Toponium at the Large Hadron Collider
A new frontier in top-quark physics

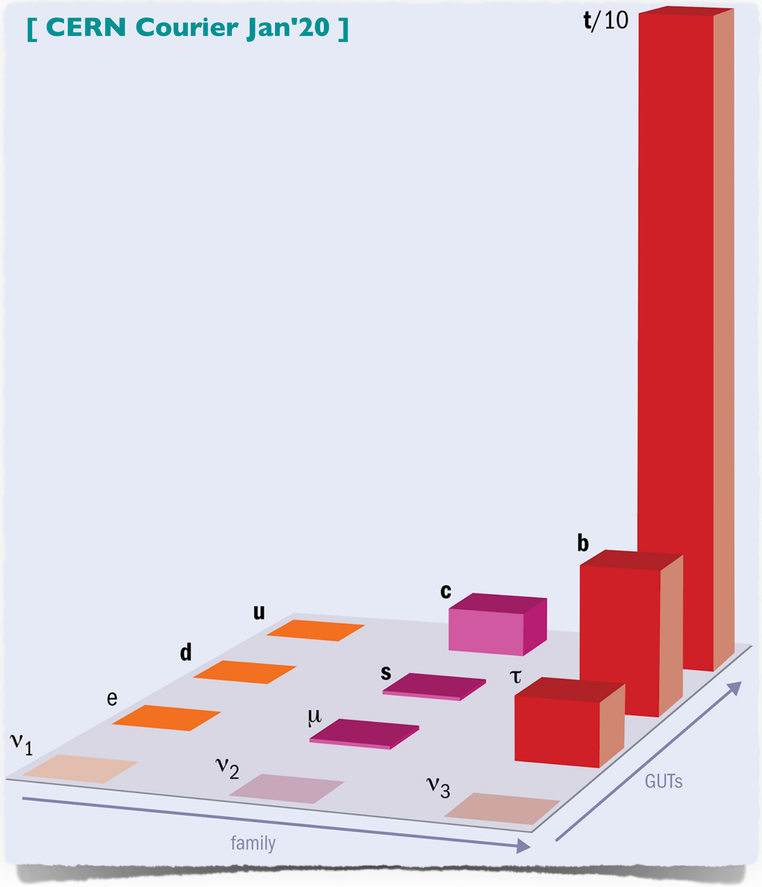
Benjamin Fuks

LPTHE / Sorbonne Université

High-Energy Physics Seminar

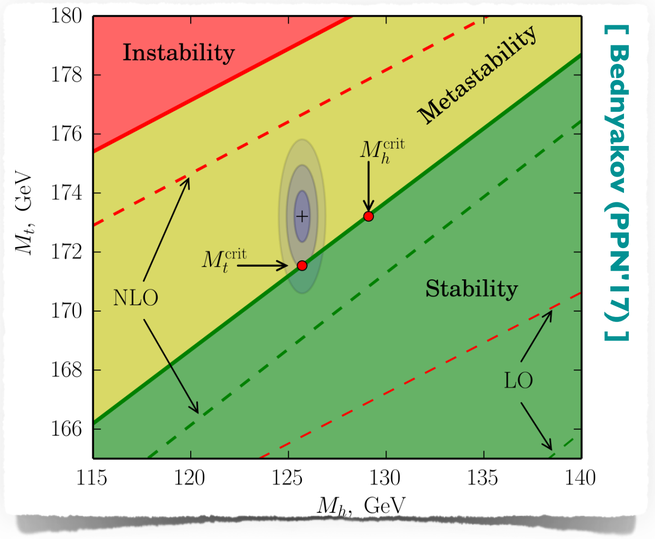
University of Vienna – 4 November 2025

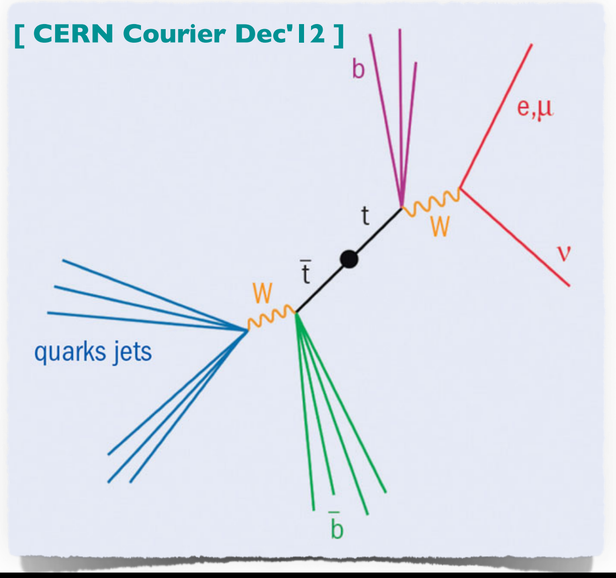
Why the top quark ?



A massive quark

- Most massive elementary particle ($m_t \sim 173$ GeV)
  - ⋆ High relevance for electroweak symmetry breaking (coupling to the Higgs boson)
  - ⋆ Key role in the (meta)stability of the vacuum
  - ⋆ Special connection with new physics beyond the Standard Model
- Extremely short lifetime ($\Gamma_t \sim 1.4$ GeV $\Leftrightarrow \tau \sim 10^{-25}$ s)
  - ⋆ No time for hadronisation
  - ⋆ Hadronic or semi-leptonic decay





30 years of top quark

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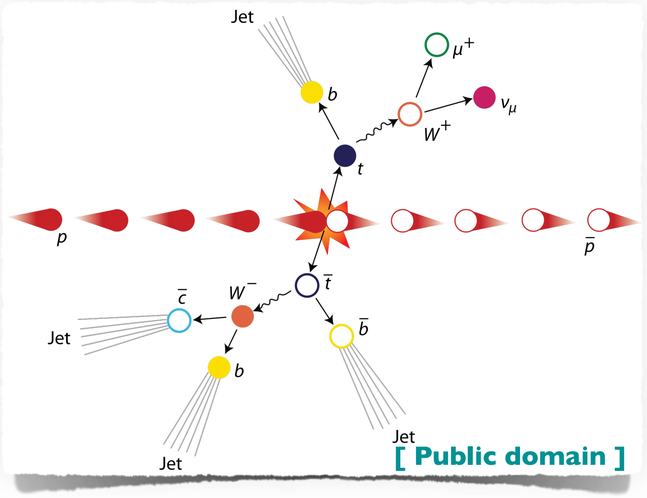
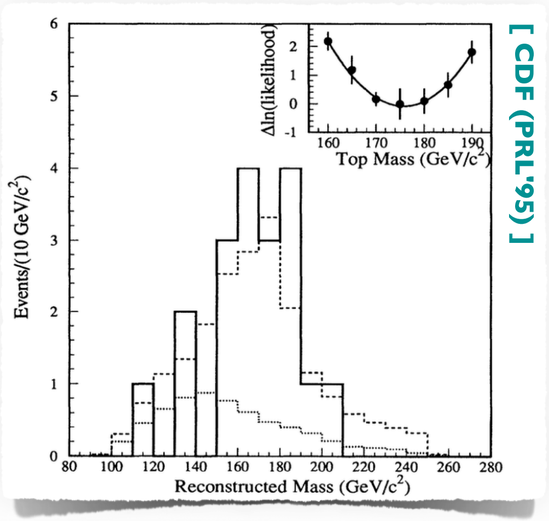
Born in 1995

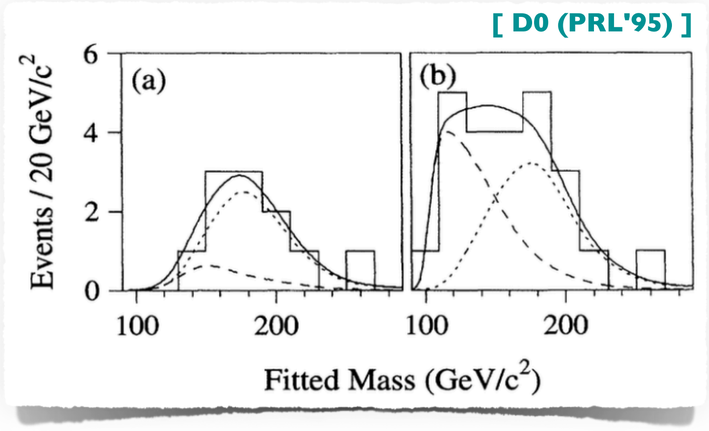
- Pair production and decay at the Tevatron (Fermilab)
  - ⋆ Excess of events (in agreement with the Standard Model predictions)
- Event reconstruction
  - ⋆ Top mass measurement from the decay products
  - ⋆ Discovery of a new quark



[Public domain]



[CDF (PRL'95)]



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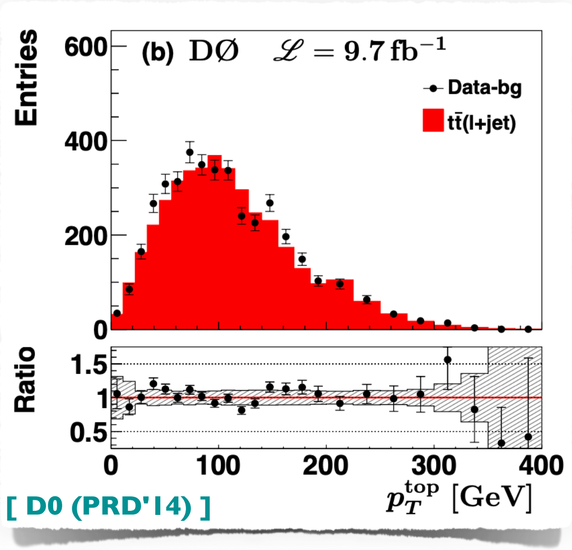
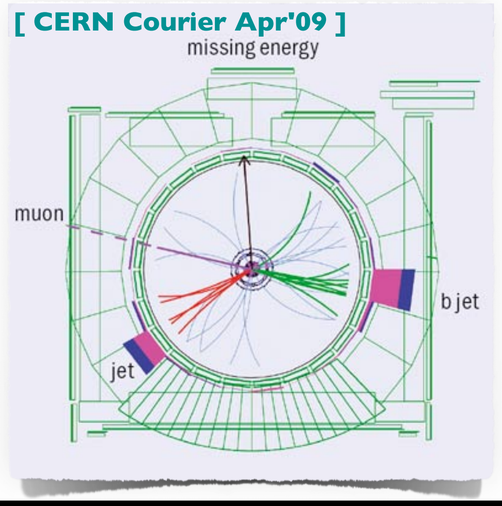


[D0 (PRD'14)]

Exploration time 🧭

- Characterisation of the top properties
  - ⋆ Production modes
  - ⋆ Differential cross sections



[CERN Courier Apr'09]

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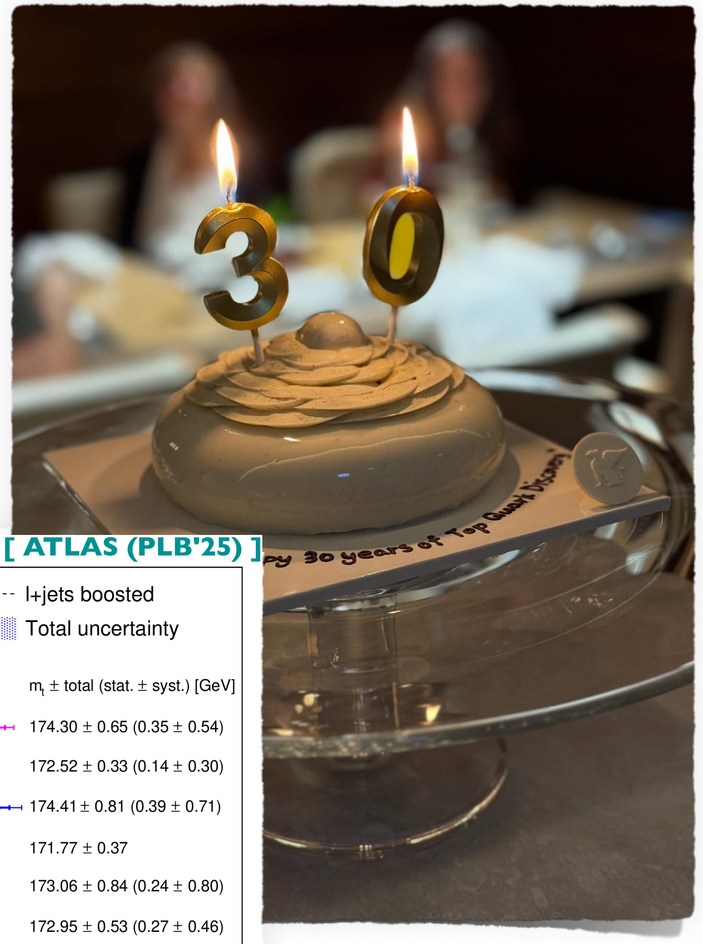
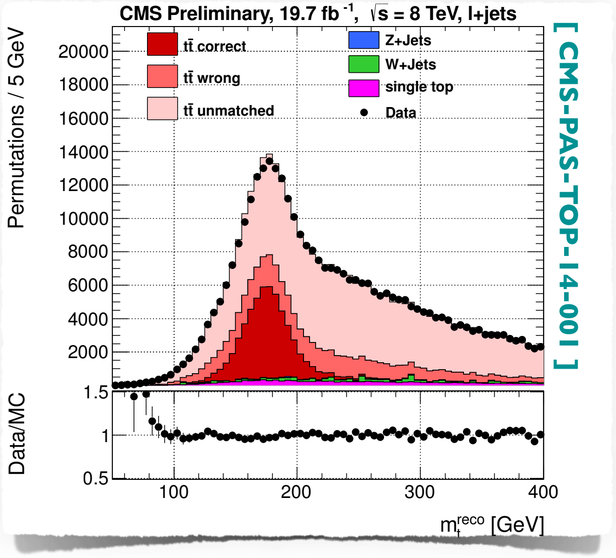
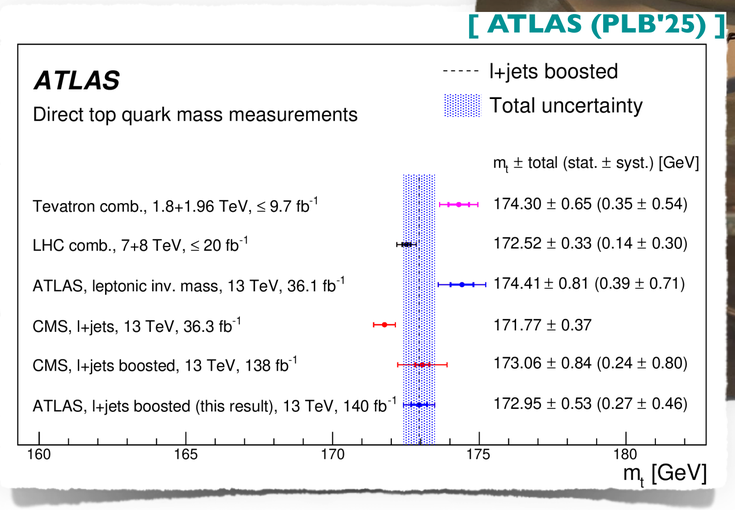
Precision era 🎯

- Top physics at the LHC
  - ⋆ Mass, width, properties
  - ⋆ New physics
  - ⋆ Rare production modes





[ATLAS (PLB'25)]



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A 30-year-old top quark

Refinement & subtleties

Excesses with partial run 2 data

Copious top quark production at the 13 TeV LHC

- LHC run 2: rate of 810 pb, luminosity of 140/fb
  - ⋆ About 100,000,000 top quark pairs
  - ⋆ Detailed analysis of the top properties (mass, width, *etc.*)
- Many differential distributions precisely measured
  - ⋆ Precision measurements as probes for anomalies

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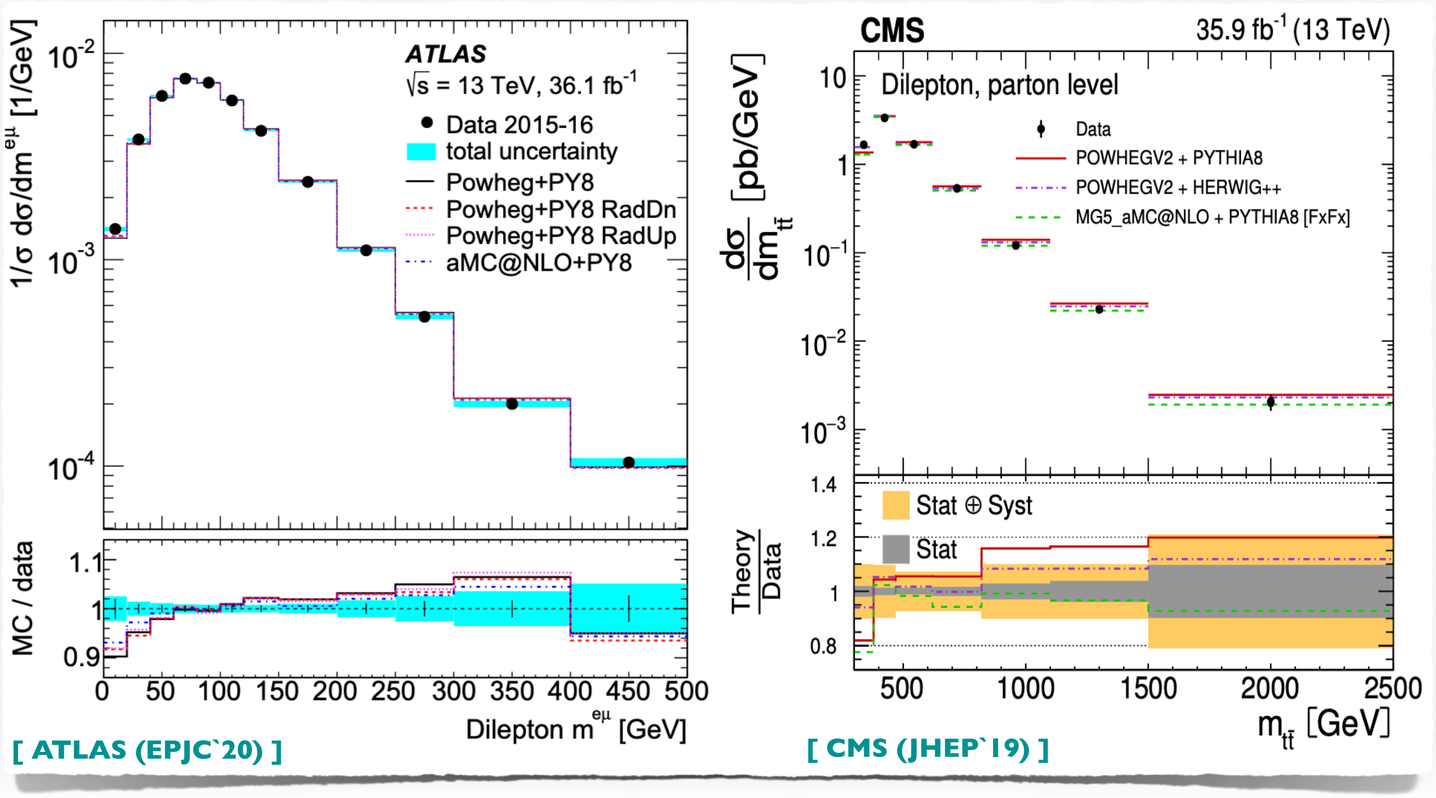
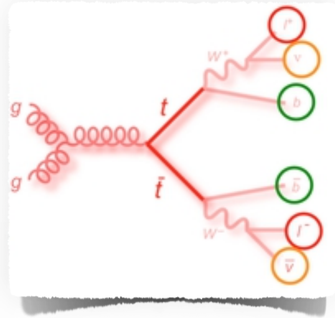


[ATLAS (EPJC`20) ] [ CMS (JHEP`19)]

