot k_i}{Q_a}, \frac{p_b \cdot k_i}{Q_b}, \frac{p_{J_1} \cdot k_i}{Q_1}, \ldots, \frac{p_{J_N} \cdot k_i}{Q_N} \right\}$$

$$\Sigma(\tau_N) = \mathrm{Prob}\left(\mathcal{T}_N < \tau_N Q\right)$$





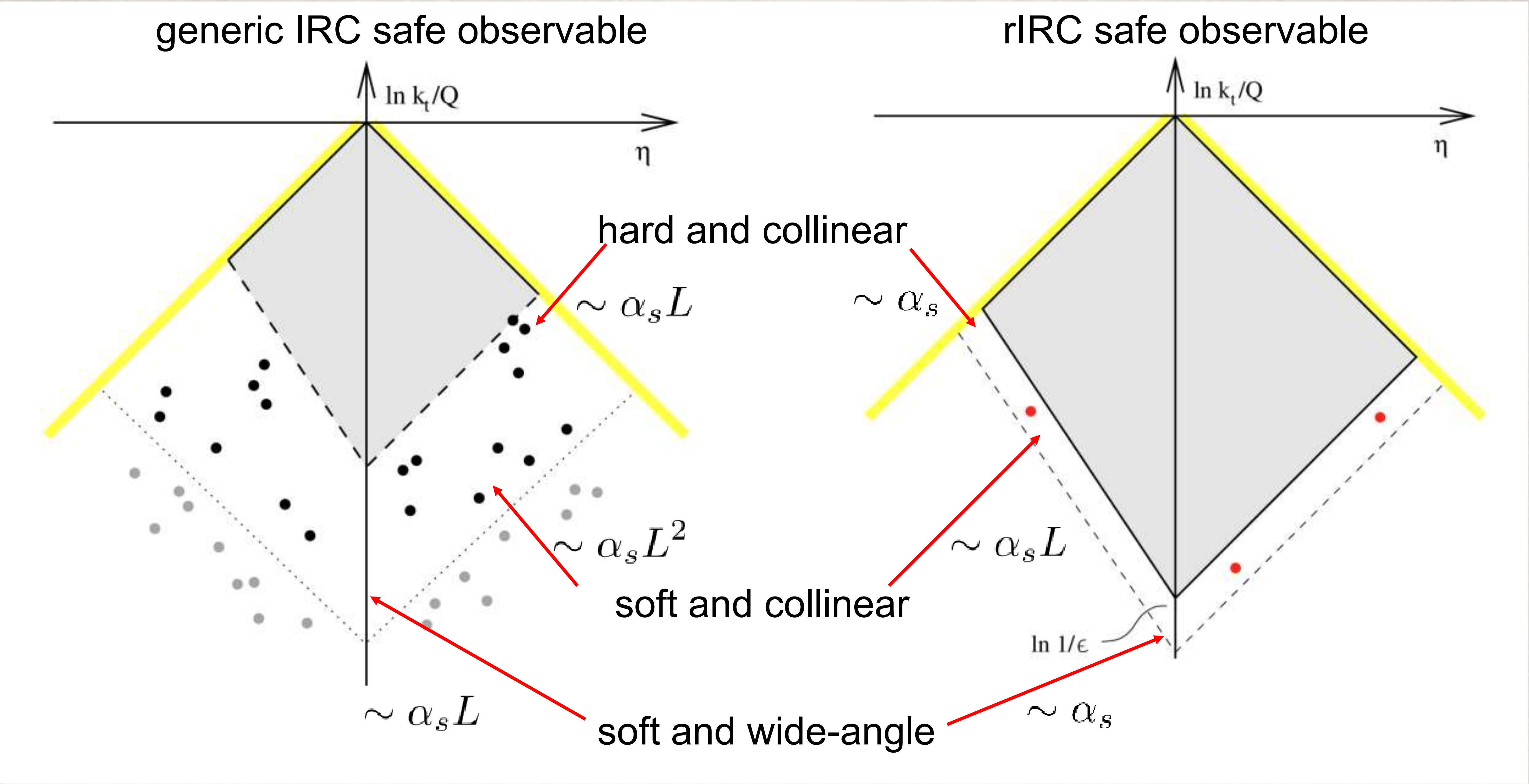
Neglecting Coulomb gluons, N-jettiness obeys an all-order factorisation

[Stewart Tackmann Waalewjin 1004.2489]

$$\frac{d\Sigma(\tau_N)}{d\tau_N} \sim \langle \underbrace{B_a \otimes B_b \otimes J_1 \otimes \ldots J_N}_{\text{numbers}} \otimes \underbrace{\mathcal{S}_{\mathbf{N}} \otimes \mathcal{H}_{\mathbf{N}}}_{\text{colour matrices}} \rangle$$

RECURSIVE IRC SAFETY

If rIRC safety **and coherence** are satisfied, the logarithms arising from real emissions is different from naïve counting based on QCD singularities

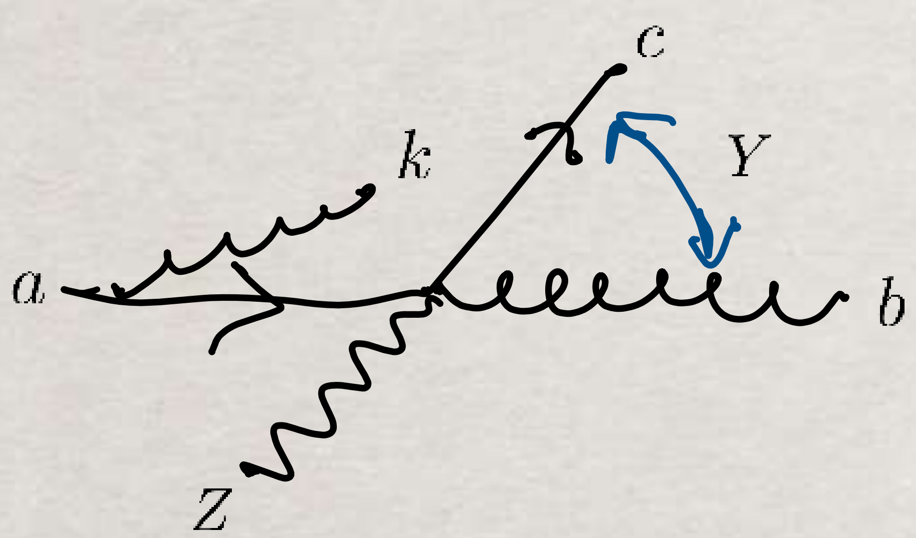


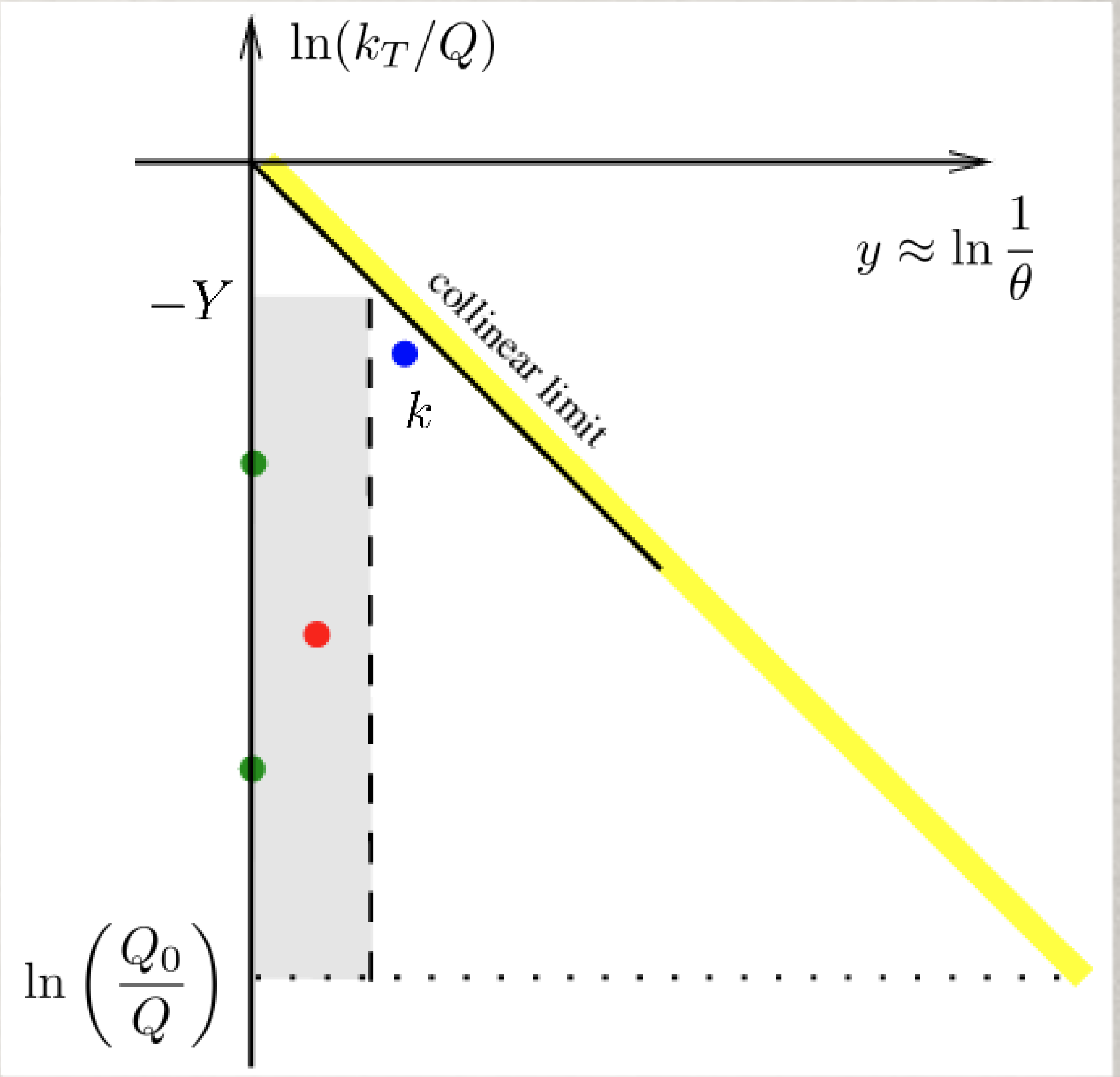
What happens to this picture if coherence is violated?

