Higgs mechanisms in supersymmetric quivers

A comparison of algorithmic approaches

Chiara-Renata Horak

Department of Physics, University of Vienna

Seminar for Particle Physics,
13.01.2026

Plan for the talk

1. Introducing supersymmetric field theories
2. 3 dimensional $\mathcal{N} = 4$ theories
3. Mathematical tools: Quivers and Hasse diagrams
4. Algorithms on both branches

Plan for the talk

1. Introducing supersymmetric field theories
2. 3 dimensional $\mathcal{N} = 4$ theories
3. Mathematical tools: Quivers and Hasse diagrams
4. Algorithms on both branches

Supersymmetric quantum field theories

Quantum field theory (QFT) = mathematical framework for diverse physics phenomena

- experimentally verified
- intractable for strong coupling

Supersymmetric quantum field theories

Quantum field theory (QFT) = mathematical framework for diverse physics phenomena

- experimentally verified
- intractable for strong coupling

$\Rightarrow$ Introduce supersymmetry

$\rightsquigarrow$ more control over quantum corrections

The supersymmetry algebra L

- extension of Poincaré algebra
- evades Coleman-Mandula theorem
- new fermionic operator $\Rightarrow$ supercharge Q

$$L = L_0 \bigoplus L_1$$

$L_0 \ldots$ Poincaré algebra
$L_1 = (Q^I_\alpha, \bar{Q}^I_{\dot{\alpha}})$ with $I = 1, \ldots, \mathcal{N}$

The supersymmetry algebra L

(Anti)commutators:

- $[P_\mu, Q^I_\alpha] = 0$
- $[P_\mu, \bar{Q}^I_{\dot{\alpha}}] = 0$
- $\{Q^I_\alpha, \bar{Q}^J_{\dot{\beta}}\} \propto P$

R-symmetry for $\mathcal{N} = 1$:
$Q_\alpha \to e^{-i\lambda} Q_\alpha$ and $\bar{Q}_{\dot{\alpha}} \to e^{i\lambda} \bar{Q}_{\dot{\alpha}}$

- $[R, Q_\alpha] = -Q_\alpha$
- $[R, \bar{Q}_{\dot{\alpha}}] = \bar{Q}_{\dot{\alpha}}$

Q|fermion⟩ = |boson⟩
Q|boson⟩ = |fermion⟩

Supersymmetric representations

Recap: Poincaré group

$$C_1 = P_\mu P^\mu$$

$$C_2 = W_\mu W^\mu \quad \text{with} \quad W^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} P_\nu M_{\rho\sigma}$$

Massive:

- mass m
- spin j

Massless:

- energy E
- helicity h

Supersymmetric representations

- supermultiplet: irrep. of supersymmetry algebra
- any irrep of susy algebra is irrep of Poincaré algebra
- C_1 still Casimir, C_2 not
  $\rightsquigarrow$ same mass, different spin

Supersymmetric representations

Massless representations in $\mathcal{N} = 1$:

- Chiral multiplet:
  - Weyl spinor and complex scalar
  - matter particles
- Vector multiplet
  - photon and Weyl spinor

Supersymmetric representations

Massive representations in $\mathcal{N} = 1$:

- Chiral multiplet:
  - massive Weyl spinor and massive complex scalar
- Vector multiplet:
  - massless vector multiplet and massless chiral multiplet
  - susy Higgs mechanism

Superfields

General superfield: 4 complex scalars, 2 left-handed spinors, 2 right-handed spinors and 1 vector

$$\Downarrow \text{restriction}$$

Chiral superfield Φ: complex scalar ϕ, Weyl spinor ψ and complex scalar F
Anti-chiral superfield $\Phi^\dagger$: complex scalar $\phi^\dagger$, Weyl spinor $\bar{\psi}$ and complex scalar $F^\dagger$

Moduli space of vacua

An example:

$$V(\phi, z) = |\lambda \phi z|^2 + \frac{1}{4}|\lambda \phi^2|^2 \overset{?}{=} 0$$

$$\Downarrow$$

moduli space of vacua

Moduli space of vacua: an example

2 chiral superfields Φ and Z with superpotential

$$W = \frac{1}{2}\lambda\Phi^2 Z$$

Potential energy:

$$V(\phi, z) = \left|\frac{\partial W}{\partial \phi}\right|^2 + \left|\frac{\partial W}{\partial z}\right|^2 = |\lambda\phi z|^2 + \frac{1}{4}|\lambda\phi^2|^2$$

No unique ground state: $V(\phi, z) = 0 \Leftrightarrow \phi = 0$ and z = anything

Plan for the talk

1. Introducing supersymmetric field theories
2. 3 dimensional $\mathcal{N} = 4$ theories
3. Mathematical tools: Quivers and Hasse diagrams
4. Algorithms on both branches

3d $\mathcal{N} = 4$ theories

Fields organized in 2 distinct multiplets:

- $\mathcal{N} = 2$ vector multiplet: $\mathcal{N} = 1$ vector and chiral multiplet
- $\mathcal{N} = 2$ hypermultiplet: two $\mathcal{N} = 1$ chiral multiplets

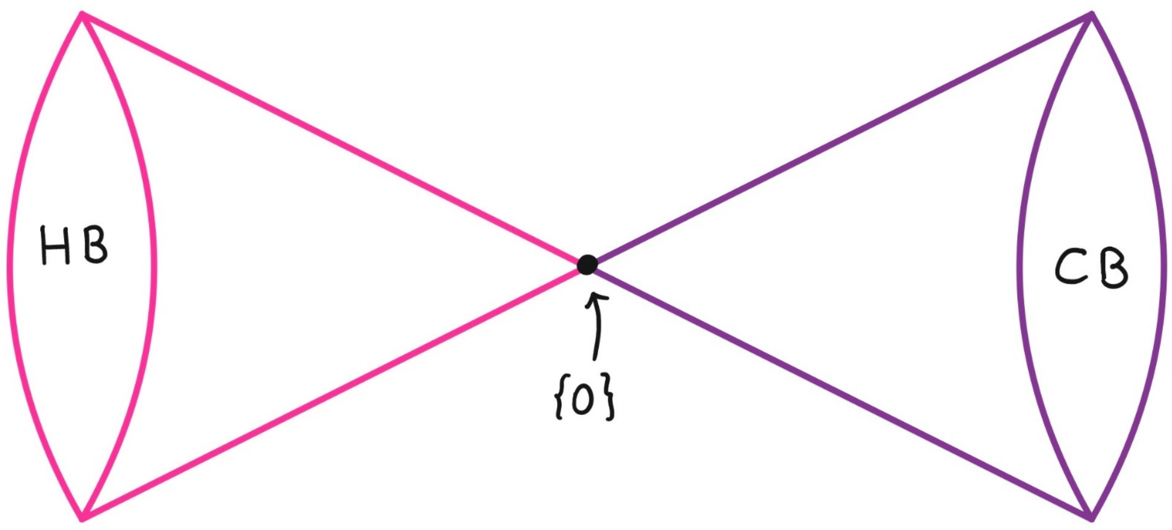
R-symmetry: $SU(2)_c \times SU(2)_H$

$\rightsquigarrow$ moduli space of vacua is hyperkähler

3d $\mathcal{N} = 4$ theories

Large, highly symmetric moduli space of vacua
$\rightsquigarrow$ has two maximal branches:

Higgs branch　　　　Coulomb branch



Higgs branch

- classical
- Hyper-Kähler quotient
- $\langle$ hypermultiplet scalars $\rangle \neq 0$
  $\langle$ vector multiplet scalars $\rangle = 0$
- at general point: gauge group completely Higgsed

3d $\mathcal{N} = 4$ theories

Coulomb branch

- quantum
- general symplectic singularity
- $\langle$ hypermultiplet scalars $\rangle = 0$
  $\langle$ vector multiplet scalars $\rangle \neq 0$
- gauge group broken to $U(1)$´s: all charged matter experiences Coulomb force

3d $\mathcal{N} = 4$ theories

- Both are singular spaces: singularities where gauge symmetry partially restored
  $\rightsquigarrow$ new massless states
- Both admit stratification
  $\rightsquigarrow$ encoded in Hasse diagram

3d $\mathcal{N} = 4$ theories

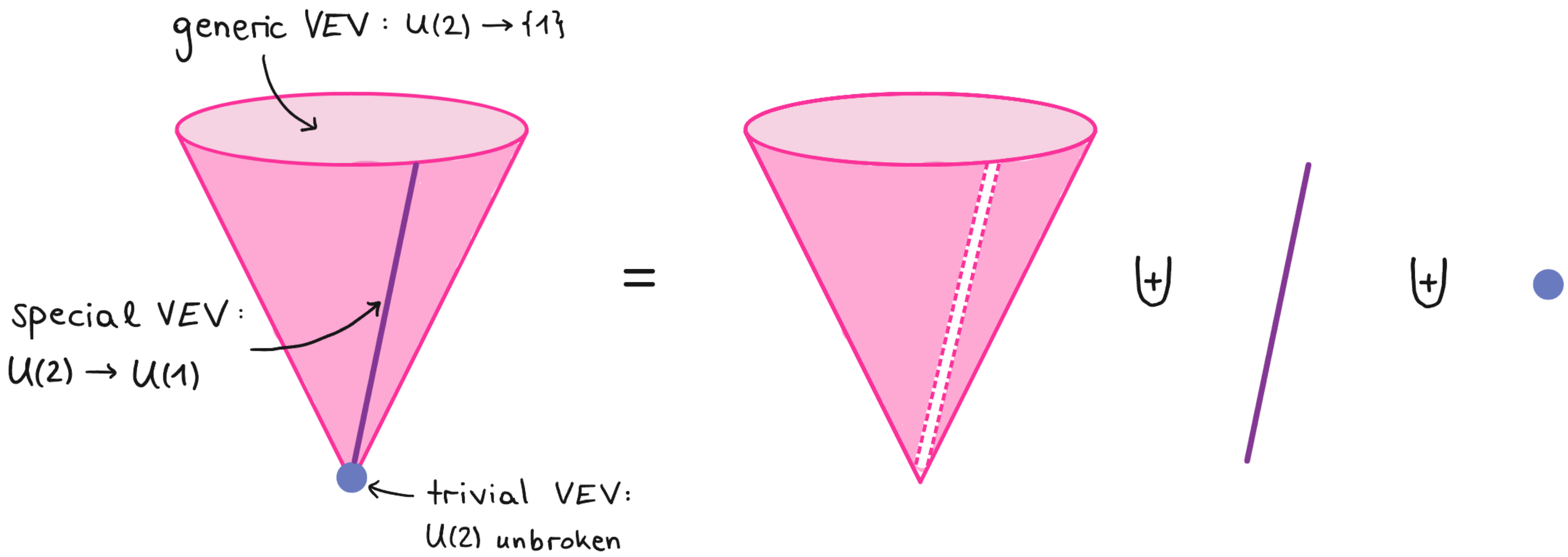
- Both are singular spaces: singularities where gauge symmetry partially restored
  $\rightsquigarrow$ new massless states

- Both admit stratification
  $\rightsquigarrow$ encoded in Hasse diagram

?

Stratification of space via example

$U(2)$ with N flavours:



$\rightsquigarrow$ decomposition into smooth strata

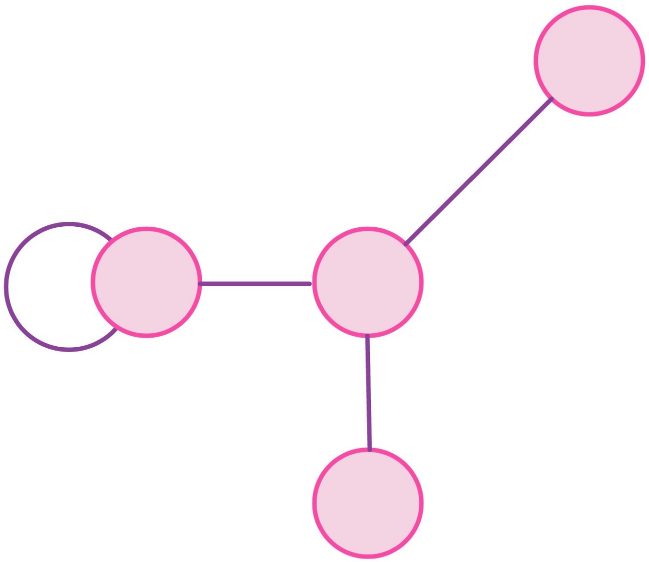
Plan for the talk

1. Introducing supersymmetric field theories
2. 3 dimensional $\mathcal{N} = 4$ theories
3. Mathematical tools: Quivers and Hasse diagrams
4. Algorithms on both branches

Quivers in mathematics

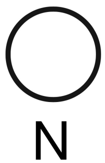
Quiver Γ consists of:

- set of vertices V of Γ
- set of edges E of Γ
- described by adjacency matrix A and rank vector K



