The renormalization of the Standard Model effective field theory

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based on *2106.05291*, *2106.05291*, *2205.03301*, *2409.15408*

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The SMEFT is the SM extended with effective operators

(Probably) the most reasonable model of new physics

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{c_i^{(6)}}{\Lambda^2}\mathcal{O}_i^{(6)} + \sum_j \frac{c_j^{(8)}}{\Lambda^4}\mathcal{O}_j^{(8)} + \cdots$$

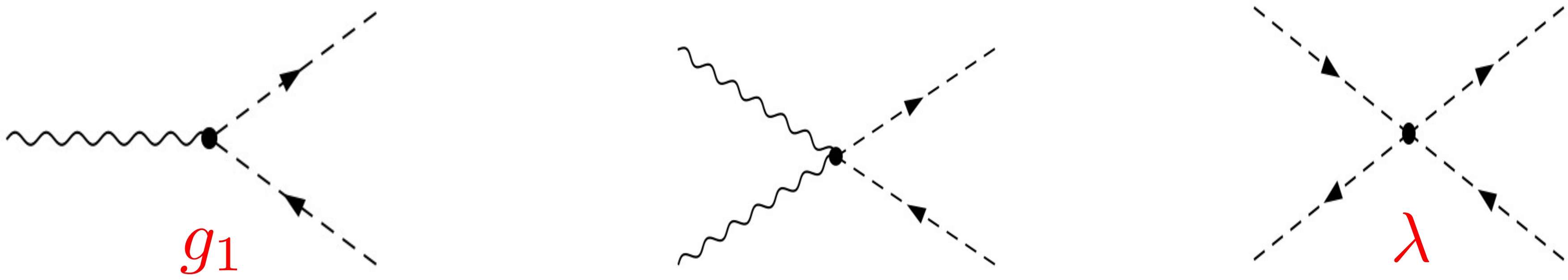
If probed by experiments at very different scales, RGEs of the theory are needed [Jenkins, Manohar, Trott, Alonso '13].

Interesting theoretical aspects at dimension-8 (positivity, tree-loop mixing, test tools, ...)

How things work at lower energy dimension

Renormalization within simplified SM at dimension-4 and dimension-6

$$\mathcal{L}_{\rm SM} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} + D_\mu\phi^\dagger D^\mu\phi - \lambda|\phi|^4 + \cdots$$



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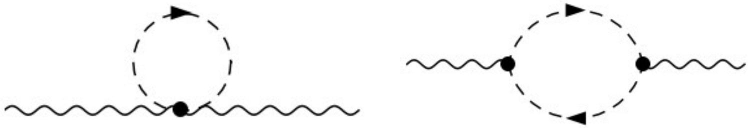
Renormalization within simplified SM at dimension-4 and dimension-6

$$\beta_{g_1} = \frac{41}{6} g_1^3 + 0 \times \lambda$$

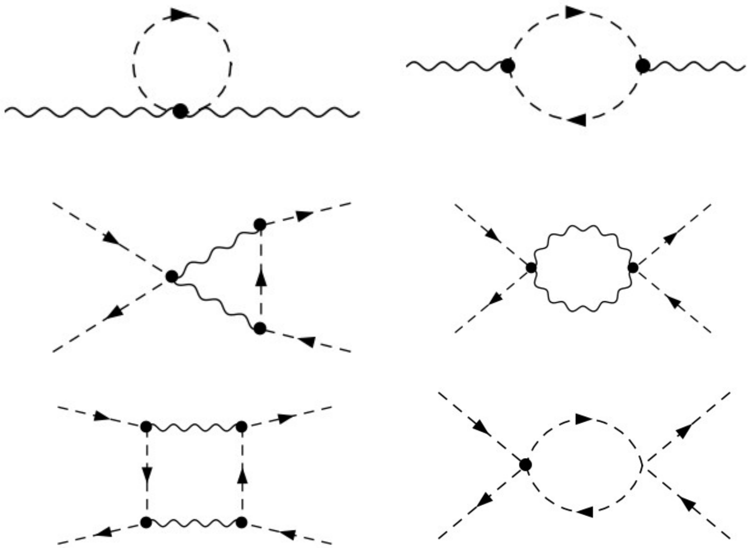


How things work at lower energy dimension

Renormalization within simplified SM at dimension-4 and dimension-6

$$\beta_{g_1} = \frac{41}{6} g_1^3 + 0 \times \lambda$$

$$\beta_\lambda = \frac{3}{8} g_1^4 + 24\lambda^2$$



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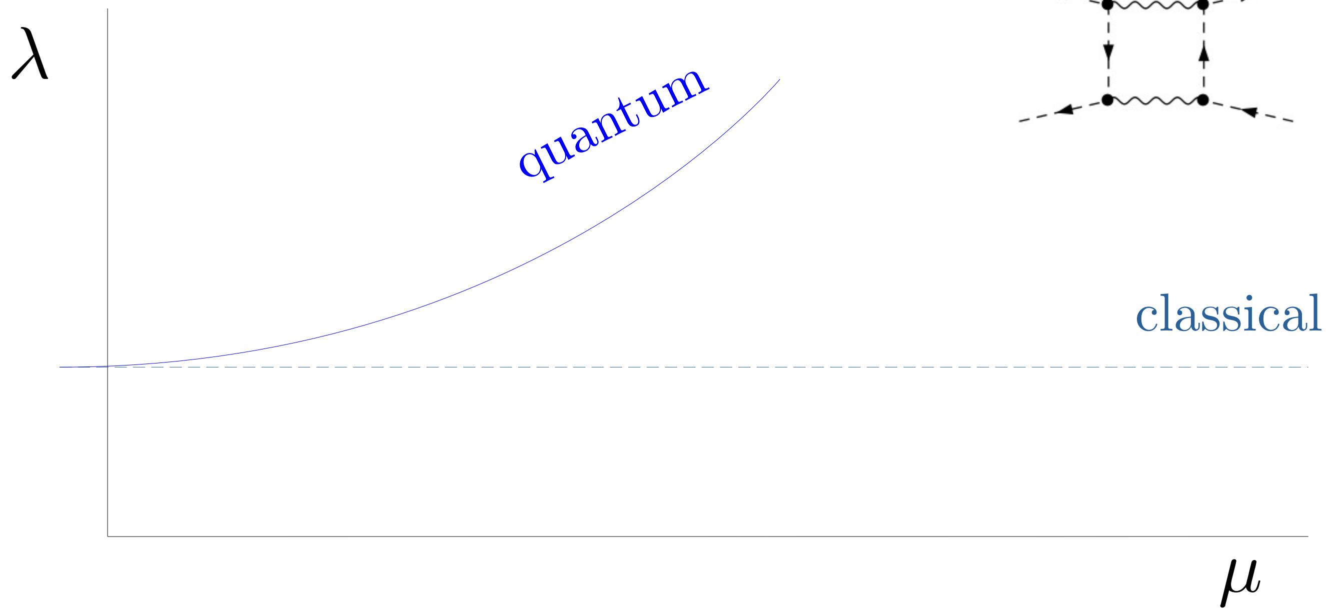
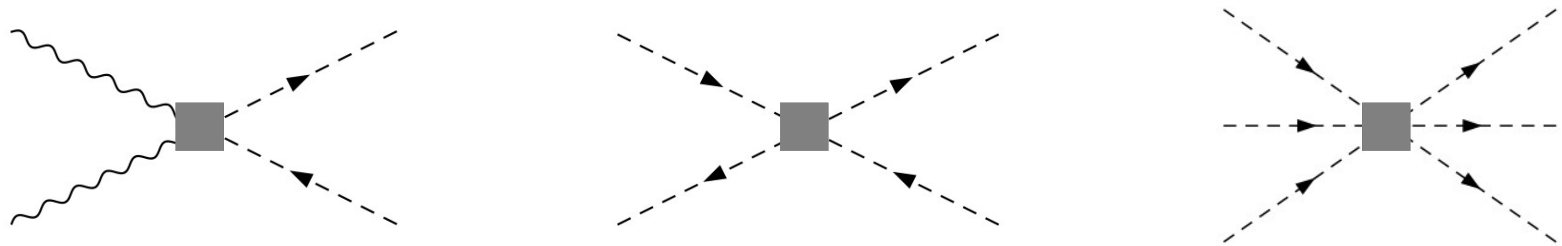


How things work at lower energy dimension

Renormalization within simplified SM at dimension-4 and dimension-6

$\mathcal{O}(\frac{1}{\Lambda^2})$



$c_{\phi^2 B^2}$ $\quad$ $c_{\phi^4 D^2}$ $\quad$ c_{ϕ^6}

$$\dot{c}_i \equiv \beta_{c_i} = \gamma_{ij} c_j$$

$$\gamma_{ij} = \begin{array}{c} \\ c_{\phi^2 B^2} \\ c_{\phi^4 D^2} \\ c_{\phi^6} \end{array} \begin{array}{c} \begin{array}{ccc} c_{\phi^2 B^2} & c_{\phi^4 D^2} & c_{\phi^6} \end{array} \\ \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix} \end{array}$$

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Renormalization within simplified SM at dimension-4 and dimension-6

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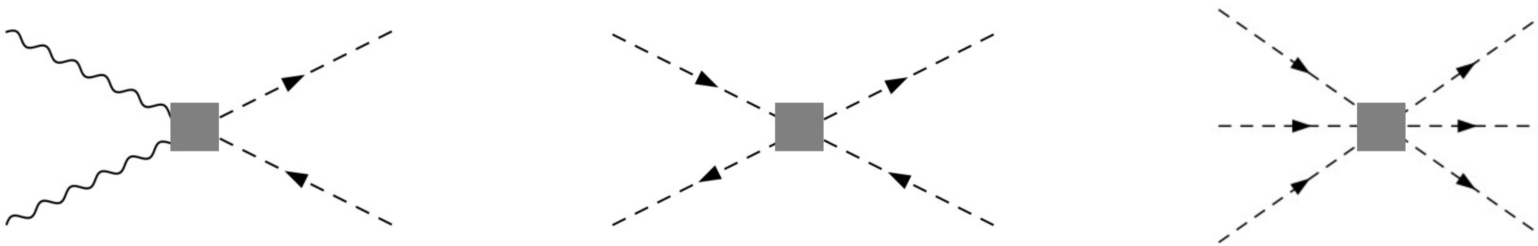
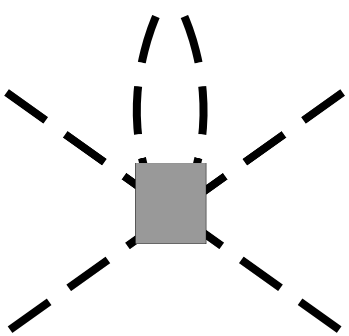


$c_{\phi^2 B^2}$ $c_{\phi^4 D^2}$ c_{ϕ^6}



$$\gamma_{ij} = \begin{array}{c} \\ c_{\phi^2 B^2} \\ c_{\phi^4 D^2} \\ c_{\phi^6} \end{array} \begin{array}{c} \begin{array}{ccc} c_{\phi^2 B^2} & c_{\phi^4 D^2} & c_{\phi^6} \end{array} \\ \left(\begin{array}{ccc} \times & \times & 0 \\ \times & \times & 0 \\ \times & \times & \times \end{array} \right) \end{array}$$

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Renormalization within simplified SM at dimension-4 and dimension-6

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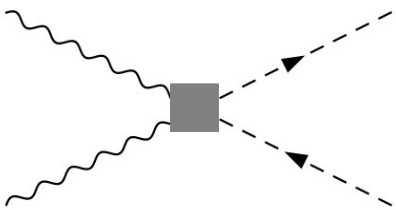
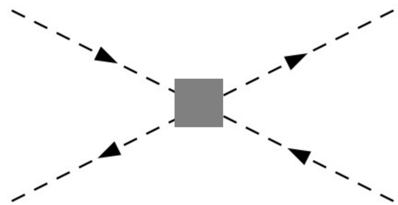
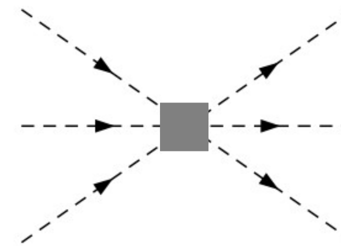


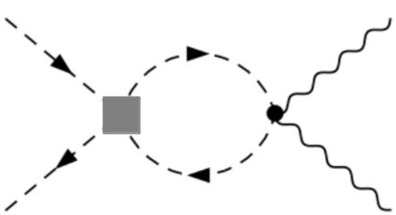
$c_{\phi^2 B^2}$

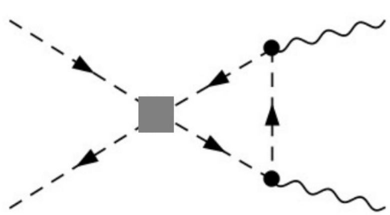


$c_{\phi^4 D^2}$



c_{ϕ^6}

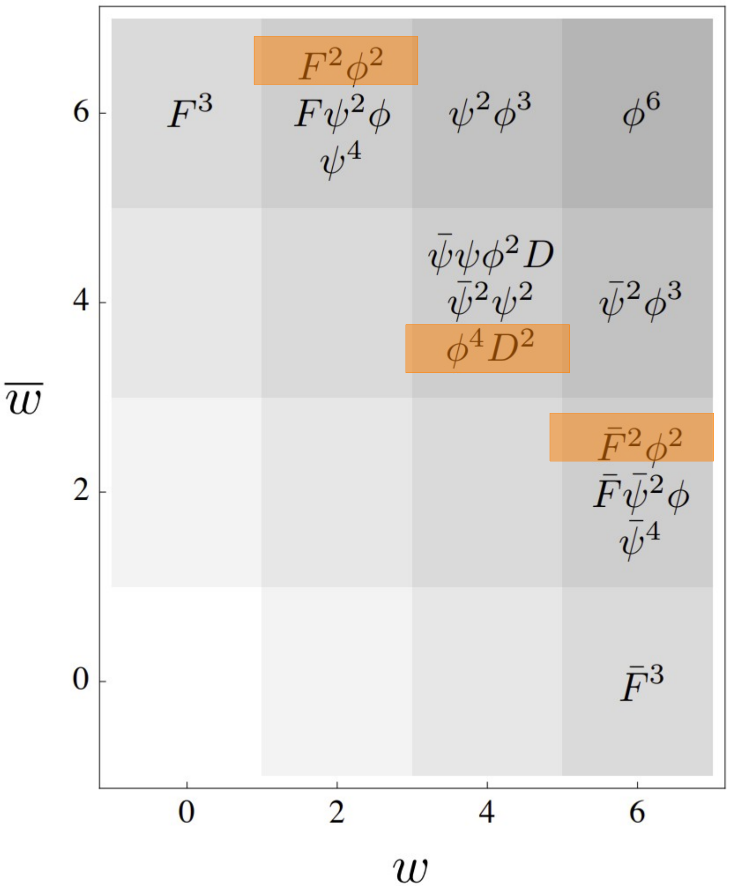




$$\gamma_{ij} = \begin{array}{c} \\ c_{\phi^2 B^2} \\ c_{\phi^4 D^2} \\ c_{\phi^6} \end{array} \begin{array}{c} \begin{array}{ccc} c_{\phi^2 B^2} & c_{\phi^4 D^2} & c_{\phi^6} \end{array} \\ \begin{pmatrix} \times & {\color{red}0} & 0 \\ \times & \times & 0 \\ \times & \times & \times \end{pmatrix} \end{array}$$

How things work at lower energy dimension

Renormalization within simplified SM at dimension-4 and dimension-6 [Cheung and Shen '15; Craig, Jiang, Li, Sutherland '20]



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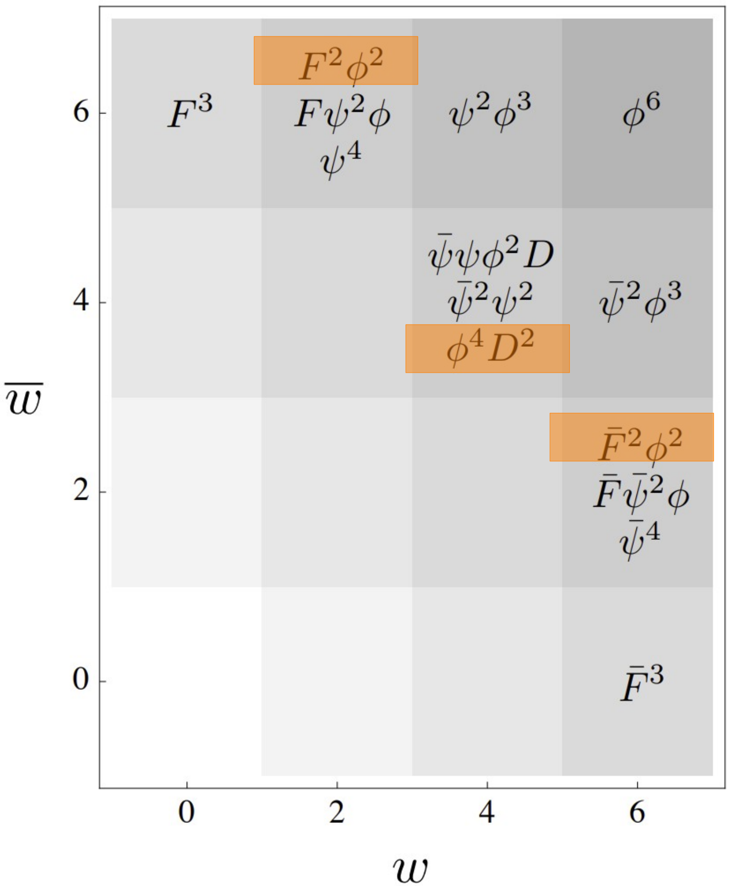