[Banfi Salam Zanderighi 1001.4082]

Large gap between jets

Let us focus on Z+jet and increase the gap between the jet and the beam…



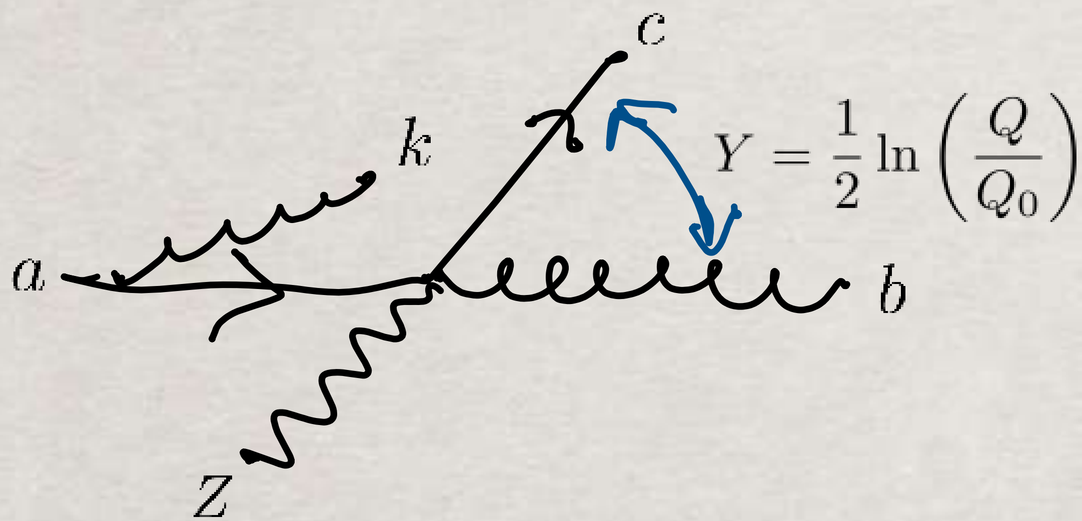


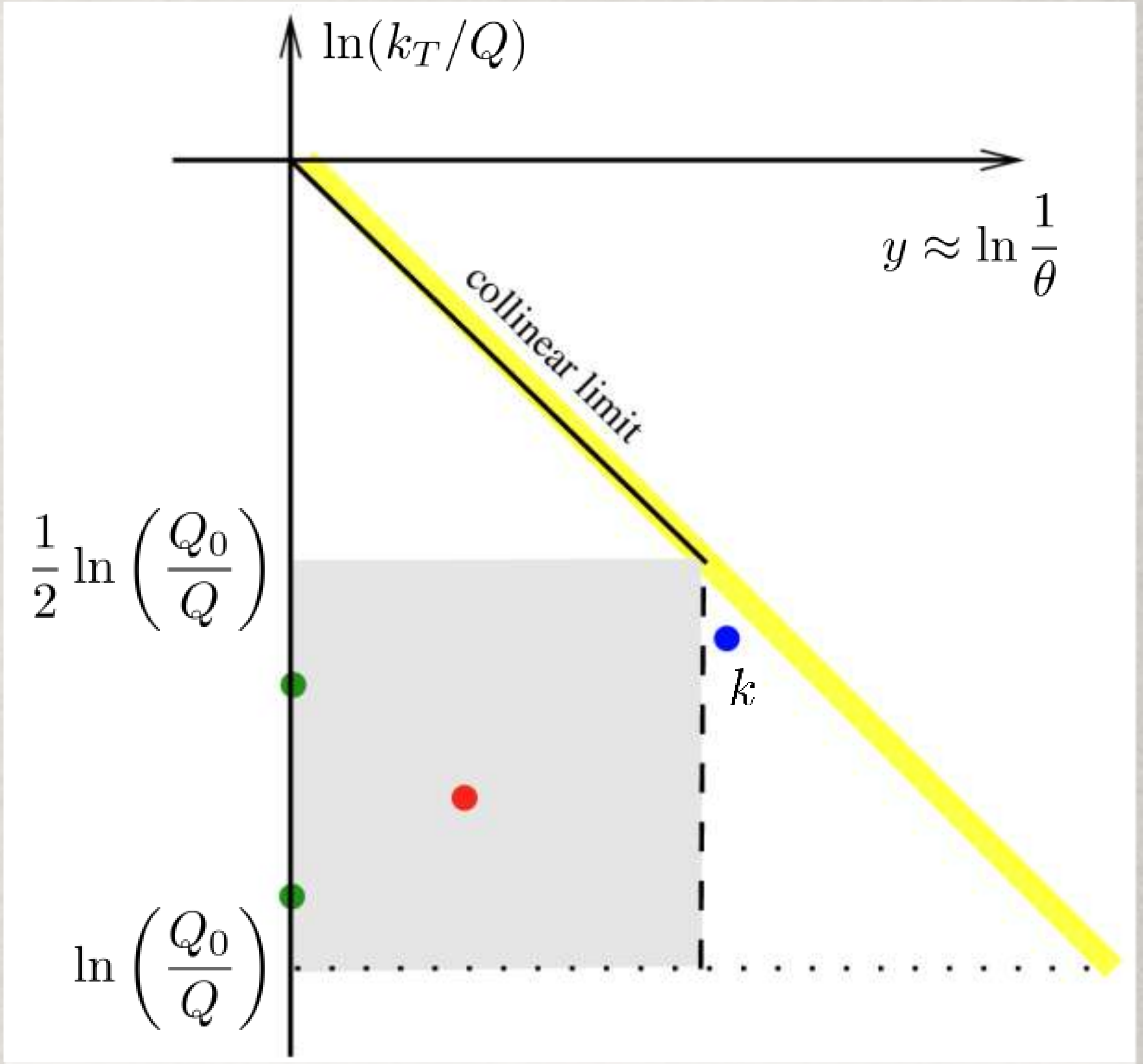
$$\Sigma^{\text{CVLs}}(Q_0) = \frac{\alpha_s}{\pi} \int_{Q_0}^{Q} \frac{dk_T}{k_T} \int_0^{\ln(Q/k_T)} dy\, \Theta\left(y - Y\right) \times$$

$$\times \operatorname{Tr}\Big(\mathbf{V}_{Q_0,k_T}\, \mathbf{T}_a\, \mathbf{V}_{k_T,Q}\, \mathbf{H}\, \mathbf{V}^\dagger_{k_T,Q}\, \mathbf{T}^\dagger_a\, \mathbf{V}^\dagger_{Q_0,k_T}\Big) \sim N_c^2 \left(\frac{\alpha_s}{\pi}\right)^4 (i\pi)^2 Y \ln^5\left(\frac{Q}{Q_0}\right)$$

Large gap between jets

… until it scales like the logarithm of the veto scale





$$\Sigma^{\text{CVLs}}(Q_0) = \frac{\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{dk_T}{k_T}\int_0^{\ln(Q/k_T)} dy\,\Theta\left(y - \frac{1}{2}\ln\left(\frac{Q}{Q_0}\right)\right) \times$$

$$\times\,\mathrm{Tr}\Big(\mathbf{V}_{Q_0,k_T}\,\mathbf{T}_a\,\mathbf{V}_{k_T,Q}\,\mathbf{H}\,\mathbf{V}^\dagger_{k_T,Q}\,\mathbf{T}^\dagger_a\,\mathbf{V}^\dagger_{Q_0,k_T}\Big) \sim N_c^2\left(\frac{\alpha_s}{\pi}\right)^4 (i\pi)^2 \ln^6\left(\frac{Q}{Q_0}\right)$$

