Non-Leptonic *B* Decays with Multi-Hadron Final States

color meets flavor

Thomas Mannel, University of Siegen

CPPS Center for Particle Physics Siegen

TP1 Theoretical Particle Physics

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Contents

Introduction

Introduction

- Nonleptonic decays are still nototiously difficult:
  - There is a lot of data!
  - ... but no reliable theoretical method.
- Many different Ansätze:
  - Naive Factortization (Bauer, Stech, Wirbel 1987)
  - QCD Factorization (Beneke, Buchalla, Neubert, Sachrajda 1999?2000)
  - Perturbative QCD (PQCD) and Soft Collinear Effective Theory
  - ...
- CP violation phenomenology is prominent in non-leptonic decays

Two-Body Decays

First ideas on non-leptonic two-body B decays

Ansatz of Bauer Stech Wirbel 1989: Naive Factorization:

$$\langle B|H_{\rm eff}|D\pi\rangle = \frac{4G_F}{\sqrt{2}} V_{\rm CKM} C_1(\mu)\langle B|(\bar b_L\gamma_\mu c_L)(\bar u_L\gamma^\mu d_L)|D\pi\rangle$$
$$\longrightarrow \frac{4G_F}{\sqrt{2}} V_{\rm CKM} C_1(\mu)\langle B|(\bar b_L\gamma_\mu c_L)|D\rangle \, \langle 0|(\bar u_L\gamma^\mu d_L)|\pi\rangle$$

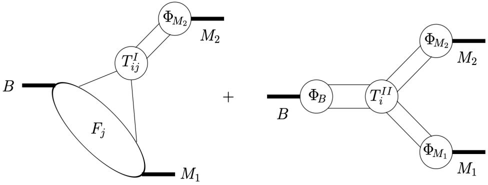
This cannot be an equation:

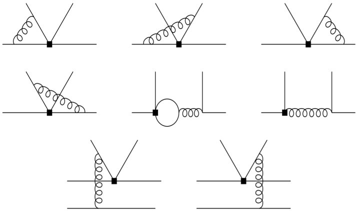
- $O_1 = (\bar b_L\gamma_\mu c_L)(\bar u_L\gamma^\mu d_L)$ has an anomalous dimension
- ... $J_\mu^{(1)} = (\bar b_L\gamma_\mu c_L)$ and $J_\mu^{(2)} = (\bar u_L\gamma_\mu d_L)$ has not!
- This only holds in the limit $N_c \to \infty$

Still: Emprirical Formulae using Naive Factorization roughly reproduce the data

QCD factorization Beneke, Buchalla, Neubert, Sachrajda 1999/2000

Observation: In the limit of large m_b naive factorization holds (BBNS 1999)





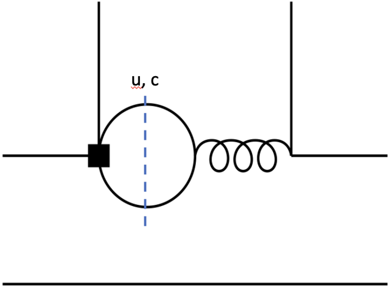
$$\begin{aligned}\mathcal{T}_p &= a_1^p(\pi\pi)\,(\bar{u}b)_{V-A}\otimes(\bar{d}u)_{V-A}\\ &+ a_2^p(\pi\pi)\,(\bar{d}b)_{V-A}\otimes(\bar{u}u)_{V-A}\\ &+ a_3(\pi\pi)\,(\bar{d}b)_{V-A}\otimes(\bar{q}q)_{V-A}\\ &+ a_4^p(\pi\pi)\,(\bar{q}b)_{V-A}\otimes(\bar{d}q)_{V-A}\\ &+ a_5(\pi\pi)\,(\bar{d}b)_{V-A}\otimes(\bar{q}q)_{V+A}\\ &+ a_6^p(\pi\pi)\,(-2)(\bar{q}b)_{S-P}\otimes(\bar{d}q)_{S+P}.\end{aligned}$$

Pros and Cons:

+ Solves the probelm of scale dependence
+ CPV is small $\mathcal{O}(\alpha_s)$ + power corrections
+ NLO Corrections have been computed for leading order
- no control over power corrections (yet?)