Example: top pair production and dileptonic decay

- Detailed analysis of the top properties
- Dilepton invariant mass spectrum
  - ⋆ $m^2_{\ell\ell} = (p_\ell + p_{\ell'})^2$
  - ⋆ 2019–2020: partial run 2 results
    - ➙ Good agreement



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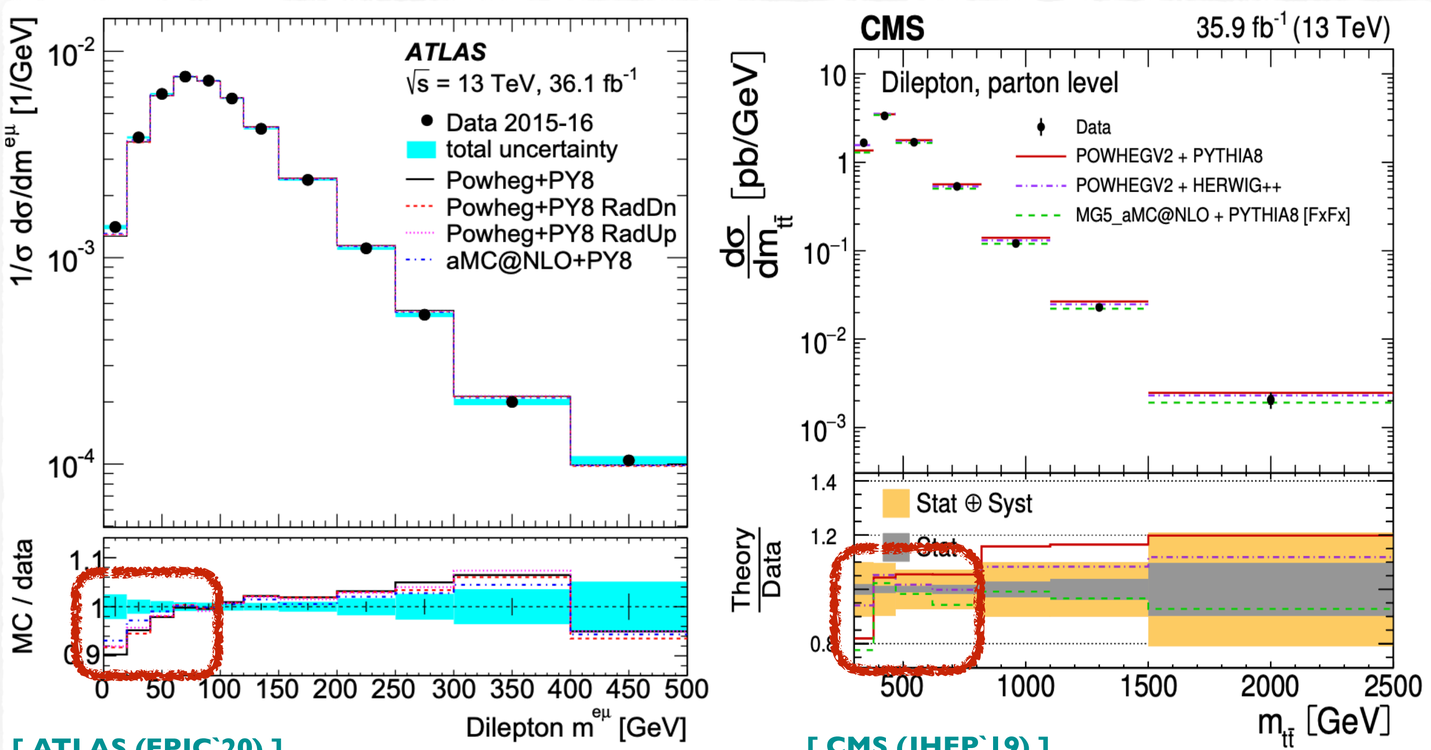

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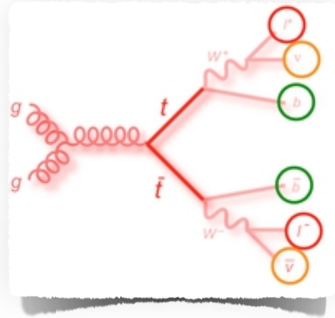

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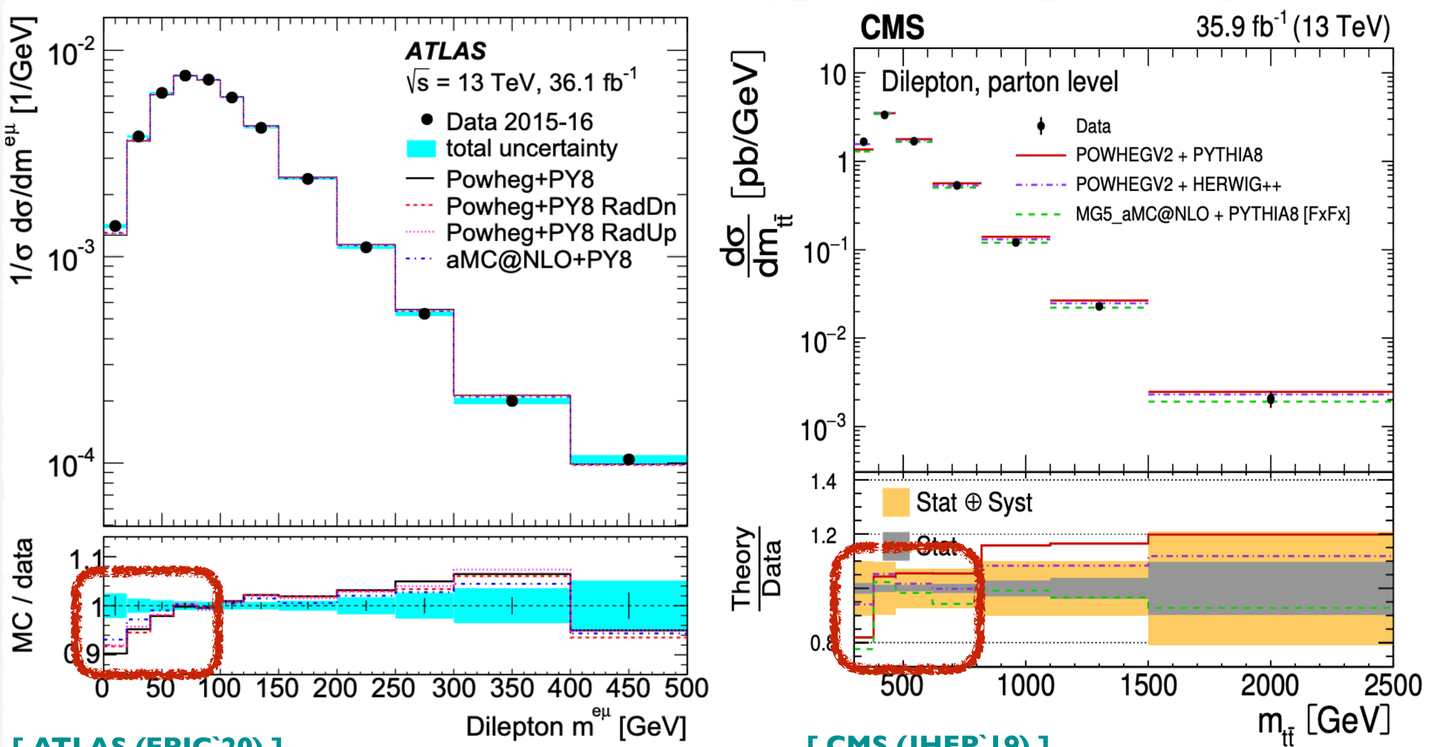

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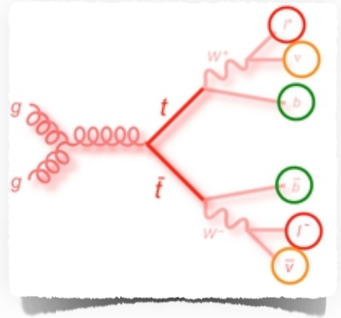
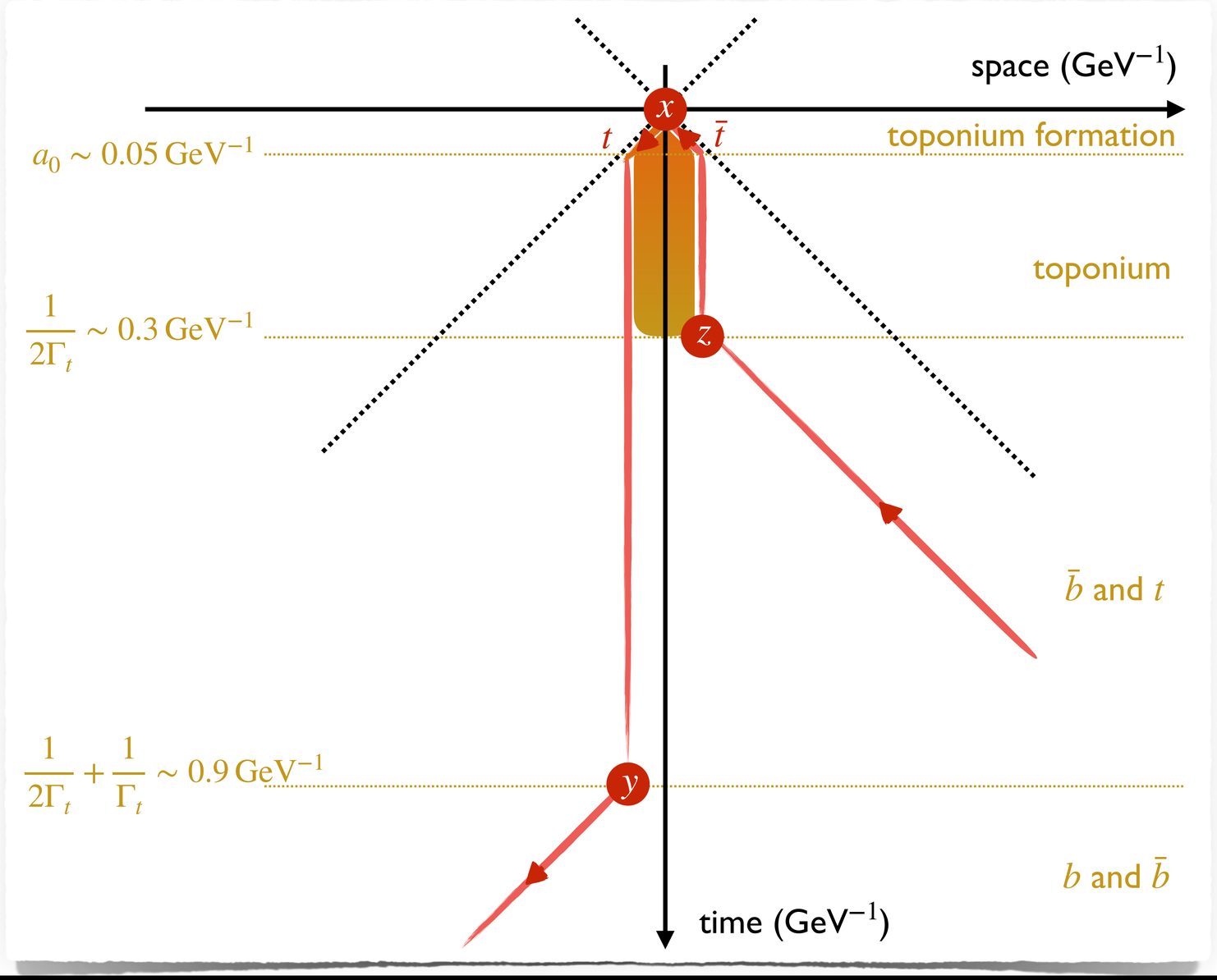


Characterisation of the excess

- New physics?
- Issues with Standard Model modelling
  - ⋆ Low invariant mass ⬌ production threshold
  - ⋆ Angular observables impacted too (small separation between the leptons)
  - ⋆ Bound-state effects / Sommerfeld enhancement

! $\Gamma_t \gg \Lambda_{\mathrm{QCD}}$!

Top-antitop production near threshold



[Fadin & Khoze (JETP Lett`87)]
[Fadin, Khoze & Sjöstrand (Z.Phys.C`90)]
[Sumino, Fujii, Hagiwara, Murayama & Ng (PRD`93)]

A position-space picture

- Two-particle state created at $x = (t_0, \vec{x})$
  - ⋆ Non-relativistic regime
- Propagation to the QCD potential barrier
  - ⋆ Typical scale: the Bohr radius

$$a_0 = \frac{1}{C_F\,\alpha_s\,m_t/2} \sim (20\ \mathrm{GeV})^{-1}$$

- Oscillations within the barrier until system decay
  - ⋆ Top [$y = (t_1, \vec{y})$] or antitop [$z = (t_2, \vec{z})$] decay
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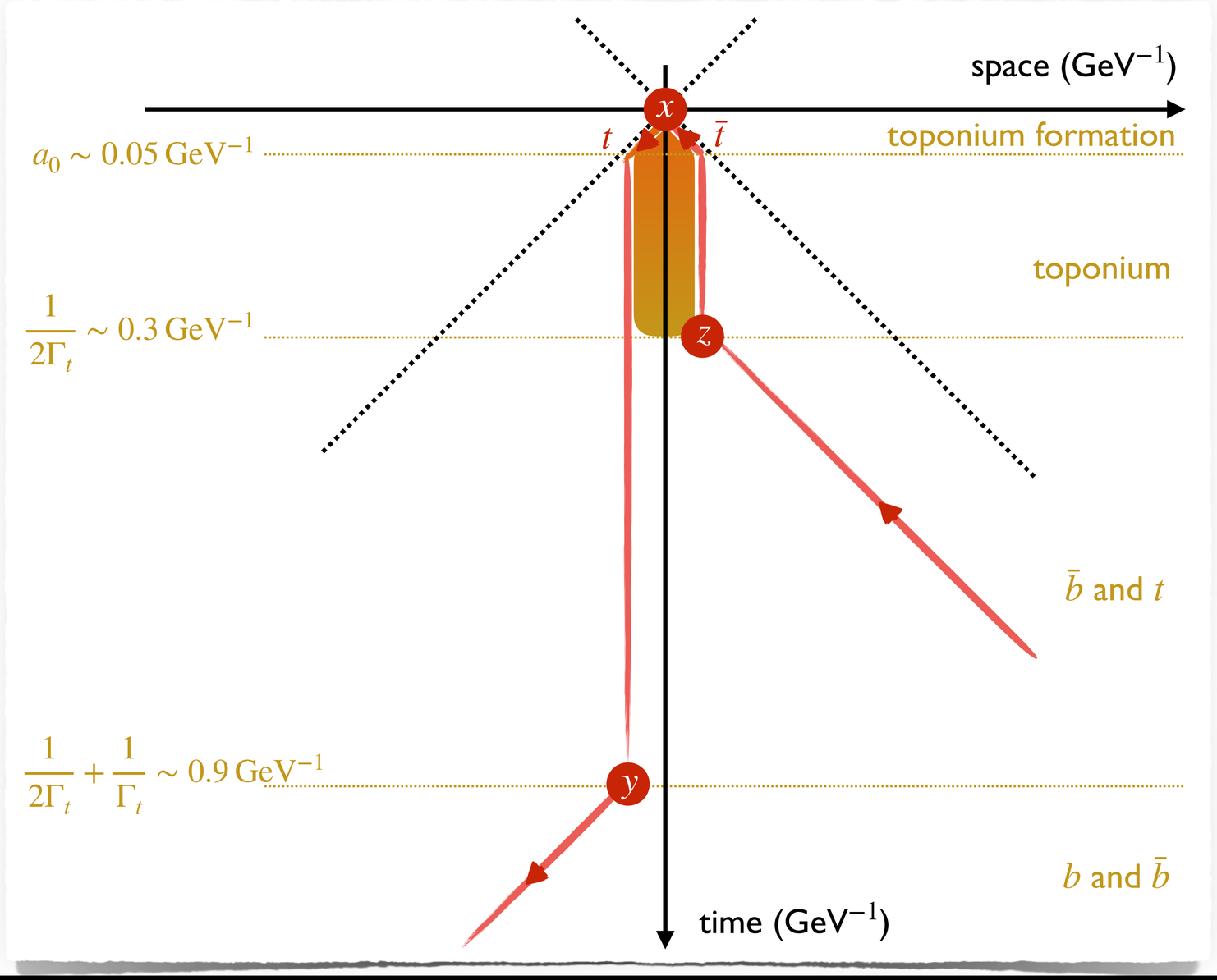

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- Expected to be 0.8% of the total rate
- Ignored and not included in the simulations

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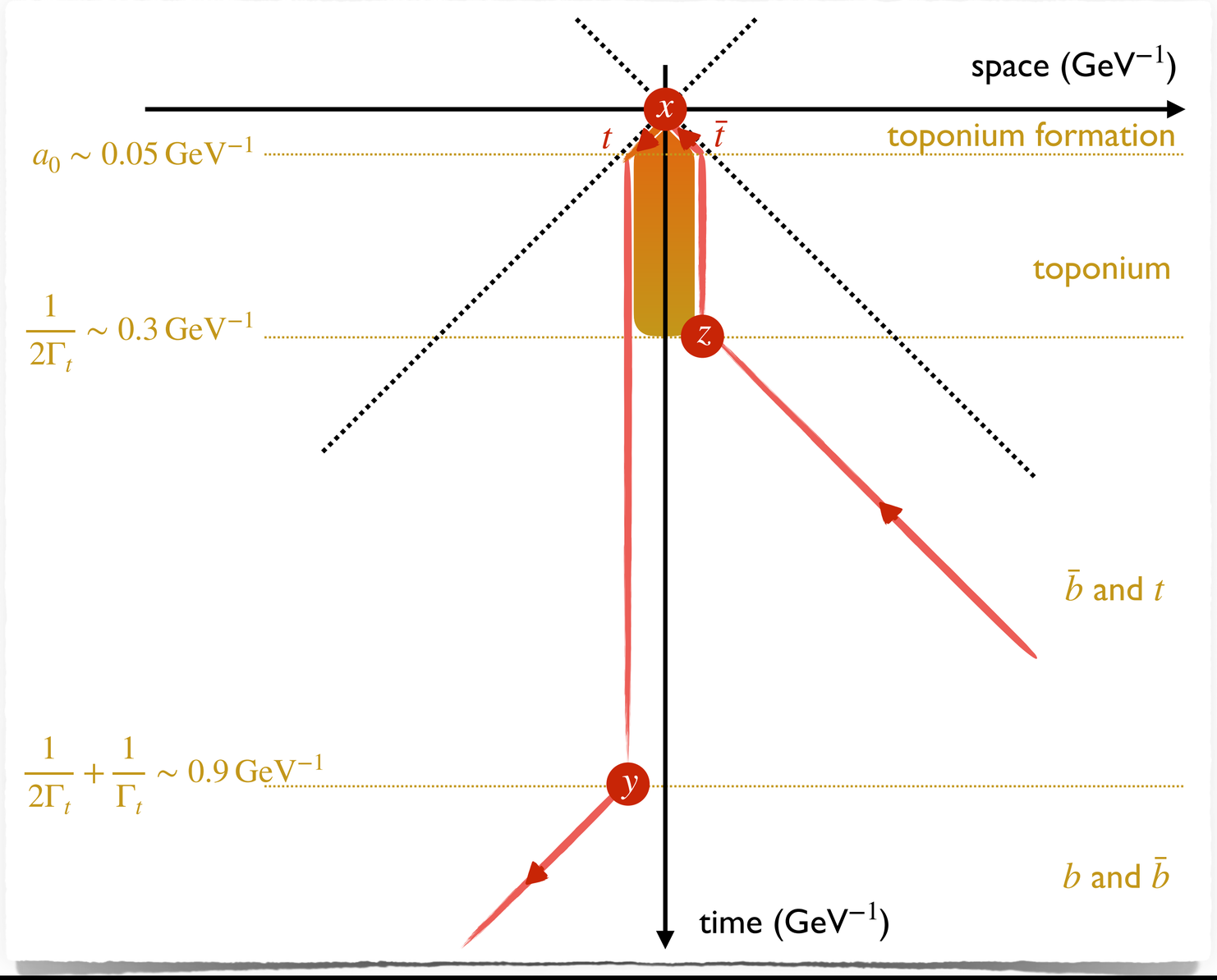

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LHC sensitivity to such a small effect?