Quivers in supersymmetric QFTs

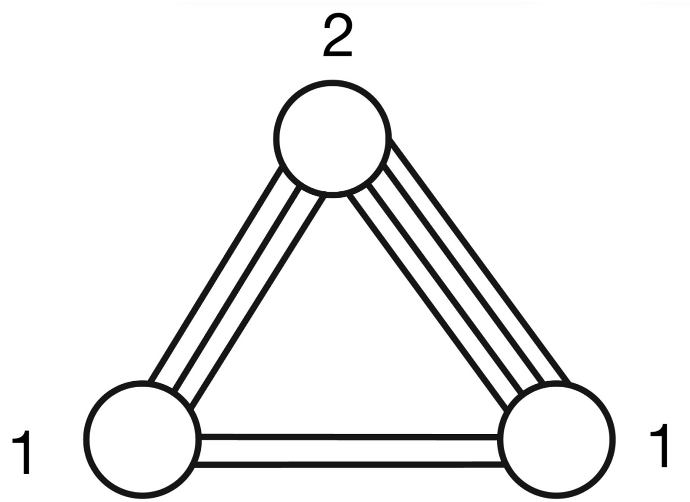
- graphical depiction of 3d $\mathcal{N} = 4$ field content

Quiver component	Field interpretation
	$\mathcal{N} = 4$ vector multiplet $\rightsquigarrow$ gauge bosons
	$\mathcal{N} = 4$ hypermultiplet $\rightsquigarrow$ matter fermions

Quivers in practice

Quiver Q consists of **matrix** A and **vector** K
Example:

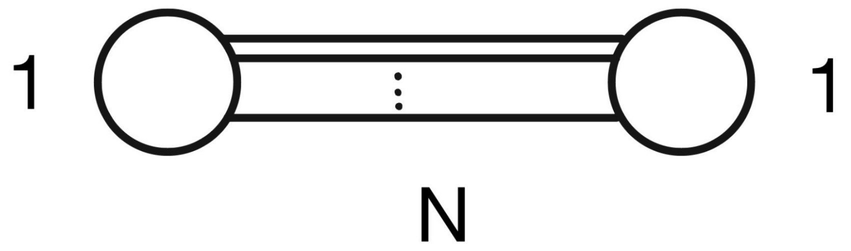
$$A = \begin{pmatrix} -2 & 3 & 2 \\ 3 & -2 & 4 \\ 2 & 4 & -2 \end{pmatrix} \quad \text{and} \quad K = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$



$$\Rightarrow U(1) \times U(2) \times U(1)$$

Quivers in practice

SQED:



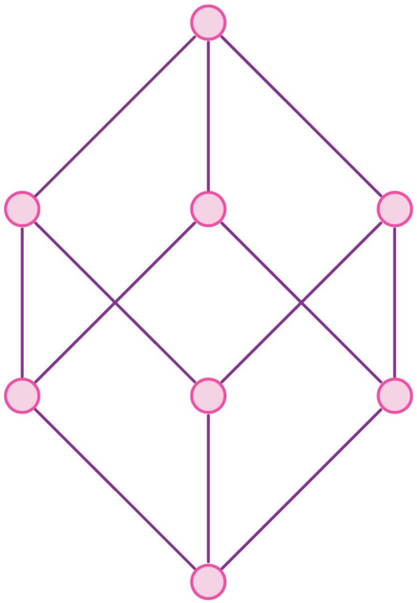
U(1) $\rightsquigarrow$ photon field
N flavours $\rightsquigarrow$ electrons

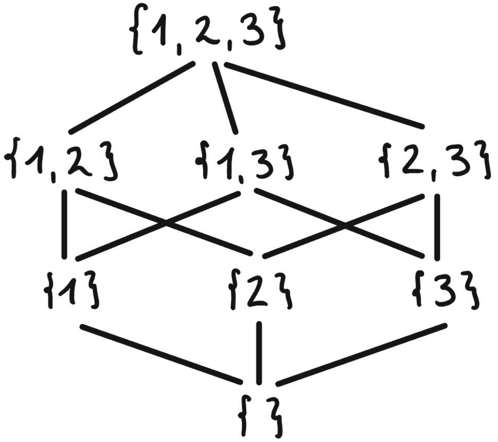
Hasse diagrams in mathematics

Hasse diagram

is graphical description of partially ordered set with:

- $x < y$ in poset: x lower than y
- line segment between x and y iff x covers y or vice versa

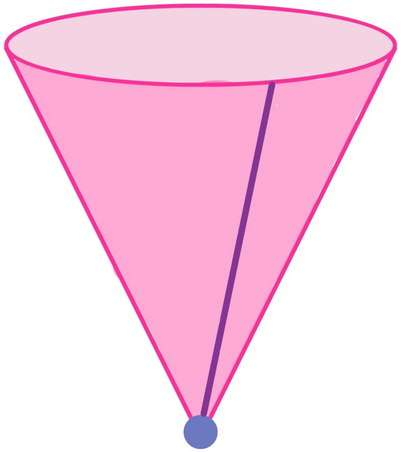




Hasse diagrams for quivers

Back to stratification of branches:

stratification of symplectic leaves $\mathcal{L}$

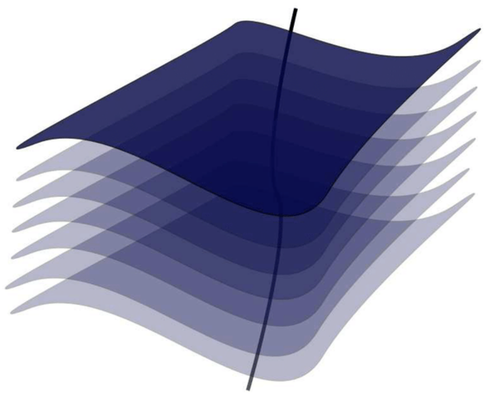


Hasse diagrams for quivers

Back to stratification of branches:

stratification of symplectic leaves $\mathcal{L}$

$\Uparrow$



M symplectic manifold $\Rightarrow$ M Poisson manifold
latter stratified into symplectic submanifolds $\mathcal{L}$