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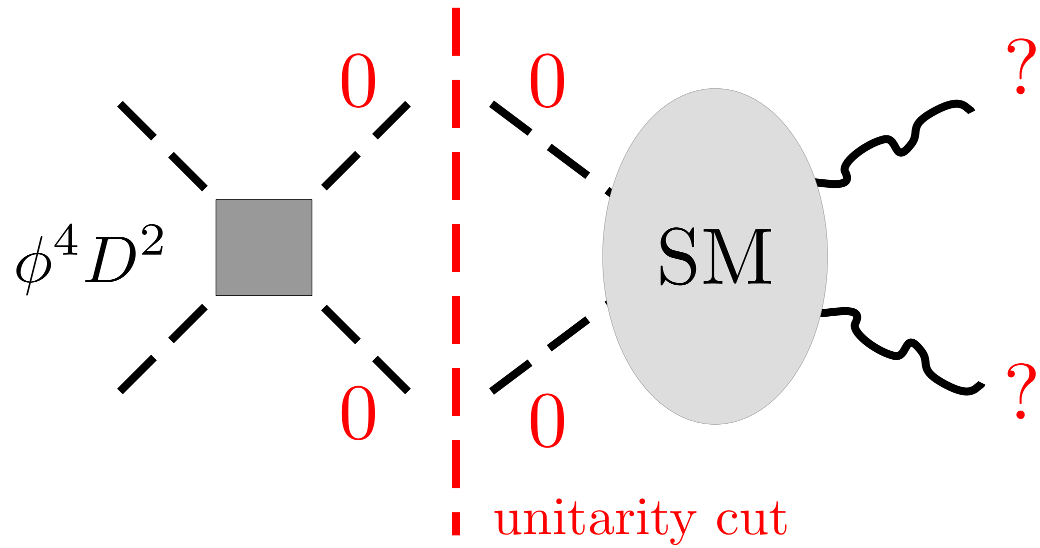

$$F^2\phi^2 \sim \frac{\langle 12\rangle^2}{\Lambda^2}$$

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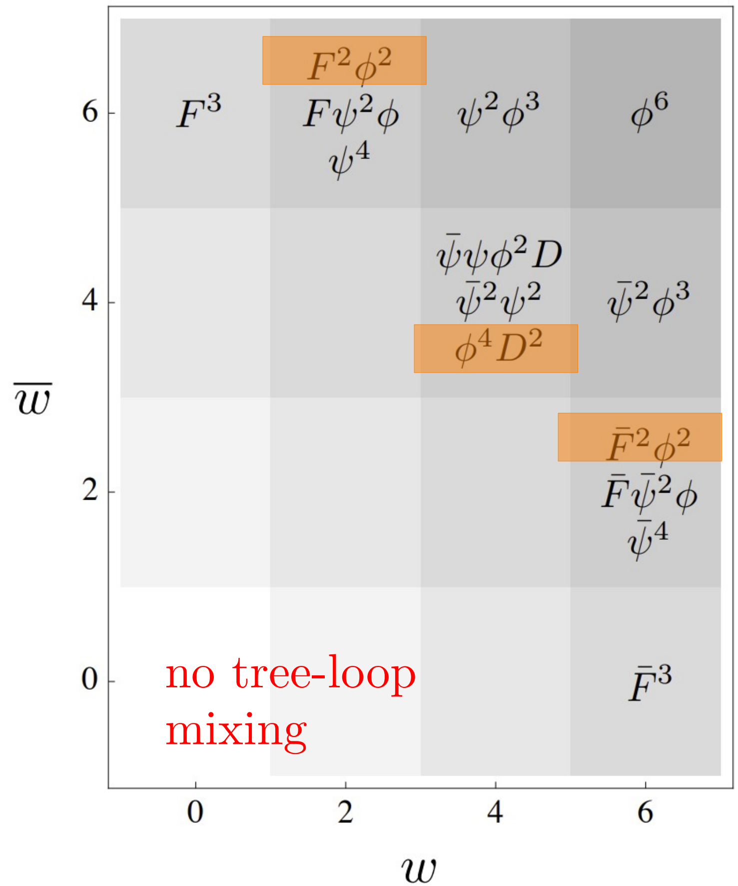

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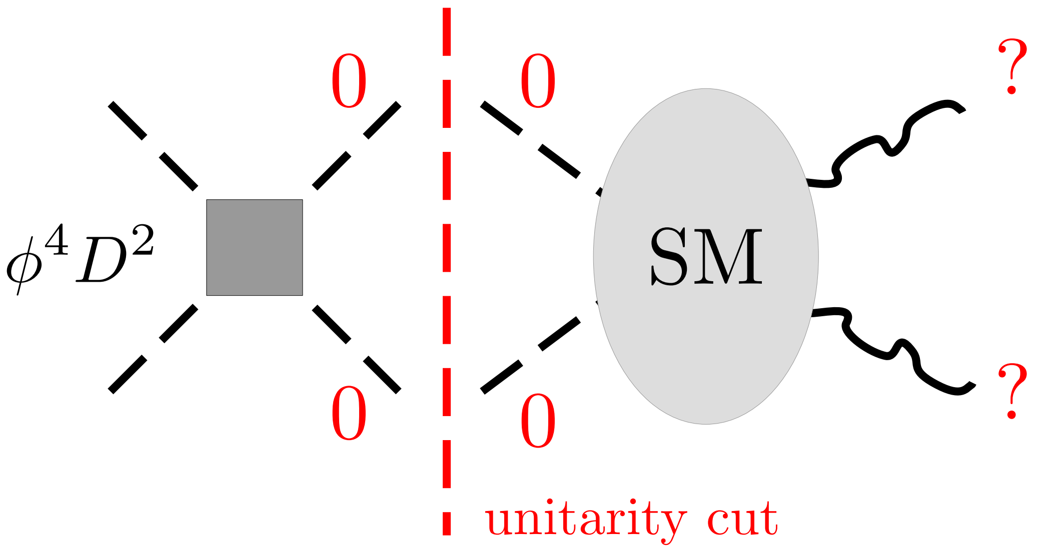
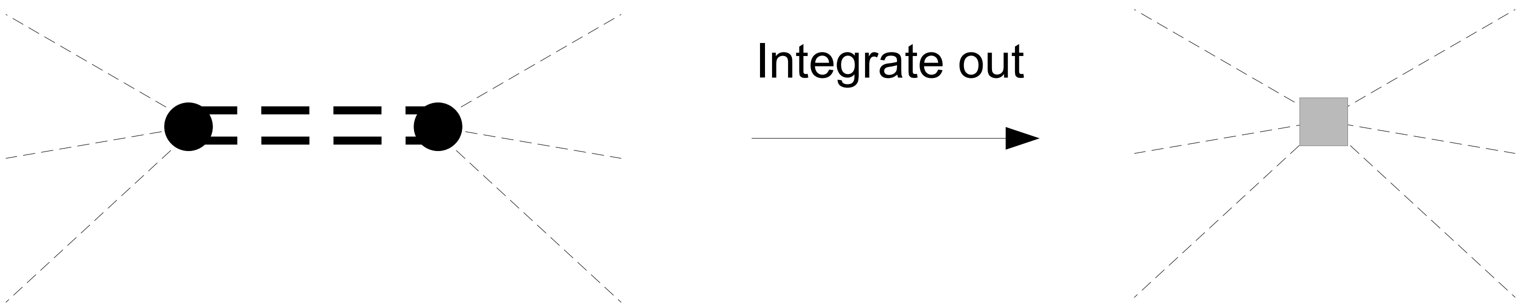


$$F^2\phi^2 \sim \frac{\langle 12\rangle^2}{\Lambda^2}$$

Besides pure theoretical considerations, anomalous dimensions of dimension-8 operators [Murphy '20; Li, Ren, Shu, Xiao, Yu, Zheng '20] not always phenomenologically irrelevant

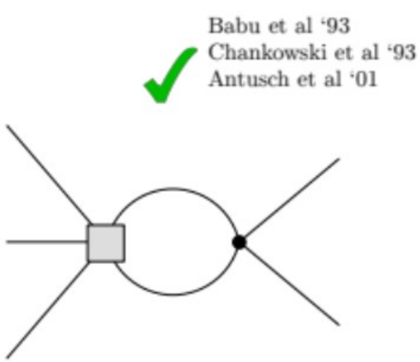
Simplest example:



Custodial symmetry violation absent at tree-level dimension-6, one-loop dimension-6 and tree-level dimension-8 [**MC**, Krause, Nardini '18; Durieux, McCullough, Salvioni '22]

The status of the SMEFT renormalization a few years back:

$$\mu\frac{d\alpha^i}{d\mu} = g_{ij}\alpha^j + g'_{ijk}\alpha^j\alpha^k + g''_{ijkl}\alpha^j\alpha^k\alpha^l + g'''_{ijklm}\alpha^j\alpha^k\alpha^l\alpha^m$$



$$\mathcal{O}^{(5)}_{\ell\phi} = \epsilon_{ij}\epsilon_{mn}\left(\ell^i C\ell^m\right)\phi^j\phi^n$$

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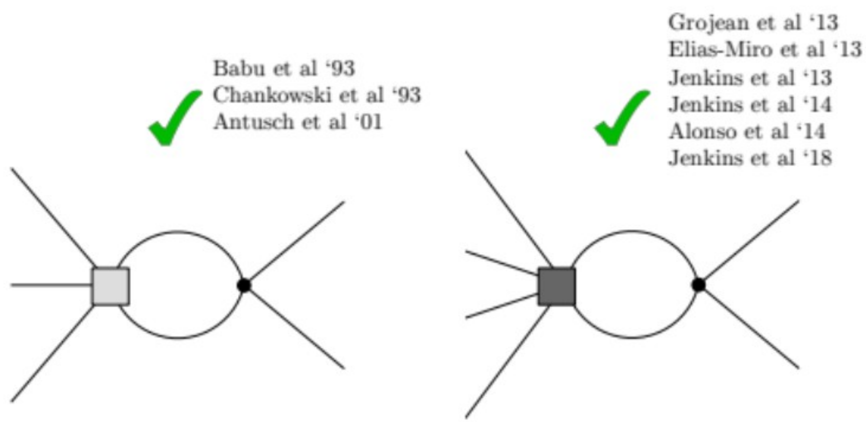

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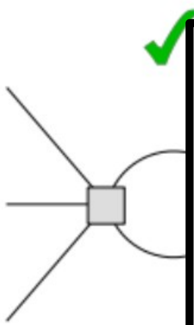

Aneesh V. Manohar

2025 recipient

For outstanding contributions to the physics of baryons, including deriving many physical properties of nucleons and hyperons in the large number of colors limit of quantum chromodynamics and deriving the renormalization group evolution of the standard model effective field theory at one loop.

Elizabeth E. Jenkins

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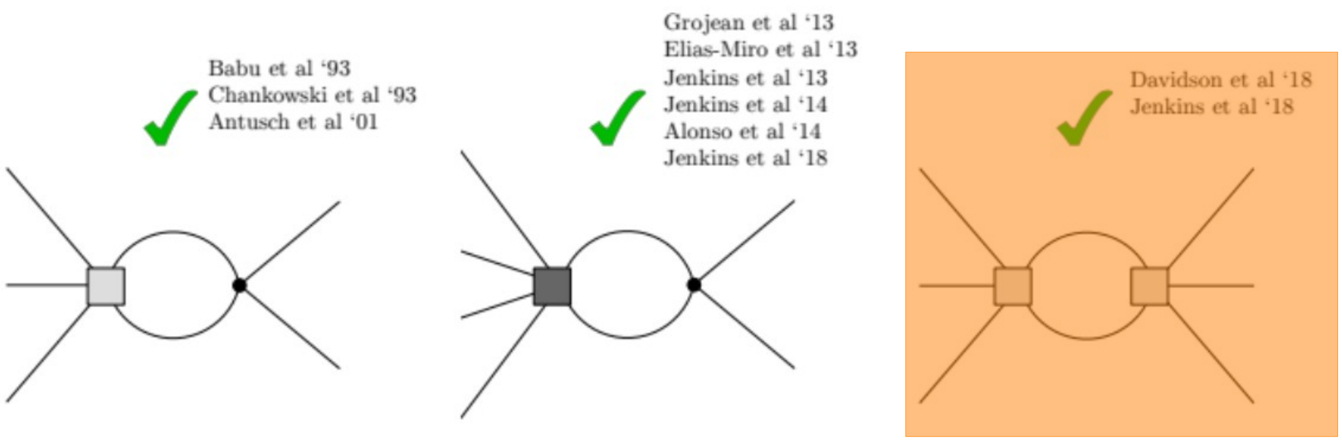

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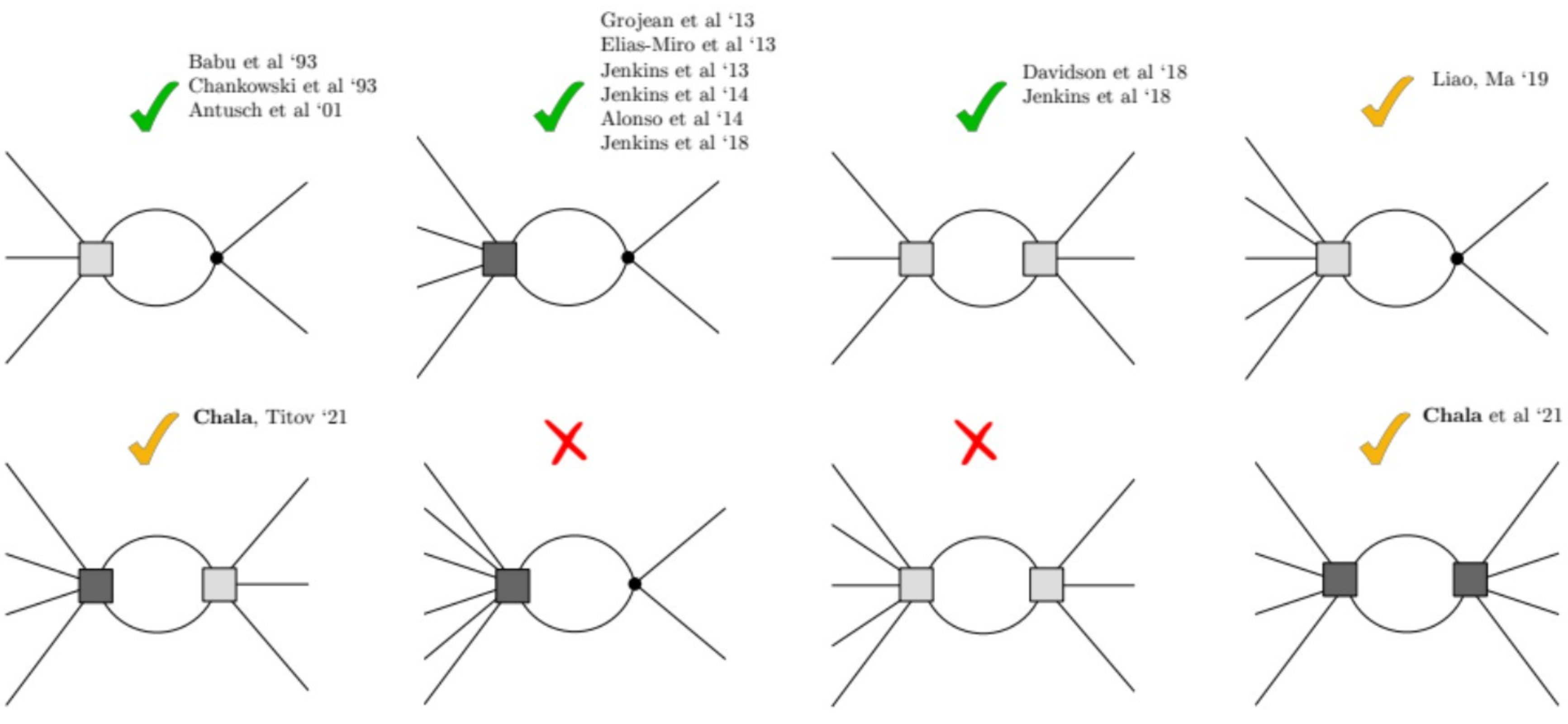


The status of the SMEFT renormalization a few years back:

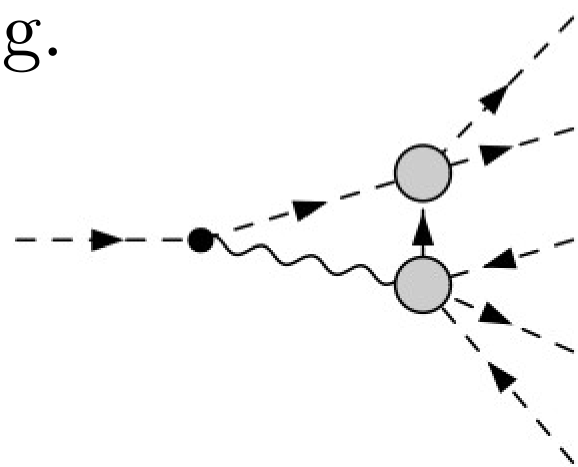
2106.05291	d_5	d_5^2	d_6	d_5^3	$d_5 \times d_6$	d_7	d_5^4	$d_5^2 \times d_6$	d_6^2	$d_5 \times d_7$	d_8
$d_{\leq 4}$ (bosonic)			✓ [7]						This work		X
$d_{\leq 4}$ (fermionic)			✓ [7]						X		X
d_5	✓ [66–68]				✓ [71]	✓ [71]					
d_6 (bosonic)		✓ [30]	✓ [7–9]					X	This work	X	X
d_6 (fermionic)		✓ [30]	✓ [7–9, 69]					X	X	X	X
d_7				✓ [71]	✓ [71]	✓ [22, 70]					
d_8 (bosonic)							X	X	This work	X	X
d_8 (fermionic)							X	X	X	X	X

The status of the SMEFT renormalization a few years back:

2106.05291

	d_5	d_5^2	d_6	d_5^3	$d_5 \times d_6$	d_7	d_5^4	$d_5^2 \times d_6$	d_6^2	$d_5 \times d_7$	d_8
$d_{\leq 4}$ (bosonic)			✓ [7]						This work		X
$d_{\leq 4}$ (fermionic)			✓ [7]						X		X
d_5	✓ [66–68]				✓ [71]	✓ [71]					
d_6 (bosonic)		✓ [30]	✓ [7–9]					X	This work	X	X
d_6 (fermionic)		✓ [30]	✓ [7–9, 69]					X	X	X	X
d_7				✓ [71]	✓ [71]	✓ [22, 70]					
d_8 (bosonic)							X	X	This work	X	X
d_8 (fermionic)							X	X	X	X	X

e.g.