Analogy with one-jettiness

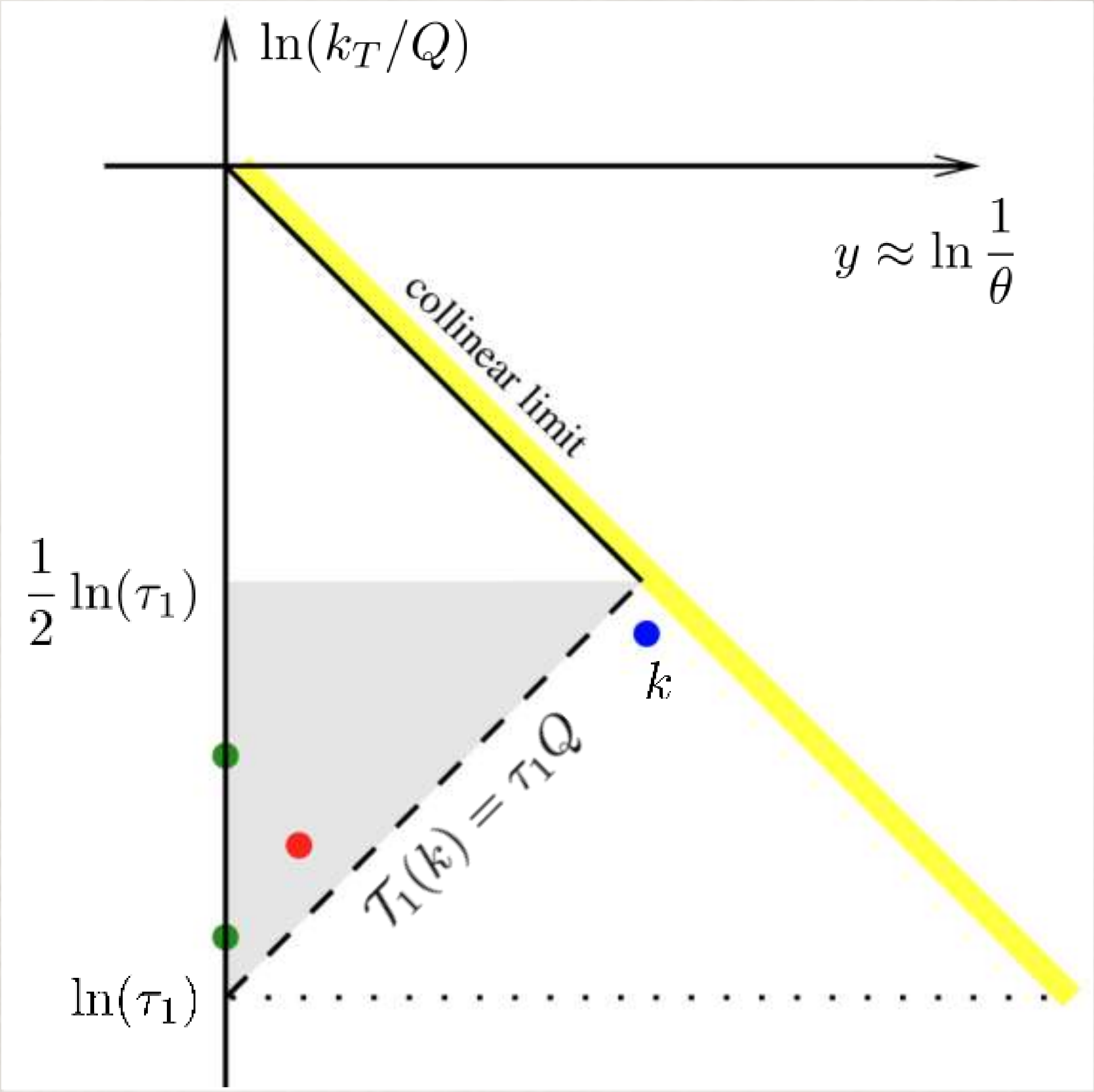
The one-jettiness constraint can be interpreted as a distorted version of the gap between jets, with a diagonal boundary

$$\mathcal{T}_1(k) = n_a \cdot k \sim k_T e^{-y}$$

$$y > \ln\left(\frac{Q}{Q_0}\right) \longrightarrow \frac{\mathcal{T}_1(k)}{Q} < \tau_1$$

$$\Downarrow$$

$$y > \ln\left(\frac{\tau_1 k_T}{Q}\right)$$



$$\Sigma^{\mathrm{CVLs}}(\tau_1) \approx \frac{\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{dk_T}{k_T}\int_0^{\ln(Q/k_T)} dy\,\Theta\left(y - \ln\left(\frac{\tau_1 k_T}{Q}\right)\right) \times$$

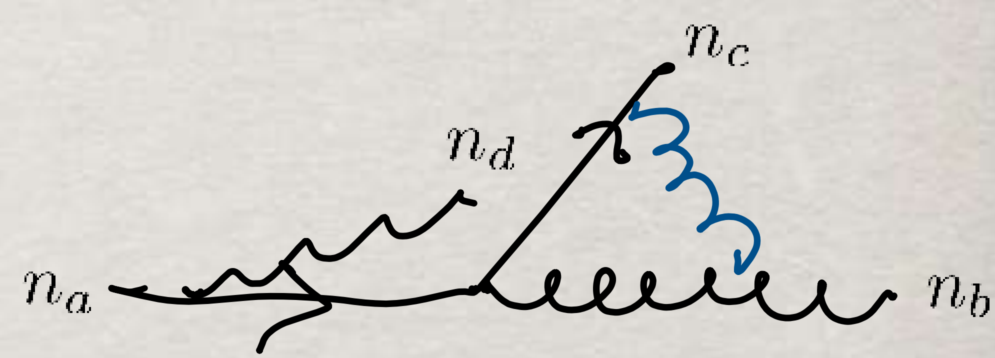
$$\times\,\mathrm{Tr}\Big(\mathbf{V}_{Q_0,k_T}\,\mathbf{T}_a\,\mathbf{V}_{k_T,Q}\,\mathbf{H}\,\mathbf{V}^\dagger_{k_T,Q}\,\mathbf{T}^\dagger_a\,\mathbf{V}^\dagger_{Q_0,k_T}\Big) \sim N_c^2\left(\frac{\alpha_s}{\pi}\right)^4 (i\pi)^2\ln^6\left(\frac{1}{\tau_1}\right) \text{ ?}$$

CVLs for one-jettiness

Without the collinear emission, the system is in a three-parton state

$$\mathbf{t}_i \cdot \mathbf{t}_j = \frac{1}{2}(C_i + C_j - C_k)\mathbf{1} \implies \mathbf{V}_{k_T,Q}\,\mathbf{H}\,\mathbf{V}^\dagger_{k_T,Q} = W_{k_T,Q}\,\mathbf{H}$$

$$\Sigma^{\mathrm{CVLs}}(\tau_1) \approx \frac{\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{dk_T}{k_T}\int_0^{\ln(Q/k_T)} dy\,\Theta\left(y - \ln\left(\frac{\tau_1 k_T}{Q}\right)\right) \times$$
$$\times\,\mathrm{Tr}\Big(\mathbf{V}_{\tau_1 Q,k_T}\,\mathbf{t}_a\,\mathbf{H}\,\mathbf{t}_a^\dagger\,\mathbf{V}^\dagger_{\tau_1 Q,k_T}\Big)$$



The additional collinear emission introduces a non-trivial four-parton colour structure

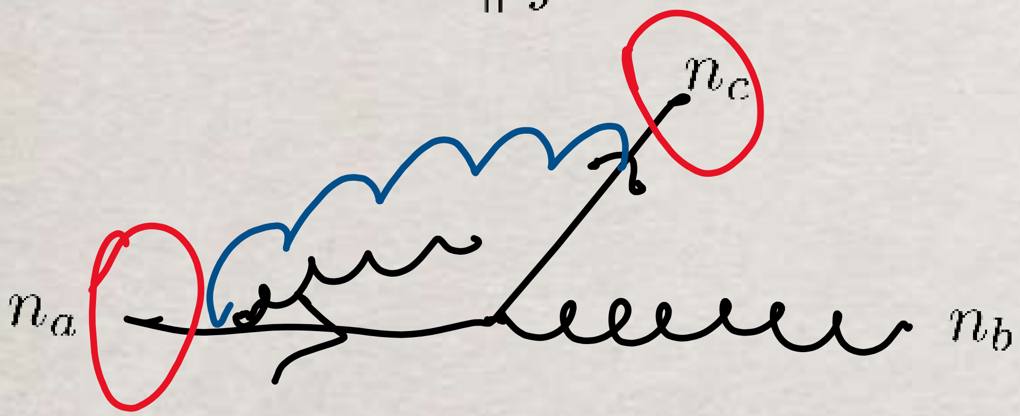
$$V_{\tau_1 Q,k_T} \approx \mathrm{Pexp}\left\{\frac{\alpha_s}{\pi}\left[\sum_i\sum_{j\neq i}\mathbf{T}_i\cdot\mathbf{T}_j\int_{\tau_1 Q}^{k_T}\frac{dq_\perp^{(ij)}}{q_\perp^{(ij)}}\int_{-\ln Q/q_\perp^{(ij)}}^{\ln Q/q_\perp^{(ij)}} d\eta^{(ij)}\int\frac{d\phi^{(ij)}}{2\pi}\Theta(q\cdot n_{i,j} - \tau_1 Q)\right.\right.$$
$$\left.\left. - i\pi\mathbf{T}_s^2\ln\left(\frac{k_T}{\tau_1 Q}\right)\right]\right\}$$