Hasse diagrams for quivers

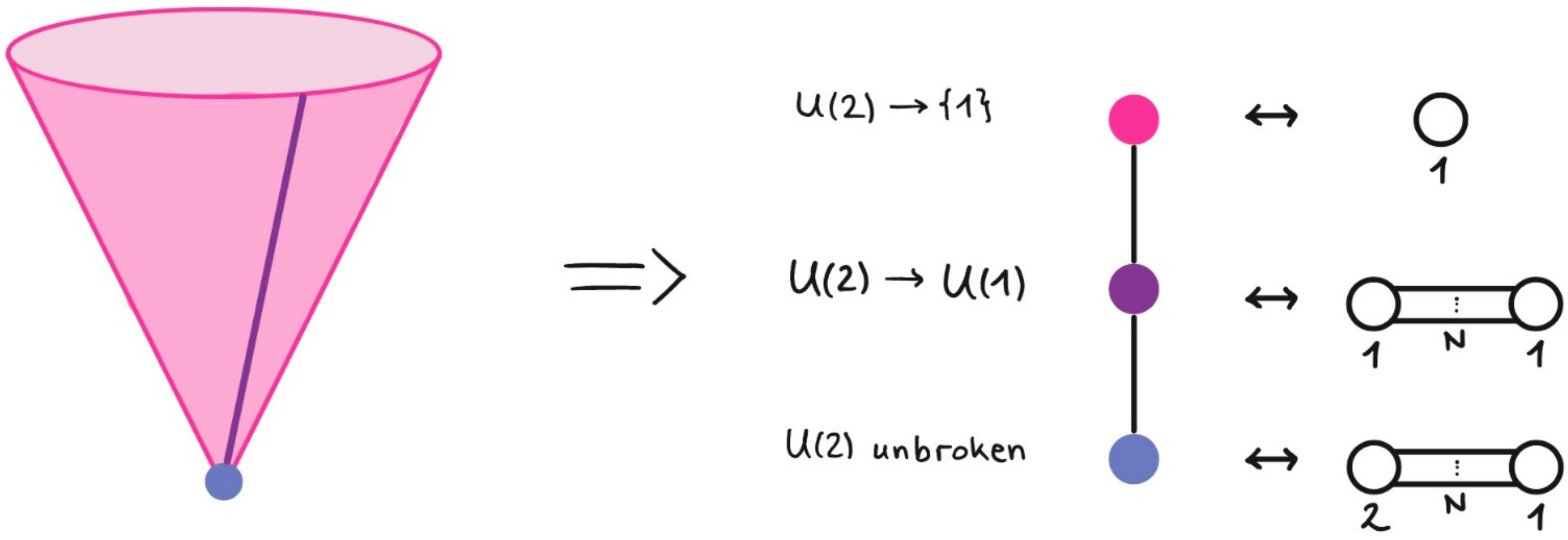
Physically:

- each leaf: set of massless states
  $\rightsquigarrow$ partially Higgsed theory
- leaves related by transverse slices
  $\rightsquigarrow$ phase transition

$\Rightarrow$ partial order in Hasse diagram

Example

From before:



Plan for the talk

1. Introducing supersymmetric field theories
2. 3 dimensional $\mathcal{N} = 4$ theories
3. Mathematical tools: Quivers and Hasse diagrams
4. Algorithms on both branches

An overview

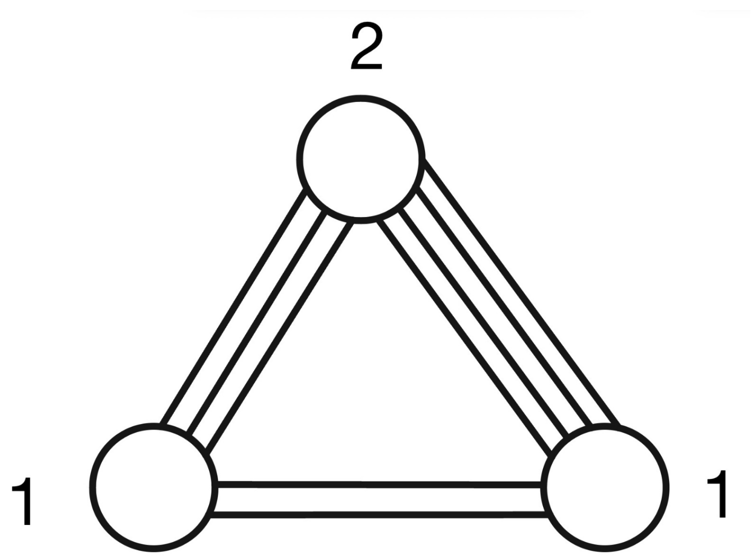
Higgs branch (HB)

- Quiver subtraction [arXiv:2409.16356]
- Ext-quivers [Crawley-Boevey '01], [Nakajima '94]

Coulomb branch (CB)

- Decay and Fission [arXiv:2401.08757]

From now on: Example

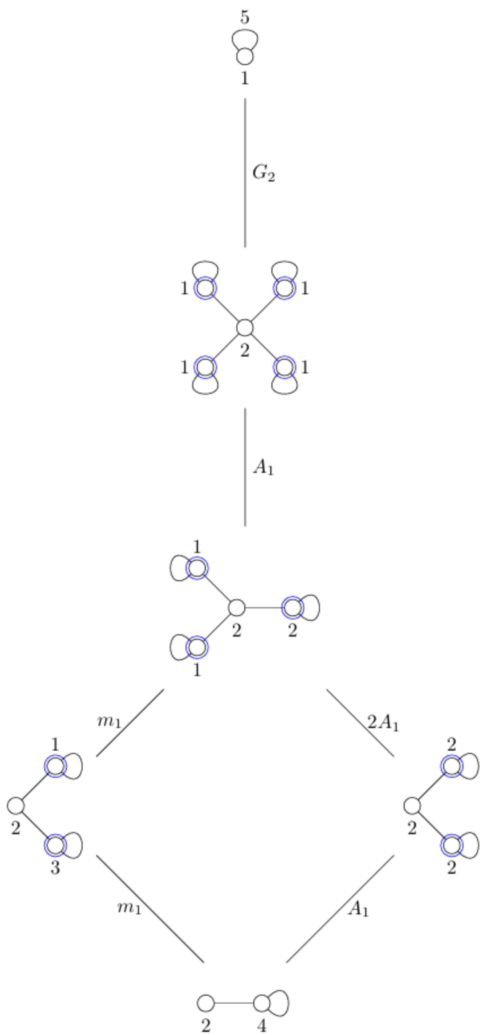


$$A = \begin{pmatrix} -2 & 3 & 2 \\ 3 & -2 & 4 \\ 2 & 4 & -2 \end{pmatrix}$$

$$K = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

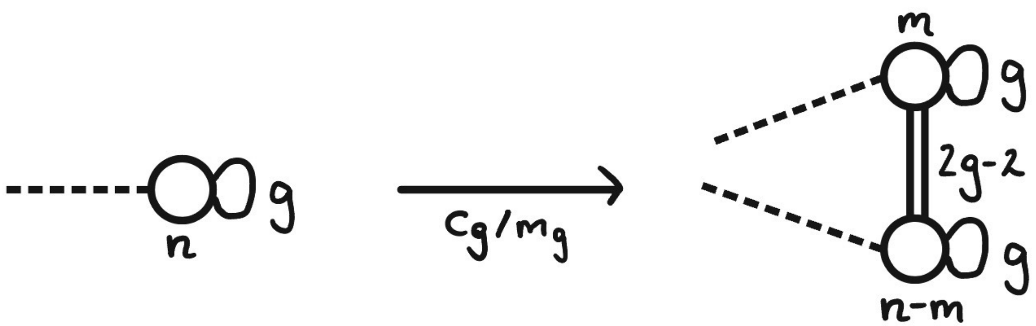
Quiver subtraction on HB [2409.16356]