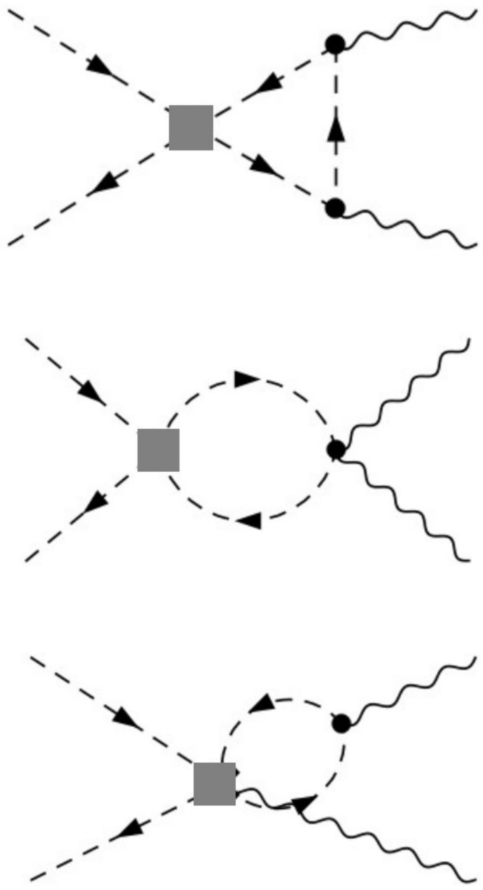


$$\frac{c^{(6)}_{\phi^4 D^2}}{\Lambda^2} \times \frac{c^{(6)}_{\phi^2 D^2}}{\Lambda^2} \to \frac{c^{(8)}_{\phi^6 D^2}}{\Lambda^4}$$

$$\left[\begin{array}{l} \mathcal{O}^{(6)}_{\phi^4 D^2} = (\phi^\dagger \phi)\Box(\phi^\dagger \phi) \\ \mathcal{O}^{(8)}_{\phi^6 D^2} = (\phi^\dagger \phi)^2 (D_\mu \phi^\dagger D^\mu \phi) \end{array}\right]$$

More generally, certain aspects of the full anomalous dimension matrix well understood [Craig, Jiang, Li, Sutherland; 2001.00017]

$\overline{w}$ \ w	0	2	4	6	8
8	X_L^4	$X_L^3H^2$, $X_L^2\psi^2H$, $X_L\psi^4$	$X_L^2H^4$, $X_L\psi^2H^3$, ψ^4H^2	ψ^2H^5	H^8
6		$X_L^2H^2D^2$, $X_L^2\psi\bar{\psi}D$, $X_L\psi^2HD^2$, ψ^4D^2	$X_LH^4D^2$, $X_L^2\bar{\psi}^2H$, $X_L\psi\bar{\psi}H^2D$, $\psi^2H^3D^2$, $X_L\psi^2\bar{\psi}^2$, $\psi^3\bar{\psi}HD$	H^6D^2, $\psi\bar{\psi}H^4D$, $\psi^2\bar{\psi}^2H^2$	$\bar{\psi}^2H^5$
4			$X_L^2X_R^2$, $X_LX_RH^2D^2$, H^4D^4, $X_LX_R\psi\bar{\psi}D$, $X_R\psi^2HD^2$, $X_L\bar{\psi}^2HD^2$, $\psi\bar{\psi}H^2D^3$, $\psi^2\bar{\psi}^2D^2$	$X_RH^4D^2$, $X_R^2\psi^2H$, $X_R\psi\bar{\psi}H^2D$, $\bar{\psi}^2H^3D^2$, $X_R\psi^2\bar{\psi}^2$, $\psi\bar{\psi}^3HD$	$X_R^2H^4$, $X_R\bar{\psi}^2H^3$, $\bar{\psi}^4H^2$
2				$X_R^2H^2D^2$, $X_R^2\psi\bar{\psi}D$, $X_R\bar{\psi}^2HD^2$, $\bar{\psi}^4D^2$	$X_R^3H^2$, $X_R^2\bar{\psi}^2H$, $X_R\bar{\psi}^4$
0					X_R^4



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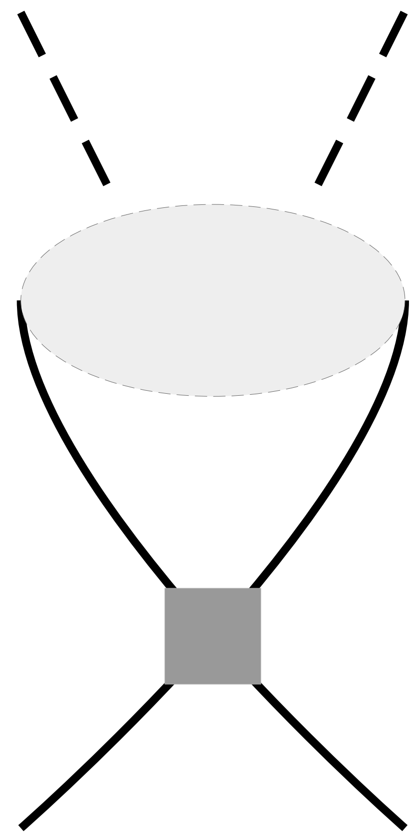
6		$X_L^2H^2D^2$, $X_L^2\psi\bar{\psi}D$, $X_L\psi^2HD^2$, ψ^4D^2	$X_LH^4D^2$, $X_L^2\bar{\psi}^2H$, $X_L\psi\bar{\psi}H^2D$, $\psi^2H^3D^2$, $X_L\psi^2\bar{\psi}^2$, $\psi^3\bar{\psi}HD$	H^6D^2, $\psi\bar{\psi}H^4D$, $\psi^2\bar{\psi}^2H^2$	$\bar{\psi}^2H^5$
4			$X_L^2X_R^2$, $X_LX_RH^2D^2$, H^4D^4, $X_LX_R\psi\bar{\psi}D$, $X_R\psi^2HD^2$, $X_L\bar{\psi}^2HD^2$, $\psi\bar{\psi}H^2D^3$, $\psi^2\bar{\psi}^2D^2$	$X_RH^4D^2$, $X_R^2\psi^2H$, $X_R\psi\bar{\psi}H^2D$, $\bar{\psi}^2H^3D^2$, $X_R\psi^2\bar{\psi}^2$, $\psi\bar{\psi}^3HD$	$X_R^2H^4$, $X_R\bar{\psi}^2H^3$, $\bar{\psi}^4H^2$
2				$X_R^2H^2D^2$, $X_R^2\psi\bar{\psi}D$, $X_R\bar{\psi}^2HD^2$, $\bar{\psi}^4D^2$	$X_R^3H^2$, $X_R^2\bar{\psi}^2H$, $X_R\bar{\psi}^4$
0					X_R^4



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6		$X_L^2 H^2 D^2$, $X_L^2 \psi\bar{\psi} D$, $X_L \psi^2 H D^2$, $\psi^4 D^2$	$X_L H^4 D^2$, $X_L^2 \bar{\psi}^2 H$, $X_L \psi\bar{\psi} H^2 D$, $\psi^2 H^3 D^2$, $X_L \psi^2 \bar{\psi}^2$, $\psi^3 \bar{\psi} H D$	$H^6 D^2$, $\psi\bar{\psi} H^4 D$, $\psi^2 \bar{\psi}^2 H^2$	$\bar{\psi}^2 H^5$
4			$X_L^2 X_R^2$, $X_L X_R H^2 D^2$, $H^4 D^4$, $X_L X_R \psi\bar{\psi} D$, $X_R \psi^2 H D^2$, $X_L \bar{\psi}^2 H D^2$, $\psi\bar{\psi} H^2 D^3$, $\psi^2 \bar{\psi}^2 D^2$	$X_R H^4 D^2$, $X_R^2 \psi^2 H$, $X_R \psi\bar{\psi} H^2 D$, $\bar{\psi}^2 H^3 D^2$, $X_R \psi^2 \bar{\psi}^2$, $\psi \bar{\psi}^3 H D$	$X_R^2 H^4$, $X_R \bar{\psi}^2 H^3$, $\bar{\psi}^4 H^2$
2				$X_R^2 H^2 D^2$, $X_R^2 \psi\bar{\psi} D$, $X_R \bar{\psi}^2 H D^2$, $\bar{\psi}^4 D^2$	$X_R^3 H^2$, $X_R^2 \bar{\psi}^2 H$, $X_R \bar{\psi}^4$
0					X_R^4

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4			$X_L^2X_R^2$, $X_LX_RH^2D^2$, H^4D^4, $X_LX_R\psi\bar{\psi}D$, $X_R\psi^2HD^2$, $X_L\bar{\psi}^2HD^2$, $\psi\bar{\psi}H^2D^3$, $\psi^2\bar{\psi}^2D^2$	$X_RH^4D^2$, $X_R^2\psi^2H$, $X_R\psi\bar{\psi}H^2D$, $\bar{\psi}^2H^3D^2$, $X_R\psi^2\bar{\psi}^2$, $\psi\bar{\psi}^3HD$	$X_R^2H^4$, $X_R\bar{\psi}^2H^3$, $\bar{\psi}^4H^2$
2				$X_R^2H^2D^2$, $X_R^2\psi\bar{\psi}D$, $X_R\bar{\psi}^2HD^2$, $\bar{\psi}^4D^2$	$X_R^3H^2$, $X_R^2\bar{\psi}^2H$, $X_R\bar{\psi}^4$
0					X_R^4

$$\mu \frac{dc^{(1)}_{B^2\phi^2D^2}}{d\mu} \sim \gamma c^{(1)}_{e^2\phi^2D^2} + \cdots$$

> 0

any sign

More generally, certain aspects of the full anomalous dimension matrix well understood [Craig, Jiang, Li, Sutherland; 2001.00017]

$\overline{w}$ \ w	0	2	4	6	8
8	X_L^4	$X_L^3H^2$, $X_L^2\psi^2H$, $X_L\psi^4$	$X_L^2H^4$, $X_L\psi^2H^3$, ψ^4H^2	ψ^2H^5	H^8
6		$X_L^2H^2D^2$, $X_L^2\psi\bar{\psi}D$, $X_L\psi^2HD^2$, ψ^4D^2	$X_LH^4D^2$, $X_L^2\bar{\psi}^2H$, $X_L\psi\bar{\psi}H^2D$, $\psi^2H^3D^2$, $X_L\psi^2\bar{\psi}^2$, $\psi^3\bar{\psi}HD$	H^6D^2, $\psi\bar{\psi}H^4D$, $\psi^2\bar{\psi}^2H^2$	$\bar{\psi}^2H^5$
4			$X_L^2X_R^2$, $X_LX_RH^2D^2$, H^4D^4, $X_LX_R\psi\bar{\psi}D$, $X_R\psi^2HD^2$, $X_L\bar{\psi}^2HD^2$, $\psi\bar{\psi}H^2D^3$, $\psi^2\bar{\psi}^2D^2$	$X_RH^4D^2$, $X_R^2\psi^2H$, $X_R\psi\bar{\psi}H^2D$, $\bar{\psi}^2H^3D^2$, $X_R\psi^2\bar{\psi}^2$, $\psi\bar{\psi}^3HD$	$X_R^2H^4$, $X_R\bar{\psi}^2H^3$, $\bar{\psi}^4H^2$
2				$X_R^2H^2D^2$, $X_R^2\psi\bar{\psi}D$, $X_R\bar{\psi}^2HD^2$, $\bar{\psi}^4D^2$	$X_R^3H^2$, $X_R^2\bar{\psi}^2H$, $X_R\bar{\psi}^4$
0					X_R^4

$$\mu \frac{dc^{(1)}_{B^2\phi^2D^2}}{d\mu} \sim \gamma c^{(1)}_{e^2\phi^2D^2} + \cdots$$

> 0

any sign

$$\gamma = 0$$

Similar results from angular-momentum conservation [Jian, Shu, Xiao, Zheng 2001.04481], but positivity further restricts the signs [**MC** 2301.09995; **MC**, Li 2309.16611]

More generally, certain aspects of the full anomalous dimension matrix well understood [**MC**, 2301.09995; **MC** and Li, 2309.16611]

	$c^{(1)}_{\phi^4D^4}$	$c^{(2)}_{\phi^4D^4}$	$c^{(3)}_{\phi^4D^4}$	$\tilde{c}^{(1)}_{e^2\phi^2D^3}$	$\tilde{c}^{(2)}_{e^2\phi^2D^3}$	$\tilde{c}^{(1)}_{l^2\phi^2D^3}$	$\tilde{c}^{(2)}_{l^2\phi^2D^3}$	$\tilde{c}^{(3)}_{l^2\phi^2D^3}$	$\tilde{c}^{(4)}_{l^2\phi^2D^3}$	$c_{e^4D^2}$	$c^{(1)}_{l^4D^2}$	$c^{(2)}_{l^4D^2}$	$c^{(1)}_{l^2e^2D^2}$	$c^{(2)}_{l^2e^2D^2}$
$c^{(1)}_{B^2\phi^2D^2}$	+	+	+	$\boxed{0}$	−	$\boxed{0}$	−	$\boxed{0}$	−	0	0	0	0	0
$c^{(1)}_{W^2\phi^2D^2}$	+	+	+	0	0	$\boxed{0}$	−	$\boxed{0}$	−	0	0	0	0	0
$\tilde{c}^{(2)}_{e^2\phi^2D^3}$	+	+	+	×	×	$\boxed{0}$	−	$\boxed{0}$	−	−	0	0	$\boxed{0}$	−
$\tilde{c}^{(2)}_{l^2\phi^2D^3}$	+	+	+	$\boxed{0}$	−	×	×	×	×	0	−	−	$\boxed{0}$	−
$\tilde{c}^{(4)}_{l^2\phi^2D^3}$	+	+	+	$\boxed{0}$	−	×	×	×	×	0	−	−	$\boxed{0}$	−
$c_{e^2B^2D}$	0	0	0	$\boxed{0}$	−	0	0	0	0	−	0	0	$\boxed{0}$	−
$c_{l^2B^2D}$	0	0	0	0	0	$\boxed{0}$	−	$\boxed{0}$	−	0	−	−	$\boxed{0}$	−
$c_{e^2W^2D}$	0	0	0	$\boxed{0}$	−	0	0	0	0	0	0	0	$\boxed{0}$	−
$c^{(1)}_{l^2W^2D}$	0	0	0	0	0	$\boxed{0}$	−	$\boxed{0}$	−	0	−	−	0	0
$c^{(2)}_{l^2e^2D^2}$	0	0	0	$\boxed{0}$	−	$\boxed{0}$	−	$\boxed{0}$	−	−	−	−	×	×

More generally, certain aspects of the full anomalous dimension matrix well understood [**MC**, 2301.09995; **MC** and Li, 2309.16611]

	$c^{(1)}_{\phi^4D^4}$	$c^{(2)}_{\phi^4D^4}$	$c^{(3)}_{\phi^4D^4}$	$\tilde{c}^{(1)}_{e^2\phi^2D^3}$	$\tilde{c}^{(2)}_{e^2\phi^2D^3}$	$\tilde{c}^{(1)}_{l^2\phi^2D^3}$	$\tilde{c}^{(2)}_{l^2\phi^2D^3}$	$\tilde{c}^{(3)}_{l^2\phi^2D^3}$	$\tilde{c}^{(4)}_{l^2\phi^2D^3}$	$c_{e^4D^2}$	$c^{(1)}_{l^4D^2}$	$c^{(2)}_{l^4D^2}$	$c^{(1)}_{l^2e^2D^2}$	$c^{(2)}_{l^2e^2D^2}$
$c^{(1)}_{B^2\phi^2D^2}$	$\frac{g^2}{3}$	$\frac{g^2}{2}$	$\frac{g^2}{6}$	$\boxed{0}$	−	$\boxed{0}$	−	$\boxed{0}$	−	0	0	0	0	0
$c^{(1)}_{W^2\phi^2D^2}$	+	+	+	0	0	$\boxed{0}$	−	$\boxed{0}$	−	0	0	0	0	0
$\tilde{c}^{(2)}_{e^2\phi^2D^3}$	+	+	+	×	×	$\boxed{0}$	−	$\boxed{0}$	−	−	0	0	$\boxed{0}$	−
$\tilde{c}^{(2)}_{l^2\phi^2D^3}$	+	+	+	$\boxed{0}$	−	×	×	×	×	0	−	−	$\boxed{0}$	−
$\tilde{c}^{(4)}_{l^2\phi^2D^3}$	+	+	+	$\boxed{0}$	−	×	×	×	×	0	−	−	$\boxed{0}$	−
$c_{e^2B^2D}$	0	0	0	$\boxed{0}$	−	0	0	0	0	−	0	0	$\boxed{0}$	−
$c_{l^2B^2D}$	0	0	0	0	0	$\boxed{0}$	−	$\boxed{0}$	−	0	−	−	$\boxed{0}$	−
$c_{e^2W^2D}$	0	0	0	$\boxed{0}$	−	0	0	0	0	0	0	0	$\boxed{0}$	−
$c^{(1)}_{l^2W^2D}$	0	0	0	0	0	$\boxed{0}$	−	$\boxed{0}$	−	0	−	−	0	0
$c^{(2)}_{l^2e^2D^2}$	0	0	0	$\boxed{0}$	−	$\boxed{0}$	−	$\boxed{0}$	−	−	−	−	×	×