CVLs for one-jettiness

Let’s take a closer look at virtual corrections

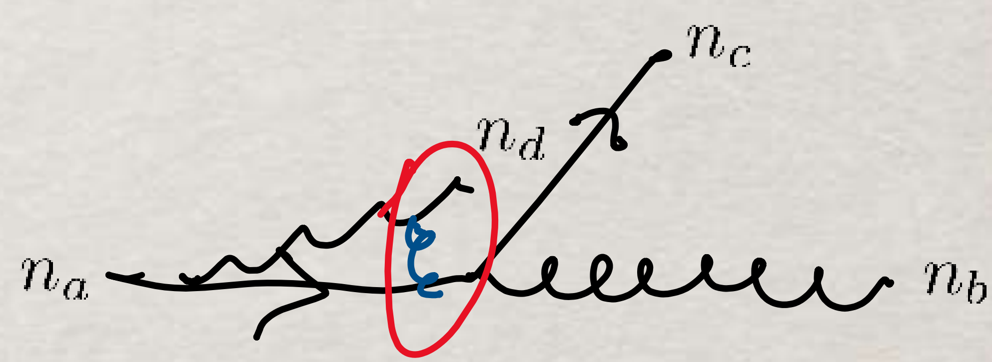
$$\mathbf{V}_{\tau_1 Q, k_T} \approx \mathrm{Pexp}\left\{ \frac{\alpha_s}{\pi} \left[\sum_i \sum_{j \neq i} \mathbf{T}_i \cdot \mathbf{T}_j \int_{\tau_1 Q}^{k_t} \frac{dq_\perp^{(ij)}}{q_\perp^{(ij)}} \int d\eta^{(ij)} \frac{d\phi^{(ij)}}{2\pi} \Theta\left(q \cdot n_i - \tau_1 Q\right) \Theta\left(q \cdot n_j - \tau_1 Q\right) - i\pi \mathbf{T}_s^2 \ln\left(\frac{k_T}{\tau_1 Q}\right) \right] \right\}$$

$i \nparallel j$



$$\int_{q \cdot n_{i,j} > \tau_1 Q} d\eta^{(ij)} \frac{d\phi^{(ij)}}{2\pi} \approx \ln\left(\frac{q_\perp^2}{\tau_1 Q^2} \frac{n_i \cdot n_j}{2}\right)$$

$i \parallel j$



$$\int_{q \cdot n_{i,j} > \tau_1 Q} d\eta^{(ij)} \frac{d\phi^{(ij)}}{2\pi} \longrightarrow 0 \qquad (\tau_1 \to 0)$$

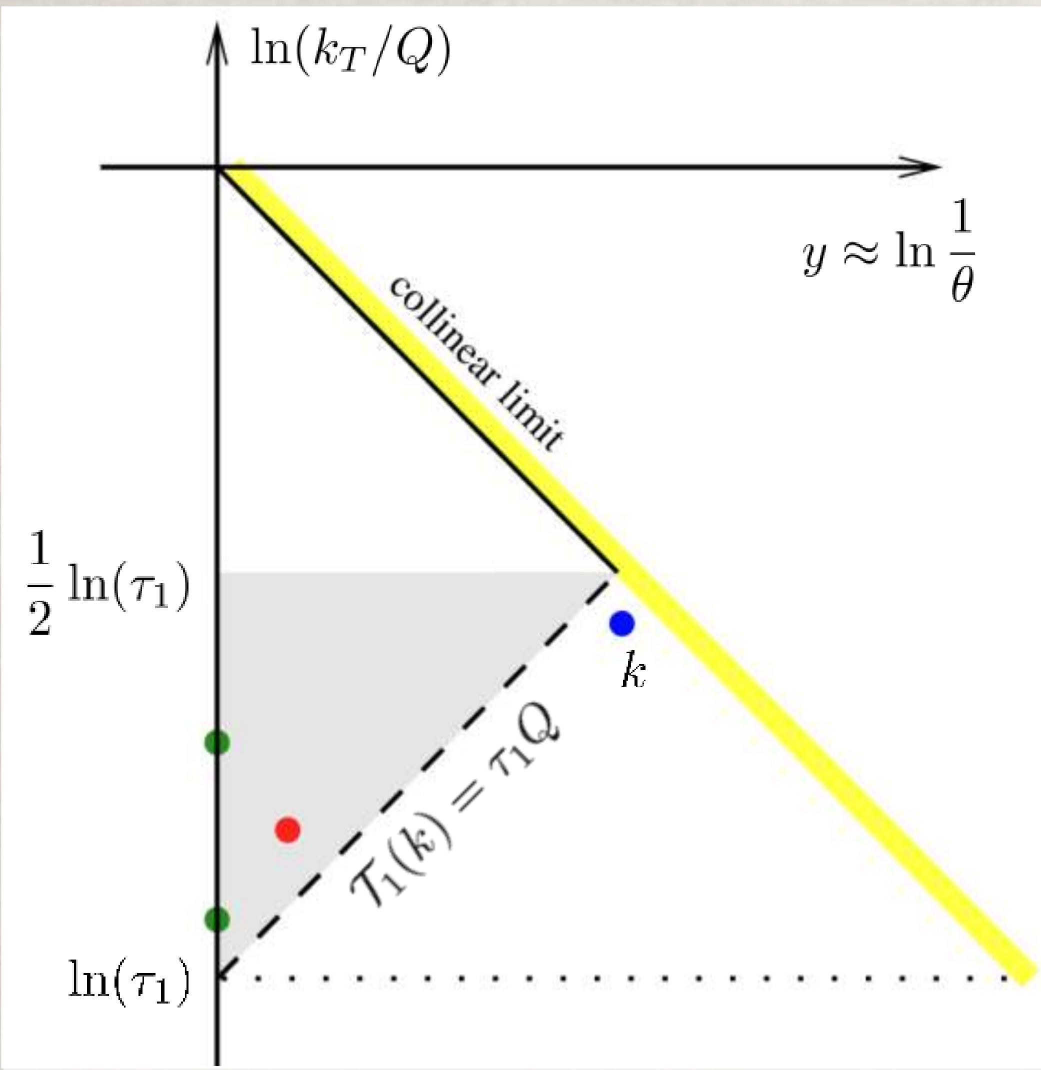
$$\sum_i \sum_{\substack{j > i \\ j \nparallel i}} \mathbf{T}_i \cdot \mathbf{T}_j = -\frac{1}{2} \sum_i \mathbf{T}_i^2 - \mathbf{T}_a \cdot \mathbf{T}_d \equiv \mathbf{T}_{ad}^2$$

$$\mathbf{V}_{\tau_1 Q, k_t} \approx \exp\left(\frac{\alpha_s}{\pi} \left[\mathbf{T}_{ad}^2 \ln^2\left(\frac{k_T}{\tau_1 Q}\right) - i\pi\, \mathbf{T}_s^2 \ln\left(\frac{k_T}{\tau_1 Q}\right) \right] \right)$$