CPV in QCDF

Need to generate a strong phase:



- Leads to a complex coefficient $\to$ strong phase
- Order $\alpha_s(m_b)$ at leading power

Three-Body Decays

Preliminaries

Kinematics: $p_B \to p_1 + p_2 + p_3$

- Two independent kinematical variables $p_i^2 = 0$

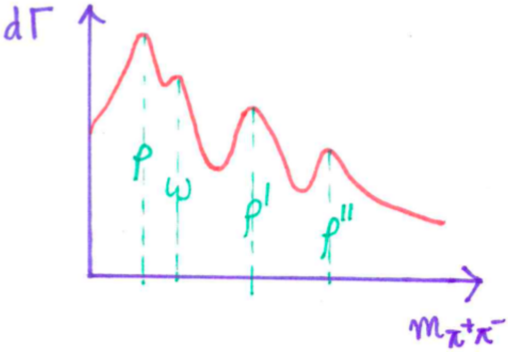
$$s_{ij}^2 = (p_i + p_j)^2 \quad s_{12} + s_{13} + s_{23} = M_B^2$$

Historically:

- "Isobar" Model:
- Description via pseudo two-particle decays:

$$(B \to M\, M_1 M_2) = (B \to M^* M \quad \text{and} \quad M^* \to M_1 M_2)$$

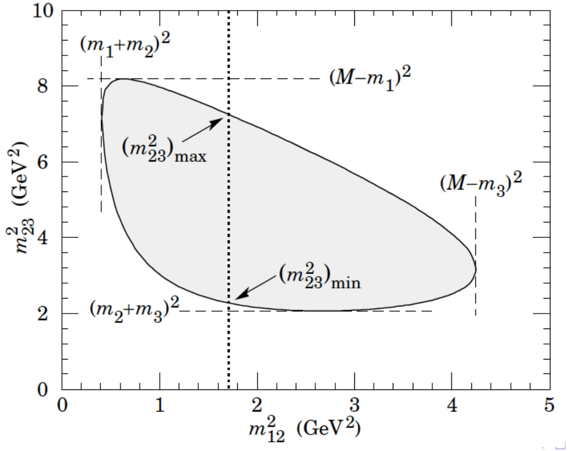
- sum over all possibilities for M^*, including $\Gamma(M^*)$
- possibly add a flat "non-resonant" backgound!



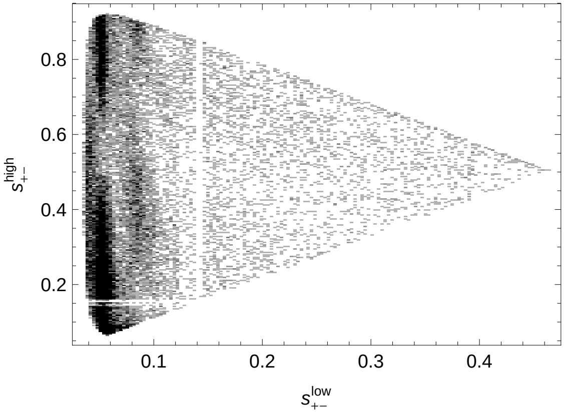
(sketch borrowed from J. Virto)

$B \to \pi\pi\pi$

Study the Dalitz Distribution:



Specifically for $B^+ \to \pi^+\pi^-\pi^+$



(Plot form LHCb arXiv:1408.5373)

Dalitz Plot is symmetric:

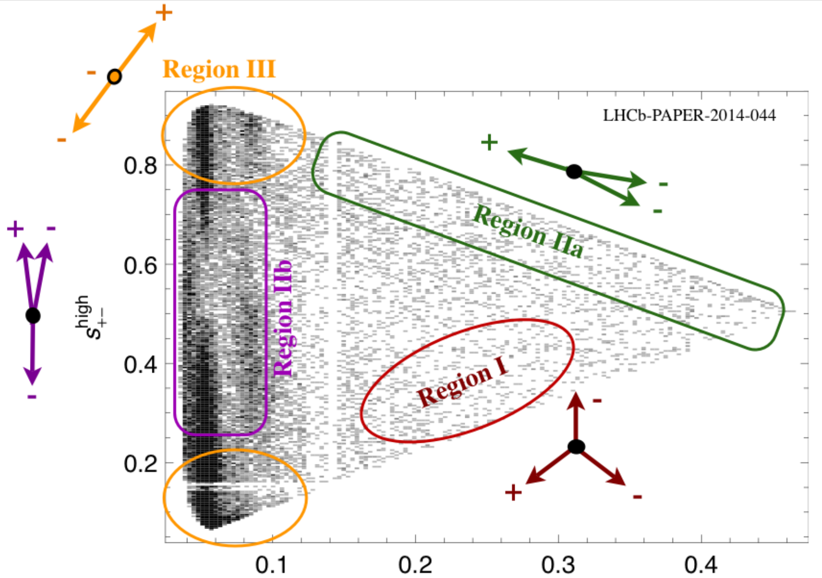
$s_{12} = s_{+-}^{\text{low}} \quad s_{23} = s_{+-}^{\text{high}}$