Toponium energy spectrum

Which two-particle state(s)?

- Non-relativistic regime: Schrödinger equation with the QCD potential

$$\left[-\frac{\Delta}{m_t} + V_{\mathrm{QCD}}(r) \right] \psi(\mathbf{r}) = E\,\psi(\mathbf{r})$$

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- QCD potential ≡ perturbative piece ⊕ confinement
  - ⋆ Short distance: Coulomb-like interactions
    - ➙ QCD coupling weak
    - ➙ One-gluon exchange dominance
    - ➙ $1/r$ structure (à la Coulomb)
    - ➙ Perturbative corrections calculable

$$V_{\mathrm{Coulomb}}(r;\mu) = -C_F \frac{\alpha_s(\mu)}{r} \left\{ 1 + \frac{\alpha_s(\mu)}{4\pi} \left[a_1 + 2\gamma_E\,\beta_0 \right) \right] + \ldots \right\}$$

  - ⋆ Large distance ≡ empirical component
    - ➙ Negligible effects (no top hadronisation)

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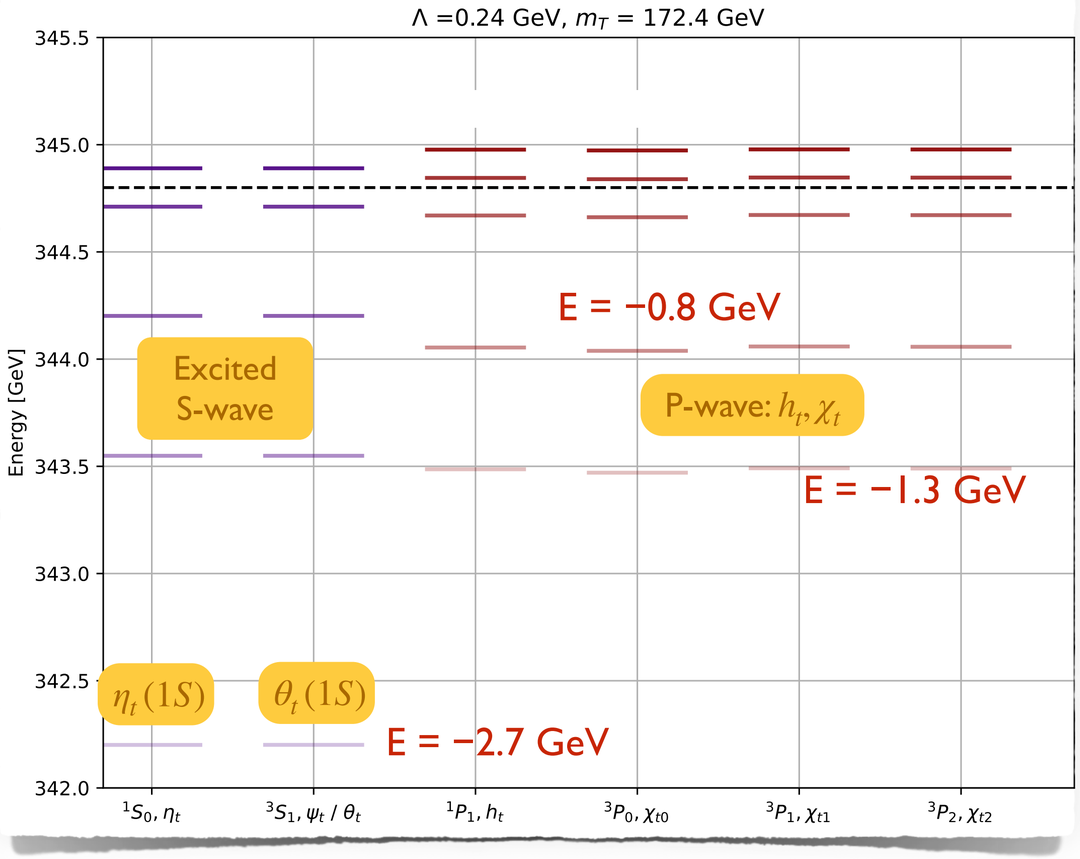

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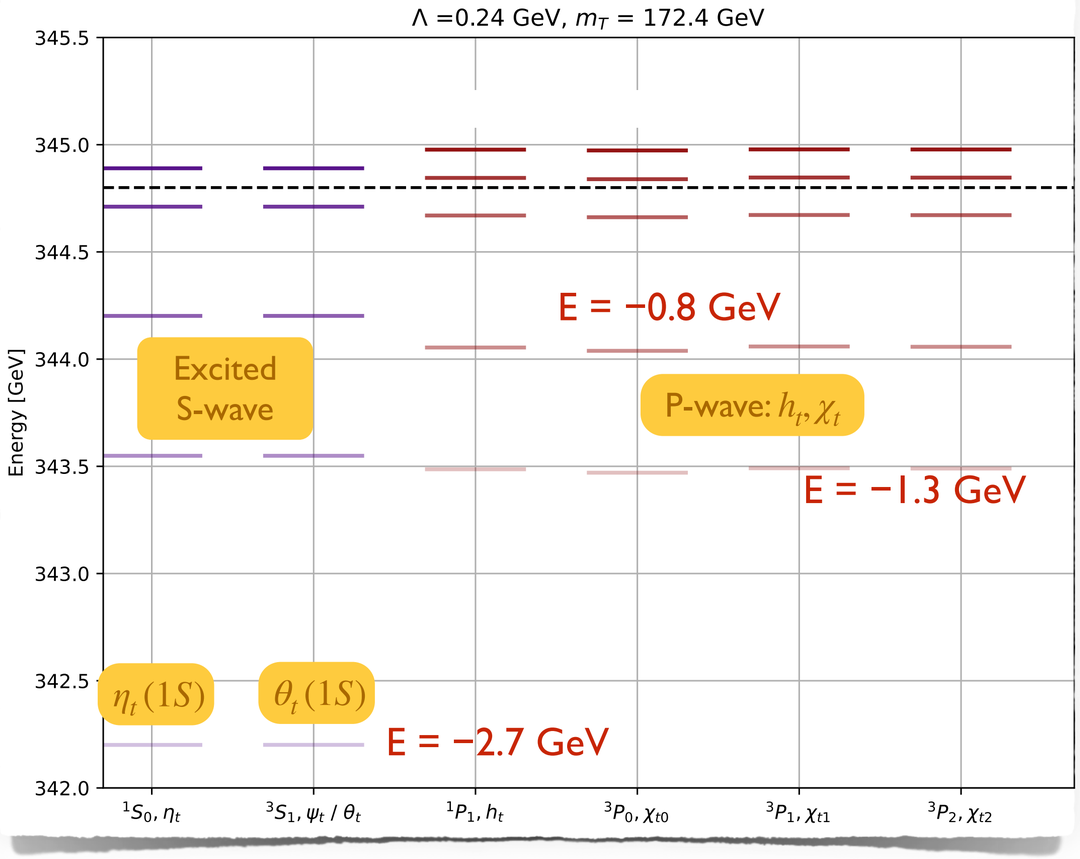
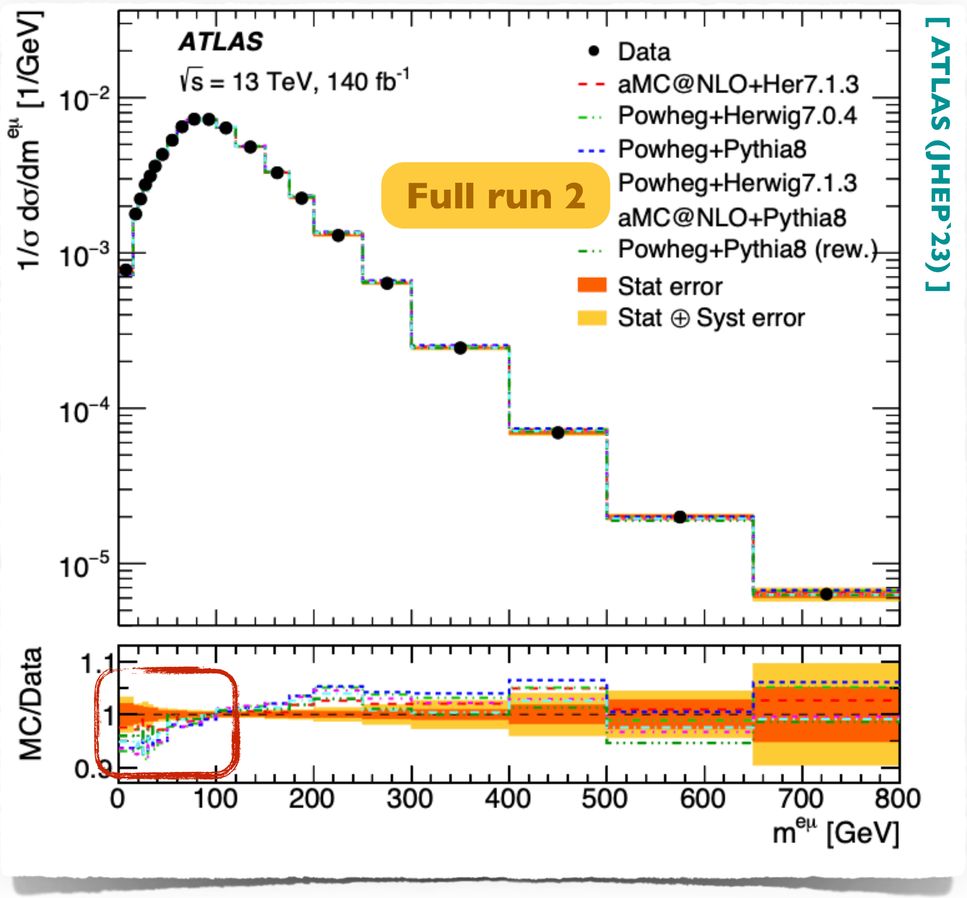


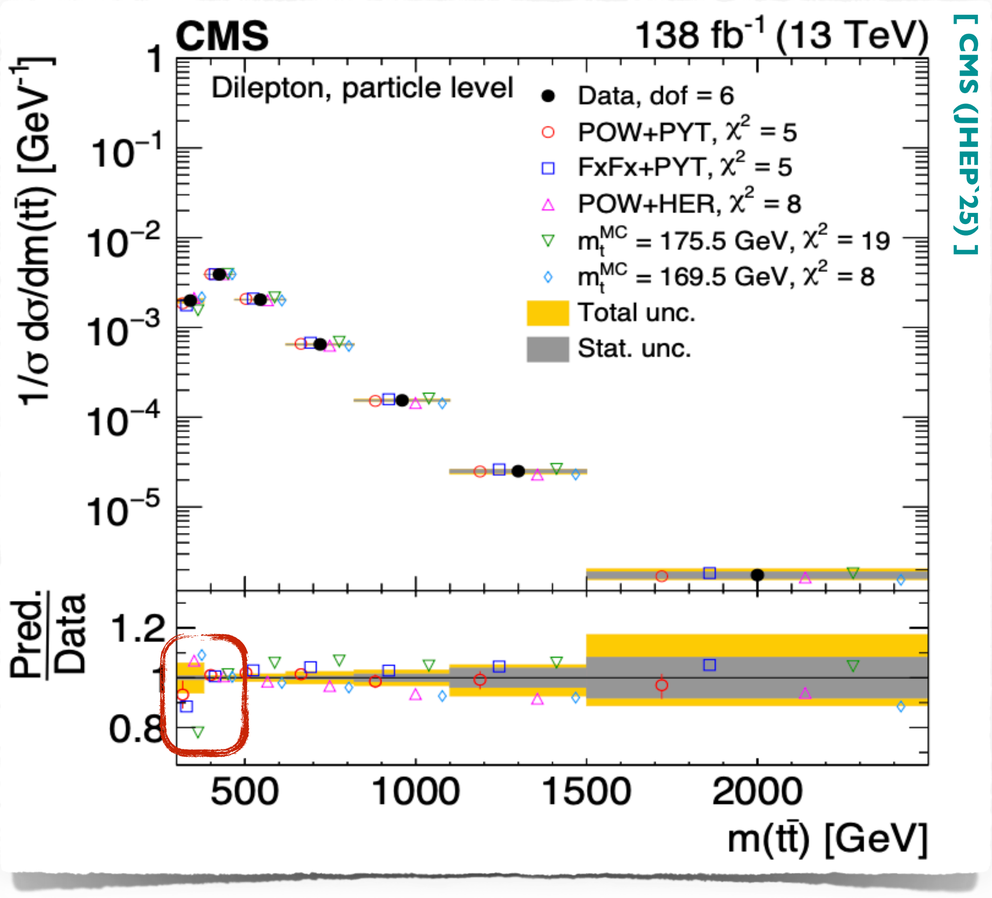
Hard to disentangle due to experimental resolution

Full run 2 analyses (2023–2025)

Excess still there…

- LHC run 2: full luminosity of 140/fb analysed
- Most accurate measurements available for the next few years





Excess confirmation (5σ) in Sep’24 – Jul’25

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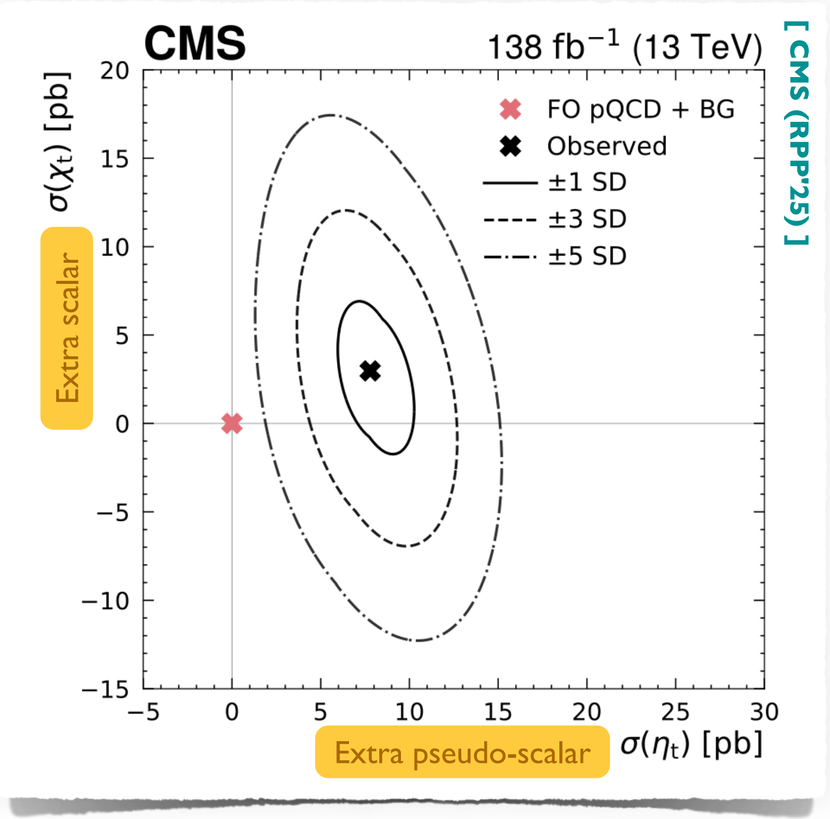
CMS interpretation from toy models

- Framework: Standard Model + extra scalar/pseudo-scalar (η_t / χ_t)
  - ⋆ Single state dominance approximation
  - ⋆ Pseudo-scalar S-wave ground state; scalar P-wave excited states
- Excess mostly pseudo-scalar (with scalar contributions)

[Hagiwara, BF, Ma & Zheng (PRD`21)]
[Maltoni, Severi, Tentori & Vryonidou (JHEP`24)]



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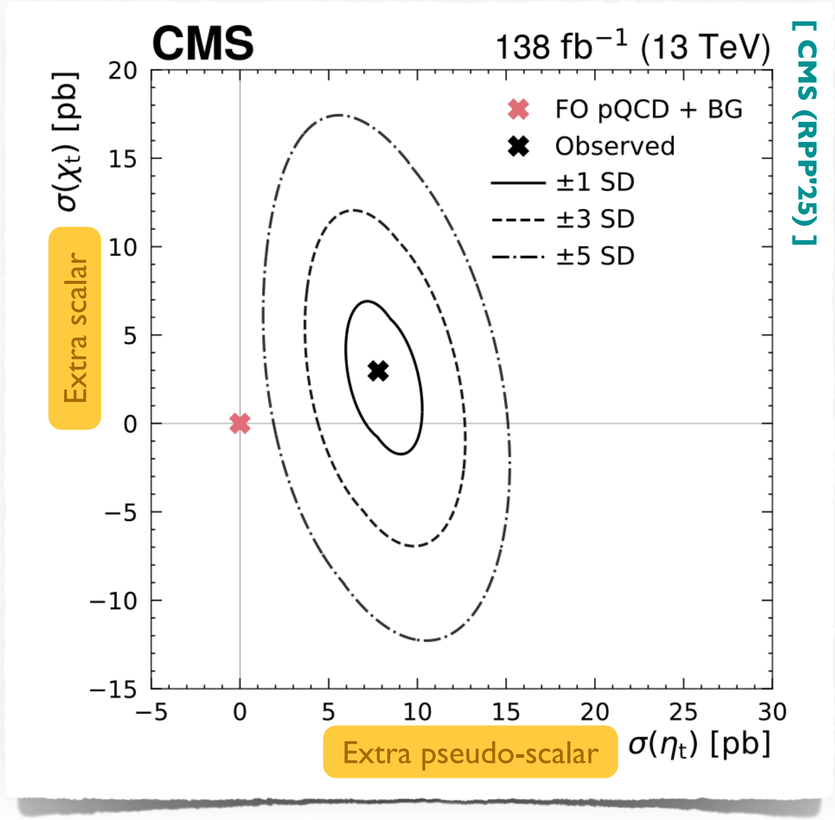

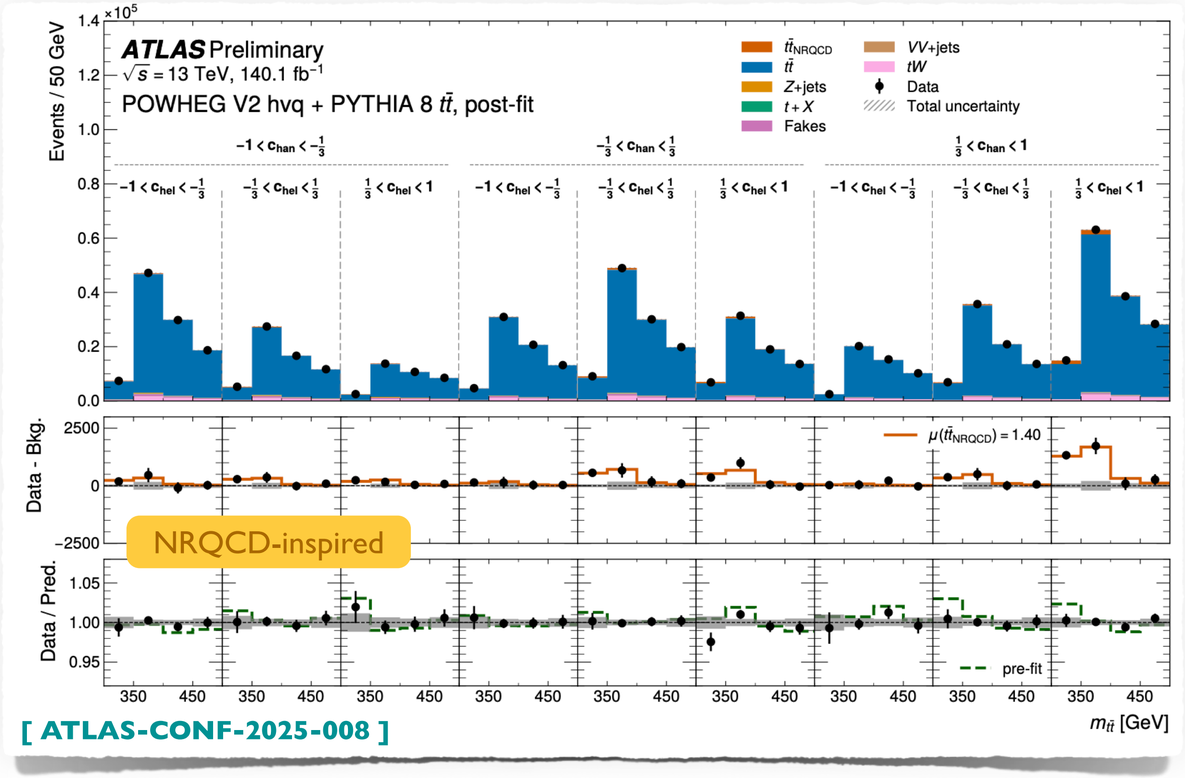

ATLAS interpretation inspired by non-relativistic QCD (NRQCD)

- Standard Model matrix elements re-weighted
  - ⋆ Ratio of S-wave Green's function ($\sim G_{\mathrm{NRQCD}}/G_{\mathrm{free}}$)
  - ⋆ Consistent resummation of all S-wave contributions

[Hagiwara, BF, Ma & Zheng (EPJC`25)]

Toy models for toponium formation

A toponium toy Lagrangian with a pseudo-scalar state

$$\mathscr{L}_{\eta_t} = \frac{1}{2}\partial_\mu \eta_t \, \partial^\mu \eta_t - \frac{1}{2} m_{\eta_t}^2 \eta_t^2 - \frac{1}{4} g_{gg} \, \eta_t \, G_a^{\mu\nu} \, \tilde{G}^a_{\mu\nu} - i g_{tt} \, \eta_t \, \bar{t} \gamma_5 t$$

How to fix the free parameters?