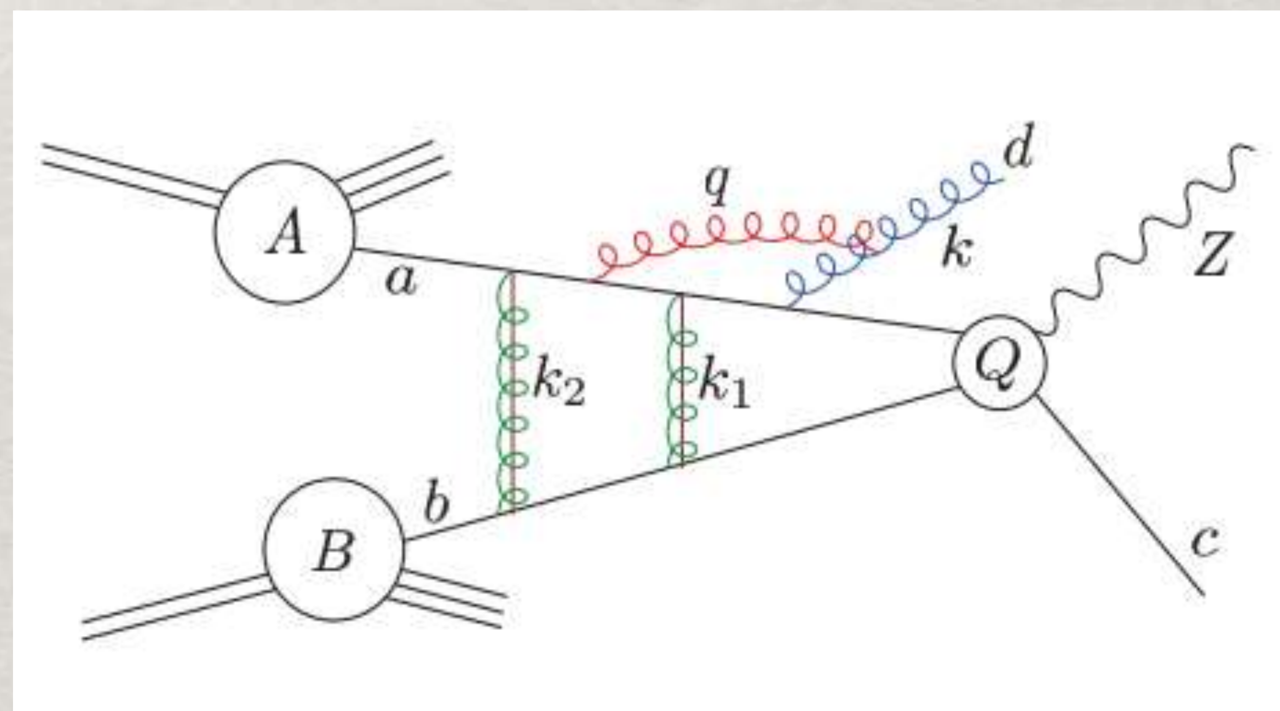
CVLs for one-jettiness

For fixed coupling, one can use the Baker-Campbell-Haussdorf formula

$$\mathbf{V}^{\dagger}_{\tau_1 Q, k_T}\mathbf{V}_{\tau_1 Q, k_t} \approx \exp\left[2\frac{\alpha_s}{\pi}\mathbf{T}^2_{ad}\ln^2\left(\frac{k_T}{\tau_1 Q}\right) + \left(\frac{\alpha_s}{\pi}\right)\ln^2\left(\frac{k_T}{\tau_1 Q}\right)\left(-i\pi\ln\left(\frac{k_T}{\tau_1 Q}\right)\right)[\mathbf{T}^2_s, \mathbf{T}^2_{ad}] + \left(\frac{\alpha_s}{\pi}\right)^3\ln^2\left(\frac{k_T}{\tau_1 Q}\right)\left(-i\pi\ln\left(\frac{k_T}{\tau_1 Q}\right)\right)^2[\mathbf{T}^2_s, [\mathbf{T}^2_s, \mathbf{T}^2_{ad}]]\right]$$



$$A_{a=q,g} \equiv \mathrm{Tr}\left([\mathbf{T}^2_s, [\mathbf{T}^2_s, \mathbf{T}^2_{ad}]]\mathbf{t}_a \mathbf{H}\,\mathbf{t}^{\dagger}_a\right) \neq 0$$

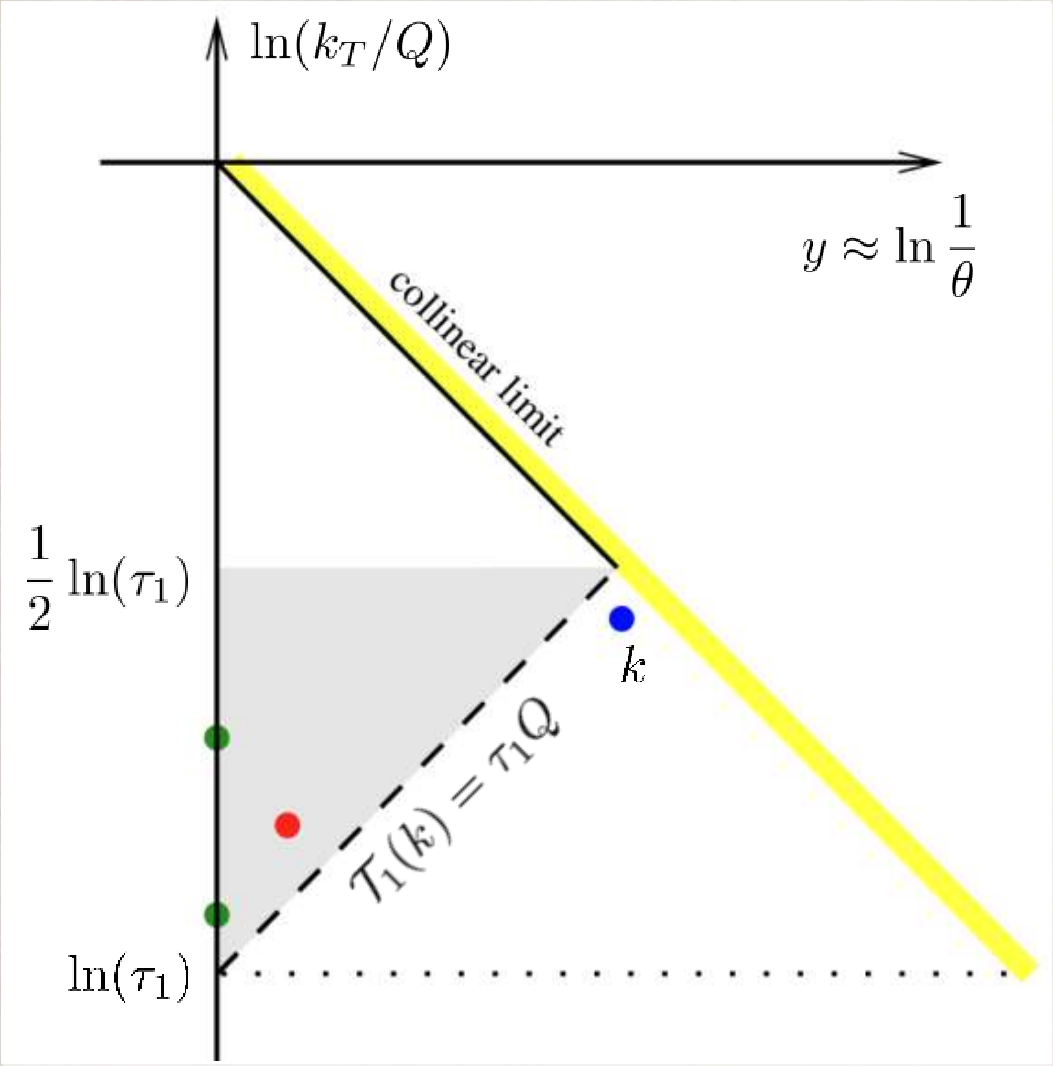


Super-super leading logarithms

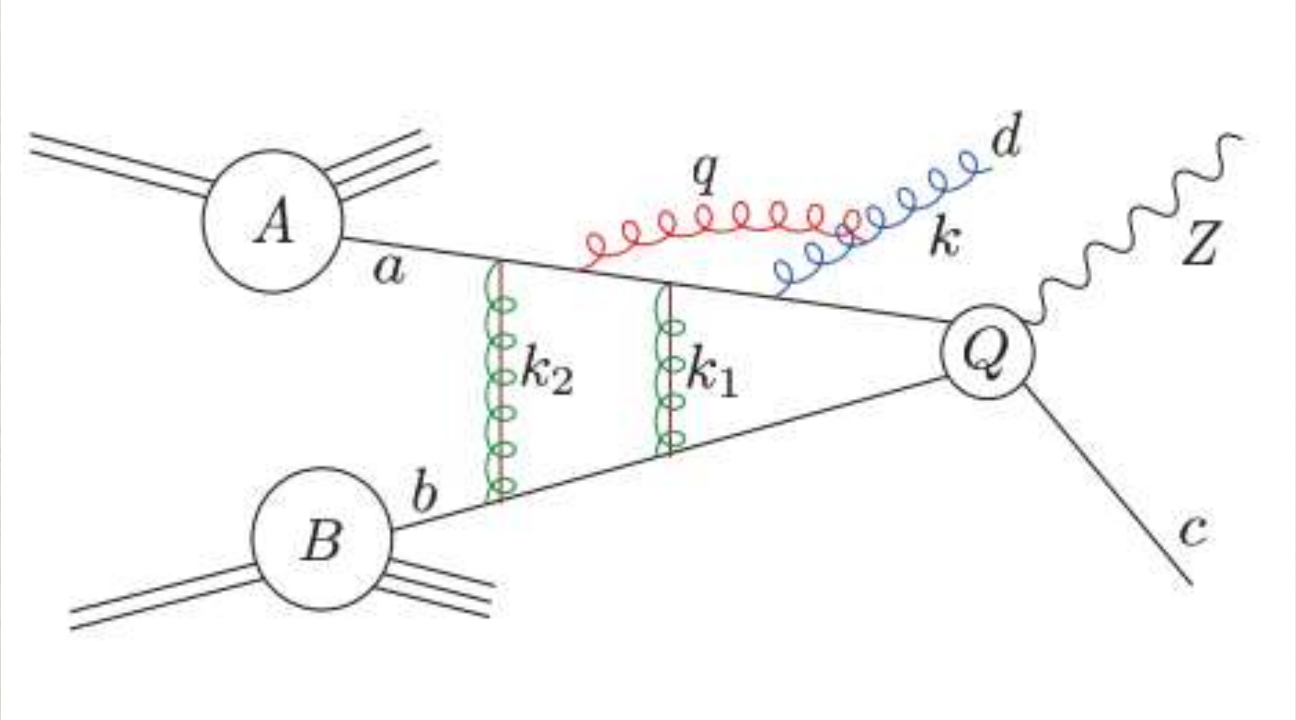
For one-jettiness, coherence violating logarithms are super-super-leading