$s_{12} = s_{++}$

Regions

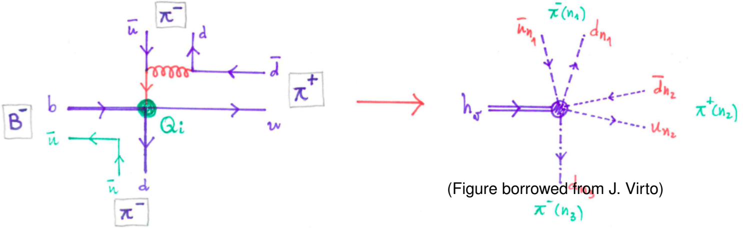
Split the Dalitz Plot into Regions:

- **Region 1:** "Mercedes Star"
  $s_{++} \sim s_{+-}^{\text{low}} \sim s_{+-}^{\text{high}} \sim 1/3$
- **Region 2:** Collinear Decay Products
  - **Region 2a:** $(\pi^+\pi^+)_{\text{coll}}$ recoil against π^- $s_{++} \sim 0, \quad s_{+-}^{\text{low}} \sim s_{+-}^{\text{high}} \sim 1/2$
  - **Region 2b:** $(\pi^+\pi^-)_{\text{coll}}$ recoil against π^+ $s_{+-}^{\text{low}} \sim 0, \quad s_{++} \sim s_{+-}^{\text{high}} \sim 1/2$
- **Region 3:** Soft Decay Products
  - **Region 3a:** Soft π^+
    $s_{++} \sim s_{+-}^{\text{low}} \sim 0 \quad s_{+-}^{\text{high}} \sim 1$
  - **Region 3b:** Soft π^-
    $s_{+-}^{\text{low}} \sim_{+-}^{\text{high}} \sim 0, \quad s_{++} \sim 1$


Region III
LHCb-PAPER-2014-044
Region IIa
Region IIb
Region I
0.8
0.6
0.4
0.2
0.1
0.2
0.3
0.4
s_{+-}^{high}

Region 1: The Center

- Three “disconnected” collinear directions: n_1 n_2 n_3



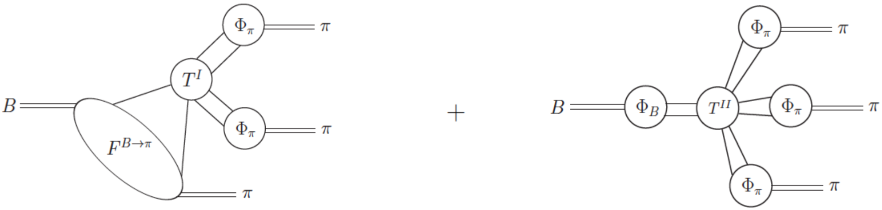
(Figure borrowed from J. Virto)

$$\langle \pi^-_{n_1} \pi^+_{n_2} \pi^-_{n_3} | O_i | B \rangle = \langle \pi^-_{n_3} | \bar{d}_{n_3} \Gamma_3 h_v | B \rangle$$

$$\times \int dudv\, T_i(u,v) \langle \pi^-_{n_1} | \bar{d}_{n_1}(\bar{u}) \Gamma_1 u_{n_1}(u) | 0 \rangle \langle \pi^+_{n_2} | \bar{u}_{n_2}(\bar{v}) \Gamma_2 d_{n_2}(v) | 0 \rangle$$

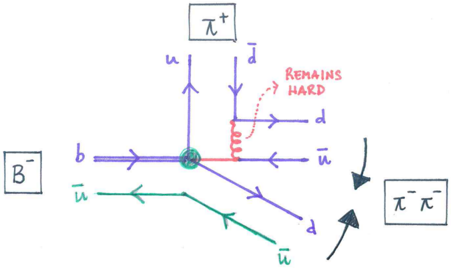
$$\sim F^{B \to \pi}\, T_i \otimes \phi_\pi \otimes \phi_\pi$$