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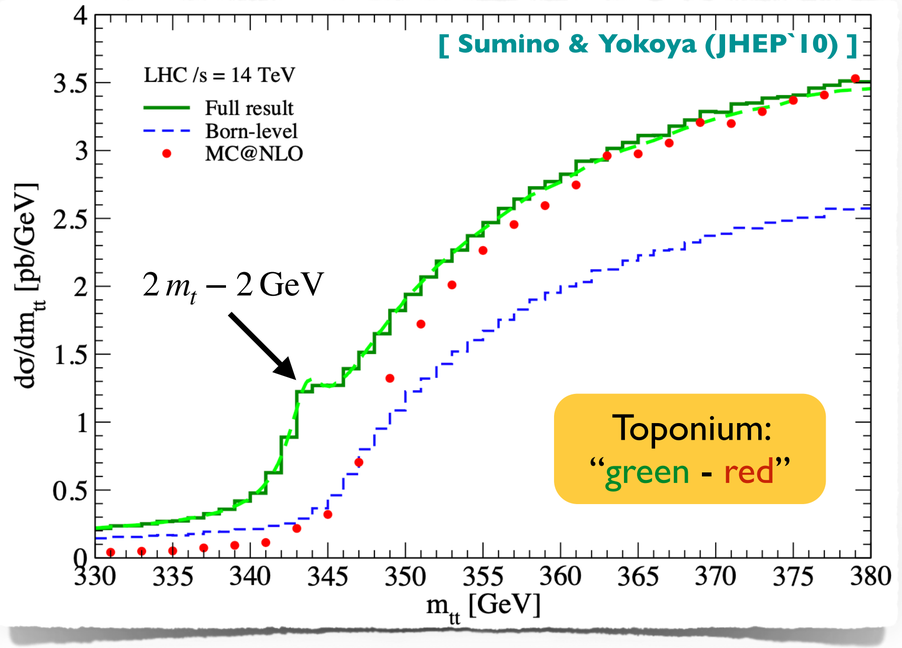
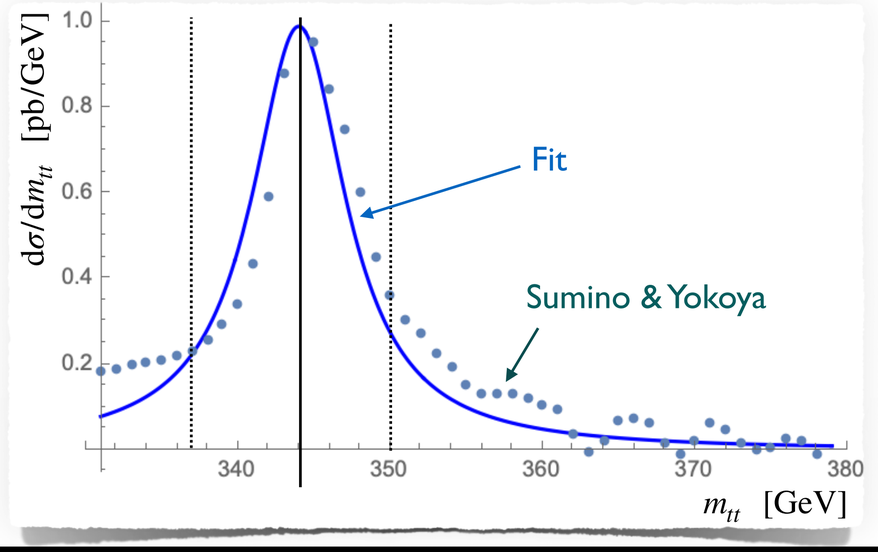
How to fix the free parameters?

- Three parameters fixed via a Breit Wigner fit
  - ⋆ Rate: 6.5 pb ➞ comparison with $\sigma_{t\bar{t}} \approx 810\,\text{pb}$
  - ⋆ Peak: 344 GeV
  - ⋆ Width: 7 GeV (capture of excited state contributions)
- NRQCD-inspired re-weighting
  - ⋆ Ratio of Green's functions
  - ⋆ Cut on $m_{t\bar{t}}$

[Hagiwara, BF, Ma & Zheng (PRD`21)]





Toy models for toponium formation

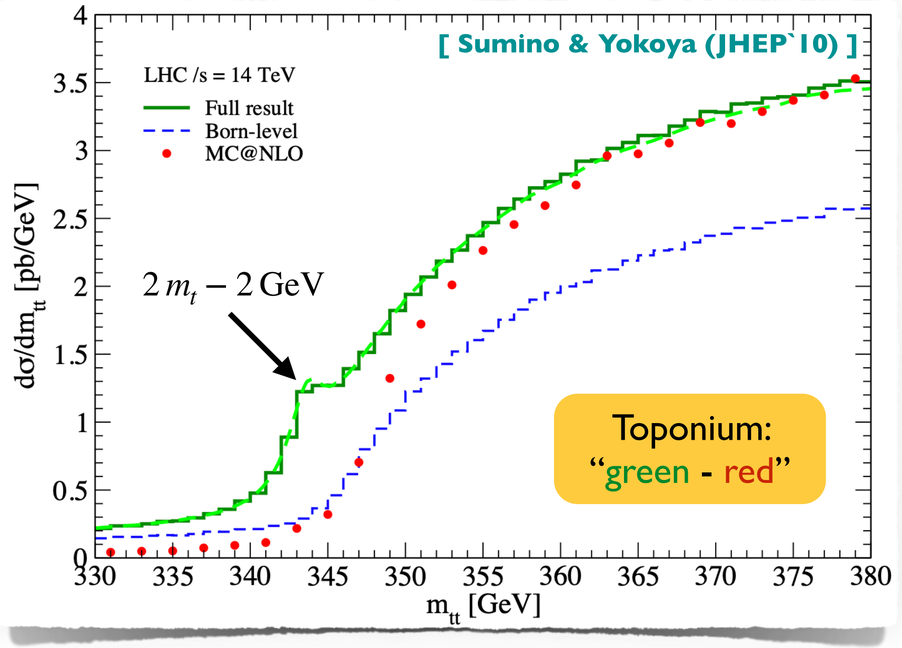
A toponium toy Lagrangian with a pseudo-scalar state

$$\mathscr{L}_{\eta_t} = \frac{1}{2}\partial_\mu \eta_t \, \partial^\mu \eta_t - \frac{1}{2} m_{\eta_t}^2 \eta_t^2 - \frac{1}{4} g_{gg} \, \eta_t \, G_a^{\mu\nu} \, \tilde{G}^a_{\mu\nu} - i g_{tt} \, \eta_t \, \bar{t} \gamma_5 t$$

How to fix the free parameters?

- Three parameters fixed via a Breit Wigner fit
  - ⋆ Rate: 6.5 pb → comparison with $\sigma_{t\bar{t}} \approx 810\,\text{pb}$
  - ⋆ Peak: 344 GeV
  - ⋆ Width: 7 GeV (capture of excited state contributions)
- NRQCD-inspired re-weighting
  - ⋆ Ratio of Green's functions
  - ⋆ Cut on $m_{t\bar{t}}$

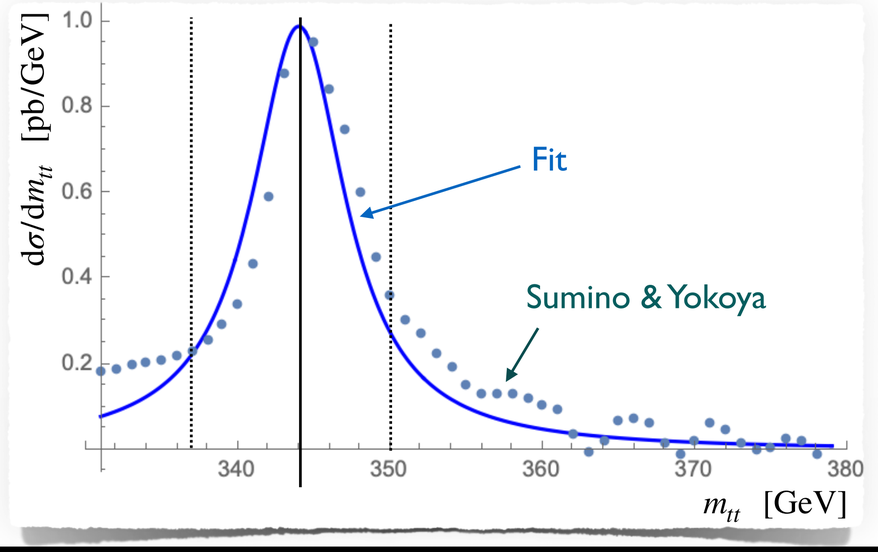
[Hagiwara, BF, Ma & Zheng (PRD`21)]



CMS parametrisation: physical toponium width of 3 GeV

- No re-weighting
- No $m_{t\bar{t}}$ cut (better matching)
- Only capturing the dominant contributions
  - ⋆ No excited-state contributions

[Maltoni, Severi, Tentori & Vryonidou (JHEP`24)]



NRQCD-inspired re-weighting

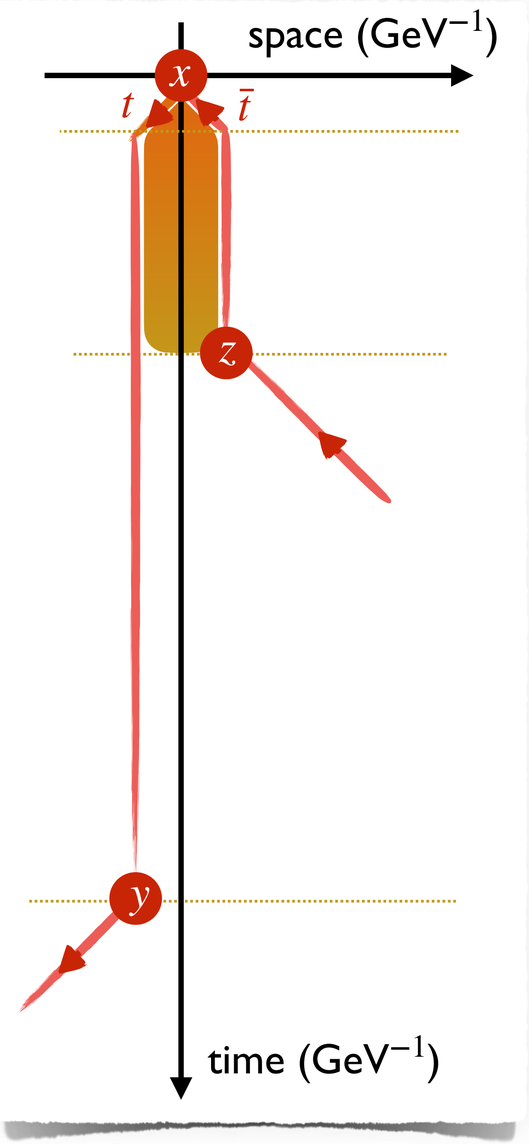
Quantum field theory: three-point correlation function in the non-relativistic limit

- Two options depending on which quark decays first

$$K_{abcd}(x,y,z) = \left\langle 0 \middle| T\left\{ t_c(y)\bar{t}_d(z) : \bar{t}_a(x)t_b(x) : \right\} \middle| 0 \right\rangle = \frac{(1+\gamma^0)_{ca}}{2}\frac{(1-\gamma^0)_{bd}}{2}$$

$$\times \int \mathrm{d}^3 r \left[K_1\left(y;(z^0,\vec{r})\right) K_2(z^0,\vec{r},\vec{z};x^0,\vec{x},\vec{x}) + K_1\left(z;(y^0,\vec{r})\right) K_2(y^0,\vec{y},\vec{r};x^0,\vec{x},\vec{x}) \right]$$

1-particle-state and 2-particle-state propagators



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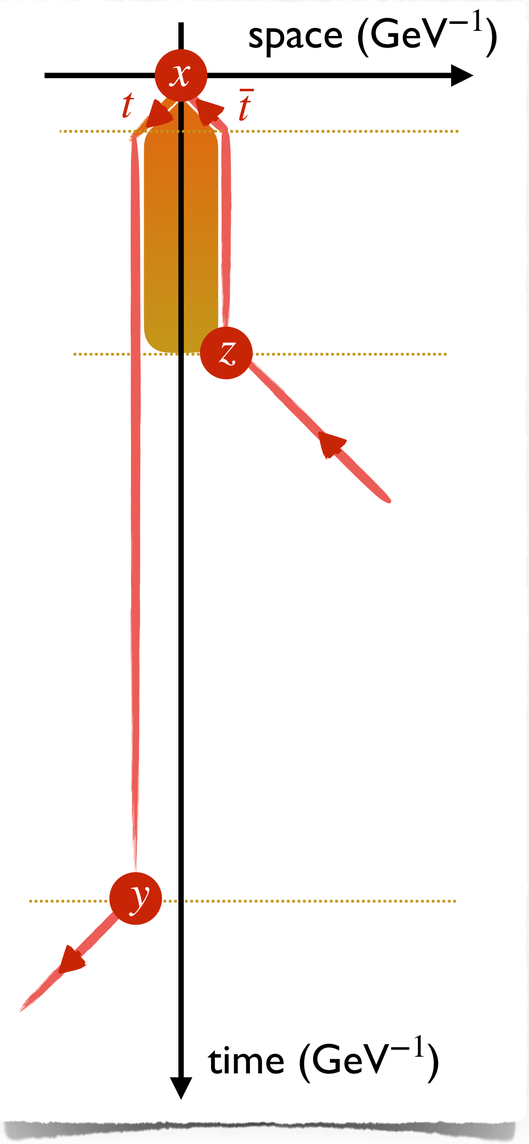
1-particle-state and 2-particle-state propagators

Separation of the centre-of-mass and relative motion

- The QCD Green's function (from K_2)
- Momentum space: solution to the Lippmann-Schwinger equation
  - ➙ Fourier transform of the QCD potential
  - ➙ Easy to solve numerically

$$\widetilde{G}(E;p) = \widetilde{G}_{\text{free}}(E;p) + \int \frac{\mathrm{d}^3 q}{(2\pi)^3}\, \widetilde{V}_{\text{QCD}}(\vec{p}-\vec{q})\, \widetilde{G}(E;q)$$

[Jezabek, Kuhn & Teubner (Z.Phys.C`92)]



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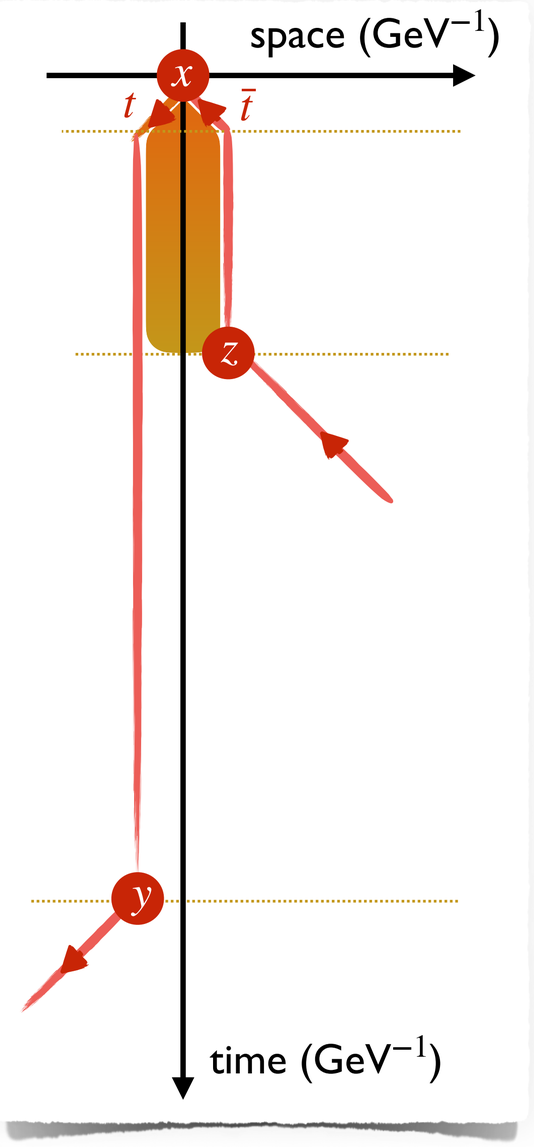
- Properties of $\widetilde{G}(E;p)$
  - ⋆ Dependence on the binding energy and the top momentum in the $t\bar{t}$ rest frame



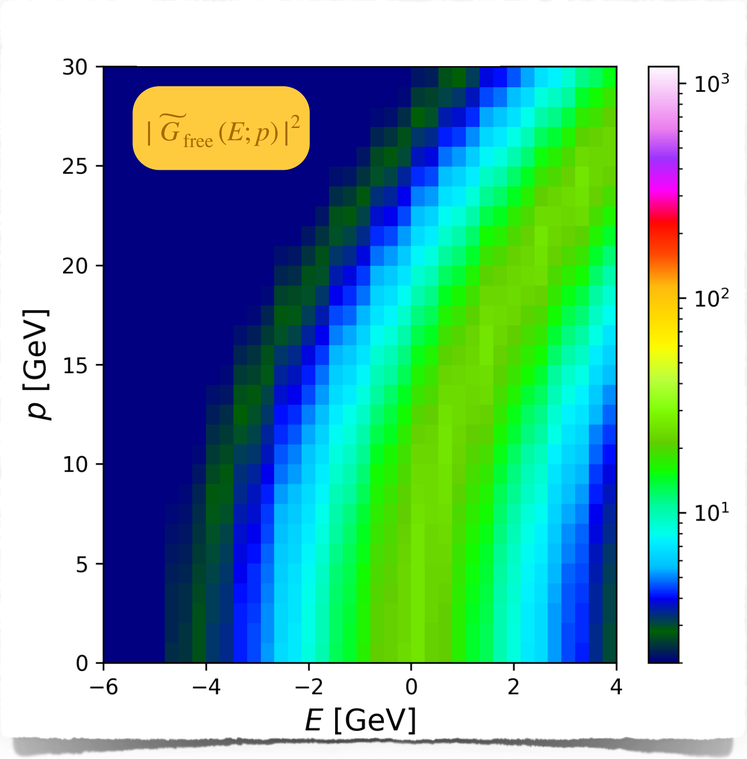
The toponium Green's function

Free Green's function

- $|\tilde{G}_0|^2$: Breit-Wigner behaviour (standard top propagator)

