Dimensional regularization and γ_5 — no-compromise* approach to the BMHV scheme

Dominik Stöckinger

TU Dresden

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Collaborators: Bélusca-Maïto, Ilakovac; Ebert, **Kühler**, Mađor-Božinović, **Weisswange**, v. Manteuffel

1-Loop: [2004.], 2-loop: [2109.], 3-loop [2312.], 4-loop [2506.] review: [2303.], 1-loop "couplings" [2411.] 2-loop YM [2504.]

* or: traditional/old-fashioned/stubborn...

parity violation . . . chiral fermions

. . . gauge invariance

fundamental observations . . . deep QFT problems

renormalization and regularization

Procedure in a nutshell — outline

0) Background: gauge invariance is vital but at stake!

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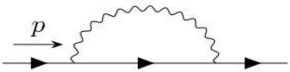

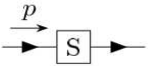

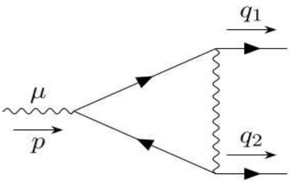

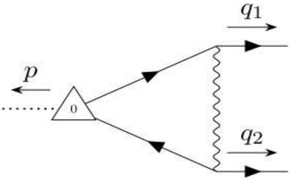

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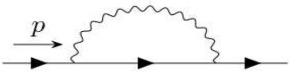

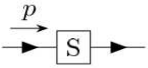

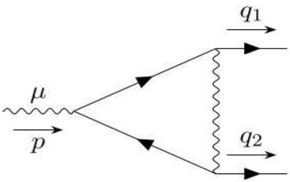

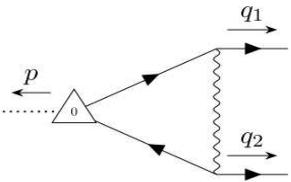

1) Ward identity
violated

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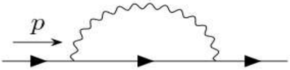

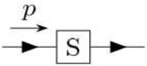

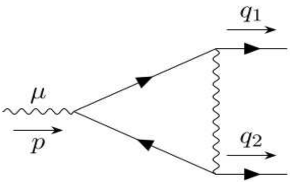

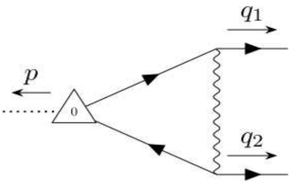

1) Ward identity violated

2) compensated by special c.t.

(our main task)

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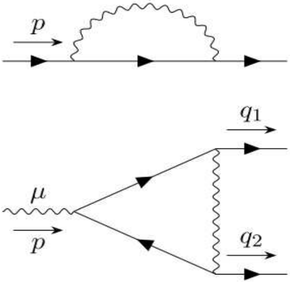

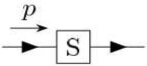

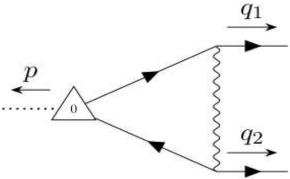

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3) Alternative: breaking via $\frac{\epsilon}{\epsilon}$ term

(tool)

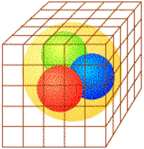
Outline

1. Introduction
   - Background: regularization and symmetry identities
   - Example breaking via γ_5 problem and required counterterm
   - Example alternative calculations
2. Detailed discussion
3. More recent developments

Necessity for regularization

Regularization is necessary to define QFT at the quantum level

- many different options



cutoff-scale Λ

$\int_{|p|<\Lambda} d^4 p$

DREG

$\mu^{4-D} \int d^D p$

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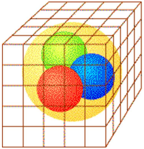

cutoff-scale Λ

$\int_{|p|<\Lambda} d^4p$

DREG

$\mu^{4-D}\int d^Dp$

- often regularizations break symmetries (Lorentz, gauge inv.. . .)

The problem: γ_5 and DReg

Three properties in 4-dimensions — inconsistent in $D \neq 4$

$$\{\gamma_5, \gamma^\mu\} = 0, \quad (1)$$

$$\mathrm{Tr}(\gamma_5 \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma) = 4i\epsilon^{\mu\nu\rho\sigma}, \quad (2)$$

$$\mathrm{Tr}(\Gamma_1 \Gamma_2) = \mathrm{Tr}(\Gamma_2 \Gamma_1)\,. \quad (3)$$

Give up at least one $\Rightarrow$ many proposals!

The problem: γ_5 and DReg

Three properties in 4-dimensions — inconsistent in $D \neq 4$

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$$\mathrm{Tr}(\gamma_5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = 4i\epsilon^{\mu\nu\rho\sigma}, \tag{2}$$
$$\mathrm{Tr}(\Gamma_1\Gamma_2) = \mathrm{Tr}(\Gamma_2\Gamma_1)\,. \tag{3}$$

Give up at least one $\Rightarrow$ many proposals!

- Often limited range of applicability
- BMHV ('t Hooft, Veltman, Breitenlohner, Maison)
  non-anticommuting, very complicated, breaks gauge inv.
  But unitary, consistent

BMHV scheme — non-anticommuting γ_5

QFT consistent, unitary; breaks symmetries, complicated

- "*D*-dim space" split into pure 4-dim space $\oplus$ (-2ϵ)-dim space

$$
\begin{aligned}
X^\mu &= \bar{X}^\mu + \hat{X}^\mu \\
\gamma_5 &= i\gamma_0\gamma_1\gamma_2\gamma_3 \\
\{\gamma_5, \bar{\gamma}^\mu\} &= 0 \\
[\gamma_5, \hat{\gamma}^\mu] &= 0
\end{aligned}
$$

Our idea: No-compromise approach to BMHV — apply it and accept/deal with its difficulties!