More generally, certain aspects of the full anomalous dimension matrix well understood [**MC**, 2301.09995; **MC** and Li, 2309.16611]

	$c^{(1)}_{\phi^4D^4}$	$c^{(2)}_{\phi^4D^4}$	$c^{(3)}_{\phi^4D^4}$	$\tilde{c}^{(1)}_{e^2\phi^2D^3}$	$\tilde{c}^{(2)}_{e^2\phi^2D^3}$	$\tilde{c}^{(1)}_{l^2\phi^2D^3}$	$\tilde{c}^{(2)}_{l^2\phi^2D^3}$	$\tilde{c}^{(3)}_{l^2\phi^2D^3}$	$\tilde{c}^{(4)}_{l^2\phi^2D^3}$	$c_{e^4D^2}$	$c^{(1)}_{l^4D^2}$	$c^{(2)}_{l^4D^2}$	$c^{(1)}_{l^2e^2D^2}$	$c^{(2)}_{l^2e^2D^2}$
$c^{(1)}_{B^2\phi^2D^2}$	$\frac{g^2}{3}$	$\frac{g^2}{2}$	$\frac{g^2}{6}$	$\boxed{0}$	$-$	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	0	0	0	0
$c^{(1)}_{W^2\phi^2D^2}$	$+$	$+$	$+$	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	0	0	0	0
$\tilde{c}^{(2)}_{e^2\phi^2D^3}$	$+$	$+$	$+$	$\times$	$\times$	$\boxed{0}$	$-\frac{4\lvert Y\rvert^2}{3}$	$\boxed{0}$	$-$	$-$	0	0	$\boxed{0}$	$-$
$\tilde{c}^{(2)}_{l^2\phi^2D^3}$	$+$	$+$	$+$	$\boxed{0}$	$-$	$\times$	$\times$	$\times$	$\times$	0	$-$	$-$	$\boxed{0}$	$-$
$\tilde{c}^{(4)}_{l^2\phi^2D^3}$	$+$	$+$	$+$	$\boxed{0}$	$-$	$\times$	$\times$	$\times$	$\times$	0	$-$	$-$	$\boxed{0}$	$-$
$c_{e^2B^2D}$	0	0	0	$\boxed{0}$	$-$	0	0	0	0	$-$	0	0	$\boxed{0}$	$-$
$c_{l^2B^2D}$	0	0	0	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	$-$	$-$	$\boxed{0}$	$-$
$c_{e^2W^2D}$	0	0	0	$\boxed{0}$	$-$	0	0	0	0	0	0	0	$\boxed{0}$	$-$
$c^{(1)}_{l^2W^2D}$	0	0	0	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	$-$	$-$	0	0
$c^{(2)}_{l^2e^2D^2}$	0	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	$\boxed{0}$	$-$	$-$	$-$	$-$	$\times$	$\times$

More generally, certain aspects of the full anomalous dimension matrix well understood [**MC**, 2301.09995; **MC** and Li, 2309.16611]

	$c^{(1)}_{\phi^4 D^4}$	$c^{(2)}_{\phi^4 D^4}$	$c^{(3)}_{\phi^4 D^4}$	$\tilde{c}^{(1)}_{e^2\phi^2 D^3}$	$\tilde{c}^{(2)}_{e^2\phi^2 D^3}$	$\tilde{c}^{(1)}_{l^2\phi^2 D^3}$	$\tilde{c}^{(2)}_{l^2\phi^2 D^3}$	$\tilde{c}^{(3)}_{l^2\phi^2 D^3}$	$\tilde{c}^{(4)}_{l^2\phi^2 D^3}$	$c_{e^4 D^2}$	$c^{(1)}_{l^4 D^2}$	$c^{(2)}_{l^4 D^2}$	$c^{(1)}_{l^2 e^2 D^2}$	$c^{(2)}_{l^2 e^2 D^2}$
$c^{(1)}_{B^2\phi^2 D^2}$	$\frac{g^2}{3}$	$\frac{g^2}{2}$	$\frac{g^2}{6}$	$\boxed{0}$	$-$	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	0	0	0	0
$c^{(1)}_{W^2\phi^2 D^2}$	$+$	$+$	$+$	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	0	0	0	0
$\tilde{c}^{(2)}_{e^2\phi^2 D^3}$	$+$	$+$	$+$	$g^2 - \lvert Y \rvert^2$	$\times$	$\boxed{0}$	$-\frac{4\lvert Y \rvert^2}{3}$	$\boxed{0}$	$-$	$-$	0	0	$\boxed{0}$	$-$
$\tilde{c}^{(2)}_{l^2\phi^2 D^3}$	$+$	$+$	$+$	$\boxed{0}$	$-$	$\times$	$\times$	$\times$	$\times$	0	$-$	$-$	$\boxed{0}$	$-$
$\tilde{c}^{(4)}_{l^2\phi^2 D^3}$	$+$	$+$	$+$	$\boxed{0}$	$-$	$\times$	$\times$	$\times$	$\times$	0	$-$	$-$	$\boxed{0}$	$-$
$c_{e^2 B^2 D}$	0	0	0	$\boxed{0}$	$-$	0	0	0	0	$-$	0	0	$\boxed{0}$	$-$
$c_{l^2 B^2 D}$	0	0	0	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	$-$	$-$	$\boxed{0}$	$-$
$c_{e^2 W^2 D}$	0	0	0	$\boxed{0}$	$-$	0	0	0	0	0	0	0	$\boxed{0}$	$-$
$c^{(1)}_{l^2 W^2 D}$	0	0	0	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	$-$	$-$	0	0
$c^{(2)}_{l^2 e^2 D^2}$	0	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	$\boxed{0}$	$-$	$-$	$-$	$-$	$\times$	$\times$

The current status of the SMEFT renormalization [**MC**, Gueds, Ramos, Santiago 2106.05291; Bakshi, **MC**, Diaz-Carmona, Guedes 2205.03301; Bakshi, **MC**, Diaz-Carmona, Ren, Vilches 2409.15408]

See also [Zhang 2310.11055; 2306.03008; Bakshi and Diaz-Carmona 2301.07151; Boughezal, Huang, Petriello 2408.15378]

	d_5	d_5^2	d_6	d_5^3	$d_5 \times d_6$	d_7	d_5^4	$d_5^2 \times d_6$	d_6^2	$d_5 \times d_7$	d_8
$d_{\leq 4}$ (bosonic)			✓ [7]						✓		✓
$d_{\leq 4}$ (fermionic)			✓ [7]						✓		✓
d_5	✓ [66–68]				✓ [71]	✓ [71]					
d_6 (bosonic)		✓ [30]	✓ [7–9]					✓	✓	✓	✓
d_6 (fermionic)		✓ [30]	✓ [7–9, 69]					✗	✓	✗	✓
d_7				✓ [71]	✓ [71]	✓ [22, 70]					
d_8 (bosonic)							✓	✓	✓	✓	✓
d_8 (fermionic)							✗	✗	✓	✗	✓

Some other partial results:

Accettulli Huber, De Angelis; 2108.03669

Helset, Jenkins, Manohar; 2212.03253

Asteriadis, Dawson, Fontes; 2212.03258

Bakshi, Diaz-Carmona; 2301.07151

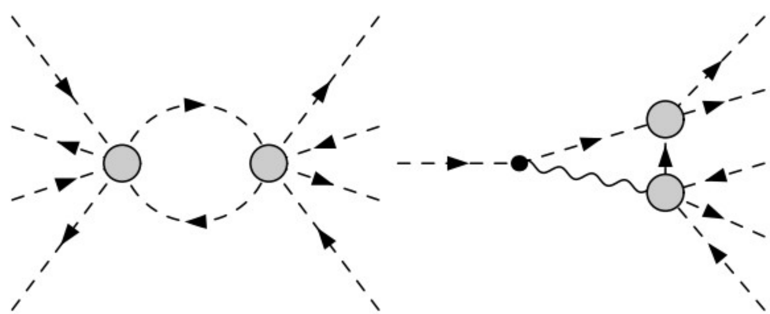
Assi, Helset, Manohar, Pagès, Chia-Hsien Shen; 2307.03187

Boughezal, Huang, Petriello; 2408.15378

...

How do we organize the computation?

We match the UV divergences of Green's functions onto a basis of off-shell-independent operators

div [ $+ \ldots$] $=$ local

But off-shellness requires introducing more operators than physically independent:

$$\mathcal{L} = c_1(\phi^\dagger\phi)^2(\phi^\dagger D^2\phi + \text{h.c.}) + c_2(\phi^\dagger\phi)^2(D^\mu\phi^\dagger D_\mu\phi) + \cdots$$

$$\phi \to c_1(\phi^\dagger\phi)^2\phi \Rightarrow \delta\mathcal{L} = c_1|\phi|^8$$

First challenge: building a basis of off-shell-independent operators [**MC**, Diaz-Carmona, Guedes 2112.12724]

$$
\begin{aligned}
\mathcal{O}_1 &= -D_\mu(\phi^\dagger\phi)D^\mu B^{\rho\nu}B_{\nu\rho} - D_\mu(\phi^\dagger\phi)D^\rho B^{\nu\mu}B_{\nu\rho} \\
&= -D_\mu(\phi^\dagger\phi)D^\mu B^{\rho\nu}B_{\nu\rho} - D_\mu(\phi^\dagger\phi)D^\rho B^{\mu\rho}B_{\nu\rho} \\
&= -D_\mu(\phi^\dagger\phi)D^\mu B^{\rho\nu}B_{\nu\rho} - \mathcal{O}_1 \\
\Rightarrow \mathcal{O}_1 &= -\frac{1}{2}D_\mu(\phi^\dagger\phi)D^\mu B^{\rho\nu}B_{\nu\rho} \\
&= \frac{1}{2}D^2(\phi^\dagger\phi)B^{\rho\nu}B^{\nu\rho} - \frac{1}{2}D_\mu(\phi^\dagger\phi)B^{\rho\nu}D^\mu B_{\nu\rho} \\
&= \frac{1}{2}D^2(\phi^\dagger\phi)B^{\rho\nu}B_{\nu\rho} - \mathcal{O}_1 \\
\Rightarrow \mathcal{O}_1 &= \frac{1}{4}D^2(\phi^\dagger\phi)B^{\rho\nu}B_{\nu\rho} \\
&= -\frac{1}{4}(D^2\phi^\dagger\phi + \phi^\dagger D^2\phi)B^{\nu\rho}B_{\nu\rho} - \frac{1}{4}(2D_\mu\phi^\dagger D^\mu\phi)B^{\nu\rho}B_{\nu\rho} \\
&= -\frac{1}{4}\mathcal{O}_2 - \frac{1}{2}\mathcal{O}_3\,.
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{O}_1 &= D_\mu(\phi^\dagger\phi)D^\nu B^{\mu\rho}B_{\nu\rho}\,, \\
\mathcal{O}_2 &= (D^2\phi^\dagger\phi + \phi^\dagger D^2\phi)B^{\nu\rho}B_{\nu\rho} \\
\mathcal{O}_3 &= D_\mu\phi^\dagger D^\mu\phi B^{\nu\rho}B_{\nu\rho}\,.
\end{aligned}
$$

First challenge: building a basis of off-shell-independent operators [**MC**, Diaz-Carmona, Guedes 2112.12724]

Strategy: compute amplitudes evaluated at off-shell momenta

Matrix of coefficients with det(M)=2 $\phi^\dagger\phi \to BB$

$$
\begin{aligned}
\mathcal{A} = &-ic_1(\kappa_{3334} + 2\kappa_{3434} + \kappa_{3444} - \kappa'_{4333} - 2\kappa'_{4334} - \kappa'_{4344}) \\
&+ 4ic_2(2\kappa_{2234} + 2\kappa_{2334} + 2\kappa_{2434} + \kappa_{3334} + 2\kappa_{3434} + \kappa_{3444} - 2\kappa'_{4322} - 2\kappa'_{4323} \\
&\qquad - 2\kappa'_{4324} - \kappa'_{4333} - 2\kappa - 2\kappa'_{4334} - \kappa_{4344}) \\
&- 4ic_3(\kappa_{2234} + \kappa_{2334} + \kappa_{2434} - \kappa'_{4322} - \kappa_{4323} - \kappa_{4324})\,;
\end{aligned}
$$

$\kappa_{ijkl} = \epsilon_3 \cdot \epsilon_4 (p_i \cdot p_j)(p_k \cdot p_l)$

A basis of off-shell-independent bosonic operators:

	Operator	Notation	Operator	Notation
ϕ^8	$(\phi^\dagger\phi)^4$	$\mathcal{O}_{\phi^8}$		
$\phi^6 D^2$	$(\phi^\dagger\phi)^2(D_\mu\phi^\dagger D^\mu\phi)$	$\mathcal{O}^{(1)}_{\phi^6}$	$(\phi^\dagger\phi)(\phi^\dagger\sigma^I\phi)(D_\mu\phi^\dagger\sigma^I D^\mu\phi)$	$\mathcal{O}^{(2)}_{\phi^6}$
	$(\phi^\dagger\phi)^2(\phi^\dagger D^2\phi+\text{h.c.})$	$\mathcal{O}^{(3)}_{\phi^6}$	$(\phi^\dagger\phi)^2 D_\mu(\phi^\dagger \mathrm{i}\overleftrightarrow{D}^\mu\phi)$	$\mathcal{O}^{(4)}_{\phi^6}$
$\phi^4 D^4$	$(D_\mu\phi^\dagger D_\nu\phi)(D^\nu\phi^\dagger D^\mu\phi)$	$\mathcal{O}^{(1)}_{\phi^4}$	$(D_\mu\phi^\dagger D_\nu\phi)(D^\mu\phi^\dagger D^\nu\phi)$	$\mathcal{O}^{(2)}_{\phi^4}$
	$(D^\mu\phi^\dagger D_\mu\phi)(D^\nu\phi^\dagger D_\nu\phi)$	$\mathcal{O}^{(3)}_{\phi^4}$	$D_\mu\phi^\dagger D^\mu\phi(\phi^\dagger D^2\phi+\text{h.c.})$	$\mathcal{O}^{(4)}_{\phi^4}$
	$D_\mu\phi^\dagger D^\mu\phi(\phi^\dagger \mathrm{i}D^2\phi+\text{h.c.})$	$\mathcal{O}^{(5)}_{\phi^4}$	$(D_\mu\phi^\dagger\phi)(D^2\phi^\dagger D_\mu\phi)+\text{h.c.}$	$\mathcal{O}^{(6)}_{\phi^4}$
	$(D_\mu\phi^\dagger\phi)(D^2\phi^\dagger \mathrm{i}D_\mu\phi)+\text{h.c.}$	$\mathcal{O}^{(7)}_{\phi^4}$	$(D^2\phi^\dagger\phi)(D^2\phi^\dagger\phi)+\text{h.c.}$	$\mathcal{O}^{(8)}_{\phi^4}$
	$(D^2\phi^\dagger\phi)(\mathrm{i}D^2\phi^\dagger\phi)+\text{h.c.}$	$\mathcal{O}^{(9)}_{\phi^4}$	$(D^2\phi^\dagger D^2\phi)(\phi^\dagger\phi)$	$\mathcal{O}^{(10)}_{\phi^4}$
	$(\phi^\dagger D^2\phi)(D^2\phi^\dagger\phi)$	$\mathcal{O}^{(11)}_{\phi^4}$	$(D_\mu\phi^\dagger\phi)(D^\mu\phi^\dagger D^2\phi)+\text{h.c.}$	$\mathcal{O}^{(12)}_{\phi^4}$
	$(D_\mu\phi^\dagger\phi)(D^\mu\phi^\dagger \mathrm{i}D^2\phi)+\text{h.c.}$	$\mathcal{O}^{(13)}_{\phi^4}$		
$X^3\phi^2$	$f^{ABC}(\phi^\dagger\phi)G^{A,\nu}_\mu G^{B,\rho}_\nu G^{C,\mu}_\rho$	$\mathcal{O}^{(1)}_{G^3\phi^2}$	$f^{ABC}(\phi^\dagger\phi)G^{A,\nu}_\mu G^{B,\rho}_\nu \tilde{G}^{C,\mu}_\rho$	$\mathcal{O}^{(1)}_{G^3\phi^2}$
	$\epsilon^{IJK}(\phi^\dagger\phi)W^{I\nu}_\mu W^{J\rho}_\nu W^{K\mu}_\rho$	$\mathcal{O}^{(1)}_{W^3\phi^2}$	$\epsilon^{IJK}(\phi^\dagger\phi)W^{I\nu}_\mu W^{J\rho}_\nu \widetilde{W}^{K\mu}_\rho$	$\mathcal{O}^{(2)}_{W^3\phi^2}$
	$\epsilon^{IJK}(\phi^\dagger\sigma^I\phi)B^{\nu}_\mu W^{J\rho}_\nu W^{K\mu}_\rho$	$\mathcal{O}^{(1)}_{W^2B\phi^2}$	$\epsilon^{IJK}(\phi^\dagger\sigma^I\phi)(\widetilde{B}^{\mu\nu}W^J_{\nu\rho}W^{K\rho}_\mu + B^{\mu\nu}W^J_{\nu\rho}\widetilde{W}^{K\rho}_\mu)$	$\mathcal{O}^{(2)}_{W^2B\phi^2}$
$X^2\phi^4$	$(\phi^\dagger\phi)^2 G^A_{\mu\nu}G^{A\mu\nu}$	$O^{(1)}_{G^2\phi^4}$	$(\phi^\dagger\phi)^2 \widetilde{G}^A_{\mu\nu}G^{A\mu\nu}$	$O^{(2)}_{G^2\phi^4}$
	$(\phi^\dagger\phi)^2 W^I_{\mu\nu}W^{I\mu\nu}$	$\mathcal{O}^{(1)}_{W^2\phi^4}$	$(\phi^\dagger\phi)^2 \widetilde{W}^I_{\mu\nu}W^{I\mu\nu}$	$\mathcal{O}^{(2)}_{W^2\phi^4}$
	$(\phi^\dagger\sigma^I\phi)(\phi^\dagger\sigma^J\phi)W^I_{\mu\nu}W^{J\mu\nu}$	$\mathcal{O}^{(3)}_{W^2\phi^4}$	$(\phi^\dagger\sigma^I\phi)(\phi^\dagger\sigma^J\phi)\widetilde{W}^I_{\mu\nu}W^{J\mu\nu}$	$\mathcal{O}^{(4)}_{W^2\phi^4}$
	$(\phi^\dagger\phi)(\phi^\dagger\sigma^I\phi)W^I_{\mu\nu}B^{\mu\nu}$	$\mathcal{O}^{(1)}_{WB\phi^4}$	$(\phi^\dagger\phi)(\phi^\dagger\sigma^I\phi)\widetilde{W}^I_{\mu\nu}B^{\mu\nu}$	$\mathcal{O}^{(2)}_{WB\phi^4}$
	$(\phi^\dagger\phi)^2 B_{\mu\nu}B^{\mu\nu}$	$\mathcal{O}^{(1)}_{B^2\phi^4}$	$(\phi^\dagger\phi)^2 \widetilde{B}_{\mu\nu}B^{\mu\nu}$	$\mathcal{O}^{(2)}_{B^2\phi^4}$
$X\phi^2D^4$	$i(D_\nu\phi^\dagger\sigma^I D^2\phi - D^2\phi^\dagger\sigma^I D_\nu\phi)D_\mu W^{I\mu\nu}$	$\mathcal{O}^{(1)}_{W\phi^2D^4}$	$(D_\nu\phi^\dagger\sigma^I D^2\phi + D^2\phi^\dagger\sigma^I D_\nu\phi)D_\mu W^{I\mu\nu}$	$\mathcal{O}^{(2)}_{W\phi^2D^4}$
	$i(D_\rho D_\nu\phi^\dagger\sigma^I D^\rho\phi - D^\rho\phi^\dagger\sigma^I D_\rho D_\nu\phi)D_\mu W^{I\mu\nu}$	$\mathcal{O}^{(3)}_{W\phi^2D^4}$		
	$i(D_\nu\phi^\dagger D^2\phi - D^2\phi^\dagger D_\nu\phi)D_\mu B^{\mu\nu}$	$\mathcal{O}^{(1)}_{B\phi^2D^4}$	$(D_\nu\phi^\dagger D^2\phi + D^2\phi^\dagger D_\nu\phi)D_\mu B^{\mu\nu}$	$\mathcal{O}^{(2)}_{B\phi^2D^4}$
	$i(D_\rho D_\nu\phi^\dagger D^\rho\phi - D^\rho\phi^\dagger D_\rho D_\nu\phi)D_\mu B^{\mu\nu}$	$\mathcal{O}^{(3)}_{B\phi^2D^4}$		
$X\phi^4D^2$	$\mathrm{i}(\phi^\dagger\phi)(D^\mu\phi^\dagger\sigma^I D^\nu\phi)W^I_{\mu\nu}$	$\mathcal{O}^{(1)}_{W\phi^4D^2}$	$\mathrm{i}(\phi^\dagger\phi)(D^\mu\phi^\dagger\sigma^I D^\nu\phi)\widetilde{W}^I_{\mu\nu}$	$\mathcal{O}^{(2)}_{W\phi^4D^2}$
	$\mathrm{i}\epsilon^{IJK}(\phi^\dagger\sigma^I\phi)(D^\mu\phi^\dagger\sigma^J D^\nu\phi)W^K_{\mu\nu}$	$\mathcal{O}^{(3)}_{W\phi^4D^2}$	$\mathrm{i}\epsilon^{IJK}(\phi^\dagger\sigma^I\phi)(D^\mu\phi^\dagger\sigma^J D^\nu\phi)\widetilde{W}^K_{\mu\nu}$	$\mathcal{O}^{(4)}_{W\phi^4D^2}$
	$(\phi^\dagger\phi)D_\nu W^{I\mu\nu}(D_\mu\phi^\dagger\sigma^I\phi+\text{h.c.})$	$\mathcal{O}^{(5)}_{W\phi^4D^2}$	$(\phi^\dagger\phi)D_\nu W^{I\mu\nu}(D_\mu\phi^\dagger \mathrm{i}\sigma^I\phi+\text{h.c.})$	$\mathcal{O}^{(6)}_{W\phi^4D^2}$
	$\epsilon^{IJK}(D_\mu\phi^\dagger\sigma^I\phi)(\phi^\dagger\sigma^J D_\nu\phi)W^{K\mu\nu}$	$\mathcal{O}^{(7)}_{W\phi^4D^2}$	$\mathrm{i}(\phi^\dagger\phi)(D^\mu\phi^\dagger D^\nu\phi)B_{\mu\nu}$	$\mathcal{O}^{(1)}_{B\phi^4D^2}$
	$\mathrm{i}(\phi^\dagger\phi)(D^\mu\phi^\dagger D^\nu\phi)\widetilde{B}_{\mu\nu}$	$\mathcal{O}^{(2)}_{B\phi^4D^2}$	$(\phi^\dagger\phi)D_\nu B^{\mu\nu}(D_\mu\phi^\dagger \mathrm{i}\phi+\text{h.c.})$	$\mathcal{O}^{(3)}_{B\phi^4D^2}$
ϕ^2D^6	$D^2\phi^\dagger D_\mu D_\nu D^\mu D^\nu\phi$	$\mathcal{O}_{\phi^2}$		

	Operator	Notation	Operator	Notation
$X^2\phi^2D^2$	$(D^\mu\phi^\dagger D^\nu\phi)W^I_{\mu\rho}W^{I\rho}_\nu$	$\mathcal{O}^{(1)}_{W^2\phi^2D^2}$	$(D^\mu\phi^\dagger D_\mu\phi)W^I_{\nu\rho}W^{I\nu\rho}$	$\mathcal{O}^{(2)}_{W^2\phi^2D^2}$
	$(D^\mu\phi^\dagger D_\mu\phi)W^I_{\nu\rho}\widetilde{W}^{I\nu\rho}$	$\mathcal{O}^{(3)}_{W^2\phi^2D^2}$	$i\epsilon^{IJK}(D^\mu\phi^\dagger\sigma^I D^\nu\phi)W^J_{\mu\rho}W^{K\rho}_\nu$	$\mathcal{O}^{(4)}_{W^2\phi^2D^2}$
	$\epsilon^{IJK}(D^\mu\phi^\dagger\sigma^I D^\nu\phi)(W^J_{\mu\rho}\widetilde{W}^{K\rho}_\nu - \widetilde{W}^J_{\mu\rho}W^{K\rho}_\nu)$	$\mathcal{O}^{(5)}_{W^2\phi^2D^2}$	$i\epsilon^{IJK}(D^\mu\phi^\dagger\sigma^I D^\nu\phi)(W^J_{\mu\rho}\widetilde{W}^{K\rho}_\nu + \widetilde{W}^J_{\mu\rho}W^{K\rho}_\nu)$	$\mathcal{O}^{(6)}_{W^2\phi^2D^2}$
	$i\epsilon^{IJK}(\phi^\dagger\sigma^I D^\nu\phi - D^\nu\phi^\dagger\sigma^I\phi)D_\mu W^{J\mu\rho}\widetilde{W}^K_{\nu\rho}$	$\mathcal{O}^{(7)}_{W^2\phi^2D^2}$	$\epsilon^{IJK}\phi^\dagger\sigma^I\phi D_\nu D_\mu W^{J\mu\rho}\widetilde{W}^{K\nu}_{\ \ \rho}$	$\mathcal{O}^{(8)}_{W^2\phi^2D^2}$
	$i(\phi^\dagger D_\nu\phi - D_\nu\phi^\dagger\phi)D_\mu W^{I\mu\rho}\widetilde{W}^{I\nu}_{\ \ \rho}$	$\mathcal{O}^{(9)}_{W^2\phi^2D^2}$	$(\phi^\dagger D_\nu\phi + D_\nu\phi^\dagger\phi)D_\mu W^{I\mu\rho}\widetilde{W}^{I\nu}_{\ \ \rho}$	$\mathcal{O}^{(10)}_{W^2\phi^2D^2}$
	$(\phi^\dagger D_\nu\phi + D_\nu\phi^\dagger\phi)D_\mu W^{I\mu\rho}W^{I\nu}_{\ \ \rho}$	$\mathcal{O}^{(11)}_{W^2\phi^2D^2}$	$i(\phi^\dagger D_\nu\phi - D_\nu\phi^\dagger\phi)D_\mu W^{I\mu\rho}W^{I\nu}_{\ \ \rho}$	$\mathcal{O}^{(12)}_{W^2\phi^2D^2}$
	$\phi^\dagger\phi D_\mu W^{I\mu\rho}D_\nu W^{I\nu}_{\ \ \rho}$	$\mathcal{O}^{(13)}_{W^2\phi^2D^2}$	$(D_\mu\phi^\dagger\phi + \phi^\dagger D_\mu\phi)W^{I\nu\rho}D^\mu W^I_{\nu\rho}$	$\mathcal{O}^{(14)}_{W^2\phi^2D^2}$
	$i(D_\mu\phi^\dagger\phi - \phi^\dagger D_\mu\phi)W^{I\nu\rho}D^\mu W^I_{\nu\rho}$	$\mathcal{O}^{(15)}_{W^2\phi^2D^2}$	$(D_\mu\phi^\dagger\phi + \phi^\dagger D_\mu\phi)D^\mu W^{I\nu\rho}\widetilde{W}^I_{\nu\rho}$	$\mathcal{O}^{(16)}_{W^2\phi^2D^2}$
	$i(D_\mu\phi^\dagger\phi - \phi^\dagger D_\mu\phi)D^\mu W^{I\nu\rho}\widetilde{W}^I_{\nu\rho}$	$\mathcal{O}^{(17)}_{W^2\phi^2D^2}$	$\epsilon^{IJK}(\phi^\dagger\sigma^I D^\nu\phi + D^\nu\phi^\dagger\sigma^I\phi)D_\mu W^{J\mu\rho}W^K_{\nu\rho}$	$\mathcal{O}^{(18)}_{W^2\phi^2D^2}$
	$i\epsilon^{IJK}(\phi^\dagger\sigma^I D^\nu\phi - D^\nu\phi^\dagger\sigma^I\phi)D_\mu W^{J\mu\rho}W^K_{\nu\rho}$	$\mathcal{O}^{(19)}_{W^2\phi^2D^2}$		
	$(D^\mu\phi^\dagger\sigma^I D_\mu\phi)B_{\nu\rho}W^{I\nu\rho}$	$\mathcal{O}^{(1)}_{WB\phi^2D^2}$	$(D^\mu\phi^\dagger\sigma^I D_\mu\phi)B_{\nu\rho}\widetilde{W}^{I\nu\rho}$	$\mathcal{O}^{(2)}_{WB\phi^2D^2}$
	$i(D^\mu\phi^\dagger\sigma^I D^\nu\phi)(B_{\mu\rho}W^{I\rho}_\nu - B_{\nu\rho}W^{I\rho}_\mu)$	$\mathcal{O}^{(3)}_{WB\phi^2D^2}$	$(D^\mu\phi^\dagger\sigma^I D^\nu\phi)(B_{\mu\rho}W^{I\rho}_\nu + B_{\nu\rho}W^{I\rho}_\mu)$	$\mathcal{O}^{(4)}_{WB\phi^2D^2}$
	$i(D^\mu\phi^\dagger\sigma^I D^\nu\phi)(B_{\mu\rho}\widetilde{W}^{I\rho}_\nu - B_{\nu\rho}\widetilde{W}^{I\rho}_\mu)$	$\mathcal{O}^{(5)}_{WB\phi^2D^2}$	$(D^\mu\phi^\dagger\sigma^I D^\nu\phi)(B_{\mu\rho}\widetilde{W}^{I\rho}_\nu + B_{\nu\rho}\widetilde{W}^{I\rho}_\mu)$	$\mathcal{O}^{(6)}_{WB\phi^2D^2}$
	$i(\phi^\dagger\sigma^I D^\mu\phi - D^\mu\phi^\dagger\sigma^I\phi)D_\mu B^{\nu\rho}W^I_{\nu\rho}$	$\mathcal{O}^{(7)}_{WB\phi^2D^2}$	$(\phi^\dagger\sigma^I D^\nu\phi + D^\nu\phi^\dagger\sigma^I\phi)D_\mu B^{\mu\rho}W^I_{\nu\rho}$	$\mathcal{O}^{(8)}_{WB\phi^2D^2}$
	$i(\phi^\dagger\sigma^I D^\nu\phi - D^\nu\phi^\dagger\sigma^I\phi)D_\mu B^{\mu\rho}W^I_{\nu\rho}$	$\mathcal{O}^{(9)}_{WB\phi^2D^2}$	$(\phi^\dagger\sigma^I\phi)D^\mu B_{\mu\rho}D_\nu W^{I\nu\rho}$	$\mathcal{O}^{(10)}_{WB\phi^2D^2}$
	$(D_\nu\phi^\dagger\sigma^I\phi + \phi^\dagger\sigma^I D_\nu\phi)B_{\mu\rho}D^\mu W^{I\nu\rho}$	$\mathcal{O}^{(11)}_{WB\phi^2D^2}$	$i(D_\nu\phi^\dagger\sigma^I\phi - \phi^\dagger\sigma^I D_\nu\phi)B_{\mu\rho}D^\mu W^{I\nu\rho}$	$\mathcal{O}^{(12)}_{WB\phi^2D^2}$
	$(\phi^\dagger\sigma^I\phi)B_{\mu\rho}D_\nu D^\mu W^{I\nu\rho}$	$\mathcal{O}^{(13)}_{WB\phi^2D^2}$	$i(D_\nu\phi^\dagger\sigma^I\phi - \phi^\dagger\sigma^I D_\nu\phi)D^\mu B_{\mu\rho}\widetilde{W}^{I\nu\rho}$	$\mathcal{O}^{(14)}_{WB\phi^2D^2}$
	$i(\phi^\dagger\sigma^I D_\mu\phi - D_\mu\phi^\dagger\sigma^I\phi)D^\mu B_{\nu\rho}\widetilde{W}^{I\nu\rho}$	$\mathcal{O}^{(15)}_{WB\phi^2D^2}$	$(\phi^\dagger\sigma^I\phi)(D^2 B^{\nu\rho})\widetilde{W}^I_{\nu\rho}$	$\mathcal{O}^{(16)}_{WB\phi^2D^2}$
	$(\phi^\dagger\sigma^I\phi)(D^\rho D_\mu W^{I\mu\nu})\widetilde{B}_{\nu\rho}$	$\mathcal{O}^{(17)}_{WB\phi^2D^2}$	$i(D^\nu\phi^\dagger\sigma^I\phi - \phi^\dagger\sigma^I D^\nu\phi)\widetilde{B}^{\mu\rho}D_\mu W^I_{\nu\rho}$	$\mathcal{O}^{(18)}_{WB\phi^2D^2}$
	$(D^\nu\phi^\dagger\sigma^I\phi + \phi^\dagger\sigma^I D^\nu\phi)\widetilde{B}^{\mu\rho}D_\mu W^I_{\nu\rho}$	$\mathcal{O}^{(19)}_{WB\phi^2D^2}$		
	$(D^\mu\phi^\dagger D^\nu\phi)B_{\mu\rho}B_\nu^{\ \rho}$	$\mathcal{O}^{(1)}_{B^2\phi^2D^2}$	$(D^\mu\phi^\dagger D_\mu\phi)B_{\nu\rho}B^{\nu\rho}$	$\mathcal{O}^{(2)}_{B^2\phi^2D^2}$
	$(D^\mu\phi^\dagger D_\mu\phi)B_{\nu\rho}\widetilde{B}^{\nu\rho}$	$\mathcal{O}^{(3)}_{B^2\phi^2D^2}$	$(D_\mu\phi^\dagger\phi + \phi^\dagger D_\mu\phi)D_\nu B^{\mu\rho}B^\nu_{\ \rho}$	$\mathcal{O}^{(4)}_{B^2\phi^2D^2}$
	$i(\phi^\dagger D_\mu D_\nu\phi - D_\mu D_\nu\phi^\dagger\phi)B^{\mu\rho}B^\nu_{\ \rho}$	$\mathcal{O}^{(5)}_{B^2\phi^2D^2}$	$\phi^\dagger\phi D_\mu D_\nu B^{\mu\rho}B^\nu_{\ \rho}$	$\mathcal{O}^{(6)}_{B^2\phi^2D^2}$
	$i(\phi^\dagger D_\nu\phi - D_\nu\phi^\dagger\phi)D_\mu B^{\mu\rho}B^\nu_{\ \rho}$	$\mathcal{O}^{(7)}_{B^2\phi^2D^2}$	$(\phi^\dagger D_\nu\phi + D_\nu\phi^\dagger\phi)D_\mu B^{\mu\rho}B^\nu_{\ \rho}$	$\mathcal{O}^{(8)}_{B^2\phi^2D^2}$
	$(\phi^\dagger D^2\phi + D^2\phi^\dagger\phi)B^{\nu\rho}\widetilde{B}_{\nu\rho}$	$\mathcal{O}^{(9)}_{B^2\phi^2D^2}$	$i(\phi^\dagger D^2\phi - D^2\phi^\dagger\phi)B^{\nu\rho}\widetilde{B}_{\nu\rho}$	$\mathcal{O}^{(10)}_{B^2\phi^2D^2}$
	$(\phi^\dagger D_\nu\phi + D_\nu\phi^\dagger\phi)D_\mu B^{\mu\rho}\widetilde{B}^\nu_{\ \rho}$	$\mathcal{O}^{(11)}_{B^2\phi^2D^2}$	$i(\phi^\dagger D_\nu\phi - D_\nu\phi^\dagger\phi)D_\mu B^{\mu\rho}\widetilde{B}^\nu_{\ \rho}$	$\mathcal{O}^{(12)}_{B^2\phi^2D^2}$
	$(D^\mu\phi^\dagger D_\nu\phi)G^A_{\mu\rho}G^{A\nu\rho}$	$\mathcal{O}^{(1)}_{G^2\phi^2D^2}$	$(D^\mu\phi^\dagger D_\mu\phi)G^A_{\nu\rho}G^{A\nu\rho}$	$\mathcal{O}^{(2)}_{G^2\phi^2D^2}$
	$(D^\mu\phi^\dagger D_\mu\phi)G^A_{\nu\rho}\widetilde{G}^{A\nu\rho}$	$\mathcal{O}^{(3)}_{G^2\phi^2D^2}$	$(D_\mu\phi^\dagger\phi + \phi^\dagger D_\mu\phi)D_\nu G^{A\mu\rho}G^{A\nu}_{\ \ \rho}$	$\mathcal{O}^{(4)}_{G^2\phi^2D^2}$
	$i(\phi^\dagger D_\mu D_\nu\phi - D_\mu D_\nu\phi^\dagger\phi)G^{A\mu\rho}G^{A\nu}_{\ \ \rho}$	$\mathcal{O}^{(5)}_{G^2\phi^2D^2}$	$\phi^\dagger\phi D_\mu D_\nu G^{A\mu\rho}G^{A\nu}_{\ \ \rho}$	$\mathcal{O}^{(6)}_{G^2\phi^2D^2}$
	$i(\phi^\dagger D_\nu\phi - D_\nu\phi^\dagger\phi)D_\mu G^{A\mu\rho}G^{A\nu}_{\ \ \rho}$	$\mathcal{O}^{(7)}_{G^2\phi^2D^2}$	$(\phi^\dagger D_\nu\phi + D_\nu\phi^\dagger\phi)D_\mu G^{A\mu\rho}G^{A\nu}_{\ \ \rho}$	$\mathcal{O}^{(8)}_{G^2\phi^2D^2}$
	$(\phi^\dagger D^2\phi + D^2\phi^\dagger\phi)G^{A\nu\rho}\widetilde{G}^{A\nu}_{\ \ \rho}$	$\mathcal{O}^{(9)}_{G^2\phi^2D^2}$	$i(\phi^\dagger D^2\phi - D^2\phi^\dagger\phi)G^{A\nu\rho}\widetilde{G}^A_{\nu\rho}$	$\mathcal{O}^{(10)}_{G^2\phi^2D^2}$
	$(\phi^\dagger D_\nu\phi + D_\nu\phi^\dagger\phi)D_\mu G^{A\mu\rho}\widetilde{G}^{A\nu}_{\ \ \rho}$	$\mathcal{O}^{(11)}_{G^2\phi^2D^2}$	$i(\phi^\dagger D_\nu\phi - D_\nu\phi^\dagger\phi)D_\mu G^{A\mu\rho}\widetilde{G}^{A\nu}_{\ \ \rho}$	$\mathcal{O}^{(12)}_{G^2\phi^2D^2}$