  ➙ $\forall E$, peak at $p_{\text{peak}} \simeq \frac{2m_t + E}{2}\sqrt{1 - \frac{4m_t^2}{(2m_t + E)^2}}$

- Toponium contributions ≠ conventional resonances
  - ⋆ Sometimes called 'quasi bound-states'
- Seed to update existing simulations
  - ⋆ Replacing the free Green's function by the QCD one



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Numerical derivation of the QCD Green's function

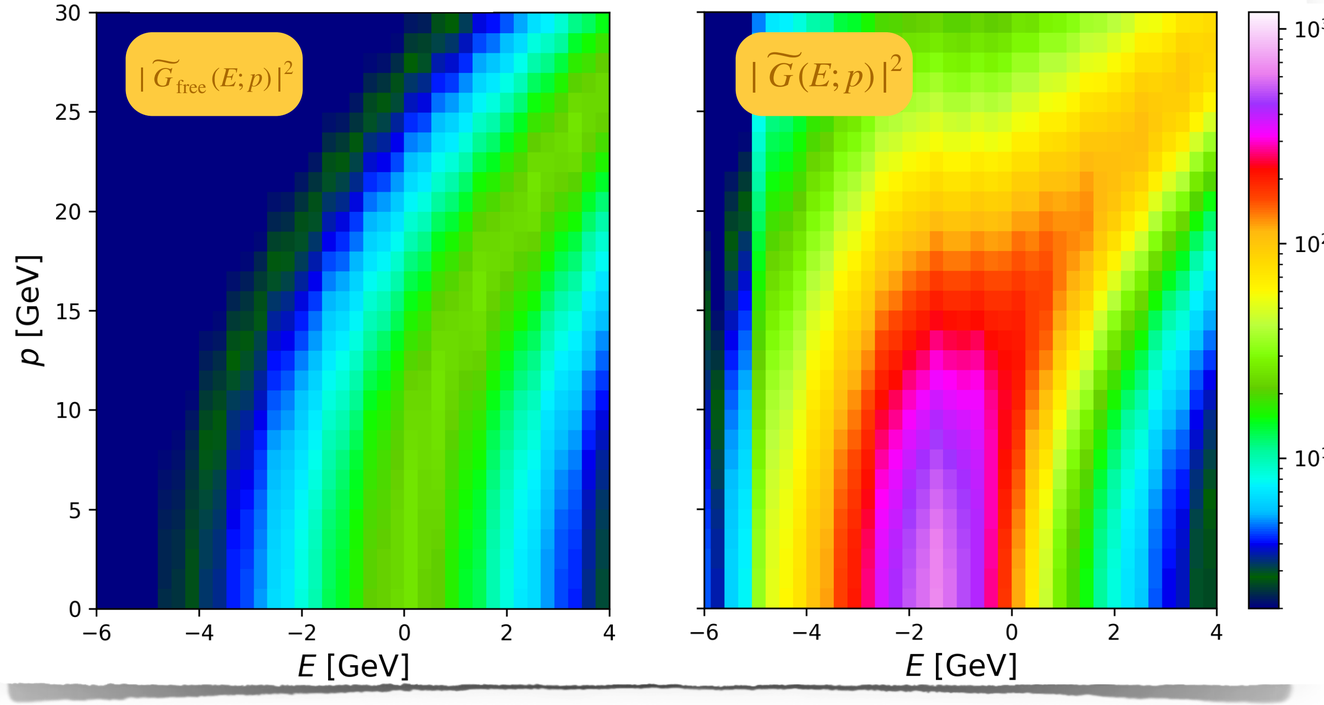
- Leading order Coulomb potential (in Fourier space):

$$\widetilde{V}_{\text{Coulomb}}(q^2) \approx -C_F \frac{4\pi\alpha_s(m_t^2)}{q^2}$$

- Large and positive energies: Breit-Wigner behaviour (different normalisation at low positive E)
- Significant toponium effects between −4 GeV and 0 GeV

  ➞ Most prominent effect at $E \approx -2\,\text{GeV}$ and low p

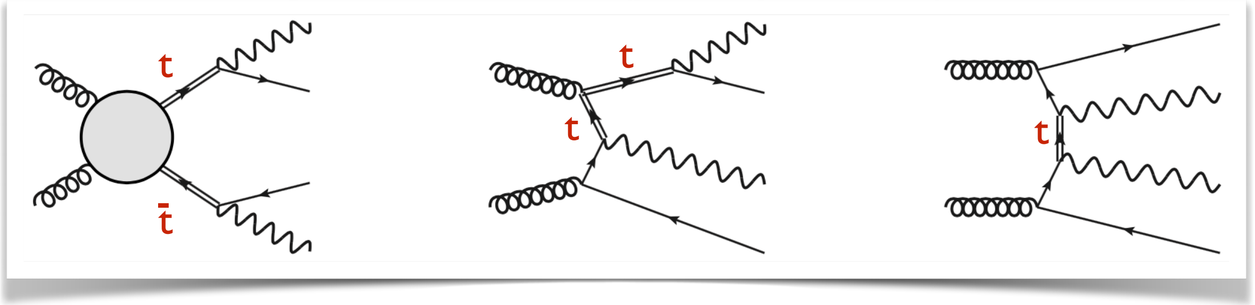
[Hagiwara, BF, Ma & Zheng (EPJC`25)]



NRQCD-inspired modelling strategy

Wb Wb production in the threshold regime

- 3 classes of diagrams with 0, 1, 2 (possibly off-shell) resonant tops



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  - ★ Scattering states: perturbative Sommerfeld enhancement
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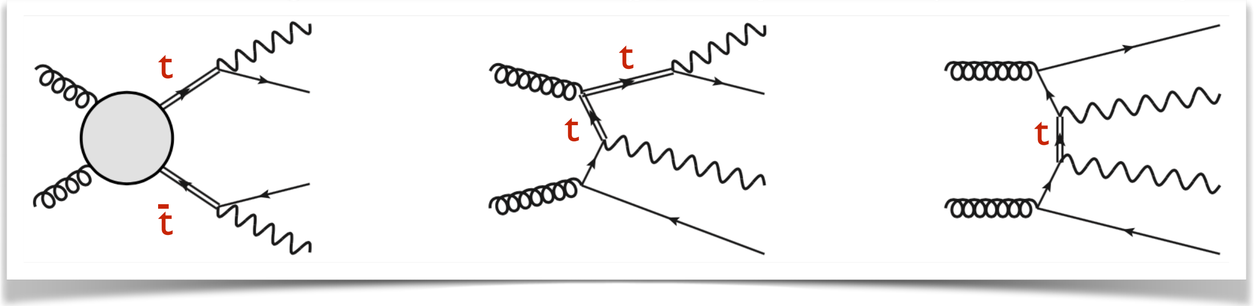

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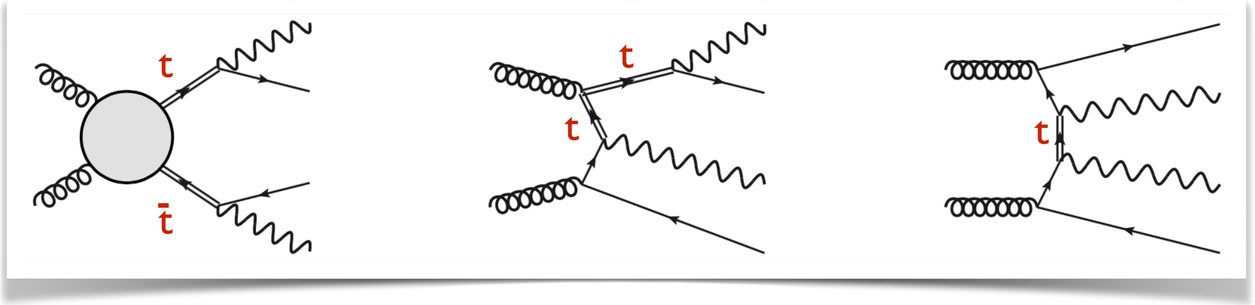

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Modelling strategy for the Large Hadron Collider

- Relying on conventional Monte Carlo event generators
  - ⋆ Computer simulations of collider collisions
  - ⋆ Traditionally used by experiments (and theorists)
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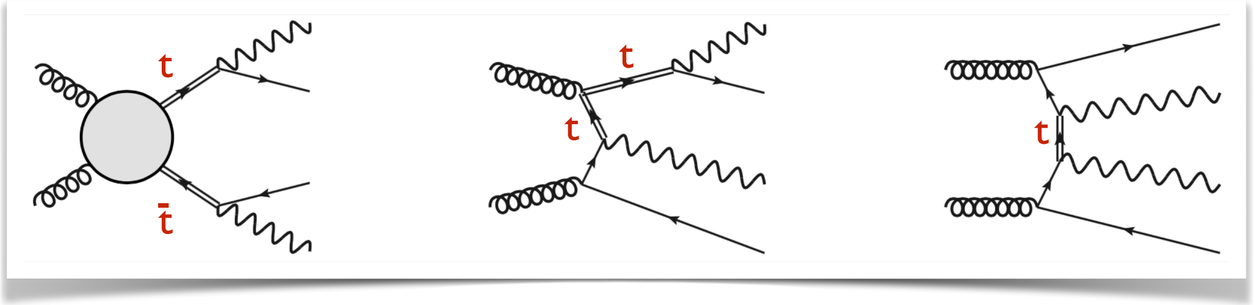

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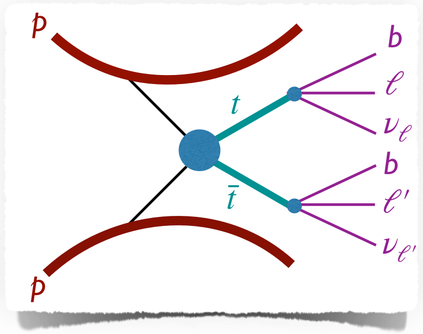
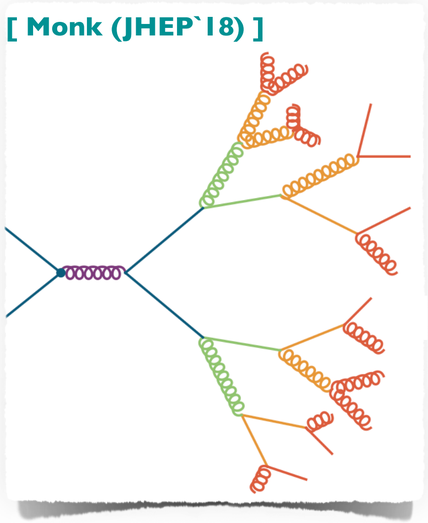
Threshold effects non-relativistic

- Conventional codes: full matrix elements
- Specific requirements
  - ➙ Removal of relativistic contributions
  - ➙ Projection on a colour-singlet two-particle state
- Two ingredients
  - ⋆ Matrix-element re-weighting
  - ⋆ Phase space cuts: $E < 4$ GeV and $p^* < 50$ GeV

Di-leptonic top-antitop production and decay at the LHC





Top-antitop production in proton-proton collisions

- Choice of the final state of interest: two leptons
  - ⋆ Decays crucial for correctly modelling below the $t\bar{t}$ threshold
- Di-leptonic top-antitop system ≡ 2 *b*-quarks + 2 leptons + 2 neutrinos
  - ⋆ Leptons ➙ $e^{\pm}$ and $\mu^{\pm}$ (cleaner experimentally than taus)
  - ⋆ Neutrinos ➙ undetected ➙ missing transverse energy (no access to longitudinal component)
  - ⋆ *b*-quarks ➙ *b*-jets (QCD collinear radiation and hadronisation processes yielding jets)
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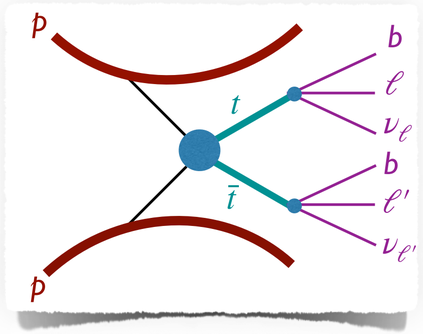

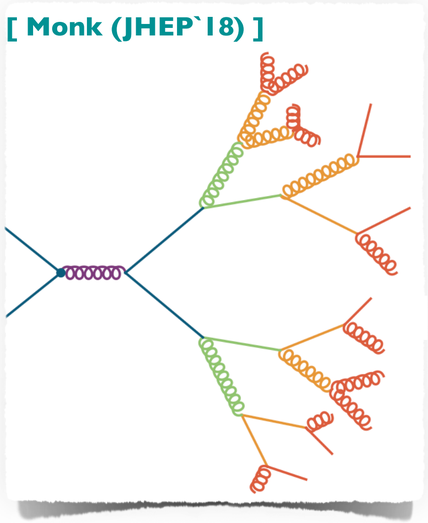

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Reconstructed-level final state: 2 $e^{\pm}/\mu^{\pm}$ + 2 *b*-jets + E_T^{miss}

- Quality objects ➙ requirements on their properties
  - ⋆ Substantial transverse momentum: e.g. $p_T > 25$ GeV
  - ⋆ Central requirements: e.g. $|\eta| < 2.5$
  - ⋆ Separated objects in the transverse plane: $\Delta R > 0.4$

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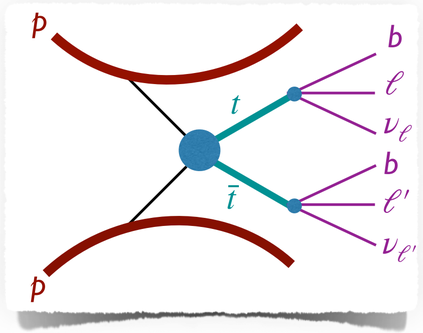
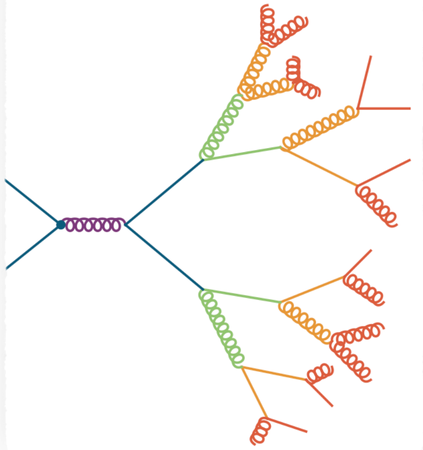


[Monk (JHEP`18)]



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Re-weighted matrix elements → **Parton showers Hadronisation** → **Reconstruction** → **Analysis of simulated collisions**

MG5_AMC
[Alwall et al. (JHEP`14)]
[Hagiwara, BF, Ma & Zheng (EPJC`25)]

PYTHIA 8
[Bierlich et al. (SciPost`22)]

FASTJET / MADANALYSIS 5
[Cacciari, Salam & Soyez (EPJC`12)]
[Araz, BF & Polykratis (EPJC`21)]

MADANALYSIS 5
[Conte & BF (IJMPA`19)]

Impact of NRQCD re-weighting on predictions

Predictions at parton-level

- Process: $gg \to (t\bar{t})_1 \to b\ell^+\nu_\ell \bar{b}\ell'^-\bar{\nu}'_\ell$
  - ⋆ No parton-showers, hadronisation, *etc.*
- Impact of the QCD Green's function
  - ⋆ Only two variables *E* and *p**
    - ➙ the binding energy
    - ➙ the top momentum in the $t\bar{t}$ rest frame
- Key doubly-differential cross section

$$\mathrm{d}^2\sigma \big/ (p^*)^2 \mathrm{d}p^* \mathrm{d}E$$

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- Rates ≡ phase space x Breit-Wigner
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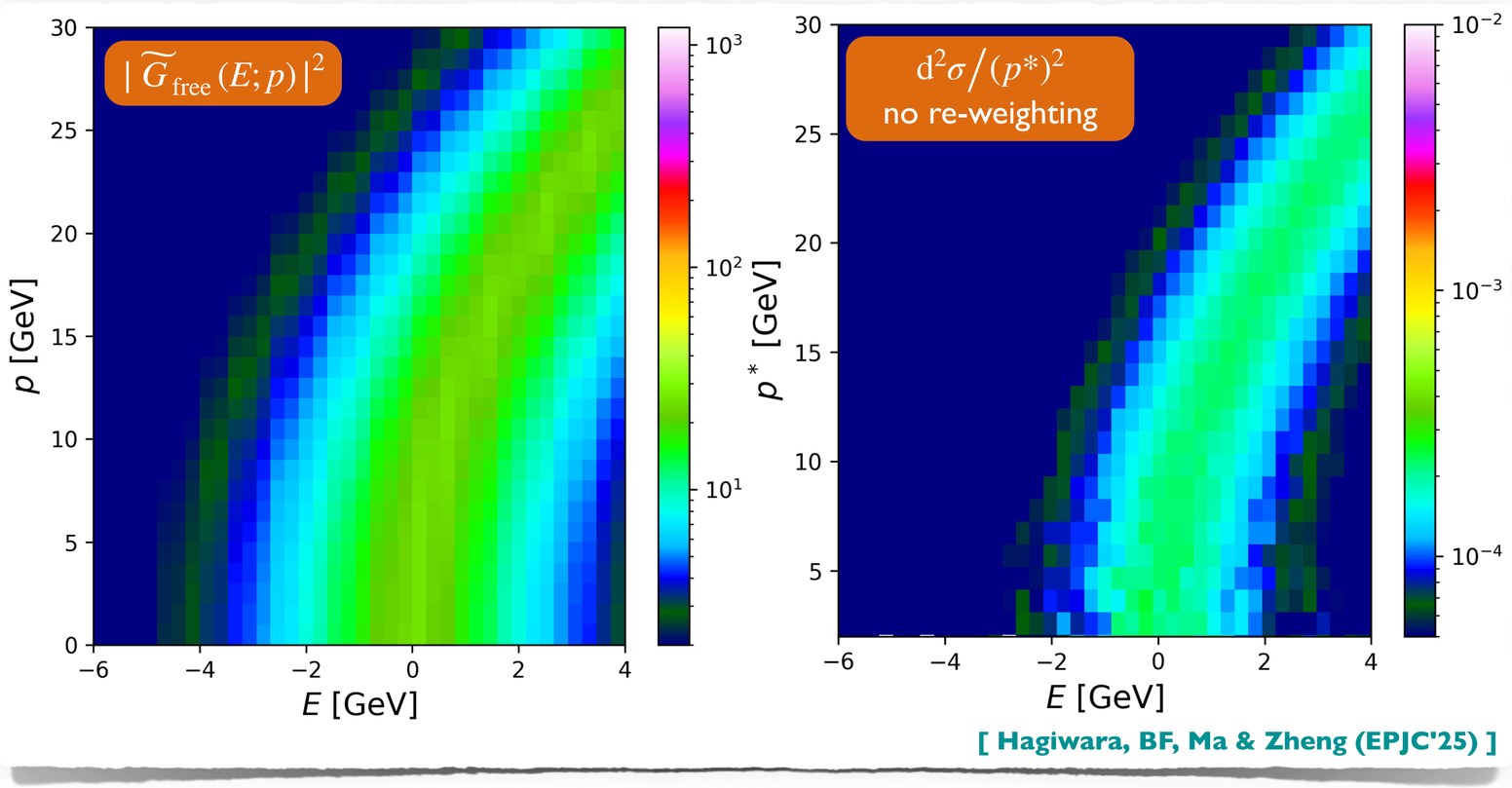