Detail/problem in dimensional regularization

Regularized diagrams $\leftrightarrow$ regularized Lagrangian!
E.g. QED or QCD:

$$\begin{aligned} D\text{-dim propagator} \leftrightarrow & \qquad \mathcal{L}_{\text{kin}} = \bar{\psi} i \gamma^\mu \partial_\mu \psi \\ \text{fully } D\text{-dim gauge interaction} \leftrightarrow & \qquad \mathcal{L}_{\text{int}} = \bar{\psi} \gamma^\mu A_\mu \psi \end{aligned}$$

$\Rightarrow$ in total: *D*-dim gauge invariance

Detail/problem in dimensional regularization

Regularized diagrams $\leftrightarrow$ regularized Lagrangian!
However, e.g. neutrino and B^μ (hypercharge):

$$\begin{aligned} D\text{-dim propagator} \leftrightarrow &\qquad \mathcal{L}_{\text{kin}} = \bar{\psi} i \gamma^\mu \partial_\mu \psi \\ \text{purely “L” gauge interaction} \leftrightarrow &\qquad \mathcal{L}_{\text{int}} = \bar{\psi} P_R \gamma^\mu B_\mu P_L \psi \end{aligned}$$

$\Rightarrow$ in total: not gauge invariant!

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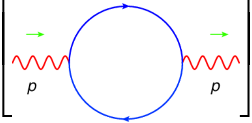
$\Rightarrow$ in total: not gauge invariant!

Leads to breaking of Ward/Slavnov-Taylor identities

p_μ [] $\neq 0$ $\Big\{$ need symmetry-restoring counterterms

Symmetry identities (Ward, Slavnov-Taylor) are vital:

$$\text{“}S(\Gamma^{\mathrm{reg}} + \Gamma^{\mathrm{ct}}) = 0\text{”}$$

- *Unphysical states/negative norm*
- *Unitary and gauge independent physical S-matrix*

They must be valid in the renormalized theory

Construction of Slavnov-Taylor identity

- Ghosts for all generators $\longrightarrow$ BRST:

$$s\varphi = c_a \delta_{\text{gauge},a}\varphi$$

- BRS transformations of ghosts $\leftrightarrow$ $s^2 = 0$:

$$sc_a = \frac{1}{2} g f_{abc} c_b c_c$$

- Slavnov–Taylor operator ($S(\Gamma) = 0$ aka "Lee identities/Zinn-Justin identity")

$$S(\Gamma) = \int d^4x \, \underbrace{\langle s\varphi_i(x)\rangle} \frac{\delta\Gamma}{\delta\varphi_i(x)}$$

- Add sources $\mathcal{L}_{\text{ext}} = K_{\varphi_i} s\varphi_i$ for composite operators

$$S(\Gamma) = \int d^4x \, \frac{\delta\Gamma}{\delta K_{\varphi_i}(x)} \frac{\delta\Gamma}{\delta\varphi_i(x)}$$

Concrete identities

In QED-like theories: Slavnov-Taylor identity $S(\Gamma) = 0 \rightsquigarrow$ "Ward identities"

$$
\begin{aligned}
p_\mu \Gamma^{\mu\nu}_{AA} &= 0 \\
p_\mu \Gamma^{\mu\nu\rho\sigma}_{AAAA} &= 0 \\
p_\mu \Gamma^{\mu}_{A\psi\bar{\psi}} &\propto eQ(\Sigma(p_\psi) - \Sigma(p_{\bar{\psi}}))
\end{aligned}
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- Case 1: regularization preserves identities

  $\rightsquigarrow$field/parameter renormalization transformation

- Case 2: regularization breaks identities

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- Case 1: regularization preserves identities

- Case 2: regularization breaks identities

  $\rightsquigarrow$add also special counterterms which satisfy "$S(\Gamma^{\text{ct}}) = -\Delta$"

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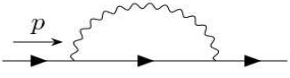

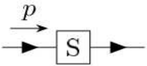

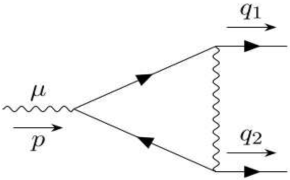

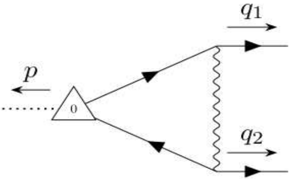

Symmetry identities (Ward, Slavnov-Taylor) are usually manifestly valid in DReg!

But sometimes not!

- *How can DReg break a symmetry?* γ_5*-problem!*
- *What does the breaking look like?*
- *How can we repair it?*

Warm-up exercise:

simple divergent one-loop integral

$$\int d^D k \frac{1}{k^2(k+p)^2} = \frac{1}{\epsilon} + \text{ finite}$$

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simple tensor integral

$$\int d^D k \frac{k^\mu k^\nu}{k^2(k+p)^2} = \frac{1}{3\epsilon} p^\mu p^\nu - \frac{1}{12\epsilon} p^2 g^{\mu\nu} + \text{ finite}$$

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integral with "evanescent numerator" (multiply with $\hat{g}_{\mu\nu}$!)

$$\int d^D k \frac{\hat{k}^2}{k^2(k+p)^2} = \frac{1}{3\epsilon} \hat{p}^2 + \frac{1}{6} p^2 + \text{ finite}$$

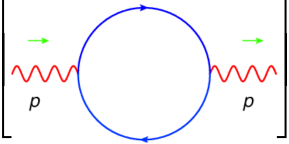
...produces div-evanescent AND finite, non-evanescent terms!