+ ... other classes

We have also worked out how physical terms are shifted upon removing redundant ones; caveat only linear terms -equations of motion [Criado, Perez-Victoria ‘18]

e.g.

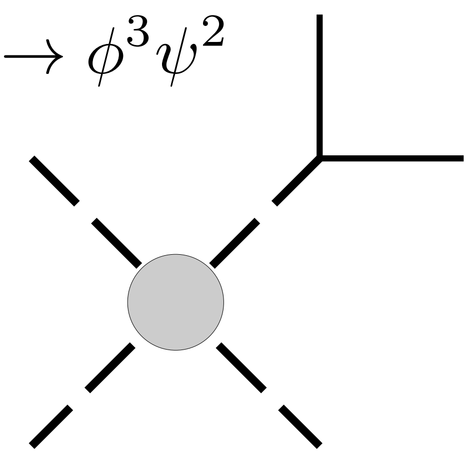
$$
\begin{aligned}
c^{(1)}_{\phi^6} \to\ & c^{(1)}_{\phi^6} + \frac{3}{2}c_{B^2D^4}g_1^2g_2^2 - \frac{3c^{(8)}_{B^2\phi^2D^2}g_1^2}{4} - 3c^{(1)}_{B\phi^2D^4}g_1\lambda - \frac{3}{4}c^{(3)}_{B\phi^2D^4}g_1g_2^2 + 3c^{(3)}_{B\phi^2D^4}g_1\lambda \\
& - \frac{3c^{(3)}_{B\phi^4D^2}g_1}{2} + \frac{3}{2}c_{\phi^2}g_1^2\lambda + \frac{5}{2}c_{\phi^2}g_2^2\lambda + 8c_{\phi^2}\lambda^2 + 4c^{(12)}_{\phi^4}\lambda - 4c^{(4)}_{\phi^4}\lambda - 2c^{(6)}_{\phi^4}\lambda \\
& + \frac{3}{2}c_{W^2D^4}g_1^2g_2^2 + \frac{5c_{W^2D^4}g_2^4}{4} - \frac{5c^{(11)}_{W^2\phi^2D^2}g_2^2}{4} + \frac{5c^{(13)}_{W^2\phi^2D^2}g_2^2}{4} + \frac{5c^{(19)}_{W^2\phi^2D^2}g_2^2}{2} \\
& + \frac{5c^{(1)}_{W^3D^2}g_2^3}{2} - \frac{5c^{(2)}_{W^3D^2}g_2^3}{4} + \frac{7c^{(10)}_{WB\phi^2D^2}g_1g_2}{4} + \frac{3c^{(11)}_{WB\phi^2D^2}g_1g_2}{4} - \frac{3c^{(13)}_{WB\phi^2D^2}g_1g_2}{2} \\
& + \frac{5c^{(8)}_{WB\phi^2D^2}g_1g_2}{4} - 5c^{(1)}_{W\phi^2D^4}g_2\lambda - \frac{3}{4}c^{(3)}_{W\phi^2D^4}g_1^2g_2 + 5c^{(3)}_{W\phi^2D^4}g_2\lambda - \frac{5c^{(6)}_{W\phi^4D^2}g_2}{2} \\
& - \frac{3c^{(7)}_{W\phi^4D^2}g_2}{2},
\end{aligned}
$$

For later works we use most modern methods for computing Green's bases [Fonseca 2307.08745; Ren, Yu, 2211.01420]

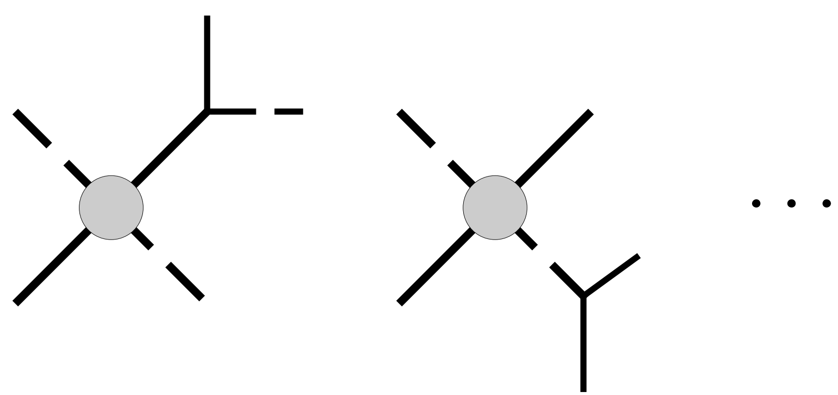
We also worked out new methods for removing unphysical terms (more on this later)

How to organize the full computation? First bosonic sector, then two-fermions, then four-fermions:

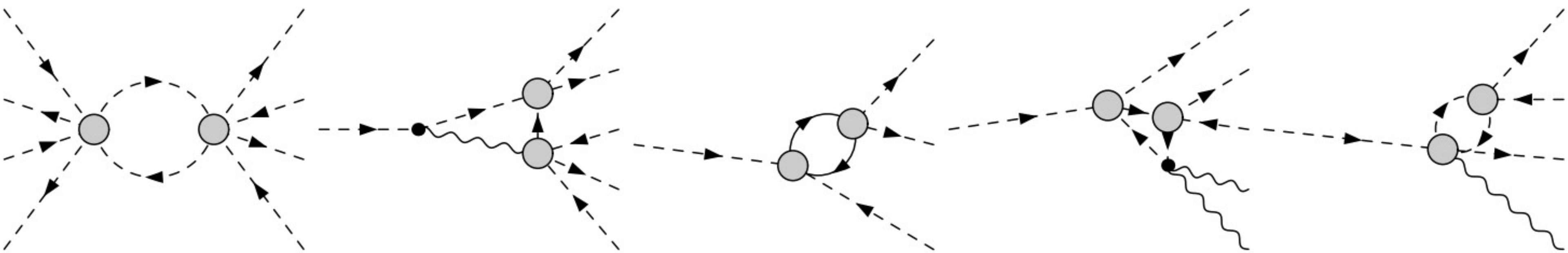
$\phi^4 D^2 \to \phi^3 \psi^2$



$\phi^2 \psi^2 D \to \psi^2 \cdots$



Some results for dim-6 x dim-6 $\rightarrow$ dim-8



$\gamma'_{c^{(1)}_{B^2\phi^4}}$	c_ϕ	$c_{\phi D}$	$c_{\phi\Box}$	$c^{(1)}_{\phi\psi_L}$	$c^{(3)}_{\phi\psi_L}$	$c_{\phi\psi_R}$	$c_{\phi ud}$	$c_{\psi_R\phi}$
c_ϕ	0	0	0	0	0	0	0	0
$c_{\phi D}$		$\times$	0	0	0	0	0	0
$c_{\phi\Box}$			0	0	0	0	0	0
$c^{(1)}_{\phi\psi_L}$				$\times$	0	0	0	0
$c^{(3)}_{\phi\psi_L}$					$\times$	0	0	0
$c_{\phi\psi_R}$						$\times$	0	0
$c_{\phi ud}$							$\times$	0
$c_{\psi_R\phi}$								0

$\gamma'_{c^8_\phi}$	c_ϕ	$c_{\phi D}$	$c_{\phi\Box}$	$c^{(1)}_{\phi\psi_L}$	$c^{(3)}_{\phi\psi_L}$	$c_{\phi\psi_R}$	$c_{\phi ud}$	$c_{\psi_R\phi}$
c_ϕ	$\times$	$\times$	$\times$	0	$\times$	0	$\times$	$\times$
$c_{\phi D}$		$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
$c_{\phi\Box}$			$\times$	0	$\times$	0	$\times$	$\times$
$c^{(1)}_{\phi\psi_L}$				$\times$	$\times$	$\times$	0	$\times$
$c^{(3)}_{\phi\psi_L}$					$\times$	$\times$	0	$\times$
$c_{\phi\psi_R}$						$\times$	0	$\times$
$c_{\phi ud}$							$\times$	0
$c_{\psi_R\phi}$								$\times$

$$\dot{\lambda} \supset \left(5c^2_{\phi D} - 24 c_{\phi D} c_{\phi\Box} + 24 c^2_{\phi\Box}\right)\frac{\mu^4}{\Lambda^4} \qquad \dot{c}_{\phi D} \supset \left(10 c^2_{\phi D} - 4 c_{\phi D} c_{\phi\Box}\right)\frac{\mu^2}{\Lambda^2}$$

Some immediate implications

1. Loop-generated operators are not renormalized

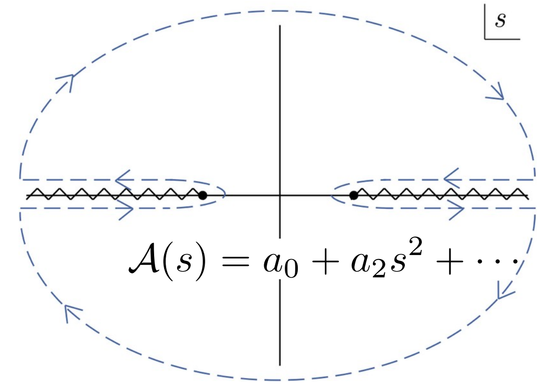
2. Peskin-Takeuchi parameters are not renormalized

$$\frac{1}{16\pi}S = \frac{v^2}{\Lambda^2}\left[c_{\phi WB} + c^{(1)}_{WB\phi^4}\frac{v^4}{\Lambda^4}\right], \quad \frac{1}{16\pi}U = \frac{v^4}{\Lambda^4}c^{(3)}_{W^2\phi^4}$$

3. Some anomalous dimensions significantly large

$$\dot{c}^{(3)}_{\phi^4} = \frac{1}{3}\Big(32\mathrm{Tr}[(c^{(3)}_{\phi l})^\dagger c^{(3)}_{\phi l}] + 96\mathrm{Tr}[(c^{(3)}_{\phi q})^\dagger c^{(3)}_{\phi q}] + 24\mathrm{Tr}[(c_{\phi ud})^\dagger c_{\phi ud}] - 16c_{\phi D}c_{\phi\Box} + 7c^2_{\phi D} - 40c^2_{\phi\Box}\Big),$$

Some immediate implications

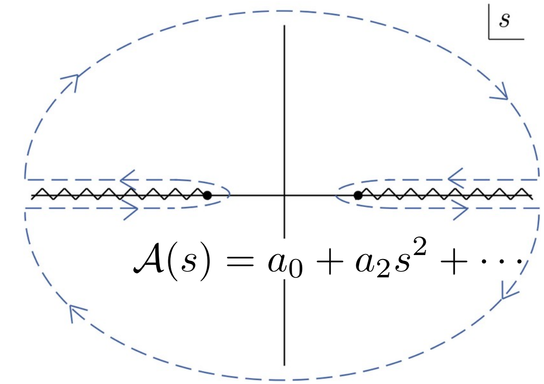


4. Positivity bounds are preserved

$$c_{\phi^4}^{(2)} \geq 0\,, \quad c_{\phi^4}^{(1)} + c_{\phi^4}^{(2)} \geq 0\,, \quad c_{\phi^4}^{(1)} + c_{\phi^4}^{(2)} + c_{\phi^4}^{(3)} \geq 0$$

$$a_2 = \frac{1}{\pi}\int_{m^2}^{\infty} \frac{\sigma s}{s^2} \geq 0$$

Some immediate implications



4. Positivity bounds are preserved

$$c_{\phi^4}^{(2)} \geq 0\,, \quad c_{\phi^4}^{(1)} + c_{\phi^4}^{(2)} \geq 0\,, \quad c_{\phi^4}^{(1)} + c_{\phi^4}^{(2)} + c_{\phi^4}^{(3)} \geq 0$$

$$a_2 = \frac{1}{\pi}\int_{m^2}^{\infty} \frac{\sigma s}{s^2} \geq 0$$

$$16\pi^2 c_{\phi^4}^{(2)} = \frac{1}{3}(5c_{\phi D}^2 + 16 c_{\phi D} c_{\phi \Box} + 16 c_{\phi \Box}^2) \log\frac{M}{\mu} > 0\,,$$

$$16\pi^2 \left[c_{\phi^4}^{(1)} + c_{\phi^4}^{(2)}\right] = \frac{16}{3}(c_{\phi D}^2 - c_{\phi D} c_{\phi \Box} + 2 c_{\phi \Box}^2) \log\frac{M}{\mu} > 0\,,$$

$$16\pi^2 \left[c_{\phi^4}^{(1)} + c_{\phi^4}^{(2)} + c_{\phi^4}^{(3)}\right] = 3(c_{\phi D}^2 + 8 c_{\phi \Box}^2) \log\frac{M}{\mu} > 0\,;$$

Some results for dimension-8 → dimension-8

	$\phi^4 D^4$	$B\phi^4 D^2$	$W\phi^4 D^2$	$B^2\phi^4$	$W^2\phi^4$	$WB\phi^4$	$G^2\phi^4$	$\phi^6 D^2$	ϕ^8
$B^2\phi^2 D^2$	g_1^2	0	0	0	0	0	0	0	0
$W^2\phi^2 D^2$	g_2^2	0	0	0	0	0	0	0	0
$WB\phi^2 D^2$	$g_1 g_2$	0	0	0	0	0	0	0	0
$G^2\phi^2 D^2$	0	0	0	0	0	0	0	0	0
$W^3\phi^2$	0	0	0	0	0	0	0	0	0
$W^2 B\phi^2$	0	0	0	0	0	0	0	0	0
$G^3\phi^2$	0	0	0	0	0	0	0	0	0
$\phi^4 D^4$	g_2^2	0	0	0	0	0	0	0	0
$B\phi^4 D^2$	$g_1 g_2^2$	λ	0	0	0	0	0	0	0
$W\phi^4 D^2$	g_2^3	0	g_2^2	0	0	0	0	0	0
$B^2\phi^4$	$g_1^2 g_2^2$	$g_1\lambda$	$g_1^2 g_2$	λ	0	$g_1 g_2$	0	0	0
$W^2\phi^4$	g_2^4	$g_1 g_2^2$	g_2^3	0	λ	$g_1 g_2$	0	0	0
$WB\phi^4$	$g_1 g_2^3$	$g_2\lambda$	$g_1\lambda$	$g_1 g_2$	$g_1 g_2$	λ	0	0	0
$G^2\phi^4$	0	0	0	0	0	0	g_3^2	0	0
$\phi^6 D^2$	g_2^4	$g_1\lambda$	$g_2\lambda$	0	0	0	0	λ	0
ϕ^8	λ^3	$g_1\lambda^2$	$g_2\lambda^2$	$g_1^2\lambda$	$g_2^2\lambda$	$g_1 g_2\lambda$	0	λ^2	λ