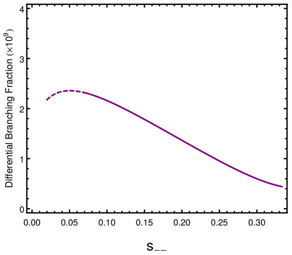
- $1/m_b^2$ and α_s supressed with repect to a two body decay
- At leading order / leading power / leading twist all convolutions are finite $\rightarrow$ factorization:



Extrapolation to collinear $\pi^- \pi^-$

- There are no resonances in this channel
- No infrared / collinear problems expected
- Perturbative result expected to be regular: No “soft” propagators

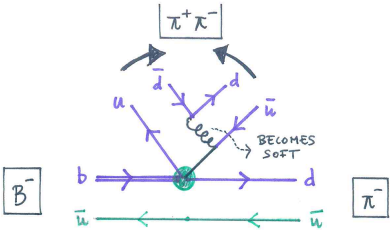


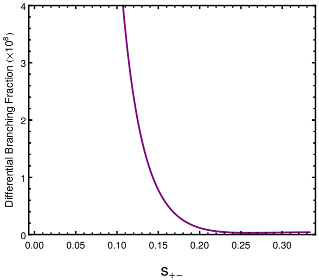


$$\frac{d\Gamma}{ds_{--}\,ds_{+-}} \simeq 0.84\,\Gamma_0\, f_+(m_B^2/2)^2 + \mathcal{O}(s_{--})$$

Extrapolation to collinear $\pi^+\pi^-$

- There are resonances in this channel: ρ. ω, ...
- Perturbative result expected to be IR singular
- “soft” propagators

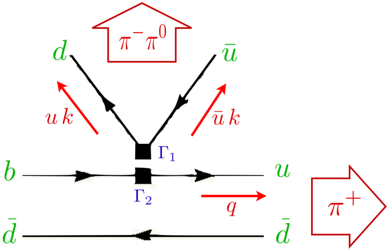


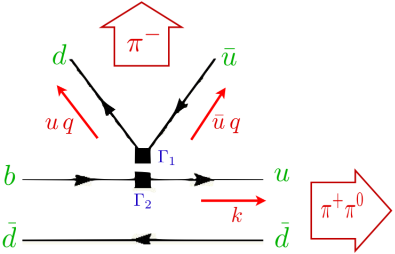


$$\frac{d\Gamma}{ds_{+-}\,ds_{--}} \simeq \frac{0.38}{s_{+-}} \Gamma_0 \, f_+(0)^2 + \text{regular}$$

Region 2b: new non-perturbative input

- Factorization breaks down in the resonance regions
- New, nonperturbative input is needed
- Three-body decay resembles two-body decay





- Operators are the same as in two-body decays ...

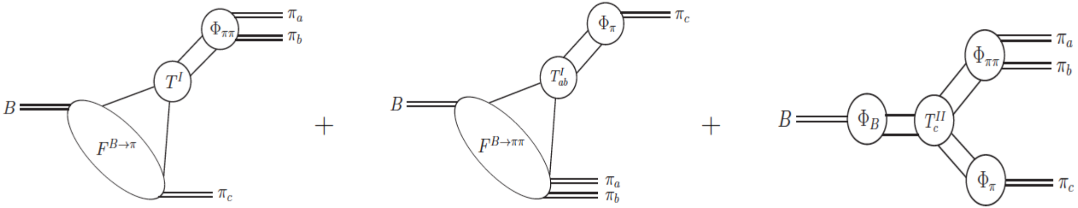
- ... but the final states are different

$$
\begin{aligned}
&\langle \pi^-_{\bar n} \pi^+_{\bar n} \pi^-_{n} | O_i | B \rangle = \\
&\langle \pi^-_{n} | \bar h_v \Gamma \xi_n | B \rangle \times \int dz \, T_1(z) \langle \pi^-_{\bar n} \pi^+_{\bar n} | \bar\chi_{\bar n}(z\bar n) \Gamma' \chi_{\bar n}(0) | 0 \rangle \\
&+ \langle \pi^-_{\bar n} \pi^+_{\bar n} | \bar h_v \Gamma \xi_{\bar n} | B \rangle \times \int dz \, T_2(z) \langle \pi^-_{n} | \bar\chi_{\bar n}(zn) \Gamma' \chi_{n}(0) | 0 \rangle
\end{aligned}
$$

$$
\sim F^{B\to\pi} \, T_1 \otimes \phi_{\pi\pi} + F^{B\to\pi\pi} \, T_2 \otimes \phi_\pi
$$

- Two-Pion light-cone distribution (Polyakov, Diehl, Gousset, ...)
- Generalized (soft) Form factor (Feldmann, Khofjamirian, van Dyk, ThM ...)