Example: Ward identity is valid in DReg in ordinary QED

$$p_\mu \Gamma^{\mu\nu}_{AA} = 0??$$

Check QED transversality of photon self energy ($\rightsquigarrow$ textbooks)

$p_\mu \Big[$



$$\Big] = p_\mu \int d^D k \frac{\mathrm{Tr}(\not{k}\gamma^\mu(\not{k}+\not{p})\gamma^\nu)}{k^2(k+p)^2}$$

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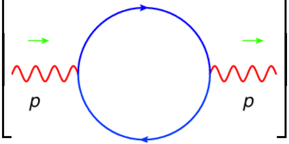
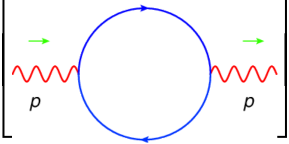


] $= p_\mu \int d^D k \frac{\mathrm{Tr}(\not{k}\gamma^\mu(\not{k}+\not{p})\gamma^\nu)}{k^2(k+p)^2}$

using $\not{p} = (\not{k}+\not{p}) - \not{k}$ gives zero:

$$= \int d^D k \frac{(k+p)^2}{(k+p)^2} \frac{\mathrm{Tr}(\not{k}\gamma^\nu)}{k^2} - \int d^D k \frac{k^2}{k^2} \frac{\mathrm{Tr}((\not{k}+\not{p})\gamma^\nu)}{(k+p)^2} = 0$$

Check the same Ward identity in “chiral QED”



$$p_\mu \left[\quad \right] = p_\mu \int d^D k \frac{\mathrm{Tr}(\not{k} P_R \gamma^\mu P_L (\not{k} + \not{p}) P_R \gamma^\nu P_L)}{k^2 (k+p)^2}$$

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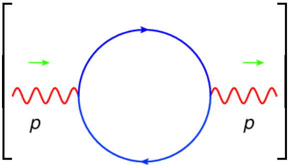
p_μ 

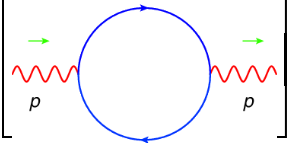
$$= p_\mu \int d^Dk \frac{\mathrm{Tr}(\not{k} P_R \gamma^\mu P_L (\not{k}+\not{p}) P_R \gamma^\nu P_L)}{k^2(k+p)^2}$$

extracts purely 4-dim parts in numerator!

$$= p_\mu \int d^Dk \frac{\mathrm{Tr}(\bar{\not{k}} \bar{\gamma}^\mu (\bar{\not{k}}+\bar{\not{p}}) \bar{\gamma}^\nu P_L)}{k^2(k+p)^2}$$

using the same method, we cannot cancel anymore

Check the same Ward identity in “chiral QED”



$$p_\mu \left[\quad \cdots \quad \right] = p_\mu \int d^D k \frac{\mathrm{Tr}(\not{k} P_R \gamma^\mu P_L (\not{k}+\not{p}) P_R \gamma^\nu P_L)}{k^2(k+p)^2}$$

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using the same method, we cannot cancel anymore

we can only simplify to a term with evanescent numerator produces div-evanescent AND finite, non-evanescent terms!

The full result for the self energy makes the breaking terms explicit:

$$\widetilde{\Gamma}^{\nu\mu}_{AA}(p)\Big|^{1}_{\text{fin}, \chi\text{QED}} \sim \frac{1}{3}\left[\left(\frac{10}{3} - 2\ln(-p^2)\right)(\overline{p}^{\mu}\overline{p}^{\nu} - \overline{p}^2\overline{g}^{\mu\nu}) - \overline{p}^2\overline{g}^{\mu\nu}\right].$$

Step 1: Regularization breaks transversality! Gauge invariance is broken by div-evan. plus finite, non-evanescent, local terms!

The full result for the self energy makes the breaking terms explicit:

$$\widetilde{\Gamma}^{\nu\mu}_{AA}(p)|^{1}_{\text{fin, }\chi\text{QED}} \sim \frac{1}{3}\left[\left(\frac{10}{3} - 2\ln(-p^2)\right)(\overline{p}^{\mu}\overline{p}^{\nu} - \overline{p}^2\overline{g}^{\mu\nu}) - \overline{p}^2\overline{g}^{\mu\nu}\right].$$

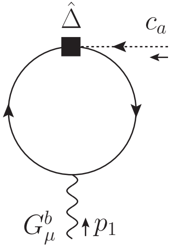
Step 2: restoring c.t. here: $\mathcal{L}^{1}_{\text{fct},\chi\text{QED}} \sim \frac{-1}{6}\bar{A}_{\mu}\bar{\partial}^2\bar{A}^{\mu} + \dots$

This is how the breaking can look like and how it can be repaired.

But what is a more efficient way to compute the breaking?