} loop-level generated operators (rows $B^2\phi^2 D^2$ through $G^3\phi^2$)

Some results for dimension-8 → dimension-8

	$\phi^4 D^4$	$B\phi^4 D^2$	$W\phi^4 D^2$	$B^2\phi^4$	$W^2\phi^4$	$WB\phi^4$	$G^2\phi^4$	$\phi^6 D^2$	ϕ^8
$B^2\phi^2 D^2$	g_1^2	0	0	0	0	0	0	0	0
$W^2\phi^2 D^2$	g_2^2	0	0	0	0	0	0	0	0
$WB\phi^2 D^2$	$g_1 g_2$	0	0	0	0	0	0	0	0
$G^2\phi^2 D^2$	0	0	0	0	0	0	0	0	0
$W^3\phi^2$	0	0	0	0	0	0	0	0	0
$W^2 B\phi^2$	0	0	0	0	0	0	0	0	0
$G^3\phi^2$	0	0	0	0	0	0	0	0	0
$\phi^4 D^4$	g_2^2	0	0	0	0	0	0	0	0
$B\phi^4 D^2$	$g_1 g_2^2$	λ	0	0	0	0	0	0	0
$W\phi^4 D^2$	g_2^3	0	g_2^2	0	0	0	0	0	0
$B^2\phi^4$	$g_1^2 g_2^2$	$g_1\lambda$	$g_1^2 g_2$	λ	0	$g_1 g_2$	0	0	0
$W^2\phi^4$	g_2^4	$g_1 g_2^2$	g_2^3	0	λ	$g_1 g_2$	0	0	0
$WB\phi^4$	$g_1 g_2^3$	$g_2\lambda$	$g_1\lambda$	$g_1 g_2$	$g_1 g_2$	λ	0	0	0
$G^2\phi^4$	0	0	0	0	0	0	g_3^2	0	0
$\phi^6 D^2$	g_2^4	$g_1\lambda$	$g_2\lambda$	0	0	0	0	λ	0
ϕ^8	λ^3	$g_1\lambda^2$	$g_2\lambda^2$	$g_1^2\lambda$	$g_2^2\lambda$	$g_1 g_2\lambda$	0	λ^2	λ

loop-level generated operators

sizable departure from NDA

Some results for dimension-8 $\to$ dimension-8

	$\phi^4 D^4$	$B\phi^4 D^2$	$W\phi^4 D^2$	$B^2\phi^4$	$W^2\phi^4$	$WB\phi^4$	$G^2\phi^4$	$\phi^6 D^2$	ϕ^8
$B^2\phi^2 D^2$	g_1^2	0	0	0	0	0	0	0	0
$W^2\phi^2 D^2$	g_2^2	0	0	0	0	0	0	0	0
$WB\phi^2 D^2$	$g_1 g_2$	0	0	0	0	0	0	0	0
$G^2\phi^2 D^2$	0	0	0	0	0	0	0	0	0
$W^3\phi^2$	0	0	0	0	0	0	0	0	0
$W^2 B\phi^2$	0	0	0	0	0	0	0	0	0
$G^3\phi^2$	0	0	0	0	0	0	0	0	0
$\phi^4 D^4$	g_2^2	0	0	0	0	0	0	0	0
$B\phi^4 D^2$	$g_1 g_2^2$	λ	0	0	0	0	0	0	0
$W\phi^4 D^2$	g_2^3	0	g_2^2	0	0	0	0	0	0
$B^2\phi^4$	$g_1^2 g_2^2$	$g_1\lambda$	$g_1^2 g_2$	λ	0	$g_1 g_2$	0	0	0
$W^2\phi^4$	g_2^4	$g_1 g_2^2$	g_2^3	0	λ	$g_1 g_2$	0	0	0
$WB\phi^4$	$g_1 g_2^3$	$g_2\lambda$	$g_1\lambda$	$g_1 g_2$	$g_1 g_2$	λ	0	0	0
$G^2\phi^4$	0	0	0	0	0	0	g_3^2	0	0
$\phi^6 D^2$	g_2^4	$g_1\lambda$	$g_2\lambda$	0	0	0	0	λ	0
ϕ^8	λ^3	$g_1\lambda^2$	$g_2\lambda^2$	$g_1^2\lambda$	$g_2^2\lambda$	$g_1 g_2\lambda$	0	λ^2	λ

loop-level generated operators

sizable departure from NDA

$$\dot{c}^{(3)}_{\phi^4} = -g_2^2\left(12c^{(1)}_{\phi^4} + \frac{29}{3}c^{(2)}_{\phi^4} + 14c^{(3)}_{\phi^4}\right) - 56\left(c^{(4)}_{q^2\phi^2 D^3}\right)_{\alpha_1,\alpha_2} y^u_{\alpha_2,\alpha_3}\left(y^u\right)^*_{\alpha_3,\alpha_1} + \cdots$$

Some results for dimension-8 → dimension-8

	$\psi^2 B\phi^3$	$\psi^2 W\phi^3$	$\psi^2 G\phi^3$	$\psi^2\phi^2 D^3$	$\psi^2\phi^5$	$\psi^2\phi^4 D$	$\psi^2 B\phi^2 D$	$\psi^2 W\phi^2 D$	$\psi^2 G\phi^2 D$	$\psi^2\phi^3 D^2$
$B^2\phi^2 D^2$	0	0	0	g_1^2	0	0	0	0	0	0
$W^2\phi^2 D^2$	0	0	0	g_2^2	0	0	0	0	0	0
$WB\phi^2 D^2$	0	0	0	$g_1 g_2$	0	0	0	0	0	0
$G^2\phi^2 D^2$	0	0	0	g_3^2	0	0	0	0	0	0
$W^3\phi^2$	0	0	0	0	0	0	0	0	0	0
$W^2 B\phi^2$	0	0	0	0	0	0	0	0	0	0
$G^3\phi^2$	0	0	0	0	0	0	0	0	0	0
$\phi^4 D^4$	0	0	0	$\lvert y^t\rvert^2$	0	0	0	0	0	0
$B\phi^4 D^2$	0	0	0	$g_1\lvert y^t\rvert^2$	0	0	$\lvert y^t\rvert^2$	0	0	$g_1 y^t$
$W\phi^4 D^2$	0	0	0	$g_2\lvert y^t\rvert^2$	0	0	0	$\lvert y^t\rvert^2$	0	$g_2 y^t$
$B^2\phi^4$	$g_1 y^t$	0	0	$g_1^2\lvert y^t\rvert^2$	0	0	$g_1\lvert y^t\rvert^2$	0	0	$g_1^2 y^t$
$W^2\phi^4$	0	$g_2 y^t$	0	$g_2^2\lvert y^t\rvert^2$	0	0	0	$g_2\lvert y^t\rvert^2$	0	$g_2^2 y^t$
$WB\phi^4$	$g_2 y^t$	$g_1 y^t$	0	$g_1 g_2\lvert y^t\rvert^2$	0	0	$g_2\lvert y^t\rvert^2$	$g_1\lvert y^t\rvert^2$	0	$g_1 g_2 y^t$
$G^2\phi^4$	0	0	$g_3 y^t$	0	0	0	0	0	0	0
$\phi^6 D^2$	0	0	0	$g_2^2\lvert y^t\rvert^2$	0	$\lvert y^t\rvert^2$	$g_1\lvert y^t\rvert^2$	$g_2\lvert y^t\rvert^2$	0	$y^t\lvert y^t\rvert^2$
ϕ^8	0	0	0	$\lambda\lvert y^t\rvert^4$	$y^t\lvert y^t\rvert^2$	$\lambda\lvert y^t\rvert^2$	$g_1\lambda\lvert y^t\rvert^2$	$g_2\lambda\lvert y^t\rvert^2$	0	$\lambda y^t\lvert y^t\rvert^2$

$$\dot{c}^{(3)}_{\phi^4} = -g_2^2\left(12 c^{(1)}_{\phi^4} + \frac{29}{3} c^{(2)}_{\phi^4} + 14 c^{(3)}_{\phi^4}\right) - 56\left(c^{(4)}_{q^2\phi^2 D^3}\right)_{\alpha_1,\alpha_2} y^u_{\alpha_2,\alpha_3} (y^u)^*_{\alpha_3,\alpha_1} + \cdots$$

Lower-dimensional operators renormalize too:

$$\dot{\lambda} = -94\lambda\mu^4 c^{(3)}_{\phi^4} - 16 g_2 \mu^4 c^{(1)}_{W\phi^4 D^2} - 28\mu^4 \left[(c^{(3)}_{q^2\phi^2 D^3} + c^{(4)}_{q^2\phi^2 D^3})_{\alpha_1,\alpha_2}\, y^u_{\alpha_2,\alpha_3} (y^u)^*_{\alpha_3,\alpha_1} \right] + \cdots$$

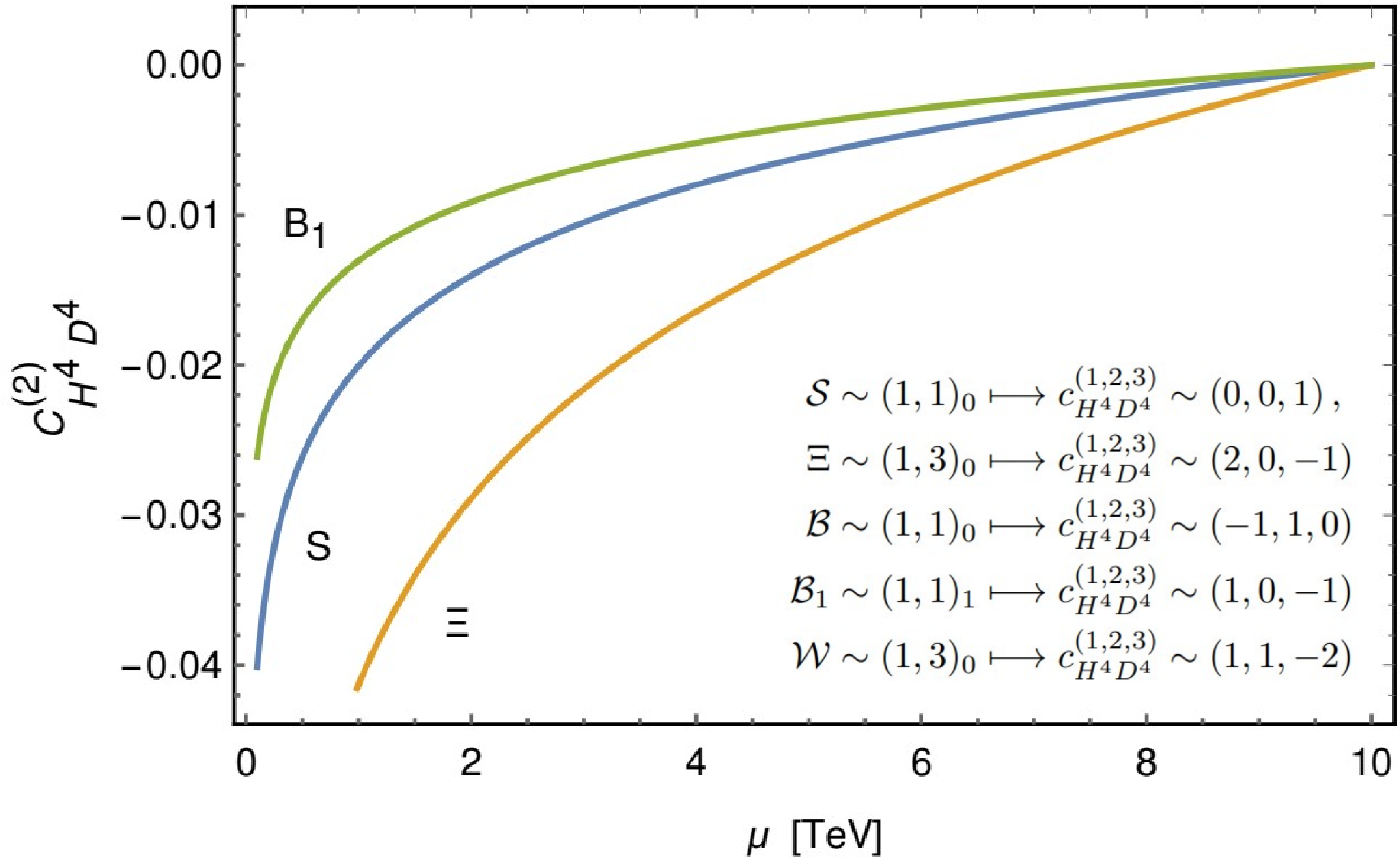
	$\phi^4 D^4$	$B\phi^4 D^2$	$W\phi^4 D^2$	$B^2\phi^4$	$W^2\phi^4$	$WB\phi^4$	$G^2\phi^4$	$\phi^6 D^2$	ϕ^8
ϕ^2	μ^6	0	0	0	0	0	0	0	0
ϕ^4	$\lambda\mu^4$	$g_1\mu^4$	$g_2\mu^4$	0	0	0	0	μ^4	0
$B^2\phi^2$	$g_1^2\mu^2$	$g_1\mu^2$	0	μ^2	0	0	0	0	0
$W^2\phi^2$	$g_2^2\mu^2$	0	$g_2\mu^2$	0	μ^2	0	0	0	0
$WB\phi^2$	$g_1 g_2\mu^2$	$g_2\mu^2$	$g_1\mu^2$	0	0	μ^2	0	0	0
$G^2\phi^2$	0	0	0	0	0	0	μ^2	0	0
$\phi^4 D^2$	$\lambda\mu^2$	$g_1\mu^2$	$g_2\mu^2$	0	0	0	0	μ^2	0
ϕ^6	$\lambda^2\mu^2$	$\lambda g_1\mu^2$	$\lambda g_2\mu^2$	$g_1^2\mu^2$	$g_2^2\mu^2$	$g_1 g_2\mu^2$	0	$\lambda\mu^2$	μ^2

Important implication: Certain positivity bounds are broken [see also **MC**, Santiago 2110.01624]

$$c_{\phi^4}^{(2)} \geq 0\,, \quad c_{\phi^4}^{(1)} + c_{\phi^4}^{(2)} \geq 0\,, \quad c_{\phi^4}^{(1)} + c_{\phi^4}^{(2)} + c_{\phi^4}^{(3)} \geq 0$$

$$\begin{aligned}
16\pi^2 \beta^{(1)}_{H^4D^4} &= \frac{1}{6}\Big[(30c^{(1)}_{H^4D^4} + 41c^{(2)}_{H^4D^4} + 15c^{(3)}_{H^4D^4})g_2^2 - (16c^{(1)}_{H^4D^4} + 7c^{(2)}_{H^4D^4} + 15c^{(3)}_{H^4D^4})g_1^2 \\
&\quad + 16(3c^{(1)}_{H^4D^4} + c^{(2)}_{H^4D^4} + c^{(3)}_{H^4D^4})\lambda\Big]\,, \\
16\pi^2 \beta^{(2)}_{H^4D^4} &= \frac{1}{6}\Big[(28c^{(1)}_{H^4D^4} + 43c^{(2)}_{H^4D^4} + 15c^{(3)}_{H^4D^4})g_2^2 + (14c^{(1)}_{H^4D^4} + 33c^{(2)}_{H^4D^4} + 15c^{(3)}_{H^4D^4})g_1^2 \\
&\quad + 16(c^{(1)}_{H^4D^4} + 3c^{(2)}_{H^4D^4} + c^{(3)}_{H^4D^4})\lambda\Big]\,, \\
16\pi^2 \beta^{(3)}_{H^4D^4} &= -\frac{1}{3}\Big[(36c^{(1)}_{H^4D^4} + 29c^{(2)}_{H^4D^4} + 42c^{(3)}_{H^4D^4})g_2^2 + (8c^{(1)}_{H^4D^4} + 2c^{(2)}_{H^4D^4} + 9c^{(3)}_{H^4D^4})g_1^2 \\
&\quad - 16(3c^{(1)}_{H^4D^4} + 2c^{(2)}_{H^4D^4} + 5c^{(3)}_{H^4D^4})\lambda\Big]\,.
\end{aligned}$$

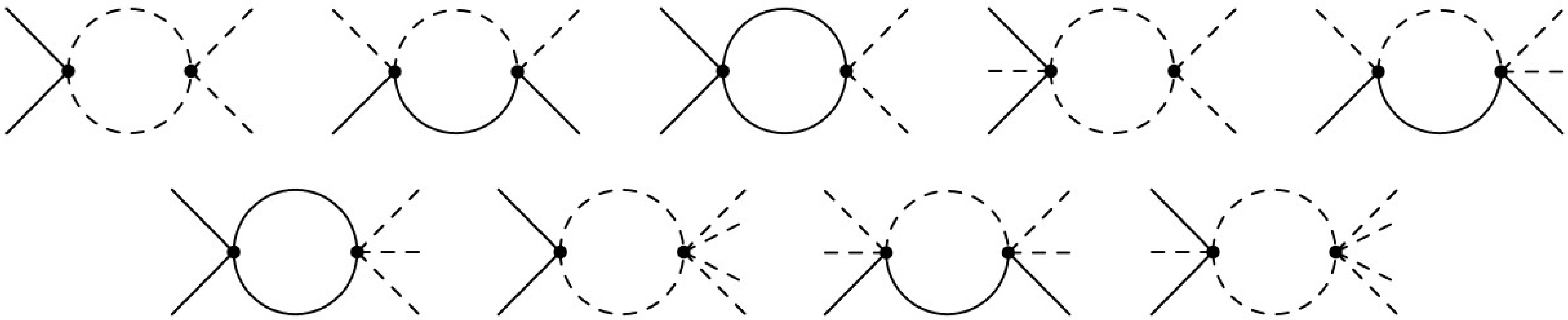
Important implication: Certain positivity bounds are broken [see also **MC**, Santiago 2110.01624]



What about two-fermion operators? [Bakshi, **MC**, Díaz-Carmona, Ren and Vilches; 2409.15408]

New complications: There're much more operators, redundant bosonic matter, on-shell relations crazy, ...