[AB Forshaw Holguin 2511.11799]

$$\Sigma^{\mathrm{CVLs}}_{a=q,g}(\tau_1) = A_a \left(\frac{\alpha_s}{\pi}\right)^4 \int_{\tau_1 Q}^{Q} \frac{dk_T}{k_T} \int_{k_T/Q}^{1} \frac{d\theta}{\theta}\, \Theta(\tau_1 Q - k_T\theta) \ln^2\left(\frac{k_T}{\tau_1 Q}\right) \left(-i\pi \ln\left(\frac{k_T}{\tau_1 Q}\right)\right)^2$$



$$A_q = \frac{8}{3} N_c^2 T_R^4 \qquad A_g = -\frac{32}{3} N_c^2 T_R^4$$



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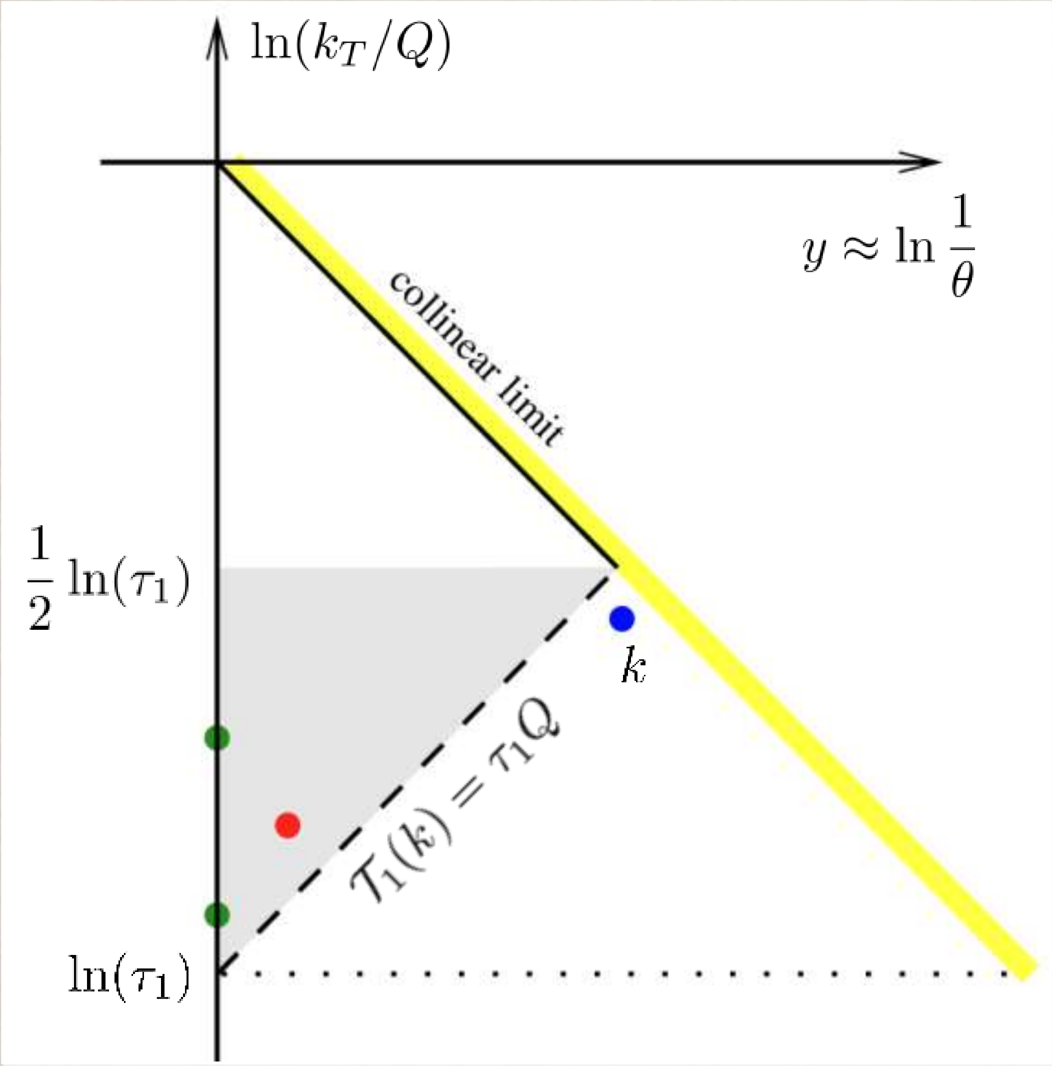
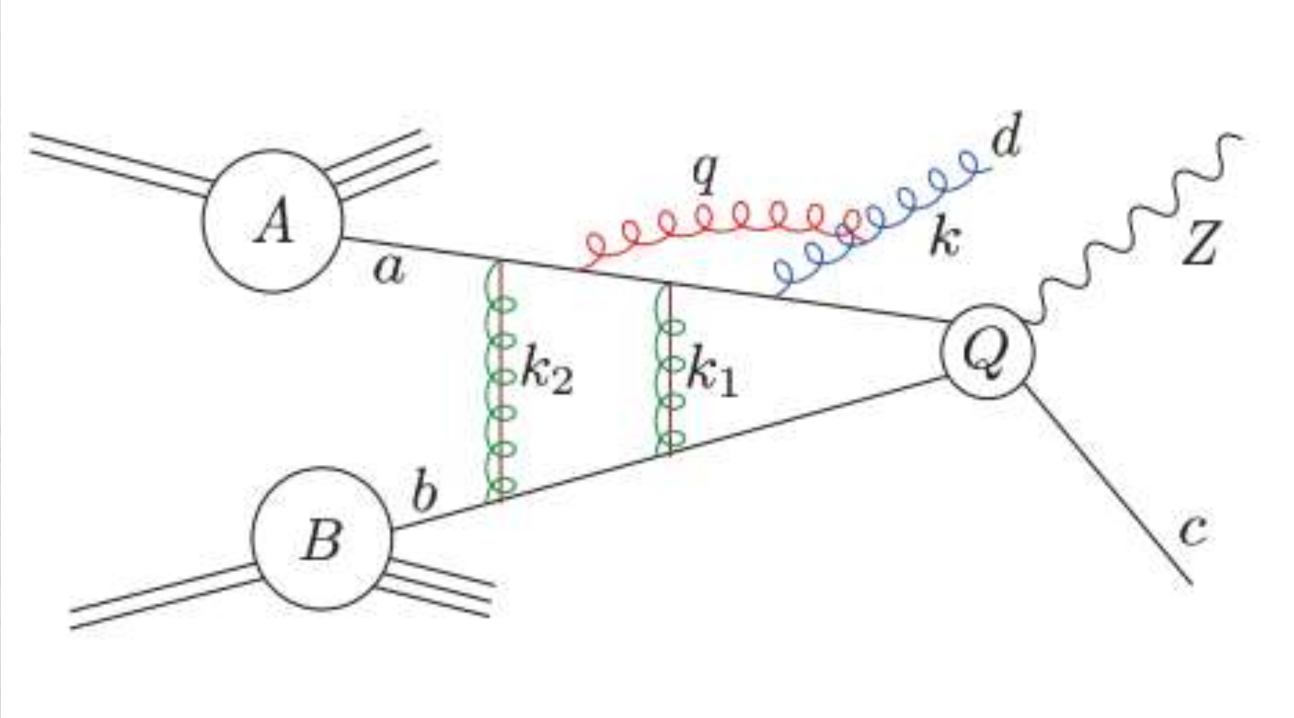


$$\Sigma_a^{\mathrm{CVLs}}(\tau_1) = A_a \left(\frac{\alpha_s}{\pi}\right)^4 \frac{(-i\pi)^2}{480} \left(\ln\frac{1}{\tau_1}\right)^6 \sim \alpha_s^4 L^6$$