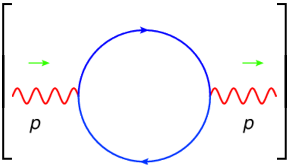
Step 3: More efficient, direct calculation of breaking
using $\widehat{\Delta}$ =operator corresponding to symmetry breaking of $\mathcal{L}_D$

$i[\widehat{\Delta}\cdot\widetilde{\Gamma}^{\mu}_{Ac}]^{(1)} =$



The result of this single diagram is equal to the breaking $\rightarrow$ (next slide)

p_μ



and thus sufficient to determine symmetry-restoring c.t.s
but the calculation is simpler and more direct

Regularized quantum action principle in DReg [Breitenlohner, Maison '77]

Regularized diagrams $\leftrightarrow$ regularized Lagrangian! $\leftrightarrow$ path integral
Simple path integral relations

$$\text{“}\quad \delta \int \mathcal{D}\phi \, e^{i\int \mathcal{L}} = \int \mathcal{D}\phi \, (i \textstyle\int \delta\mathcal{L}) e^{i\int \mathcal{L}} \quad\text{”}$$

are literally valid if path integral is defined by DReg diagrams **and** *D*-dim Lagrangian

Technical tool therefore:

$$\mathcal{S}(\textstyle\int \mathcal{L}) = \Delta \qquad \Rightarrow \qquad \mathcal{S}(\Gamma_{\text{reg}}) = \Delta \cdot \Gamma \qquad \text{in } D \text{ dimensions}$$

Procedure in a nutshell — outline

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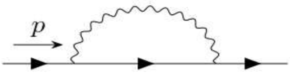
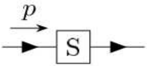
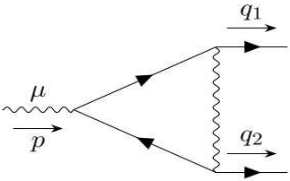
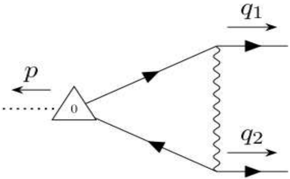








1) Ward identity violated

2) compensated by special c.t.

3) Alternative: breaking via $\frac{\hat{\epsilon}}{\epsilon}$ term

(our main task)

(tool)

Systematic task therefore

- Find all such symmetry breakings by regularized Green functions
- Show that they can be “repaired” by adding suitable counterterms
- Determine these counterterms

Tools: Slavnov-Taylor identities, quantum action principle

Outline

Details: 1-loop calculation in abelian chiral model

$$\mathcal{L}_{\text{kin+int}} = \bar{\psi} i \gamma^\mu \partial_\mu \psi + \bar{\psi} P_L \gamma^\mu P_R A_\mu \psi + \dots$$

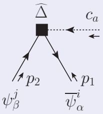
- As stated, this is not gauge invariant

Details: 1-loop calculation in abelian chiral model

$$\mathcal{L}_{\text{kin+int}} = \bar{\psi} i \gamma^\mu \partial_\mu \psi + \bar{\psi} P_L \gamma^\mu P_R A_\mu \psi + \dots$$

- As stated, this is not gauge invariant
  $\Rightarrow$ and leads to breaking of tree-level Slavnov-Taylor identity

$$\mathcal{S}_d(S_0) = \widehat{\Delta} \equiv \int \mathrm{d}^d x \, (e\mathcal{Y}_{Ri}) c \left\{ \overline{\psi}_i \left(\overleftarrow{\widehat{\not\partial}} P_\mathrm{R} + \overrightarrow{\widehat{\not\partial}} P_\mathrm{L} \right) \psi_i \right\} .$$



$$= (e\mathcal{Y}_{Ri}) \left(\widehat{\not p}_1 P_\mathrm{R} + \widehat{\not p}_2 P_\mathrm{L} \right)_{\alpha\beta}$$

Insertions $\widehat{\Delta} \cdot \Gamma$ determine STI breaking
Only $\frac{1}{\epsilon^n}$ can contribute!

Details: Two ways to check symmetry identities

QED-like Ward identities

$$p_\mu \Gamma^{\mu\nu}_{AA} = 0$$
$$p_\mu \Gamma^{\mu\nu\rho\sigma}_{AAAA} = 0$$
$$p_\mu \Gamma^{\mu}_{A\psi\bar\psi} \propto eQ(\Sigma(p_\psi) - \Sigma(p_{\bar\psi}))$$

all summarized by desired STI

$$\mathcal{S}(\mathrm{LIM}_{D\to 4}\Gamma_{\mathrm{ren}}) = 0$$

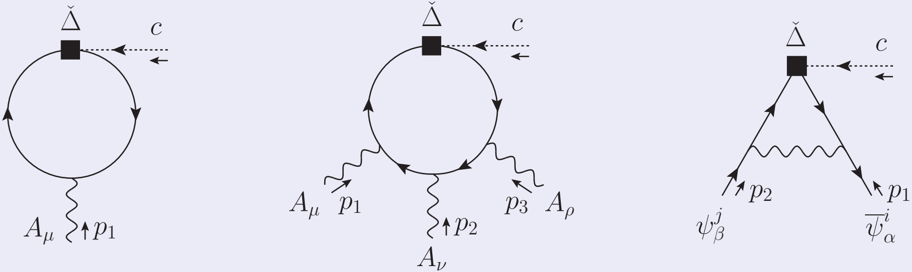
but the STI is broken on the regularized level (and q.a.p. provides relation):

$$\mathcal{S}_d(\Gamma_{\mathrm{reg}}) = \Delta \cdot \Gamma \neq 0$$

- Two alternative ways to compute breaking
- crosscheck! But $\Delta \cdot \Gamma$ simpler (completeness!)

Details: full 1-loop calculation

The complete set of power-counting divergent 1-loop diagrams with insertion of $\widehat{\Delta}$:



Results mean: breaking of three concrete WI/STIs.

They have the form $\frac{\epsilon/\text{evanescent}}{\epsilon} \times$ (local)

$\rightsquigarrow$ local counterterms can repair the symmetry!