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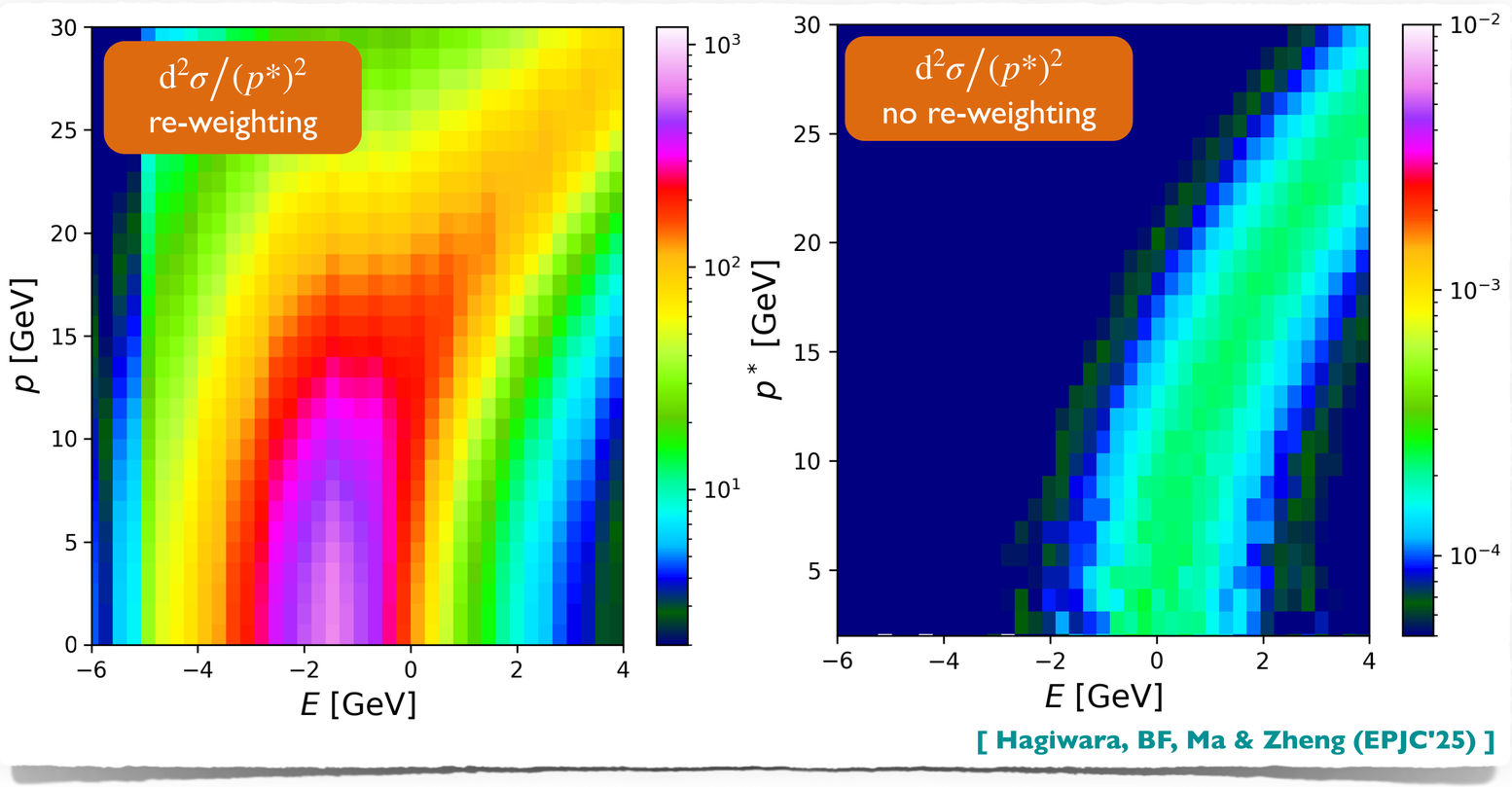

[Hagiwara, BF, Ma & Zheng (EPJC'25)]

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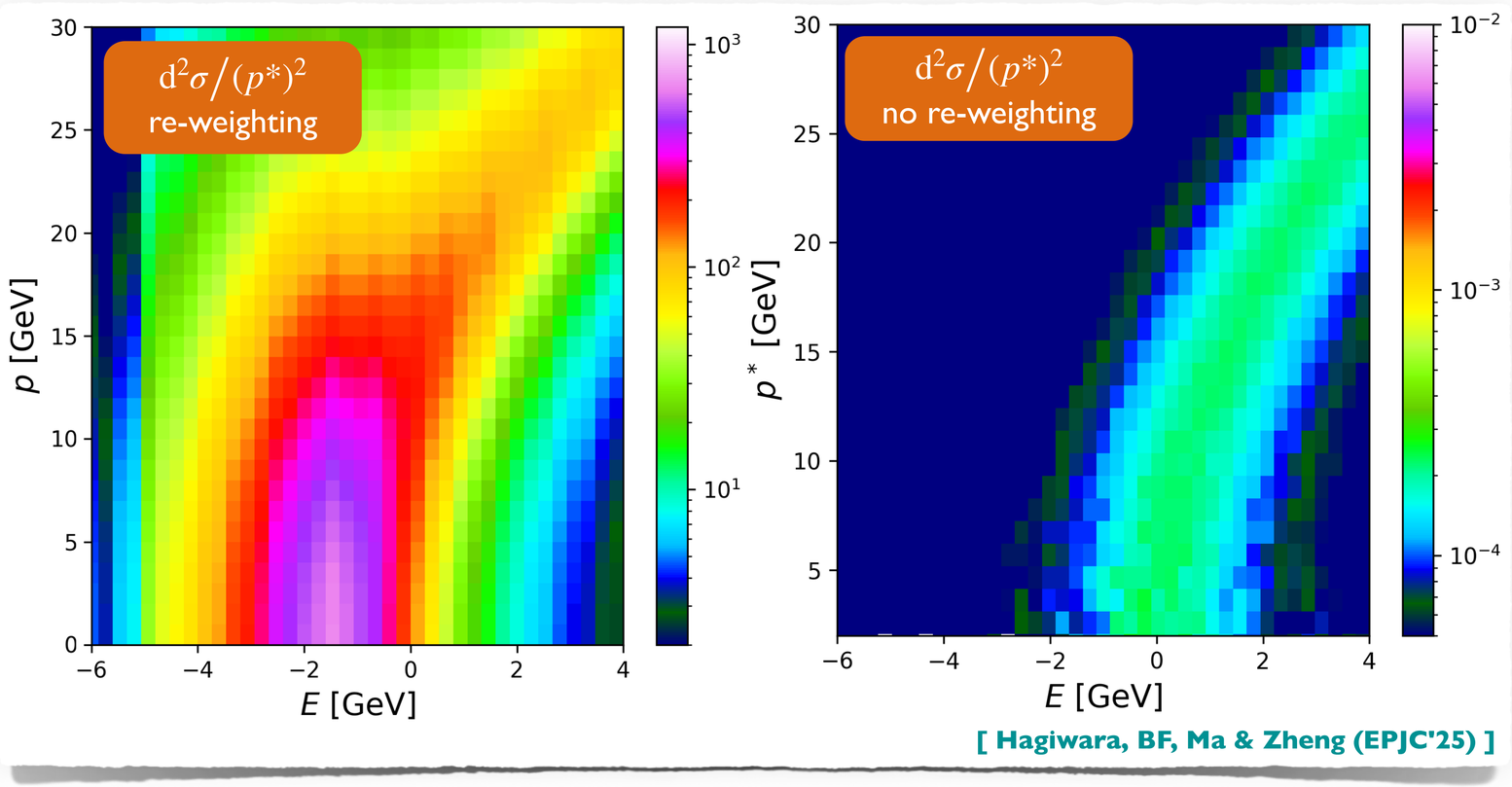

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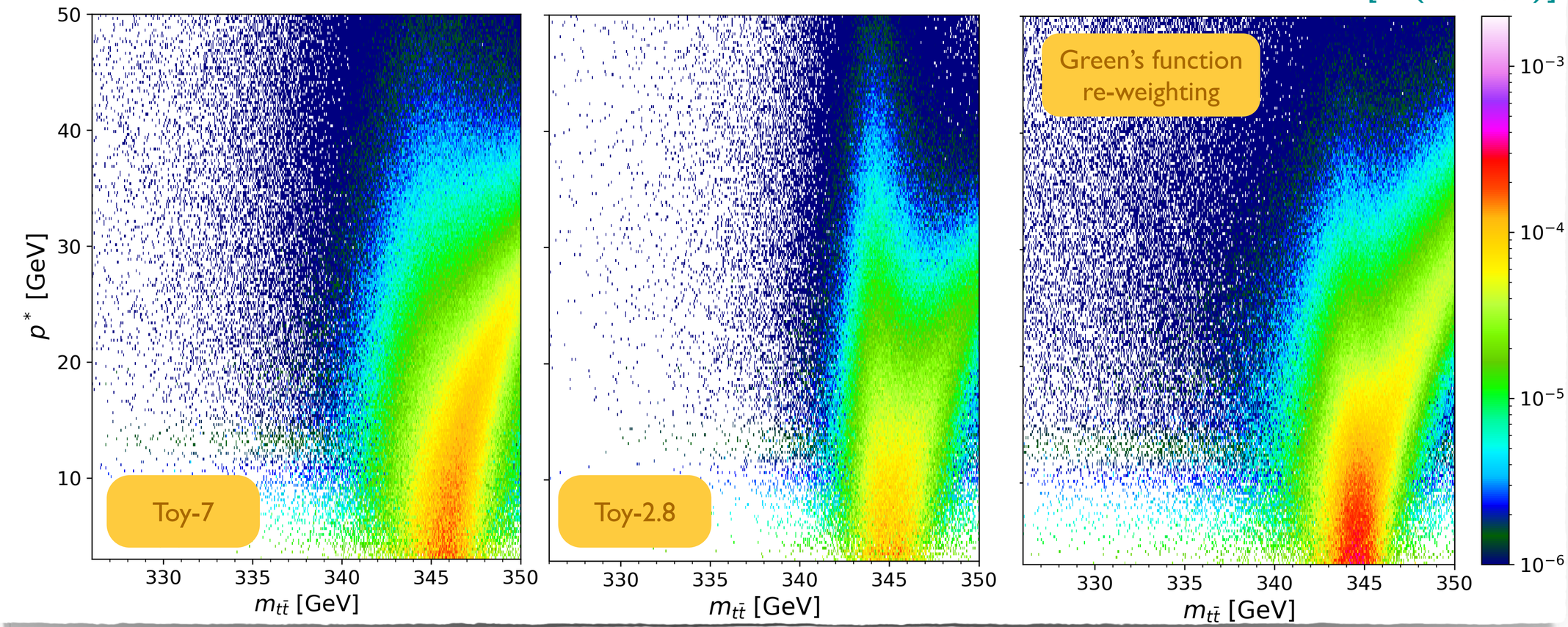
Typical top momentum in the toponium rest frame

$$\langle p(E)\rangle = \frac{\int p\,\mathrm{d}^3p\,\frac{\mathrm{d}\sigma}{p^2\mathrm{d}p\mathrm{d}E}}{\int \mathrm{d}^3p\,\frac{\mathrm{d}\sigma}{p^2\mathrm{d}p\mathrm{d}E}}$$

- For $E \simeq -2$ GeV: 20 GeV (inverse Bohr radius)

Toy models *versus* NRQCD: $\mathrm{d}^2\sigma/(p^*)^2$

[BF (2505.03869)]



Imperfections of the toy model visible in the key distribution

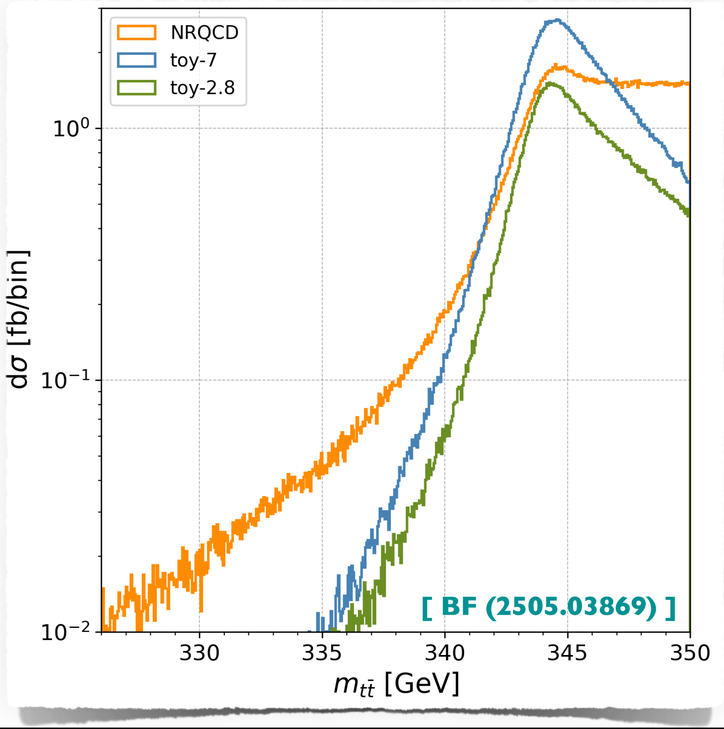
- The m_{tt} spectrum OK in all cases ➙ peak more or less reproduced
  - ⋆ OK today *cf.* experimental resolution
- Strong distortion with the p^* dependence

Toy models or not ?

Toy models and NRQCD: invariant masses

$m_{t\bar{t}}$ invariant mass distribution

- Peak at $E \simeq -2$ GeV well reproduced
- Bound state effects around the peak important
  - ⋆ Today: one bin ➙ the integral matters
- The toy models different from NRQCD
  - ⋆ Run 3: more bins?



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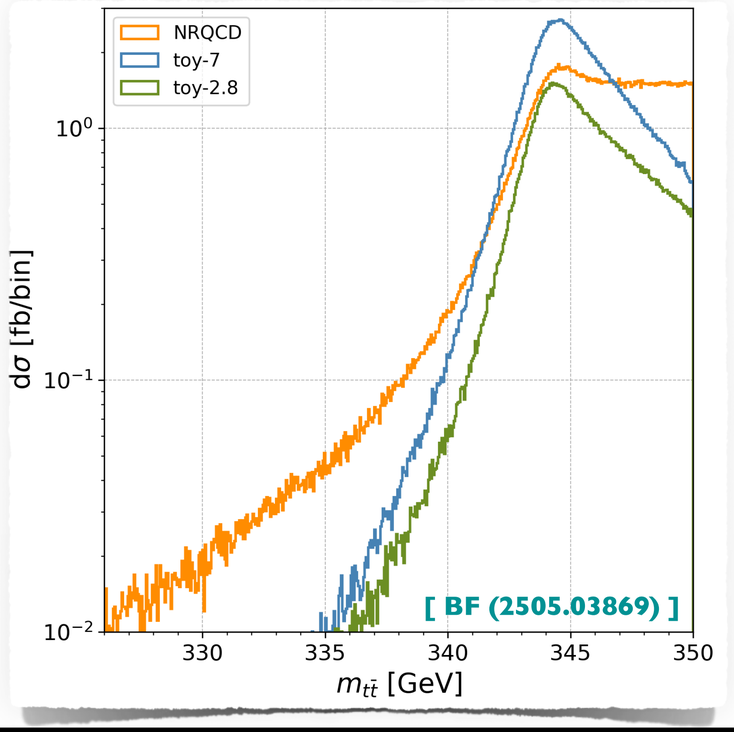
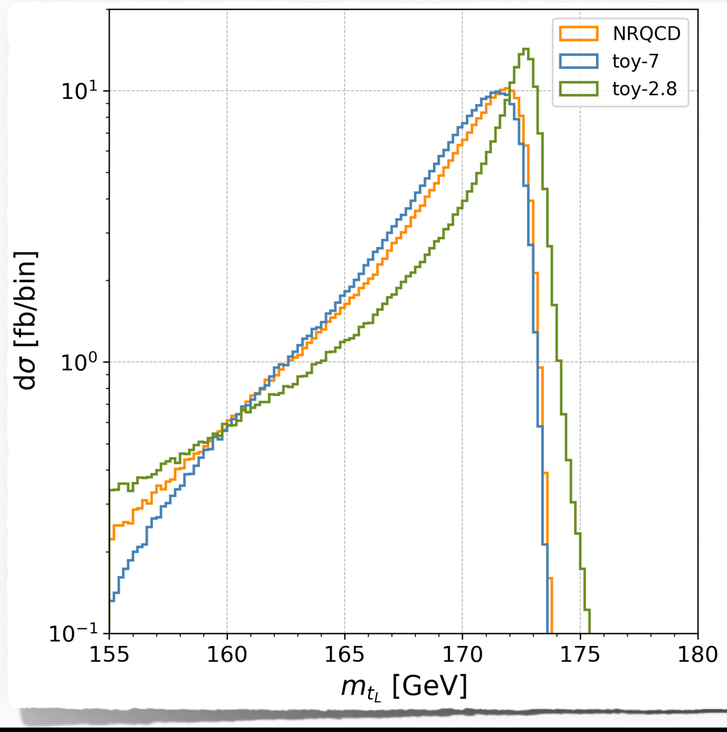
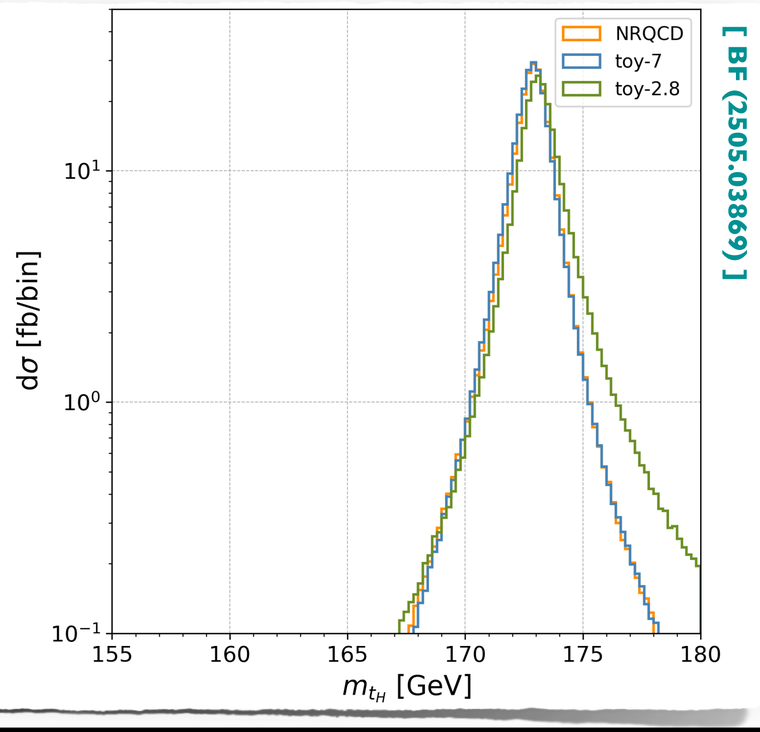
Reconstructed top mass distributions

- Light top ⬌ invariant mass shifted to lower values
  - ➙ Governed by the QCD Green's function (distortion of the shape)
  - ➙ Bound state effects within the Coulomb potential generated by t_H
- Heavy top ⬌ Breit-Wigner
  - ➙ Effectively stable until t_L decays
  - ➙ No QCD potential from the decayed top (free propagation)
- Toy models quite off!