This result has been confirmed using EFT techniques

[Becher Hager Neubert Schwienbacher 2603.12383]

Remark on log counting

There are two different ways of counting logarithms

$$\Sigma = 1 + g_{\rm DL}(\alpha_s L^2) + \frac{1}{L} g_{\rm NDL}(\alpha_s L^2) + \dots$$

- Useful the all-order structure is unknown
- Smaller range, expansion in $\sqrt{\alpha_s}$ when $\alpha_s L^2 \sim 1$

$$\ln \Sigma = L g_{\rm LL}(\alpha_s L) + g_{\rm NLL}(\alpha_s L) + \dots$$

- Useful when LL exponentiate (e.g. rIRC observables)
- Larger range, expansion in α_s when $\alpha_s L \sim 1$

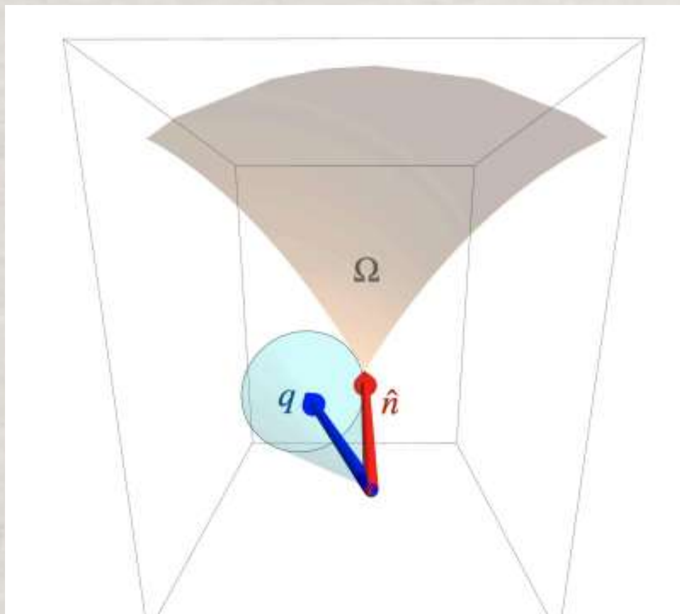
Accuracy in $\ln \Sigma$ translates into an accuracy in Σ. For instance, NNLL resummation

- gives all terms $\alpha_s^n L^{n-1}$ in $\ln \Sigma$
- gives all terms $\alpha_s^n L^{2n-2}$ (NNDL) in Σ

Coherence violating logarithms $\alpha_s^4 L^6$ compete with NNDL

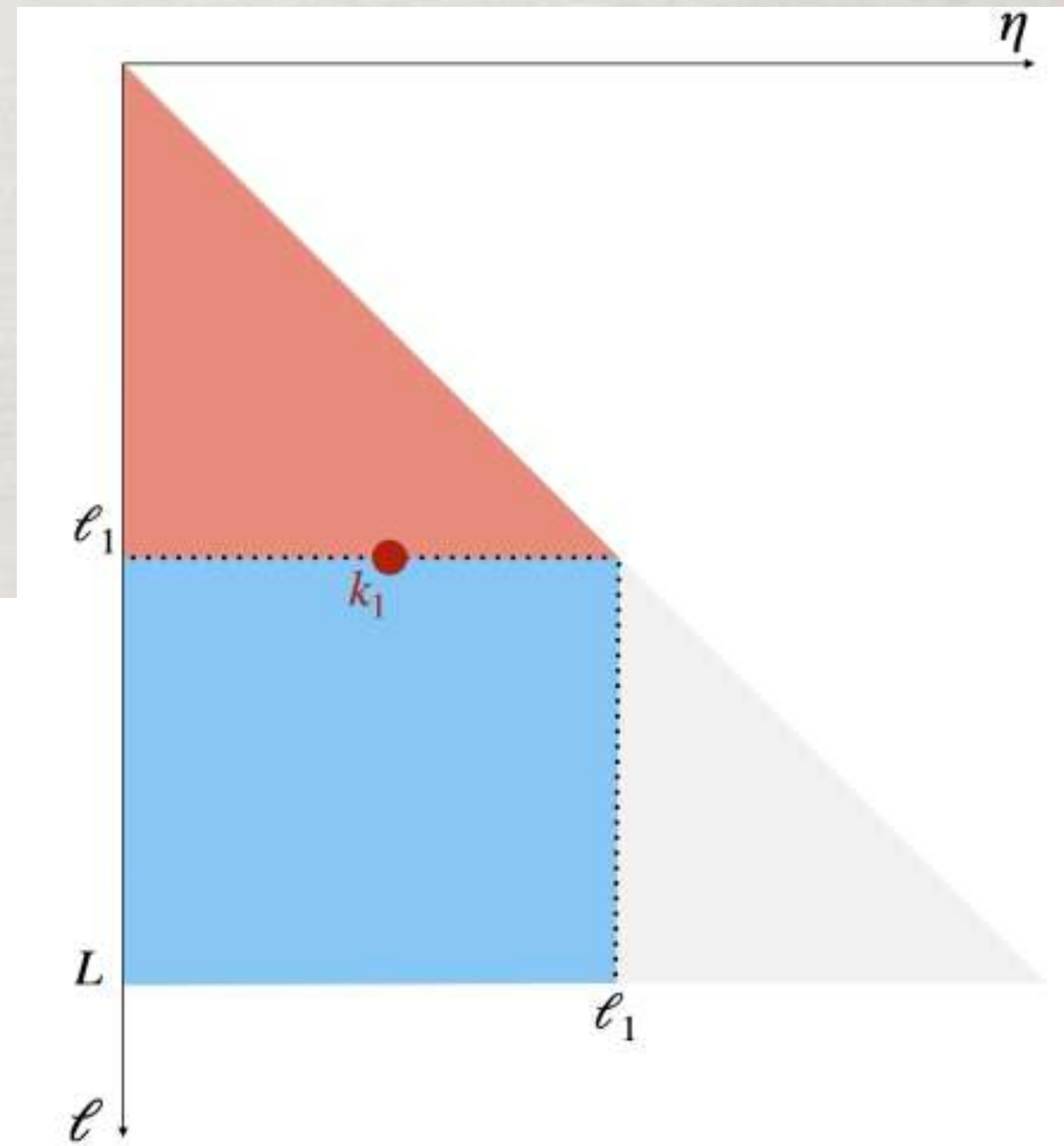
Transverse energy flow

Super-super leading logarithms have been found also in the transverse energy flow into an azimuthal patch of size $\Delta\phi = 2\pi f$ [Dasgupta Fraley Monni Nabeebaccus 2511.14681]



$$\hat{\sigma}_{\rm CVL} = \left(-\frac{2\alpha_s}{\pi}\right)^5 (1-f)^2 \int_{E_T}^{Q} \frac{dk_{t,3}}{k_{t,3}} \int_{-\eta_3^{\rm max}}^{\eta_3^{\rm max}} d\eta_3 \int_{E_T}^{k_{t,3}} \frac{dk_{t,4}}{k_{t,4}} \int_{-\eta_4^{\rm max}}^{\eta_4^{\rm max}} d\eta_4 \int_{E_T}^{k_{t,4}} \frac{dk_t}{k_t}$$
$$\left[\frac{1}{3}\ln^2\left(\frac{k_{t,4}}{E_T}\right) \langle m_0 | \boldsymbol{D}^{\mu_3\,\dagger}_{a_3,(2)} \boldsymbol{D}^{\mu_4\,\dagger}_{a_4,(3)} \left[\boldsymbol{\Gamma}^{\rm I}_{(4)}, \left[\boldsymbol{\Gamma}^{\rm I}_{(4)}, \boldsymbol{\Gamma}^{\rm R}_{(4)}\right]\right] \boldsymbol{D}^{\mu_4}_{a_4,(3)} \boldsymbol{D}^{\mu_3}_{a_3,(2)} | m_0 \rangle\right]$$

$$\Sigma_{\rm DY}^{\rm (CVL)} = -\left(\frac{\alpha_s}{\pi}\right)^5 \pi^2 C_F N_c^2 f\,(1-f)^2 \frac{L^8}{180}$$



In DY, one needs two collinear emissions for to have a non-trivial colour structure $\Longrightarrow$ CVLs start at $(\alpha_s L)^2(\alpha_s L)^3 = \alpha_s^5 L^8$