(There is an additional diagram corresponding to the fermion triangle loop and the true anomaly (assumed absent))

Details: Results for counterterms

$$0 \stackrel{!}{=} \underbrace{\mathcal{S}\Big(\mathrm{LIM}_{D\to 4}\Gamma^{(1)}_{\mathrm{reg}}}_{\substack{\text{from}\\ \hat{\Delta}\cdot\Gamma^{1}_{\mathrm{reg}}}} + \underbrace{S^{1}_{\mathrm{sct}}}_{\text{finiteness}} + \underbrace{S^{1}_{\mathrm{fct}}\Big)}_{\text{sym.restoration}}$$

Full 1-loop symmetry-restoring counterterms:

$$S^{1}_{\mathrm{fct}} = \frac{e^2}{16\pi^2}\int d^4x \left\{ \frac{-\mathrm{Tr}(\mathcal{Y}_R^2)}{6}\bar{A}_\mu \bar{\partial}^2 \bar{A}^\mu + \frac{e^2\,\mathrm{Tr}(\mathcal{Y}_R^4)}{12}(\bar{A}^2)^2 + \left(\frac{5+\xi}{6}\right)(\mathcal{Y}_R^j)^2 \left(\bar{\psi}_j i\bar{\partial\!\!\!/}\, P_{\mathrm{R}}\,\psi_j\right) \right\}.$$

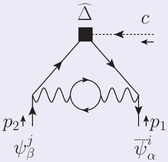
$\rightsquigarrow$ Correspondence to the three diagrams/three Ward identities

Outline

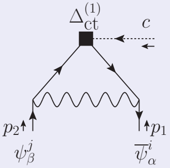
Three dimensions of difficulty

1. higher loops
2. abelian/non-abelian
3. intricacies of the EWSM

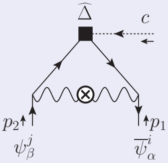
Higher orders (subrenormalization!) [Paul Kühler. . . Matthias Weisswange. . .]

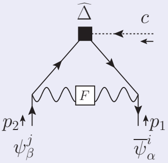


2-loop insertion of $\widehat{\Delta}$



1-loop insertion of Δ^1_{ct}





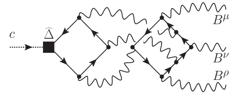
insertion of $\widehat{\Delta}$ into 1-loop diagram with 1-loop ct insertion

Checks: 1. The sum gives $\mathcal{S}_d(\Gamma^{(n)})|_{\text{finite}}$ =local.

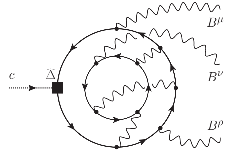
2. We can solve $\mathcal{S}_d(\Gamma^{(n)})|_{\text{finite}} + \mathcal{S}_d S^n_{\text{fct}} \stackrel{!}{=} 0$ for local CTs.

Higher orders (subrenormalization!) [Paul Kühler…Matthias Weisswange…]

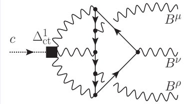
4-loop insertion of $\widehat{\Delta}$



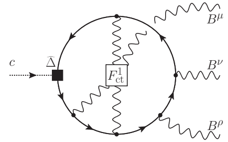
4-loop insertion of $\widehat{\Delta}$



3-loop insertion of Δ^1_{ct}



3-loop counterterm diagram



Checks: 1. The sum gives $\mathcal{S}_d(\Gamma^{(n)})|_{\text{finite}} =$local.

2. We can solve $\mathcal{S}_d(\Gamma^{(n)})|_{\text{finite}} + \mathcal{S}_d S^n_{\text{fct}} \stackrel{!}{=} 0$ for local CTs.

General pattern at n =2-, 3-, 4-loop

Requiring the renormalized STI to hold leads to the result

$$S^n_{\text{fct}} = \frac{e^{2n}}{(16\pi^2)^n} \int \mathrm{d}^4 x \left\{ \mathcal{F}^{n,\text{break}}_{AA} \frac{1}{2} \bar{A}_\mu \bar{\partial}^2 \bar{A}^\mu - \frac{e^2}{8} \mathcal{F}^{n,\text{break}}_{AAAA} (\bar{A}^2)^2 - \mathcal{F}^{n,\text{break}}_{\psi\bar{\psi},ji} \left(\bar{\psi}_j i \bar{\not{\partial}} P_{\text{R}} \psi_j \right) \right\}$$

$$\mathcal{F}^{2,\text{break}}_{AA} = \tfrac{11}{48} \text{Tr}\left(\mathcal{Y}^4_R\right)$$

$$\mathcal{F}^{3,\text{break}}_{AA} = -\left(\tfrac{35242}{21600} + \tfrac{8448}{21600}\zeta_3\right) \text{Tr}\left(\mathcal{Y}^6_R\right) - \tfrac{1639}{21600} \text{Tr}\left(\mathcal{Y}^4_R\right) \text{Tr}\left(\mathcal{Y}^2_R\right)$$

$$\mathcal{F}^{4,\text{SM}}_{AA} = \frac{105574465087}{604661760} + \frac{2665684621}{6998400}\zeta(3) - \frac{499349}{174960}\zeta(4) - \frac{99009133}{209952}\zeta(5)$$

Technicalities: (A) Mathematica+FeynArts+FeynCalc up to 3-loop, (B) QGraf+FORM+Fire/Reduze2 at 4-loop

[Bélusca-Maïto, Ilakovac, **Kühler**, Mađor-Božinović, DS '21], [DS, Weisswange '23], [v. Manteuffel, DS, Weisswange '25]

Three dimensions of difficulty

1. higher loops
2. abelian/non-abelian
3. intricacies of the EWSM

Non-abelian details (STI with loop corrections)