Towards discovery…

Toponium phenomenology: selection of toponium events and excess characterisation

- Selection of toponium events (signal) from conventional top-antitop events (background)
- Tool: spin polarisation density matrices
- Observable: angles of the leptons in their parent top rest frames

$$\sum_{\sigma,\bar{\sigma},\sigma',\bar{\sigma}'} \rho^{\sigma\bar{\sigma},\sigma'\bar{\sigma}'}_{gg\to(t\bar{t})_1} \, \mathrm{d}\rho^{t\to b\ell^+\nu_\ell}_{\sigma,\sigma'} \, \mathrm{d}\rho^{\bar{t}\to\bar{b}\ell'^-\bar{\nu}'_\ell}_{\bar{\sigma}',\bar{\sigma}'} \quad \Rightarrow \quad \frac{\mathrm{d}\Gamma}{\mathrm{d}\cos\theta \, \mathrm{d}\varphi \, \mathrm{d}\cos\bar{\theta} \, \mathrm{d}\bar{\varphi}} \propto \frac{1 + \sin\bar{\theta}\sin\theta\cos(\bar{\varphi}-\varphi) + \cos\bar{\theta}\cos\theta}{2}$$

[Hagiwara, Yokoya & Zheng (JHEP'18)]

★ Small angles favoured

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Toponium characterisation: small $\Delta\varphi_{\ell\ell'}$, $|\Delta\eta_{\ell\ell'}|$ and $m_{\ell\ell'}$

- Measurement of lepton properties in the lab frame
- Small azimuthal angle separation (survives boost to the lab frame)
- Small di-lepton invariant mass

$$m^2_{\bar{\ell}\ell'} = 2E_{\bar{\ell}}E_{\ell'}\left(1-\sin\bar{\theta}\sin\theta\cos(\bar{\varphi}-\varphi)-\cos\bar{\theta}\cos\theta\right)$$

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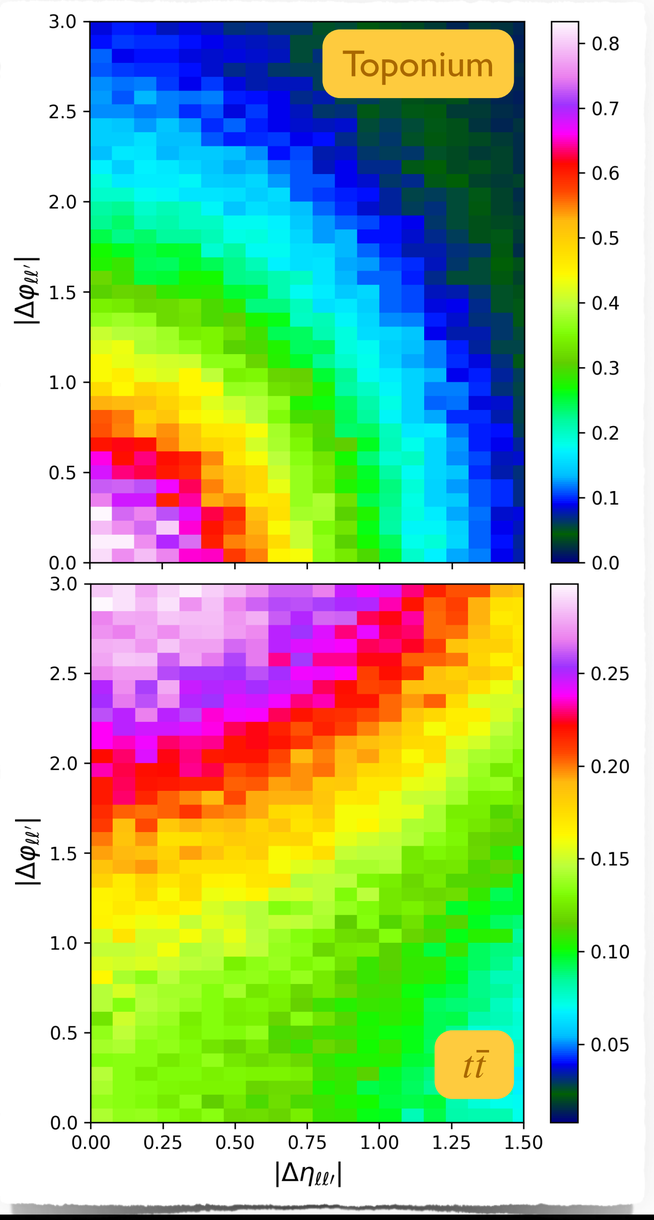
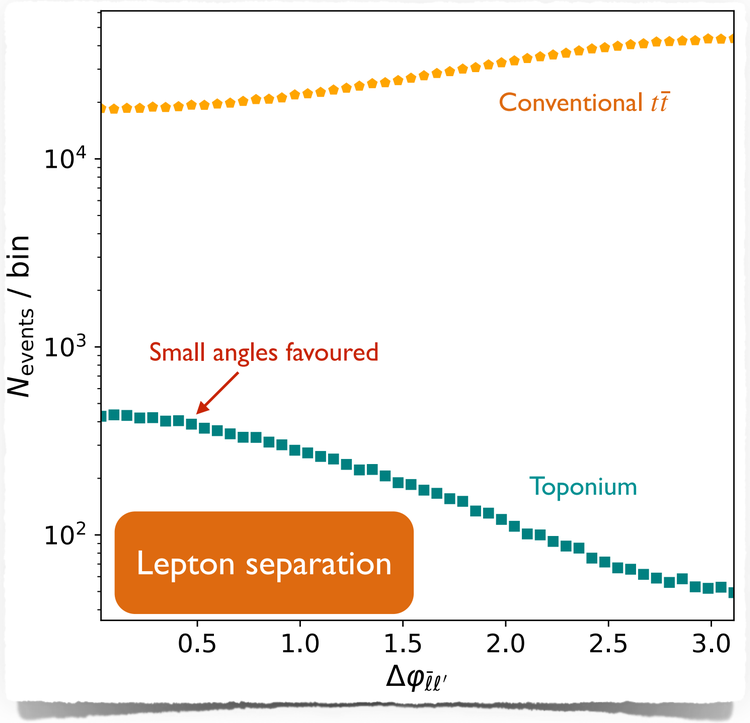
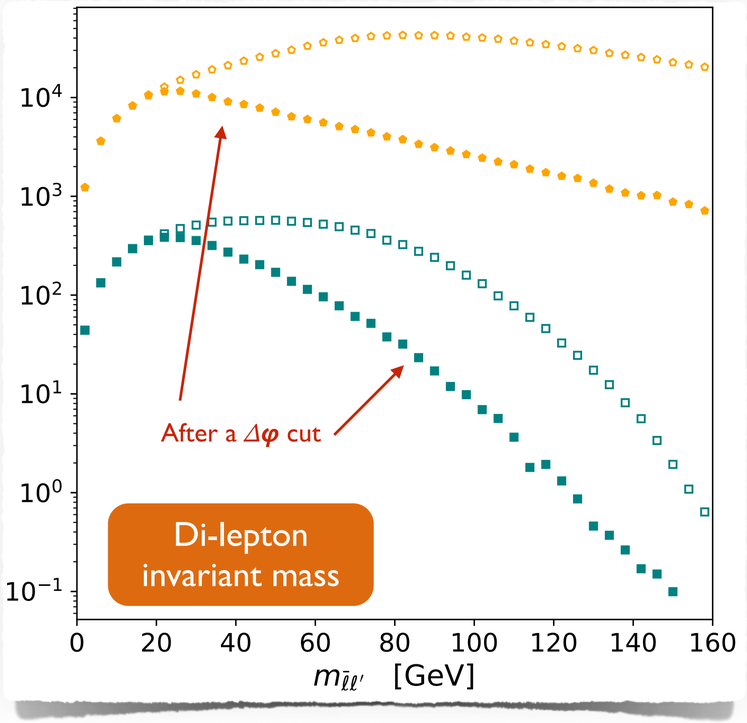
Three potential handles on the signal



Observables at reconstructed level

[Hagiwara, BF, Ma & Zheng (PRD`21)]





Top reconstruction

- Determination of the neutrino momenta
  → W mass ⊕ top mass ⊕ ν mass ⊕ p_T assumptions
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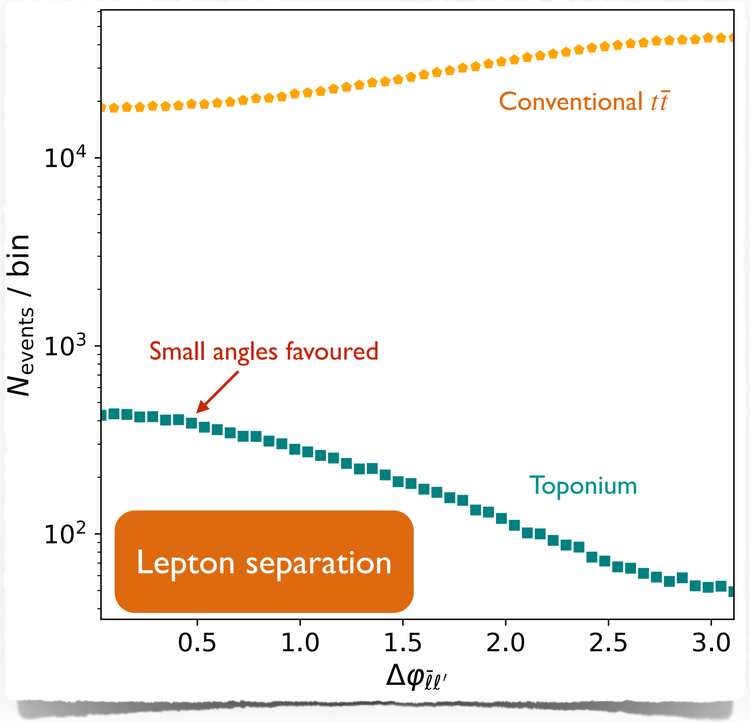

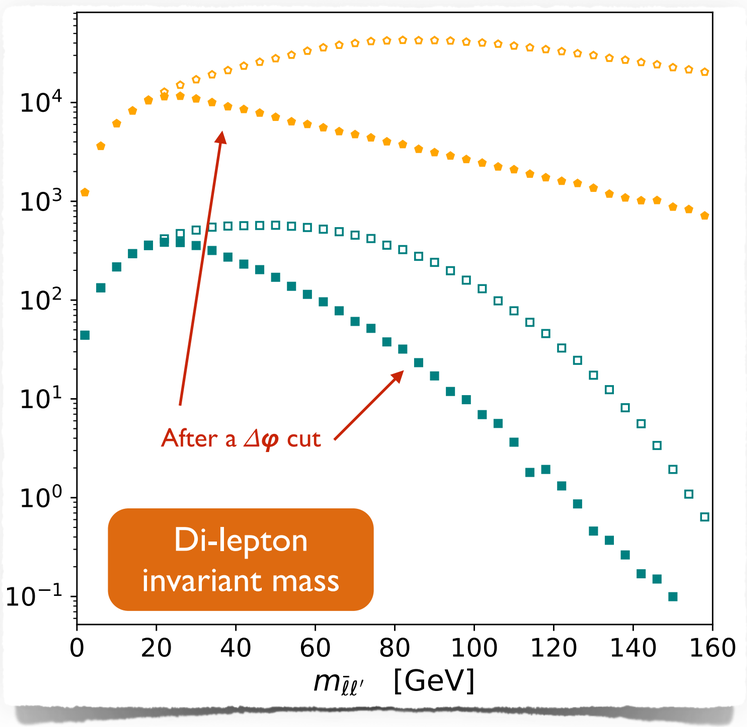

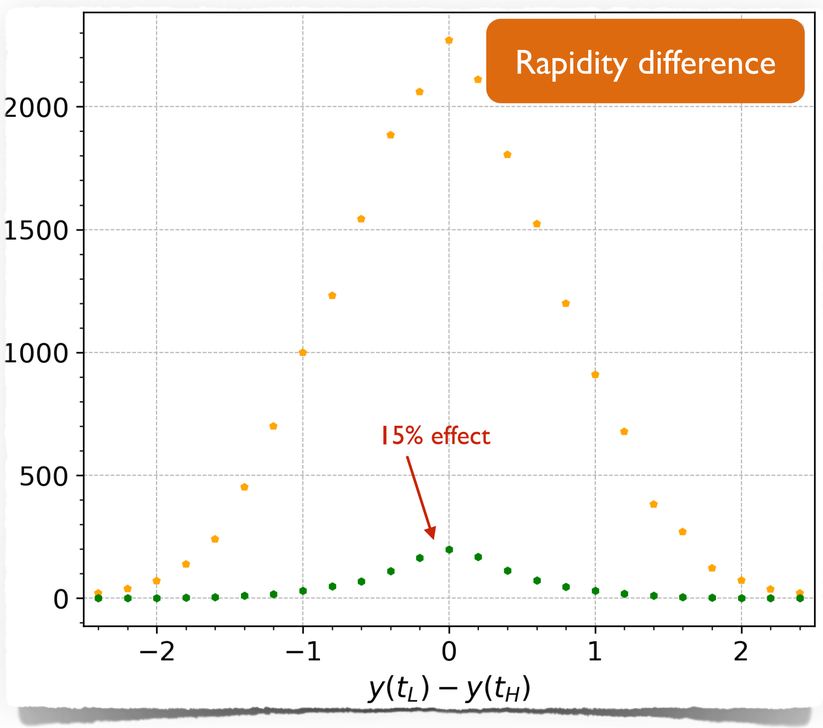

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13 TeV, 140/fb

Cut	$t\bar{t}$	Toponium	Ratio
Initial	113,000,000	900,000	0.0079
Di-lepton	1,370,000	10,300	0.0076
$\Delta\varphi_{\bar{\ell}\ell'}$	178,000	4,060	0.023
$m_{\bar{\ell}\ell'}$	77,900	2,760	0.035
$m_T(\bar{\ell}\ell' b\bar{b}; \nu_\ell\bar{\nu}_{\ell'})$	40,800	2,460	0.060
$t\bar{t}$ kinematical fit	20,400	1,420	0.070

More (better?) selection possibilities…

Toponium at the LHC run 2 (140/fb of pp collisions at 13 TeV)

- Basic selection (2 leptons and 2 *b*-jets)

- *Option A:* $\Delta\varphi_{\ell\ell'} < 0.9; \Delta\eta_{\ell\ell'} < 0.9$
  ➙ $N_{t\bar{t}} = 127{,}000; N_{\text{toponium}} = 3{,}520$ [sensitivity of 9.7σ (S/B ~ 2.8%)]

- *Option B:* $\Delta\varphi_{\ell\ell'} < \pi/5; m_{\ell\ell'} < 40$ GeV
  ➙ $N_{t\bar{t}} = 77{,}100; N_{\text{toponium}} = 2{,}200$ [sensitivity of 7.8σ (S/B ~ 2.9%)]

- *Option C:* $\Delta R^2_{\ell\ell'} < 0.8$
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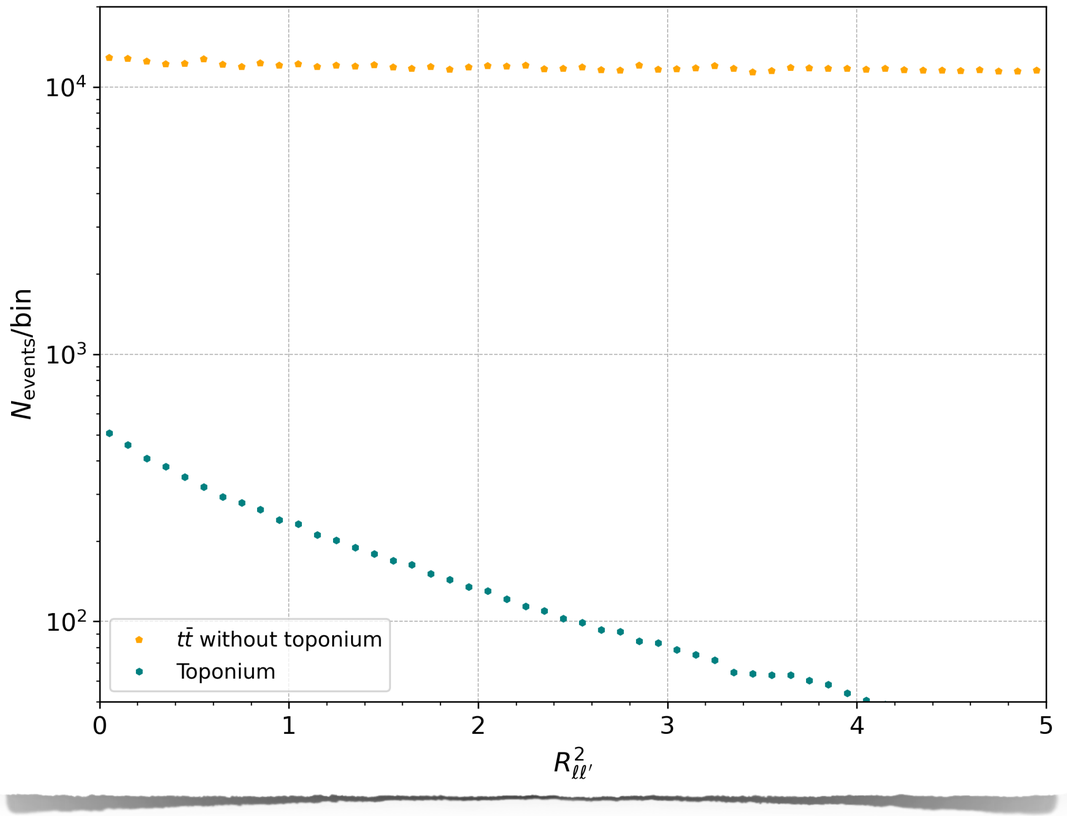

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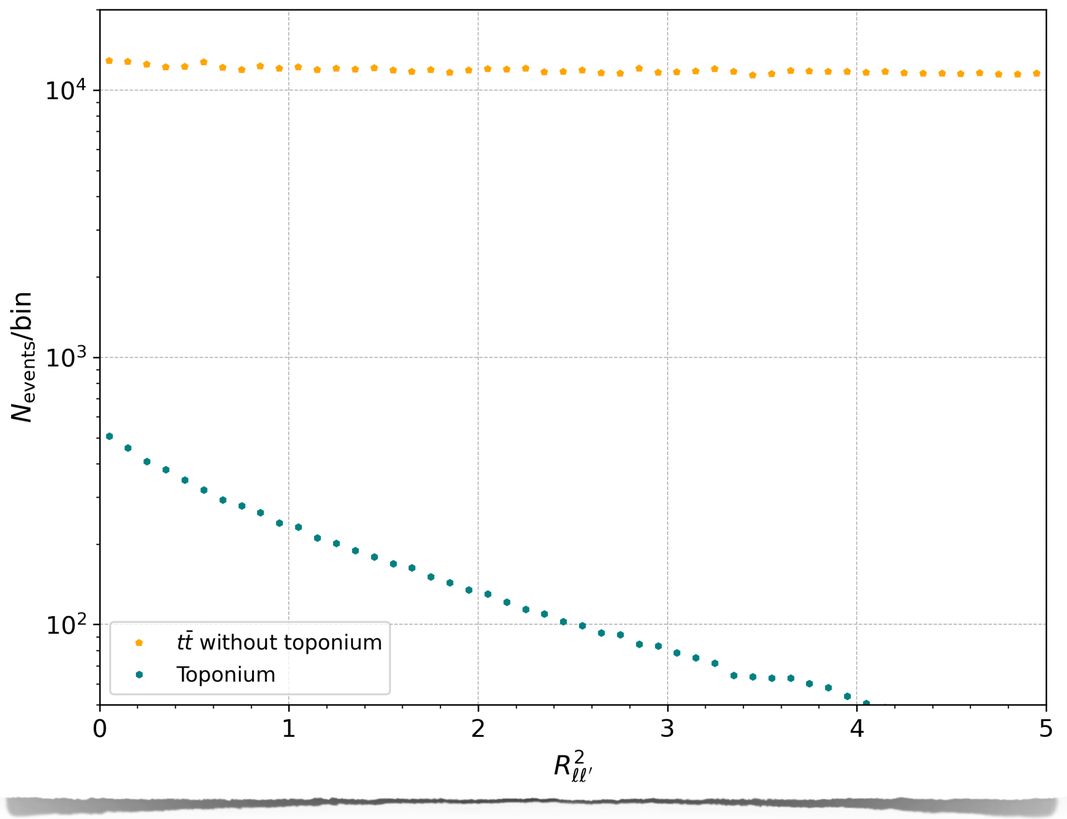
Excellent (theoretical) sensitivity with almost no cuts

[Hagiwara, BF, Ma & Zheng (EPJC'25)]



And the data?