- transition data at every step
- intrinsic partial order
- local and global rules
- list of all possible elementary slices



Quiver subtraction - local rules

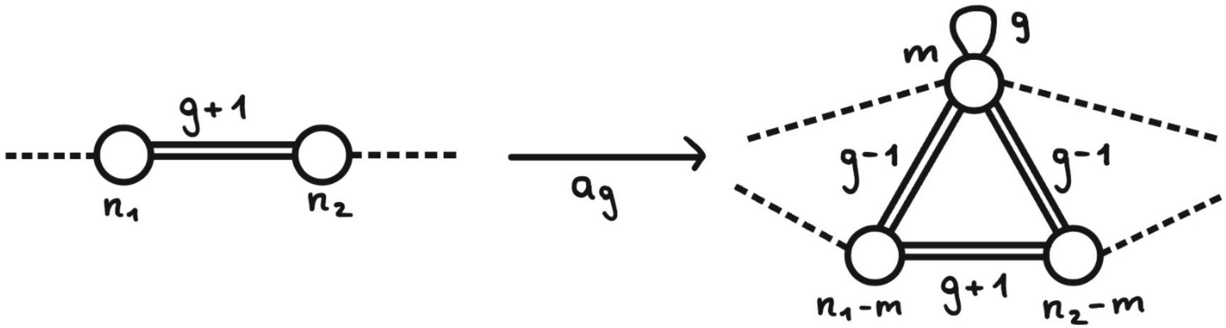
Rule 1 - Adjoint Higgsing:



- $U(n)$ with g loops
- $U(n) \to U(n-m) \times U(m)$
- all vector multiplets not in adjoint turn massive

Quiver subtraction - local rules

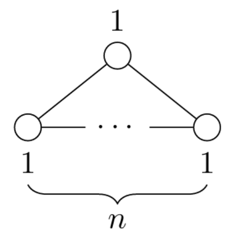
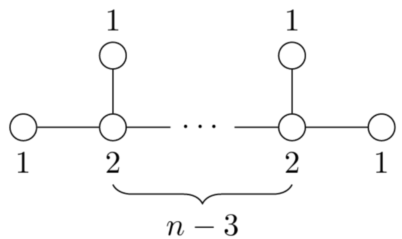
Rule 2 - Bi-fundamental Higgsing:

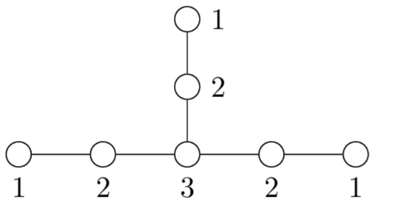
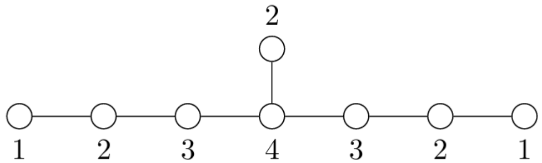
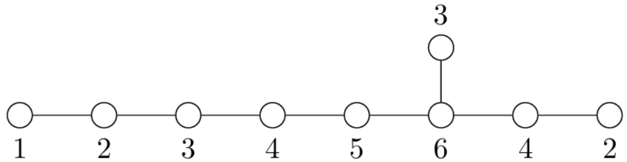


- $U(n_1)$ and $U(n_2)$ with $g+1$ edges
- $U(n_1) \times U(n_2) \to U(n_1 - m) \times U(n_2 - m) \times U(m)$

Quiver subtraction - local rules

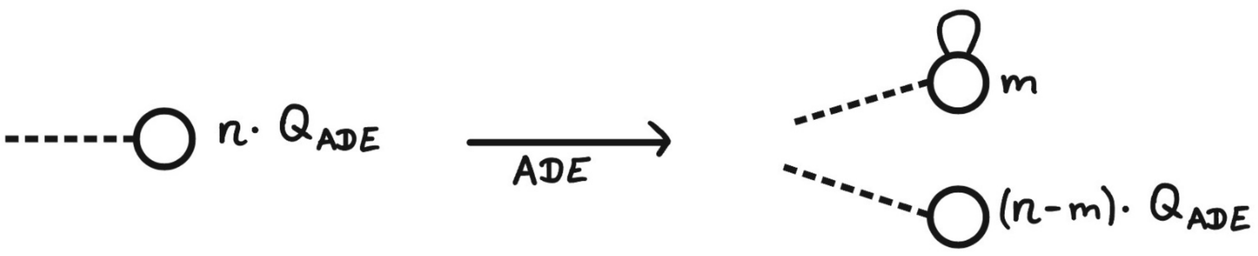
Rule 3 - ADE Higgsing:

Label	Affine Dynkin Diagrams
A_n	
D_n $n \geq 4$	

E_6	
E_7	
E_8	

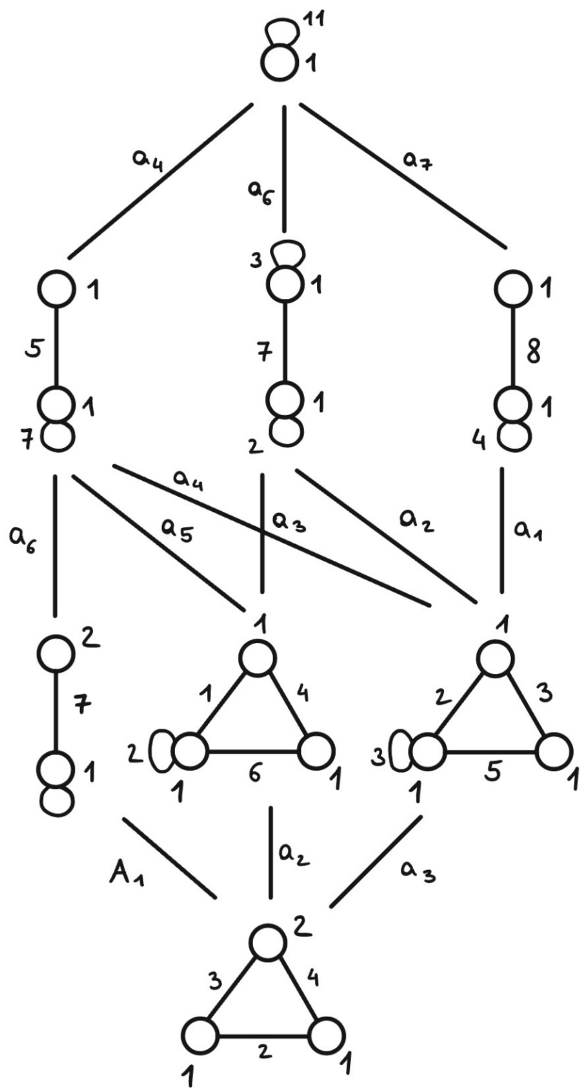
Quiver subtraction - local rules

Rule 3 - ADE Higgsing:

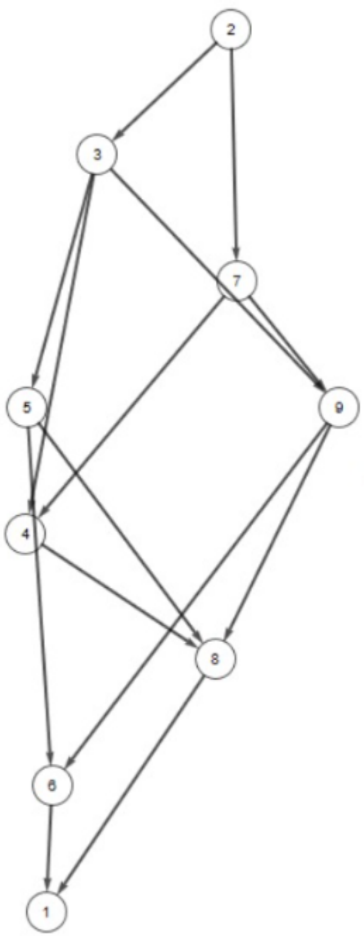


- affine ADE subquiver
- $U(n_0) \times \cdots \times U(n_k) \rightarrow$
  $U(n_0 - mh_1^\vee) \times \cdots \times U(n_k - mh_1^\vee) \times U(m)$

Hasse diagram via quiver subtraction



Ext-quiver [Crawley-Boevey '01]



1	1*{0, 0, 0}
2	1*{3, 1, 1}
3	1*{2, 1, 1}
4	1*{2, 0, 1}
5	1*{1, 1, 1}
6	1*{1, 1, 0}
7	1*{1, 1, 0} + 1*{2, 0, 1}
8	1*{1, 0, 1}
9	1*{1, 0, 1} + 1*{1, 1, 0}

- purely mathematical approach
- uses real and imaginary roots
- partial order on them

Building the Ext-quiver [Crawley-Boevey '01]

Quiver data (A, K):
quadratic form

$$p(\alpha) = 1 - \frac{1}{2}\,\alpha^{\mathsf{T}} \cdot A \cdot \alpha$$

- α real root $\Leftrightarrow p(\alpha) = 0$
- reflection $s_i(\alpha) = \alpha - (\alpha^{\mathsf{T}} \cdot A \cdot \epsilon_i)\epsilon_i$ and $p(\epsilon_i) = 0$
- α imaginary root $\Leftrightarrow$ reflection of real root

Building the Ext-quiver [Crawley-Boevey '01]

Representation type of $\mathcal{L}$: $\tau = (n_1, \beta^{(1)}; \ldots; n_k, \beta^{(k)})$

n_i ... multiplicity

$\beta^{(i)} \in \Sigma^A$ ... root

so that $K = \sum_{i=1}^{k} n_i \beta^{(i)}$ and

$$\Sigma^A = \left\{ b \in R^+ | \, \forall \, b = \sum_i \beta^{(i)} : p(b) > \sum_i p(\beta^{(i)}) \right\}$$

Building the Ext-quiver [Crawley-Boevey ’01]

Theorem (Crawley-Boevey)

- symplectic leaves labelled by τ of K
- construct local quiver on a leaf = Ext-quiver

Building the Ext-quiver [Crawley-Boevey '01]

Ext-quiver for decomposition $K = \sum_{i=1}^{k} n_i \beta^{(i)}$

- each $\beta^{(i)}$ node of rank n_i
- $p(\beta^{(i)})$ number of loops at i-th node
- $\beta^{(i)\intercal} \cdot A \cdot \beta^{(j)}$ number of edges between i-th and j-th node

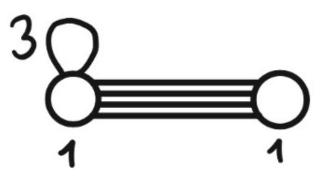
Building the Ext-quiver [Crawley-Boevey '01]

each $\beta^{(i)}$ node of rank n_i
$p(\beta^{(i)})$ number of loops at i-th node
$\beta^{(i)\top} \cdot A \cdot \beta^{(j)}$ number of edges between i-th and j-th node

$$A = \begin{pmatrix} -2 & 1 & 0 \\ 1 & -2 & 3 \\ 0 & 3 & -2 \end{pmatrix}, \quad K = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$$

decomposition: $K = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

↳ local quiver:

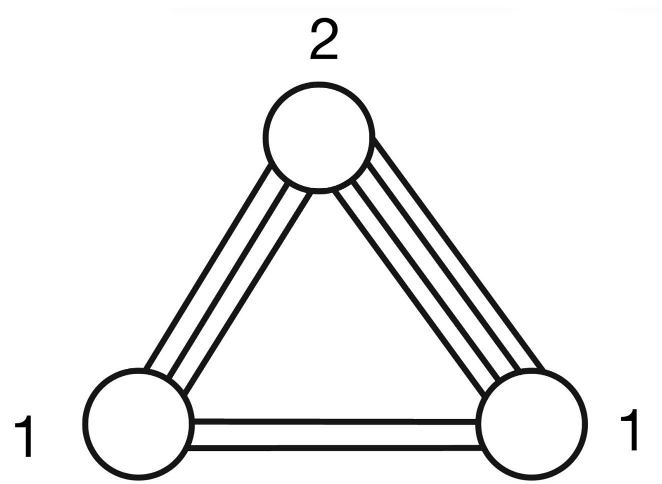


$$p\left(\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}\right) = 3$$

$$p\left(\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right) = 0$$

$$(1 \;\; 2 \;\; 1) \cdot A \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 4$$