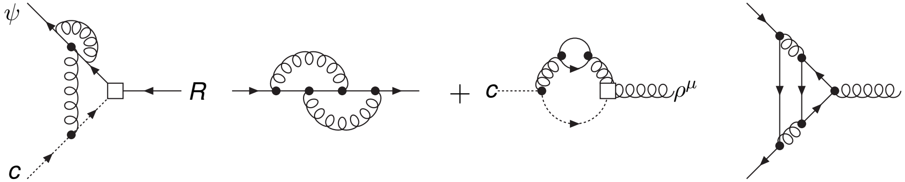
Example abelian Ward identity

$$eQ\;\left(\Gamma_{\psi\bar\psi}(p') - \Gamma_{\psi\bar\psi}(p)\right) \;+\; q^\mu\;\Gamma_{\psi\bar\psi A^\mu} \overset{!}{=} 0$$

Non-abelian STI: $\downarrow$ $\downarrow$

$$\Gamma_{\psi c\bar R}\;\Gamma_{\psi\bar\psi}(p') - \ldots + \qquad \Gamma_{c\rho_\mu}\;\Gamma_{\psi\bar\psi A^\mu} \overset{!}{=} 0$$

loop-corrected BRST prefactors $\rightsquigarrow$ special vertices/diagrams



(new structures, worse power-counting, “strange” vertices: R =bosonic spinor, ρ^μ =fermionic vector with antighost number)

Implementation in FeynArts allows both approaches of calculation

Result for non-abelian chiral gauge theory

[Bélusca-Maïto, Ilakovac, Mađor-Božinović, DS, 2020] [2-loop result: Kühler, DS 2025]

2-loop calculation implemented in FeynArts, many cross checks

symmetry-restoring counterterm for YM+fermions+scalars (1-loop)

$$S^1_{\text{fct,restore}} =$$

$$\frac{\hbar}{16\pi^2}\Bigg\{g^2\frac{S_2(R)}{6}\left(5S_{GG}+S_{GGG}-\int d^4x\, G^{a\,\mu}\partial^2 G^a_\mu\right)+\frac{Y_2(S)}{3}\overline{S_{\Phi\Phi}}$$

$$+g^2\frac{(T_R)^{abcd}}{3}\int d^4x\,\frac{g^2}{4}G^a_\mu G^{b\,\mu}G^c_\nu G^{d\,\nu}-\frac{(\mathcal{C}_R)^{ab}_{mn}}{3}\int d^4x\,\frac{g^2}{2}G^a_\mu G^{b\,\mu}\Phi^m\Phi^n$$

$$+g^2\left(1+\frac{\xi-1}{6}\right)C_2(R)S_{\overline{\psi}\psi}-\frac{\left((Y^m_R)^* T_{\overline{R}}{}^a Y^m_R\right)_{ij}}{2}\int d^4x\, g\overline{\psi}_i \not{G}^a P_R\psi_j$$

$$-g^2\frac{\xi C_2(G)}{4}\left(S_{\bar{R}c\psi_R}+S_{Rc\overline{\psi_R}}\right)\Bigg\},$$

Three dimensions of difficulty

1. higher loops
2. abelian/non-abelian
3. intricacies of the EWSM

SM difficulties — highlight

- chiral fermions with different L/R gauge interactions, e.g.

$$\psi = \psi_L + \psi_R, \qquad \mathcal{Y}_L \neq \mathcal{Y}_R \neq 0$$

such that kinetic terms, which must contain

$$\overline{\psi_R}\hat{\partial\!\!\!/}\psi_L$$

now even break global gauge invariance and require e.g.

$$\mathcal{L}_{\text{fct}} \supset (\Phi^\dagger\Phi)^2 \text{ and } (\Phi\Phi)^2$$

- can try to optimize *D*-dimensional Lagrangian by adding evanescent terms, e.g.

$$g\mathcal{Y}_{RL}\overline{\psi_R}\hat{B\!\!\!/}\psi_L\overline{\psi_R}\hat{\partial\!\!\!/}\psi_L$$

- studied impact of such options [Ebert, Kühler, DS, Weisswange '24]

General exploration — U(1) part of SM

General framework containing all above as special cases
$\Rightarrow$ option 1 ist most promising [Ebert, Kühler, DS, Weisswange '24]

$$
\begin{aligned}
S^1_{\text{fct}} = \frac{1}{16\pi^2}\Bigg[& -\frac{g^2}{3}\overline{B}_\mu \Box \overline{B}^\mu + \frac{g^4}{24}\overline{B}_\mu\overline{B}^\mu\overline{B}_\nu\overline{B}^\nu \\
& - i\frac{g}{3}\delta y_2\left[\left(\overline{\partial}^\mu\phi_2^\dagger\right)\phi_2 - \phi_2^\dagger\left(\overline{\partial}^\mu\phi_2\right)\right]\overline{B}_\mu - \frac{g^2}{4}\delta y_2\left[\phi_1^\dagger\phi_1\overline{B}^\mu\overline{B}_\mu + \frac{5}{3}\phi_2^\dagger\phi_2\overline{B}^\mu\overline{B}_\mu\right] \\
& + g\,\overline{\psi}_f\overline{\not{B}}\left[\delta F^{\text{opt1}}_{\mathrm{R},f}\mathbb{P}_\mathrm{R} + \delta F^{\text{opt1}}_{\mathrm{L},f}\mathbb{P}_\mathrm{L}\right]\psi_f \\
& - \left\{\delta Y^{\text{opt1}}_{e,2}\,y_e\overline{e_L}\phi_2^\dagger e_R + \delta Y^{\text{opt1}}_{u,2}\,y_u\overline{u_L}\phi_2 u_R + \delta Y^{\text{opt1}}_{d,2}\,y_d\overline{d_L}\phi_2^\dagger d_R + \mathrm{h.c.}\right\} \\
& + \frac{1}{6}\delta y_2\left[\overline{\partial}_\mu\phi_2\overline{\partial}^\mu\phi_2 - \frac{3}{2}ig\overline{\partial}_\mu\left(\phi_2\phi_2\right)\overline{B}^\mu + \frac{3}{4}g^2\phi_2\phi_2\overline{B}_\mu\overline{B}^\mu + \mathrm{h.c.}\right] \\
& - \left(\frac{1}{12}\delta y_4\,\phi_2\phi_2\phi_2\phi_2 + \frac{2}{3}\left(\delta y_4 - \delta y_{ud}\right)\phi_1^\dagger\phi_1\phi_2\phi_2 + \frac{2}{3}\delta y_4\,\phi_2^\dagger\phi_2\phi_2\phi_2 + \mathrm{h.c.}\right)\Bigg],
\end{aligned}
$$