Combined analysis of all released run 2 distributions (ATLAS + CMS)

- Single and double differential distributions in $m_{\ell\ell}$, $\Delta\varphi_{\ell\ell}$, $\Delta\eta_{\ell\ell}$
- Particle-level observables **[ATLAS (EPJC`20); CMS (JHEP`19)]**
- Available Rivet routines **[Bierlich et al. (SciPost'24)]**

Investigations

- Location of toponium contributions
- Impact related to uncertainties
- Fit under the H_0 (conventional SM) and $H_{toponium}$ (SM+toponium) hypotheses
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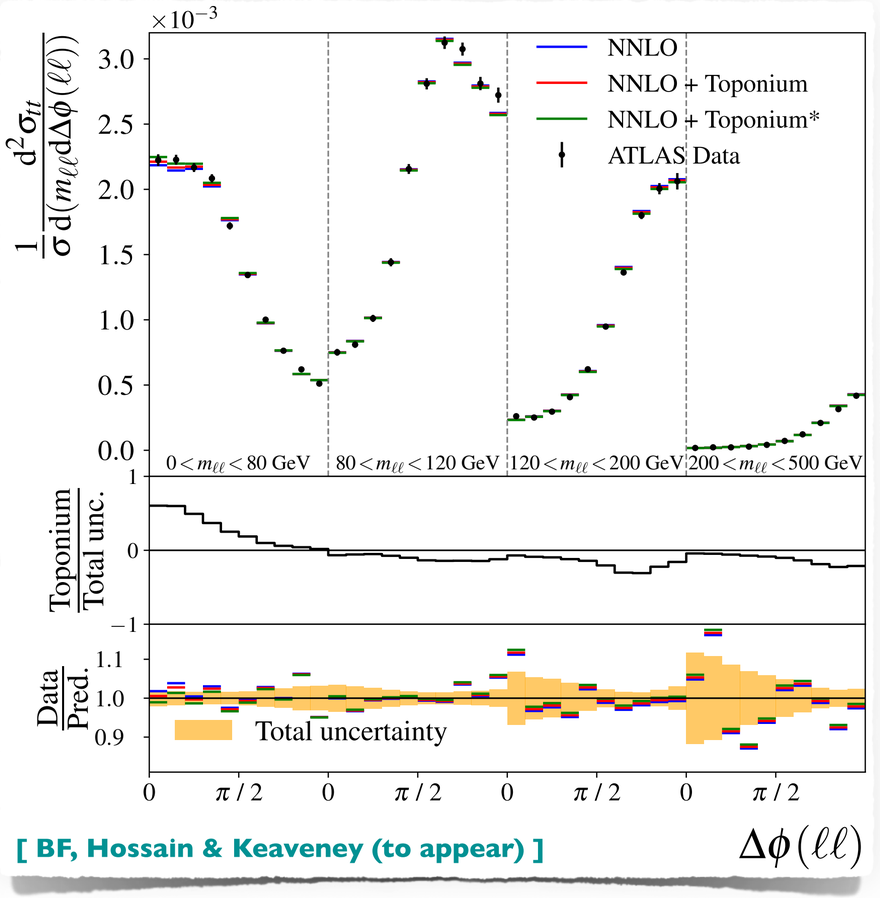
Observable	Exp.	H_0		$H_{toponium}$		B_{10}
		p	Z	p	Z	
$\frac{d^2\sigma}{dm_{\ell\ell}\, d\Delta\phi(\ell\ell)}$	ATLAS	$8{\cdot}10^{-4}$	3	0.04	2	27
$\frac{d\sigma}{dm_{\ell\ell}}$	ATLAS	$4{\cdot}10^{-5}$	4	$9{\cdot}10^{-4}$	3	20
$\frac{d\sigma}{dm_{\ell\ell}}$	CMS	0.002	3	0.02	2	7
$\frac{d\sigma}{d\Delta\phi(\ell\ell)}$	ATLAS	0.02	2	0.1	1	4
$\frac{d\sigma}{d\Delta\phi(\ell\ell)}$	CMS	0.3	0.5	0.3	0.4	1
$\frac{d\sigma}{d\Delta\eta(\ell\ell)}$	CMS	$5{\cdot}10^{-4}$	3	0.003	3	4



[BF, Hossain & Keaveney (to appear)]

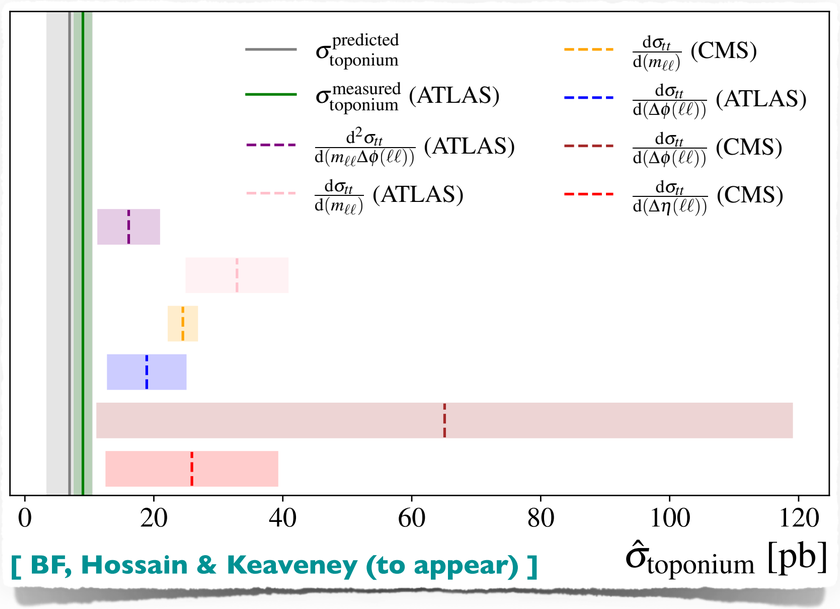
Strong evidence for $H_{toponium}$!

- Bayes factors > 10
- $p(H_0) < p(H_{toponium})$
  ➞ Toponium improves the theory-data agreement
  ➞ $p(H_{toponium})$ still small…

And BSM states?

Room for new physics close to the $t\bar{t}$ threshold?

- Yes: fitted cross sections larger than expectations (but errors still very large)
- Care with interpretations



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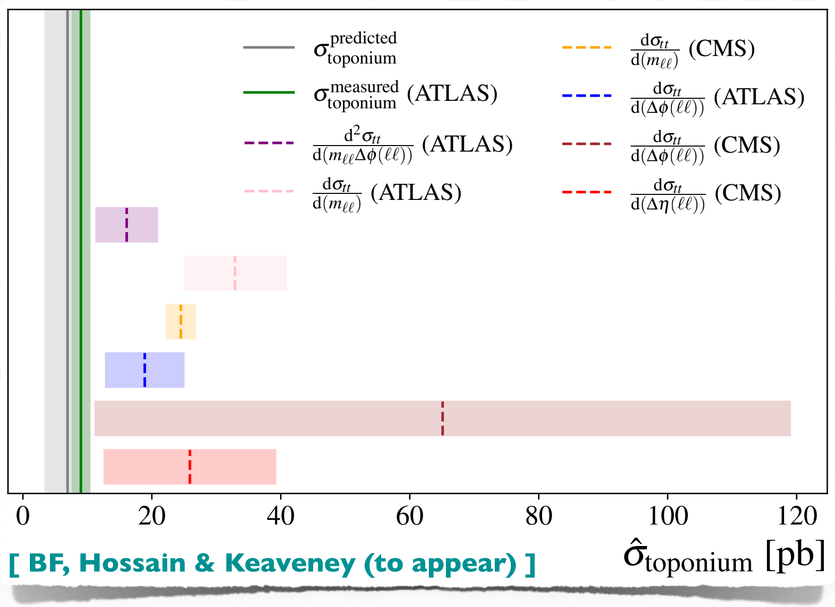

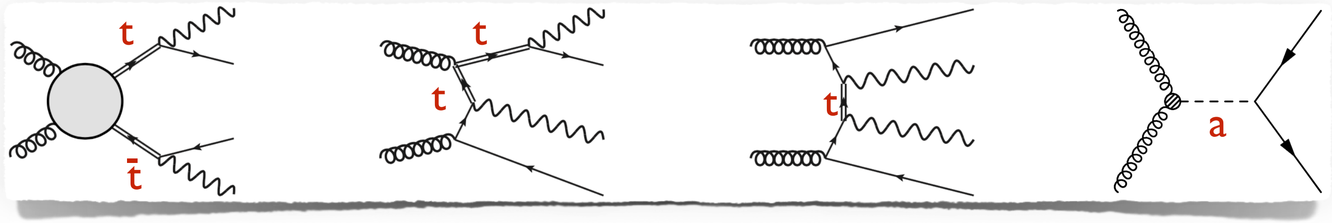

Light pseudo-scalars a

- Ubiquitous of composite constructions
- 3D parameter space: $\{m_a, c_t, \kappa_G\}$ ➙ fixed Γ_a/m_a to 1%
- Copiously produced at the LHC: $m_a \sim 2m_t$
  - ⋆ Interference of toponium and conventional SM+BSM $t\bar{t}$ production
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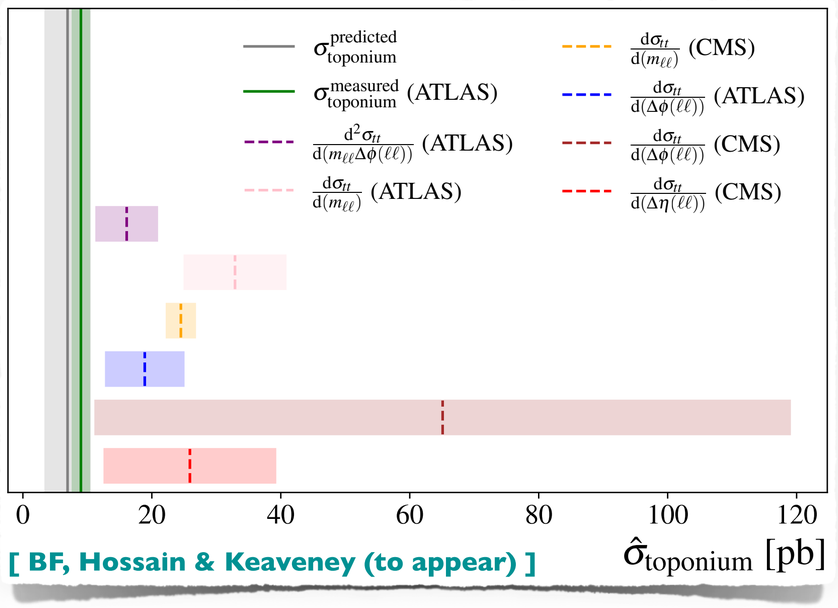
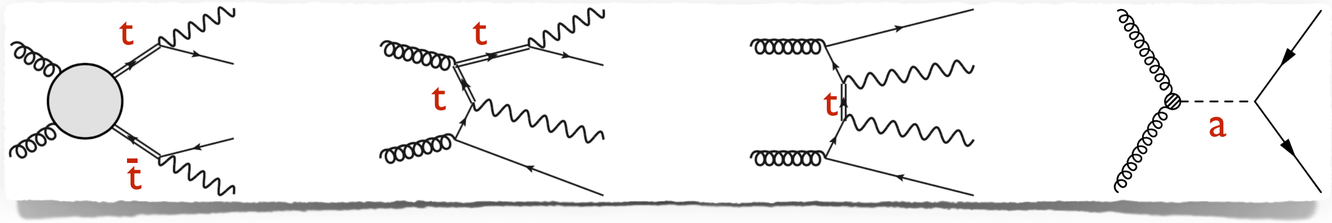


[BF, Hossain & Keaveney (to appear)]

Opening of the parameter space

- Significant viable regions
- Toponium effects cannot be neglected



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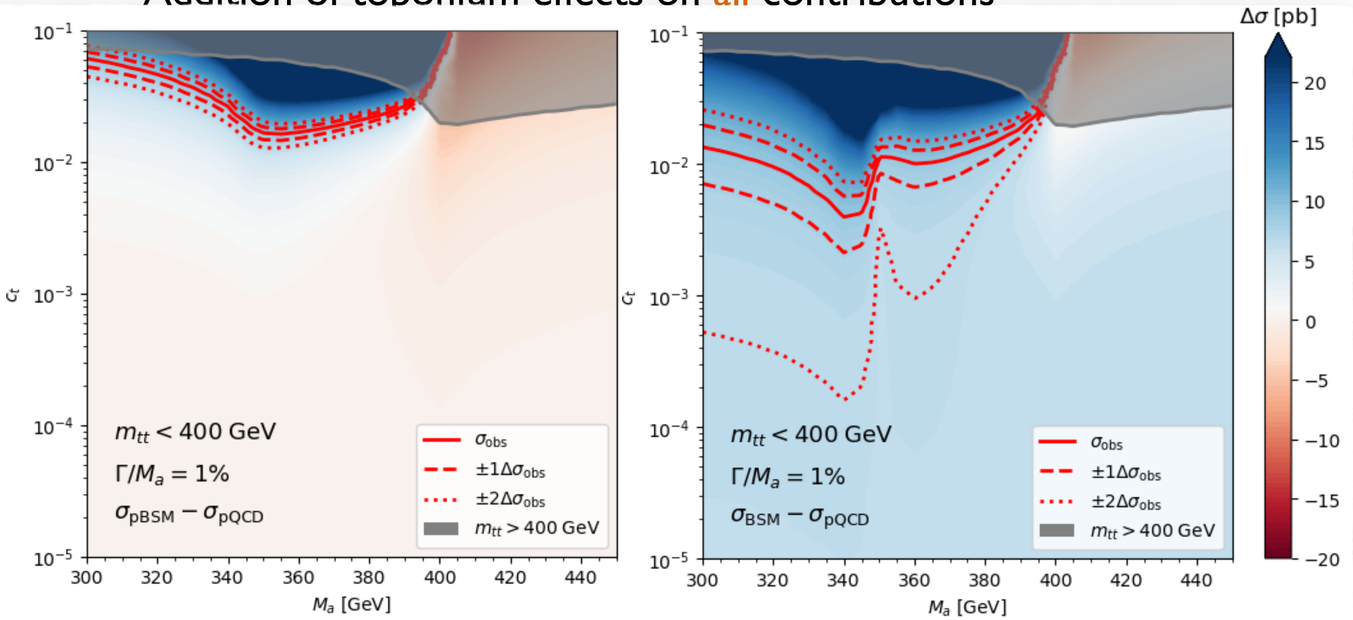
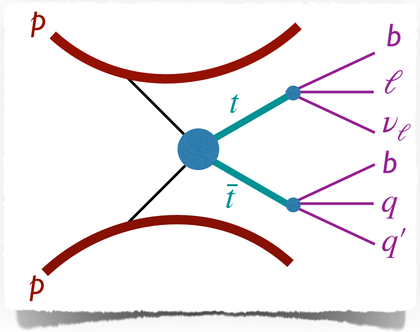


[Flacke, BF, Kim, Kim, Lee & Munoz-Aillaud (to appear)]

Single-leptonic mode



Top-antitop production in proton-proton collisions

- Other final state: one single lepton
- About 20% of all top-antitop events (more statistics)
- Easier reconstruction (one single neutrino)

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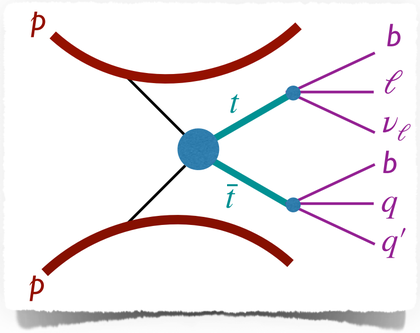

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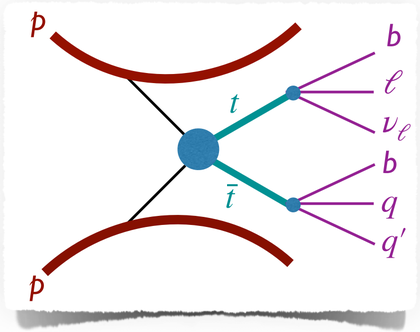

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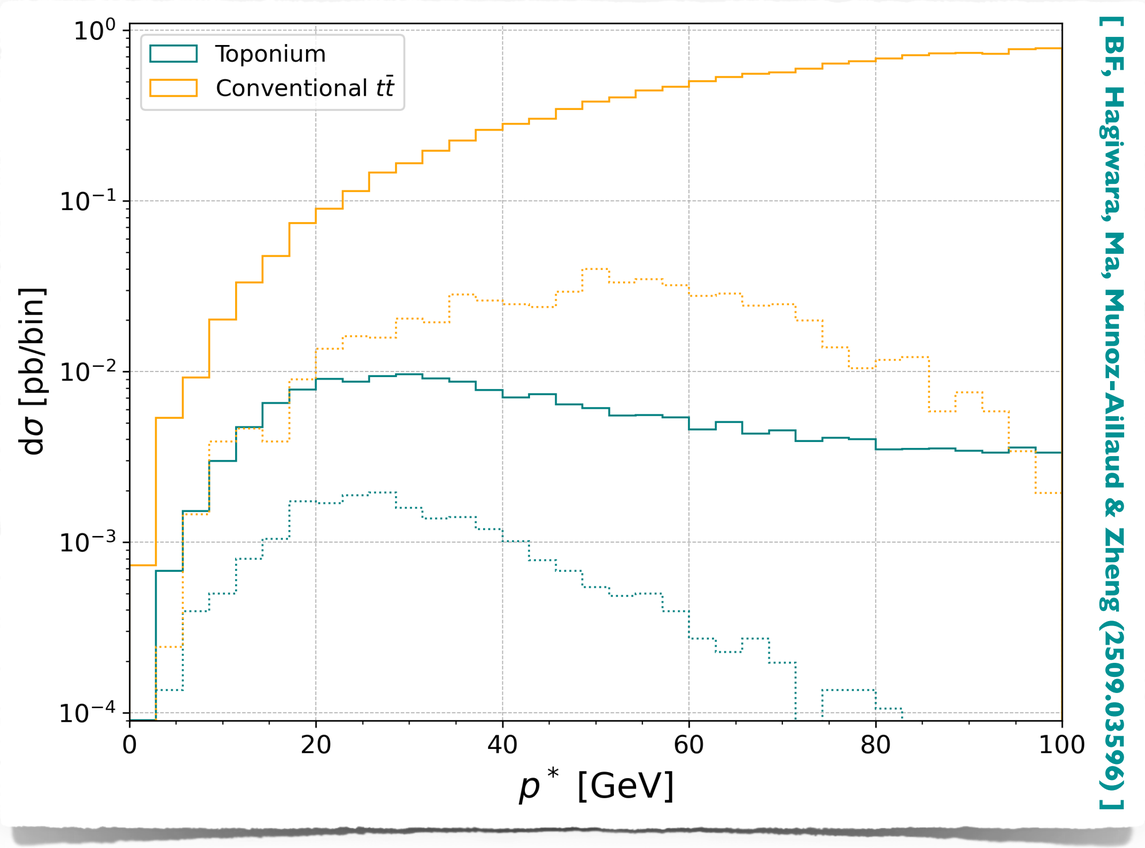
Exploiting the large stats for toponium characterisation

- p* distribution with correct behaviour
- Bound-state dynamics of the top quark



A few last words…

Top-antitop production near the threshold

- Emergence of a toponium system at a scale of 0.05 GeV^{-1}
- Decay at a time scale of ~ 0.3 GeV^{-1}
- Occurs well before hadronisation at 5 GeV^{-1}

Possibility to probe the toponium wave function

- ‘ The smallest hadron in the Standard Model ’

To-do list (for the next four years!)

- Better potential modelling
- Colour octet (no resonant enhancement)
- Matching NRQCD to NLO-QCD + parton showers
- Impact on top mass measurements