Back to the example



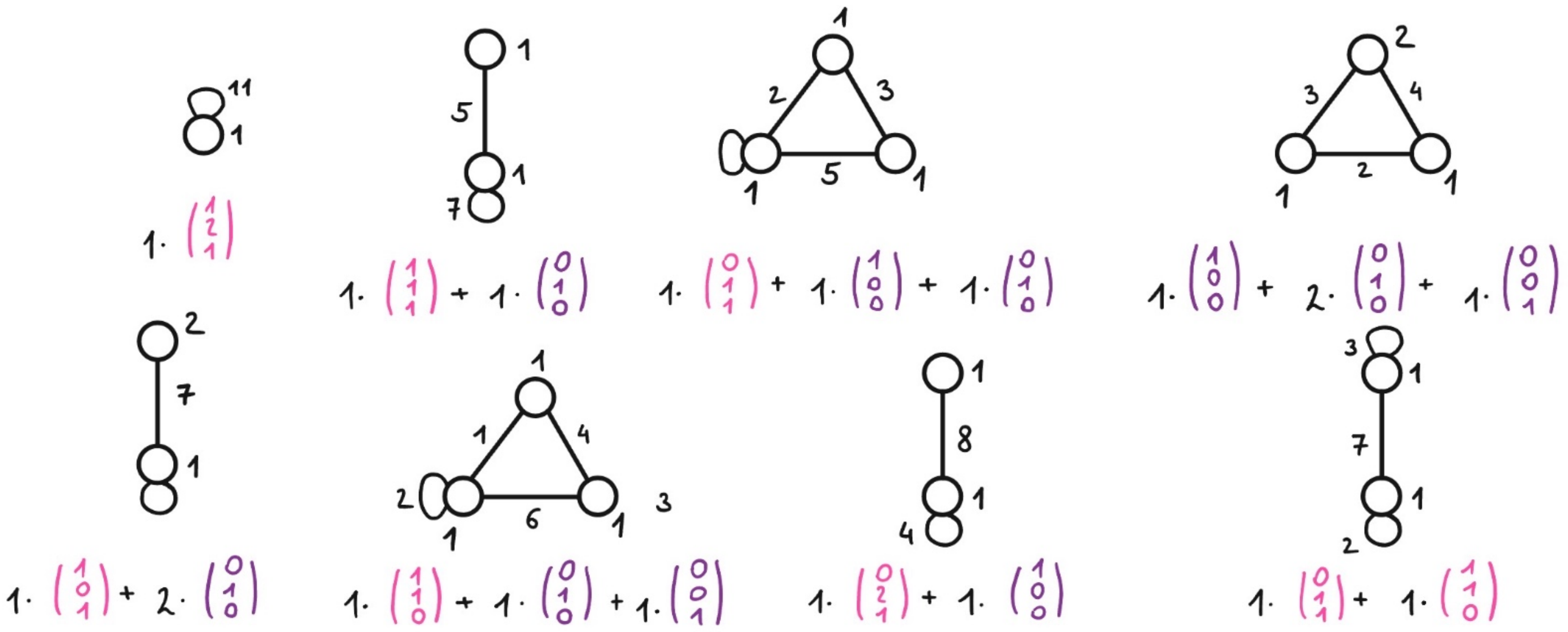
Real roots:

$R_R = \{(1,0,0),(0,1,0),(0,0,1)\}$

Imaginary roots:

$I_R = \{(0,1,1),(0,2,1),(1,0,1),(1,1,0),(1,1,1),(1,2,1)\}$

Back to the example



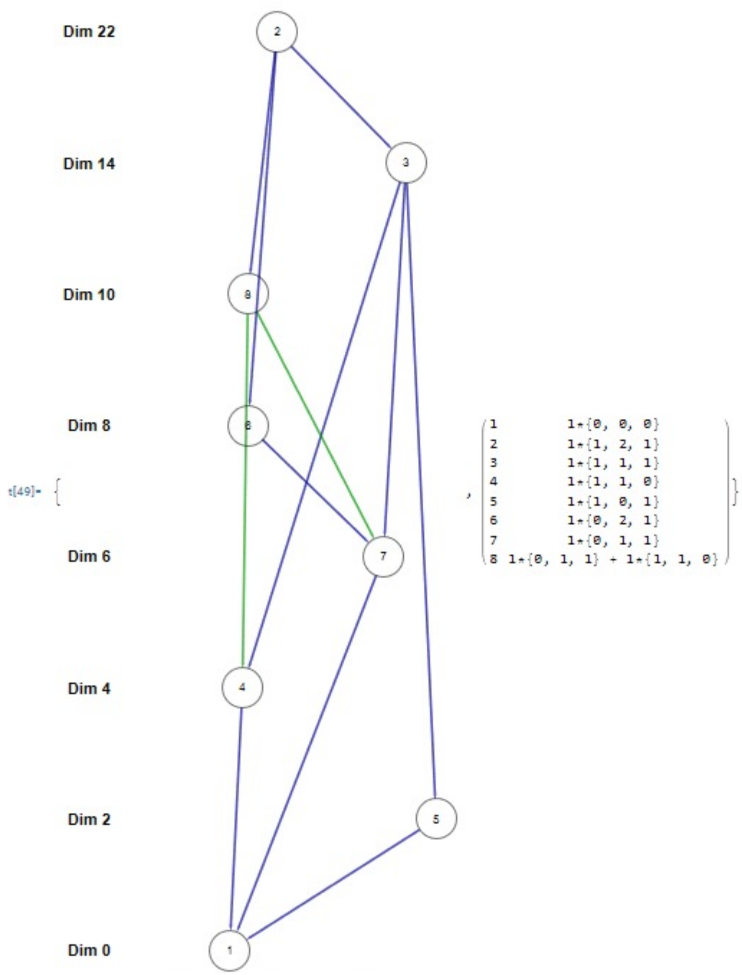
Partial order of Ext-quivers

Realisation:

Ext-quivers for specific (A,K) $\Leftrightarrow$ Quivers through Quiver subtraction of (A,K)