Summary and Conclusions

Background:

- γ_5 is problematic in DReg, BMHV scheme is rigorous
- γ_5 non-anticommuting, distinguish 4-dim and ϵ-dim quantities
- gauge invariance broken already in $\mathcal{L}_D$ and at loop level

Renormalization in general: $\Gamma_{\text{ren}} = \Gamma_{\text{reg}} + \Gamma_{\text{ct}}$

- Γ_{ren} should be finite
- $\mathcal{S}(\Gamma_{\text{ren}}) = 0$ should hold
- this fixes divergent and symmetry-restoring counterterms
- in addition, counterterms derived from field/parameter renormalization may be added

Conclusions

Apply BMHV non-anticommuting γ_5 scheme to chiral gauge theories

- gauge/BRST invariance broken via $\Delta \cdot \Gamma$
- need symmetry-restoring and evanescent divergent counterterms

Status:

- Method established, many crosschecks
- two computations of breaking, locality, cancellability
- Feynman rules implemented in FeynArts and QGRAF setups

Results and outlook:

- 4-loop abelian model, 2-loop non-abelian
- 1-loop general fermions+scalars exploration
- further goals: EWSM, SMEFT, RGEs

General Summary

Results:

- Symmetry-restoring counterterms: 1-/2-loop YM, 3/4-loop “QED”
- Method established: determine violation of Ward/Slavnov-Taylor identities from $\widehat{\Delta}$-diagrams
- Result has compact simple structure

Outlook:

- 1-, 2-loop EWSM, 3-loop YM ...
- many details not talked about relevant for SM!
- automatize, implement in FeynArts, FeynRules etc
- alternative $\mathcal{L}_D$, schemes (FDH, DRed, etc, other γ_5 schemes)
- RGEs
- Fierz problem...

Symmetry transformations of Green functions

$$\phi_i(x) \to \phi_i(x) + \delta\phi_i(x), \qquad \mathcal{L}(x) \to \mathcal{L}(x) + \delta\mathcal{L}(x)$$

How do Green functions behave?

Symmetry transformations of Green functions

$$\phi_i(x) \to \phi_i(x) + \delta\phi_i(x), \qquad \mathcal{L}(x) \to \mathcal{L}(x) + \delta\mathcal{L}(x)$$

Path integral:

Symmetry transformations of Green functions

$$\phi_i(x) \to \phi_i(x) + \delta\phi_i(x), \qquad \mathcal{L}(x) \to \mathcal{L}(x) + \delta\mathcal{L}(x)$$

Path integral:

$$Z(J) = \int \mathcal{D}\phi \; e^{i\int \mathcal{L}+J\phi}$$

Symmetry transformations of Green functions

$$\phi_i(x) \to \phi_i(x) + \delta\phi_i(x), \qquad \mathcal{L}(x) \to \mathcal{L}(x) + \delta\mathcal{L}(x)$$

$$Z(J) = \int \mathcal{D}\phi \; e^{i\int \mathcal{L}+J\phi}$$

(measure invariant)

$$= \int \mathcal{D}\phi \; e^{i\int \mathcal{L}+\delta\mathcal{L}+J\phi+J\delta\phi}$$

Symmetry transformations of Green functions

$$\phi_i(x) \to \phi_i(x) + \delta\phi_i(x), \qquad \mathcal{L}(x) \to \mathcal{L}(x) + \delta\mathcal{L}(x)$$

$$
\begin{aligned}
Z(J) &= \int \mathcal{D}\phi \; e^{i\int \mathcal{L}+J\phi} \\
\text{(measure invariant)} \qquad &= \int \mathcal{D}\phi \; e^{i\int \mathcal{L}+\delta\mathcal{L}+J\phi+J\delta\phi} \\
\text{(1st order in } \delta\text{)} \qquad &= \int \mathcal{D}\phi \; \left(1 + i\int \delta\mathcal{L} + J\delta\phi\right) e^{i\int \mathcal{L}+J\phi}
\end{aligned}
$$

Symmetry transformations of Green functions

$$\phi_i(x) \to \phi_i(x) + \delta\phi_i(x), \qquad \mathcal{L}(x) \to \mathcal{L}(x) + \delta\mathcal{L}(x)$$

$$Z(J) = \int \mathcal{D}\phi \; e^{i\int \mathcal{L}+J\phi}$$

(measure invariant) $$= \int \mathcal{D}\phi \; e^{i\int \mathcal{L}+\delta\mathcal{L}+J\phi+J\delta\phi}$$