- Factorization formula similar to the two-body case



Two-Pion Light Cone Distribution

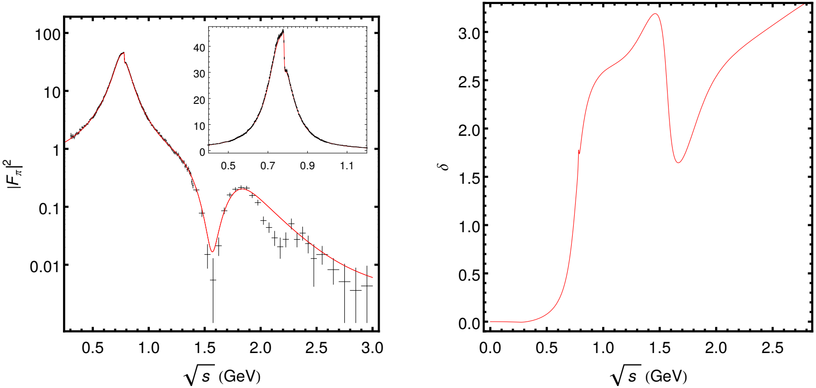
- Definition: $s = (k_1 + k_2)^2$, $k_1 = \zeta k_{12}$, $k_2 = \bar{\zeta} k_{12}$]

$$\phi^q_{\pi\pi}(z, \zeta, s) = \int \frac{dx^-}{2\pi} e^{iz(k^+_{12} x^-)} \langle \pi^+(k_1) \pi^-(k_2) | \bar{q}(x^- n_-) \not{n}_+ q(0) | 0 \rangle$$

- Normalization from the local limit:

$$\int dz\, \phi_{\pi\pi}(z, \zeta, s) = (2\zeta - 1) F_\pi(s) \quad \text{(pion time-like FF)}$$

- $F_\pi(s)$: Data (BaBar) + Theory (χPT, Asymptotics...)
- z and ζ dependence asymptotically known



(Hanhart, Kubis, ...)

Timelike Pion Form Factor known from Data

Generalized (soft) Form factor

- Relevant Form factor:

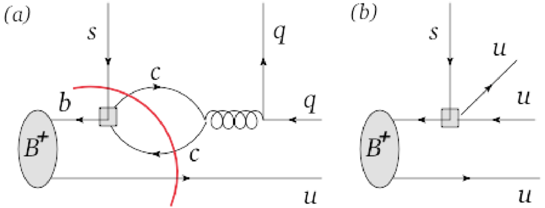
$$\langle \pi^+(k_1)\pi^-(k_2)|\,\bar{u}\not{k}_3 P_{L,R} b\,|B^-(p)\rangle = \mp \frac{m_\pi}{2} F_t(\zeta, s)$$

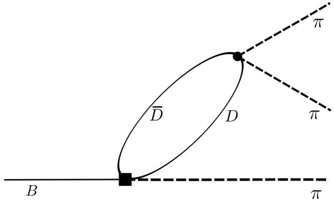
- $F_t(\zeta, s)$ can be related to the two-pion light-cone distribution via a Light-Cone Sum Rule (Khodjamirian, Hambrock)

$$F_t(\zeta, s) = \frac{m_b^2}{\sqrt{2}\hat{f}_B m_\pi} \int_{u_0}^{1} \frac{du}{u} \exp\left[\frac{(1+s\bar{u})m_B^2}{M^2} - \frac{m_b^2}{uM^2}\right] \phi_{\pi\pi}(u, \zeta, s)$$

CP Violation in nonleptonic three body decays

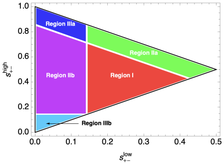
- CPV in QCDF is drive by the phase induced by the cut in the quark loop
- This is a constant over the phase space
- Cannot explain the structures observed in the CP Dalitz distribution
- A precise quantitive understanding of *CP* asymmetries is still not reached

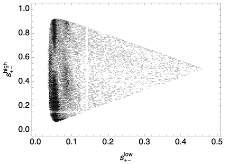




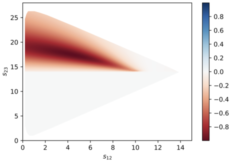
Dalitz Distributions in three-body decays I

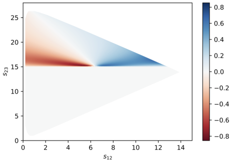
- Rates can be described relatively well in QCDF (Kränkl, Klein, Virto, Vos, ThM, ...)