Avenues for resummation

SLL resummation

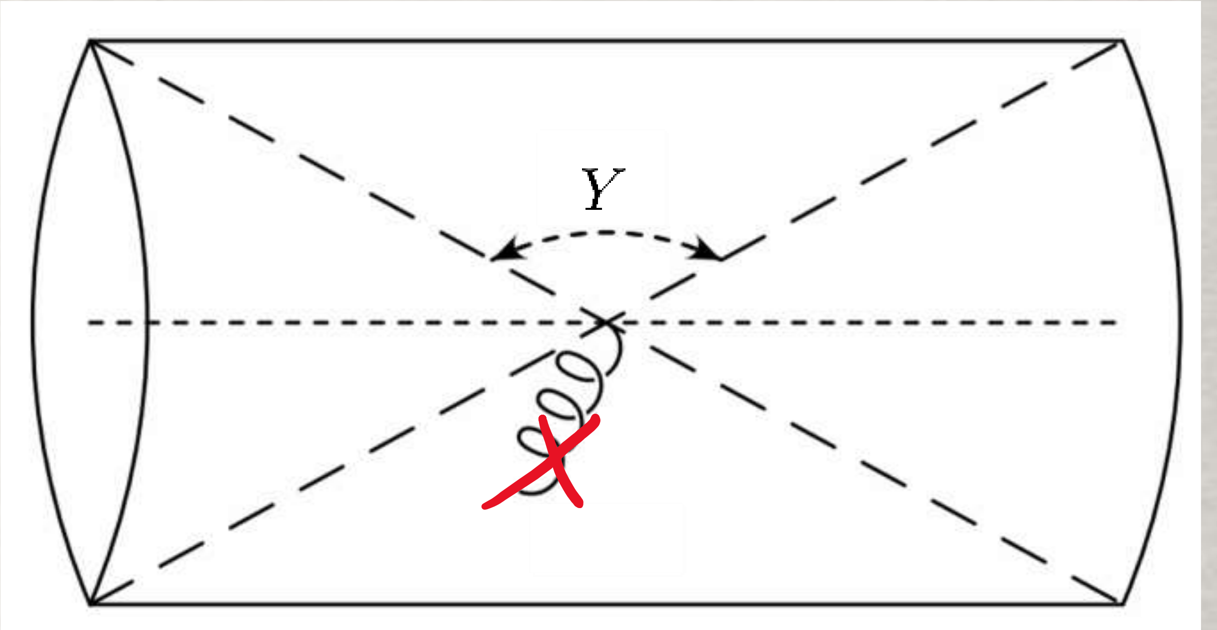
The gap-between-jets (GBJs) cross section admits an all-order factorisation formula

[Becher Neubert Shao 2107.01212]

$$\frac{d\sigma}{dY} = \int dx_a dx_b \sum_{n=2+J}^{\infty} \langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\}, Q, x_a, x_b, \mu) \otimes \boldsymbol{\mathcal{W}}_m(\{\underline{n}\}, Q_0, x_a, x_b, \mu) \rangle$$

Resummations of logarithms of $Q_0 \Longleftrightarrow$ RGE equation for the hard functions $\boldsymbol{\mathcal{H}}_m$

$$\mu \frac{d}{d\mu} \boldsymbol{\mathcal{H}}_m(Q, \mu) = -\sum_{l=2}^{m} \boldsymbol{\mathcal{H}}_l(Q, \mu) \otimes \boldsymbol{\Gamma}_{lm}$$



$\boldsymbol{\Gamma}^{(1)}(x_a, x_b) = \delta(1-x_a)\delta(1-x_b)\boldsymbol{\Gamma}^S$ + hard collinear part

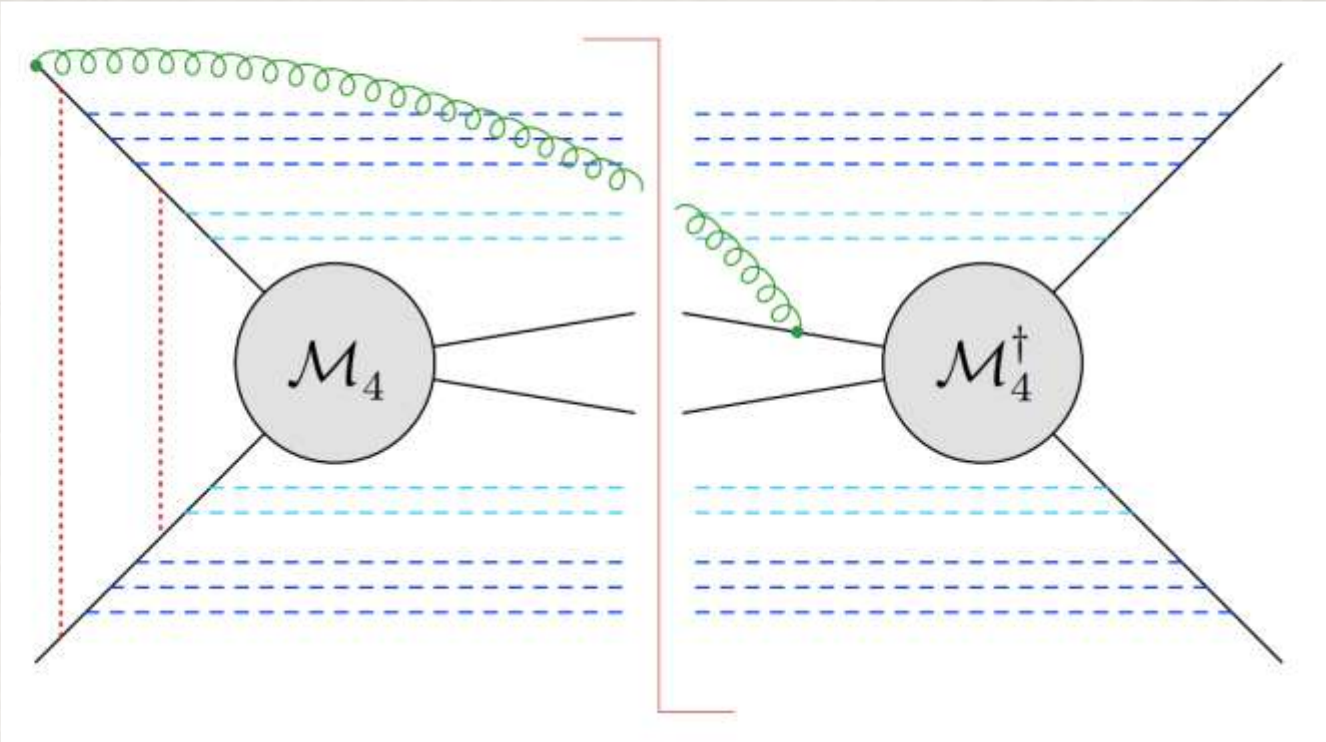
$$\boldsymbol{\Gamma}^S = \bar{\boldsymbol{\Gamma}} + \boldsymbol{\Gamma}^G + \boldsymbol{\Gamma}^c \ln \frac{\mu^2}{\hat{s}}$$

wide-angle Glauber collinear

SLL RESUMMATION

The largest super-leading logarithms can be obtained by resumming collinear emissions at all orders

[Becher Neubert Shao 2107.01212]



$L \equiv \ln(Q/Q_0)$

$$\langle \boldsymbol{\mathcal{H}}_4 (\boldsymbol{\Gamma}^c)^r \boldsymbol{\Gamma}^G (\boldsymbol{\Gamma}^c)^{n-r} \boldsymbol{\Gamma}^G \bar{\boldsymbol{\Gamma}} \rangle \to \left(\pi^2 (\alpha_s L)^2\right) (N_c \alpha_s L^2)^n (N_c Y \alpha_s L)$$

The all-order resummation can be expressed in term of seven colour structures

[Becher Neubert Shao Stillger 2307.06359]

$$\hat{\sigma}_{2\to M}^{\rm SLL} \sim \frac{1}{N_c^2} (\pi^2 Y)(N_c \alpha_s L)^3 \left[k_0 \Sigma_0 (N_c \alpha_s L^2) + \sum_{i=1}^{6} k_i \Sigma(v_i, N_c \alpha_s L^2) \right]$$

General structure of CVLs

Coherence violating logarithms have a different structure according to the considered class of observables

- At least two Coulomb gluons must be present
- Collinear emissions must always be resummed

Observable	**Soft virtual gluons**	**Colour space**	**Logarithms**
Gap between jets	One is enough	Grows with the number of soft virtual gluons	SLL $\alpha_s^n L^{2n-3}$
N-jet rIRC safe	Must be resummed if rapidity suppressed (e.g. one-jettiness)	N+4 parton-like	SSLL $\alpha_s^n L^{2n-2}$
Interjet energy flow	Must be resummed	Grows with the number of soft virtual gluons	SSLL $\alpha_s^n L^{2n-2}$

Conclusions

- Coherence is an important property of QCD radiation
- Coulomb gluons transfer colour between initial-state partons, thus modifying the structure of initial-state collinear splitting
- Coherence violations lead to super-leading logarithms in non-global observables
- Global observables can be affected by coherence-violating super-super-leading logarithms

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Two main areas of research

- Show with explicit calculations that collinear factorisation still holds
- Perform the resummation of coherence-violating logarithms

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Thank you for your attention!