(1st order in δ) $$= \int \mathcal{D}\phi \; (1 + i\int \delta\mathcal{L} + J\delta\phi) e^{i\int \mathcal{L}+J\phi}$$

result: $$0 = \int \mathcal{D}\phi \; (i\int \delta\mathcal{L} + J\delta\phi) e^{i\int \mathcal{L}+J\phi}$$

Symmetry transformations of Green functions

$$\phi_i(x) \to \phi_i(x) + \delta\phi_i(x), \qquad \mathcal{L}(x) \to \mathcal{L}(x) + \delta\mathcal{L}(x)$$

$$\begin{aligned}
Z(J) &= \int \mathcal{D}\phi \; e^{i\int \mathcal{L}+J\phi} \\
\text{(measure invariant)} \qquad &= \int \mathcal{D}\phi \; e^{i\int \mathcal{L}+\delta\mathcal{L}+J\phi+J\delta\phi} \\
\text{(1st order in } \delta\text{)} \qquad &= \int \mathcal{D}\phi \; (1 + i\int \delta\mathcal{L} + J\delta\phi) e^{i\int \mathcal{L}+J\phi} \\
\text{result:} \qquad 0 &= \int \mathcal{D}\phi \; (i\int \delta\mathcal{L} + J\delta\phi) e^{i\int \mathcal{L}+J\phi}
\end{aligned}$$

Formal "derivation" for $\delta\mathcal{L} = 0$ gives form of ST identities

$$\langle (\delta\phi_1)\phi_2 \ldots \rangle + \langle \phi_1 (\delta\phi_2) \ldots \rangle + \ldots = 0$$

Symmetry transformations of Green functions

$$\phi_i(x) \to \phi_i(x) + \delta\phi_i(x), \qquad \mathcal{L}(x) \to \mathcal{L}(x) + \delta\mathcal{L}(x)$$

$$Z(J) = \int \mathcal{D}\phi \; e^{i\int \mathcal{L}+J\phi}$$

(measure invariant)
$$= \int \mathcal{D}\phi \; e^{i\int \mathcal{L}+\delta\mathcal{L}+J\phi+J\delta\phi}$$

(1st order in δ)
$$= \int \mathcal{D}\phi \; (1 + i\int \delta\mathcal{L} + J\delta\phi) e^{i\int \mathcal{L}+J\phi}$$

result:
$$0 = \int \mathcal{D}\phi \; (i\int \delta\mathcal{L} + J\delta\phi) e^{i\int \mathcal{L}+J\phi}$$

"derivation" is valid in DReg and gives breaking DREG: [Breitenlohner, Maison '77], DRED: [DS '05],review[2303.09120]

$$\langle(\delta\phi_1)\phi_2\ldots\rangle + \langle\phi_1(\delta\phi_2)\ldots\rangle + \ldots = -i\langle\phi_1\phi_2\ldots(\int\delta\mathcal{L})\rangle$$

Symmetry transformations of Green functions

$$\phi_i(x) \to \phi_i(x) + \delta\phi_i(x), \qquad \mathcal{L}(x) \to \mathcal{L}(x) + \delta\mathcal{L}(x)$$

$$Z(J) = \int \mathcal{D}\phi \; e^{i\int \mathcal{L}+J\phi}$$

(measure invariant) $$= \int \mathcal{D}\phi \; e^{i\int \mathcal{L}+\delta\mathcal{L}+J\phi+J\delta\phi}$$

(1st order in δ) $$= \int \mathcal{D}\phi \; (1 + i\int \delta\mathcal{L} + J\delta\phi) e^{i\int \mathcal{L}+J\phi}$$

result: $$0 = \int \mathcal{D}\phi \; (i\int \delta\mathcal{L} + J\delta\phi) e^{i\int \mathcal{L}+J\phi}$$

This is exactly true in DREG (where $\delta\mathcal{L}$ might be $\neq 0$)

$$\langle(\delta\phi_1)\phi_2\ldots\rangle + \langle\phi_1(\delta\phi_2)\ldots\rangle + \ldots = -i\langle\phi_1\phi_2\ldots(\int\delta\mathcal{L})\rangle$$

Symmetry transformations of Green functions — really

Regularized quantum action principle

$$\langle(\delta\phi_1)\phi_2\ldots\rangle + \langle\phi_1(\delta\phi_2)\ldots\rangle + \ldots = -i\langle\phi_1\phi_2\ldots(\int\delta\mathcal{L})\rangle$$

Interpret this as an identity between regularized Feynman diagrams

- becomes a property of regularization scheme, does not necessarily hold (no fundamental QFT requirement)
- if desired, must be proven for each regularization
- valid in BPHZ: [Lowenstein et al '71], DREG: [Breitenlohner, Maison '77], DRED: [DS '05]

Symmetry transformations of Green functions — really

Regularized quantum action principle

$$\langle(\delta\phi_1)\phi_2\ldots\rangle + \langle\phi_1(\delta\phi_2)\ldots\rangle + \ldots = -i\langle\phi_1\phi_2\ldots(\int\delta\mathcal{L})\rangle$$

Interpret this as an identity between regularized Feynman diagrams

Idea of proof in DREG/DRED: look at possible Wick contractions

- $\delta\mathcal{L} = \delta\mathcal{L}_{\text{quadratic}} + \delta\mathcal{L}_{\text{int}}, \qquad \delta\mathcal{L}_{\text{quadratic}} = (\delta\phi_i)D_{ij}\phi_j$
- Use properties of DREG/DRED: D is inverse propagator even on regularized level, scaleless integrals vanish
- then, combinatorics leads to above identity