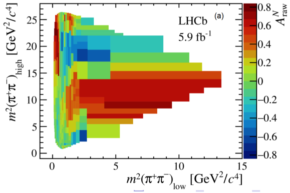




- CP Distributions are difficult: Requites modelling! (Klein, Virto, Vos, Olschewsky, ThM, ...)







Dalitz Distributions in three-body decays II

New (model!) Ansatz: (Heuser, Reyes-Torrecilla, Hanhart, Kubis, Magalhães, Peláez, ThM, 2506.XXXXX)

Study Case is $B^+ \to K^+\pi^+\pi^-$ at small invariant mass of the $\pi\pi$ system

- Small $m_{\pi\pi}$ means a large recoil against the BK system, neglect final state interactions
- Assume the final state hadrons are produced at “small scales”, i.e. the phases of the amplitude is determined by the soft and universal final-state resattering among these hadrons.

Use the Discontinuity relation (partial-wave projected)

$$\text{Disc}\,\mathcal{A}(s)_a^{(\mathcal{I})} = 2i\sum_c \mathcal{M}(s)_{ca}^{(\mathcal{I})\,*}\rho_c\mathcal{A}(s)_c^{(\mathcal{I})} \quad \text{with} \quad \rho(s)_c = \frac{\lambda^{1/2}(s, M_{1,c}^2, M_{2,c}^2)}{16\pi s}$$

We only have to deal with $LI = S0, S2$ and $P1$ (Isospin and partial waves)

CP Dalitz Distribution

The effective Hamiltonian

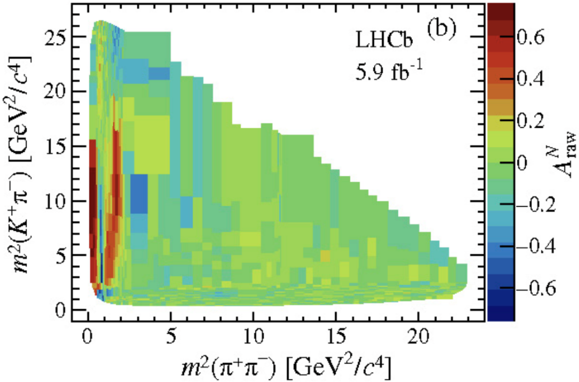
$$H_{\rm eff} = \frac{G_F}{\sqrt{2}}\left(|V_{cb}^* V_{cs}|(\bar{b}c)(\bar{c}s) + e^{i\gamma}|V_{ub}^* V_{us}|(\bar{b}u)(\bar{u}s)\right)$$

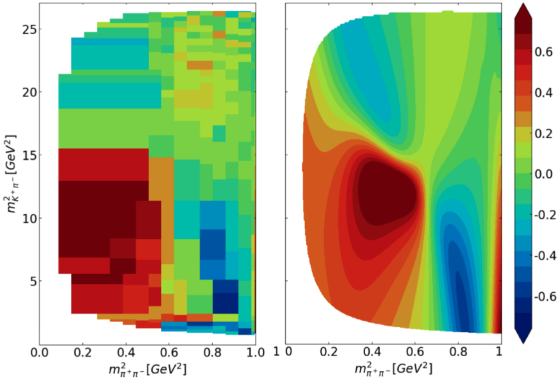
The full Amplitude

$$\mathcal{A}^{\pm}(s,t) = \sum_i f_i(s,t) P_i(s) \Omega_i(s) \bar{\mathcal{A}}_i^{\pm} \, ,$$

Parametrization of the amplitude

$$\bar{\mathcal{A}}_i^{\pm} = \hat{A}_i + e^{\pm i\gamma}\hat{B}_i = a_i + ic_i \pm ib_i$$





We might be getting closer to a useful modelling of multibody amplitudes

Conclusions

- Dalitz distributions of multibody final states have a rich phenomenology
- No sound theoretical descrition exists
- CPV yields a map of the strong phases of the amplitudes
- ... assuming SM CPV through CKM