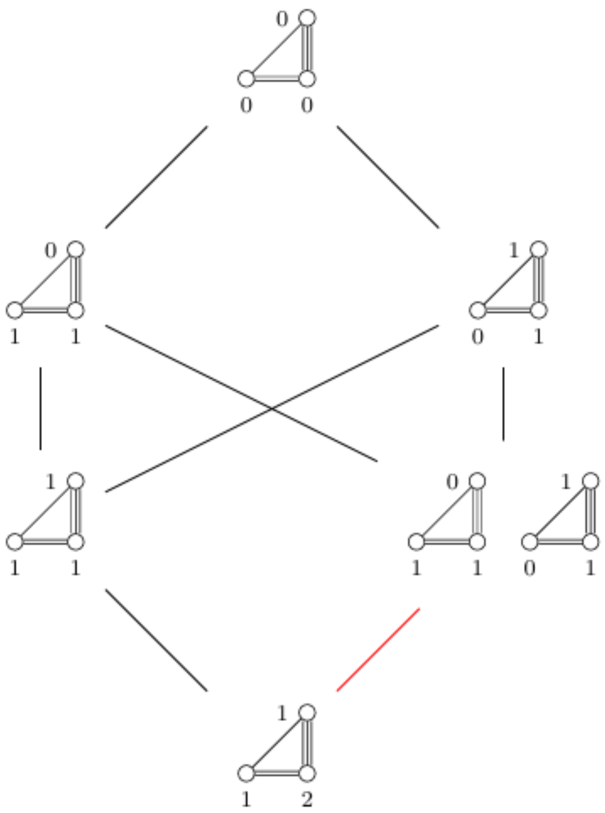
We have: Partial order in Quiver subtraction
But: Partial order for Ext-quivers ?

Partial order of Ext-quivers



- Mathematica code
- leaves and transverse slices
- only on imaginary roots

Decay & Fission algorithm on CB [arXiv:2401.08757]

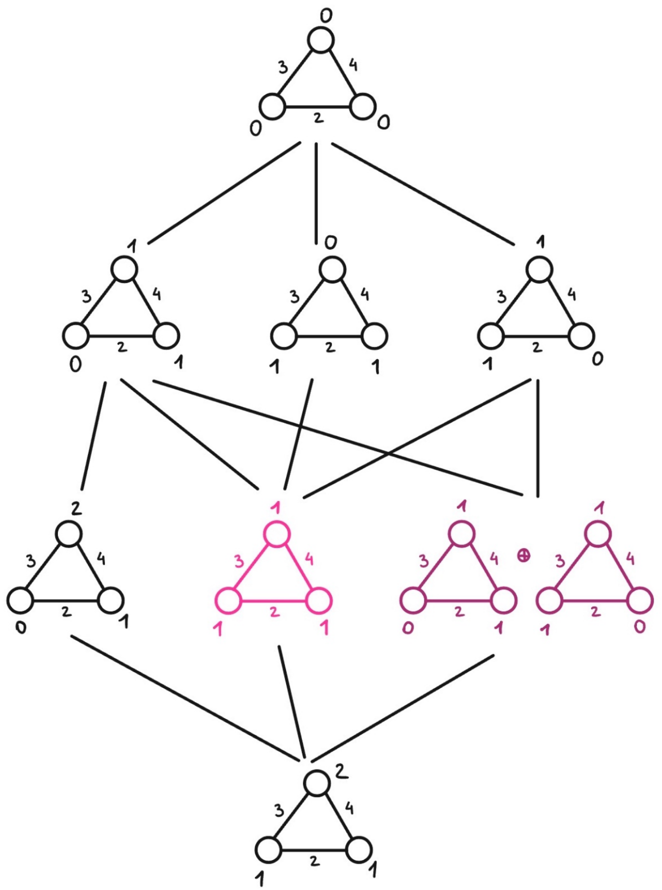


- principles of linear algebra
- always the same nodes
- only two options for partial Higgsing

Hasse diagram via Decay & Fission

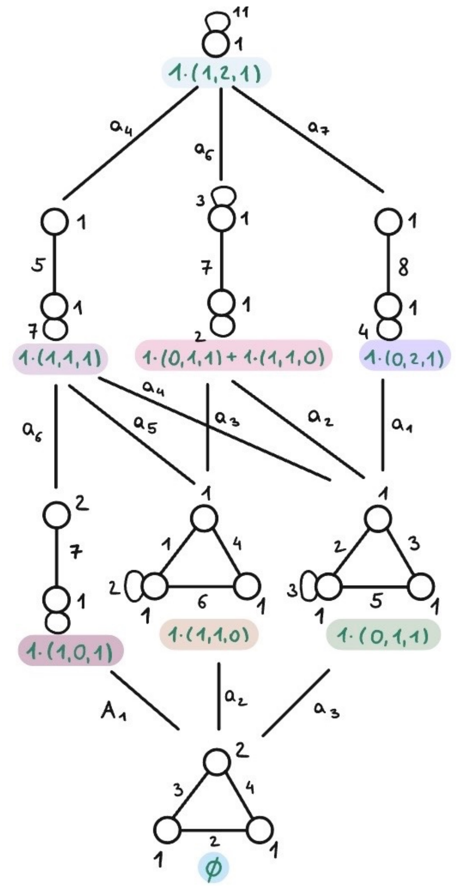
Decay: smaller rank

Fission: splits into two parts but total rank preserved

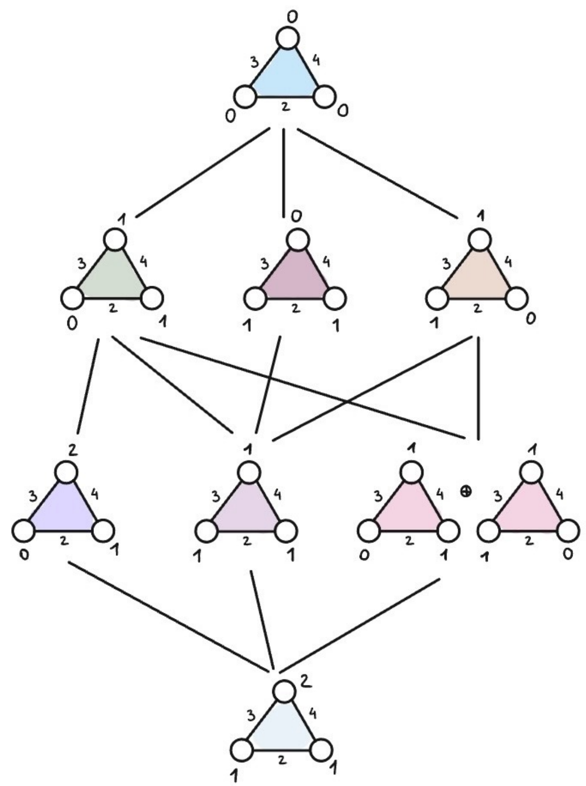


Comparison of both branches

Higgs branch (HB)



Coulomb branch (CB)



Comparison of both branches

Goal:

- map between partial order on Higgs branch and Coulomb branch
- Symplectic duality?:
  - Same amount of leaves on HB and CB
  - isometries of HB describe resolutions of CB and vice versa

Thank you!