So far, we have addressed dim-6×dim-6→dim-8, while dim-8→dim-8 is ongoing



What about two-fermion operators? [Bakshi, **MC**, Díaz-Carmona, Ren and Vilches; 2409.15408]

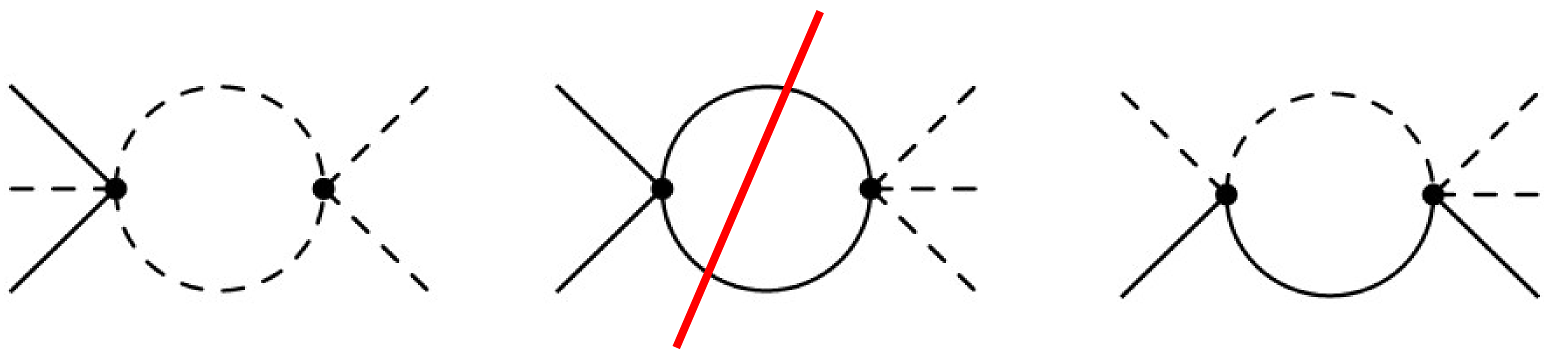
New complications: There're much more operators, redundant bosonic matter, on-shell relations crazy, ...

```
(* Dim-8 Green's basis *)

(* Psi^2 H^5 *)
(*****************************************************************************************)
OleH5     := ExpandIndices[aleH5[f1,f2] Phibar[i1] Phi[i1] Phibar[i2] Phi[i2] LLbar[sp1,i3,f1] ER[sp1,f2] Phi[i3]]
OquH5     := ExpandIndices[aquH5[f1,f2] Eps[i3,i4] Phibar[i1] Phi[i1] Phibar[i2] Phi[i2] QLbar[sp1,i3,c1,f1] UR[sp1,c1,f2] Phibar[i4]]
OqdH5     := ExpandIndices[aqdH5[f1,f2] Phibar[i1] Phi[i1] Phibar[i2] Phi[i2] QLbar[sp1,i3,c1,f1] DR[sp1,c1,f2] Phi[i3]]

(* Psi^2 H^2 D^3 *)
(*****************************************************************************************)
Ol2H2D31   := ExpandIndices[al2H2D31[f1,f2] I LLbar[sp1,i1,f1] Ga[mu,sp1,sp2] DC[LL[sp2,i1,f2],nu] (DC[DC[Phibar[i2],nu],mu]+DC[DC[Phibar[i2],mu],nu]) Phi[i2]]
Ol2H2D32   := ExpandIndices[al2H2D32[f1,f2] I LLbar[sp1,i1,f1] Ga[mu,sp1,sp2] DC[LL[sp2,i1,f2],nu] Phibar[i2] (DC[DC[Phi[i2],nu],mu]+DC[DC[Phi[i2],mu],nu])]
Ol2H2D33   := ExpandIndices[al2H2D33[f1,f2] I LLbar[sp1,i1,f1] 2 Ta[j1,i1,i2] Ga[mu,sp1,sp2] DC[LL[sp2,i2,f2],nu] (DC[DC[Phibar[i3],nu],mu]+DC[DC[Phibar[i3],mu],nu]) 2 Ta[j1,i3,i4] Ph>
Ol2H2D34   := ExpandIndices[al2H2D34[f1,f2] I LLbar[sp1,i1,f1] 2 Ta[j1,i1,i2] Ga[mu,sp1,sp2]  DC[LL[sp2,i2,f2],nu] Phibar[i3] 2 Ta[j1,i3,i4] (DC[DC[Phi[i4],nu],mu]+DC[DC[Phi[i4],mu],n>

Oe2H2D31   := ExpandIndices[ae2H2D31[f1,f2] I ERbar[sp1,f1] Ga[mu,sp1,sp2] DC[ER[sp2,f2],nu] (DC[DC[Phibar[i2],nu],mu]+DC[DC[Phibar[i2],mu],nu]) Phi[i2]]
Oe2H2D32   := ExpandIndices[ae2H2D32[f1,f2] I ERbar[sp1,f1] Ga[mu,sp1,sp2] DC[ER[sp2,f2],nu] Phibar[i2] (DC[DC[Phi[i2],nu],mu]+DC[DC[Phi[i2],mu],nu])]

Oq2H2D31   := ExpandIndices[aq2H2D31[f1,f2] I QLbar[sp1,i1,c1,f1] Ga[mu,sp1,sp2] DC[QL[sp2,i1,c1,f2],nu] (DC[DC[Phibar[i2],nu],mu]+DC[DC[Phibar[i2],mu],nu]) Phi[i2]]
Oq2H2D32   := ExpandIndices[aq2H2D32[f1,f2] I QLbar[sp1,i1,c1,f1] Ga[mu,sp1,sp2] DC[QL[sp2,i1,c1,f2],nu] Phibar[i2] (DC[DC[Phi[i2],nu],mu]+DC[DC[Phi[i2],mu],nu])]
Oq2H2D33   := ExpandIndices[aq2H2D33[f1,f2] I QLbar[sp1,i1,c1,f1] 2 Ta[j1,i1,i2] Ga[mu,sp1,sp2] DC[QL[sp2,i2,c1,f2],nu] (DC[DC[Phibar[i3],nu],mu]+DC[DC[Phibar[i3],mu],nu]) 2 Ta[j1,i3,>
Oq2H2D34   := ExpandIndices[aq2H2D34[f1,f2] I QLbar[sp1,i1,c1,f1] 2 Ta[j1,i1,i2] Ga[mu,sp1,sp2]  DC[QL[sp2,i2,c1,f2],nu] Phibar[i3] 2 Ta[j1,i3,i4] (DC[DC[Phi[i4],nu],mu]+DC[DC[Phi[i4]>

Ou2H2D31   := ExpandIndices[au2H2D31[f1,f2] I URbar[sp1,c1,f1] Ga[mu,sp1,sp2] DC[UR[sp2,c1,f2],nu] (DC[DC[Phibar[i2],nu],mu]+DC[DC[Phibar[i2],mu],nu]) Phi[i2]]
Ou2H2D32   := ExpandIndices[au2H2D32[f1,f2] I URbar[sp1,c1,f1] Ga[mu,sp1,sp2] DC[UR[sp2,c1,f2],nu] Phibar[i2] (DC[DC[Phi[i2],nu],mu]+DC[DC[Phi[i2],mu],nu])]

Od2H2D31   := ExpandIndices[ad2H2D31[f1,f2] I DRbar[sp1,c1,f1] Ga[mu,sp1,sp2] DC[DR[sp2,c1,f2],nu] (DC[DC[Phibar[i2],nu],mu]+DC[DC[Phibar[i2],mu],nu]) Phi[i2]]
Od2H2D32   := ExpandIndices[ad2H2D32[f1,f2] I DRbar[sp1,c1,f1] Ga[mu,sp1,sp2] DC[DR[sp2,c1,f2],nu] Phibar[i2] (DC[DC[Phi[i2],nu],mu]+DC[DC[Phi[i2],mu],nu])]

(* OudH2D31   := ExpandIndices[audH2D31[f1,f2] I URbar[sp1,c1,f1] Ga[mu,sp1,sp2] DC[DR[sp2,c1,f2],nu] Eps[i3,i2] Phi[i2] (DC[DC[Phi[i3],nu],mu]+DC[DC[Phi[i3],mu],nu])] *)

OudH2D31   := Sum[ExpandIndices[audH2D31[f1,f2] I URbar[sp1,c1,f1] Ga[mu,sp1,sp2] DC[DR[sp2,c1,f2],nu] Eps[i3,i2] Phi[i2] (DC[DC[Phi[i3],nu],mu]+DC[DC[Phi[i3],mu],nu])],{i2,1,2},{i3,1>


(* Redundant *)
Ol2H2D35   := ExpandIndices[rl2H2D35[f1,f2] LLbar[sp1,i1,f1] Ga[mu,sp1,sp2] DC[LL[sp2,i1,f2],nu] (DC[DC[Phibar[i2],nu],mu]+DC[DC[Phibar[i2],mu],nu]) Phi[i2]]
Ol2H2D36   := ExpandIndices[rl2H2D36[f1,f2] LLbar[sp1,i1,f1] Ga[mu,sp1,sp2] DC[LL[sp2,i1,f2],nu] Phibar[i2] (DC[DC[Phi[i2],nu],mu]+DC[DC[Phi[i2],mu],nu])]
Ol2H2D37   := ExpandIndices[rl2H2D37[f1,f2] I LLbar[sp1,i1,f1] Ga[mu,sp1,sp2] DC[LL[sp2,i1,f2],nu] DC[Phibar[i2],mu] DC[Phi[i2],nu]]
Ol2H2D38   := ExpandIndices[rl2H2D38[f1,f2] I LLbar[sp1,i1,f1] Ga[mu,sp1,sp2] DC[LL[sp2,i1,f2],nu] DC[Phibar[i2],nu] DC[Phi[i2],mu]]
Ol2H2D39   := ExpandIndices[rl2H2D39[f1,f2] LLbar[sp1,i1,f1] Ga[mu,sp1,sp2] DC[LL[sp2,i1,f2],nu] DC[Phibar[i2],mu] DC[Phi[i2],nu]]
Ol2H2D310  := ExpandIndices[rl2H2D310[f1,f2] LLbar[sp1,i1,f1] Ga[mu,sp1,sp2] DC[LL[sp2,i1,f2],nu] DC[Phibar[i2],nu] DC[Phi[i2],mu]]
                                                                              [ Read 484 lines ]
```

What about two-fermion operators? [Bakshi, **MC**, Díaz-Carmona, Ren and Vilches; 2409.15408]

New complications: There're much more operators, redundant bosonic matter, on-shell relations crazy, ...

Example: renormalization of $\mathcal{O}_{e\phi} = (\bar{l}\phi e)\phi^\dagger\phi$



$$(\dot{c}_{e\phi,mn})^{\text{dir}} = -\mu^2\Big[8(3c_{\phi\Box} - c_{\phi D})c_{e\phi,mn} + 2c_{e\phi,mp}c_{\phi e,pn} - 2(c^{(1)}_{\phi l,mp} + 3c^{(3)}_{\phi l,mp})c_{e\phi,pn}$$
$$- 4c_{\phi D}(c^{(1)}_{\phi l,mp} + c^{(3)}_{\phi l,mp})y^e_{pn} + 4c_{\phi D}y^e_{mp}c_{\phi e,pn} - 4(c^{(1)}_{\phi l,mr} + c^{(3)}_{\phi l,mr})y^e_{rp}c_{\phi e,pn}\Big]$$

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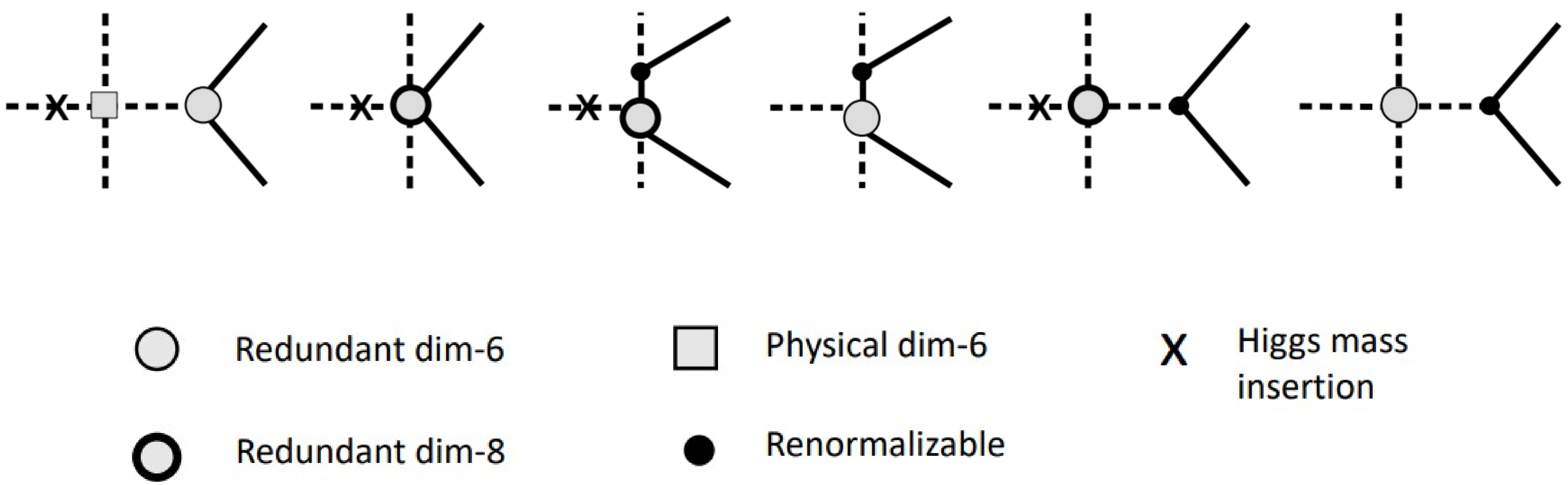

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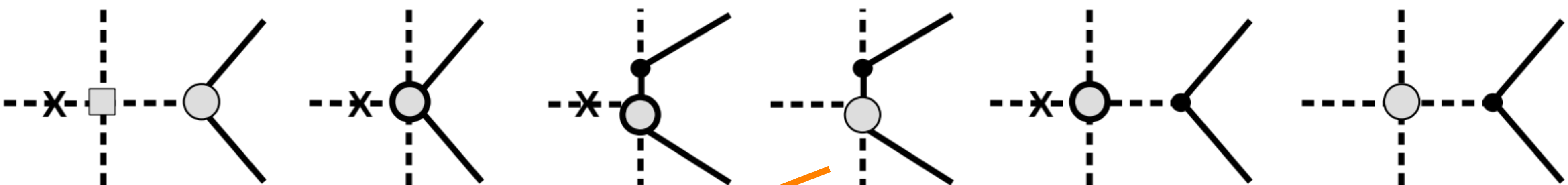
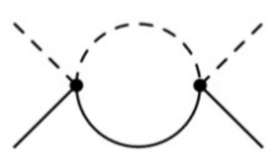


$$(\dot{c}_{e\phi,mn})^{\text{ind}} = \frac{1}{2}r'_{\phi D}y^e_{mn} + r'_{\phi e,pn}y^e_{mp} + r'^{(1)}_{\phi l,mp}y^e_{pn} + r'^{(3)}_{\phi l,mp}y^e_{pn} - \mu^2\Big[- 4c_{\phi\Box}r^{(1)}_{e\phi D,mn} + c_{\phi D}r^{(1)}_{e\phi D,mn}$$

$$\cdots r^{(2)}_{e\phi D,mn} + 2c_{\phi\Box}r^{(4)}_{e\phi D,mn} - \frac{1}{2}c_{\phi D}r^{(4)}_{e\phi D,mn} + r^{(7)}_{le\phi^3 D^2,mn}$$

$$\cdots {}_{D^2,mn} - r^{(14)}_{le\phi^3 D^2,mn} + \frac{1}{2}r^{(15)}_{le\phi^3 D^2,mn} + \frac{3}{2}ir^{(16)}_{le\phi^3 D^2,mn} - r^{(4)}_{\phi^4 D^4}y^e_{mn}$$

$$\cdots y^e_{mn} + r^{(11)}_{\phi^4 D^4}y^e_{mn} + \frac{1}{2}r^{(33)}_{l^2\phi^2 D^3,mp}y^e_{pn} + \frac{1}{2}r^{(35)}_{l^2\phi^2 D^3,mp}y^e_{pn}\Big] .$$

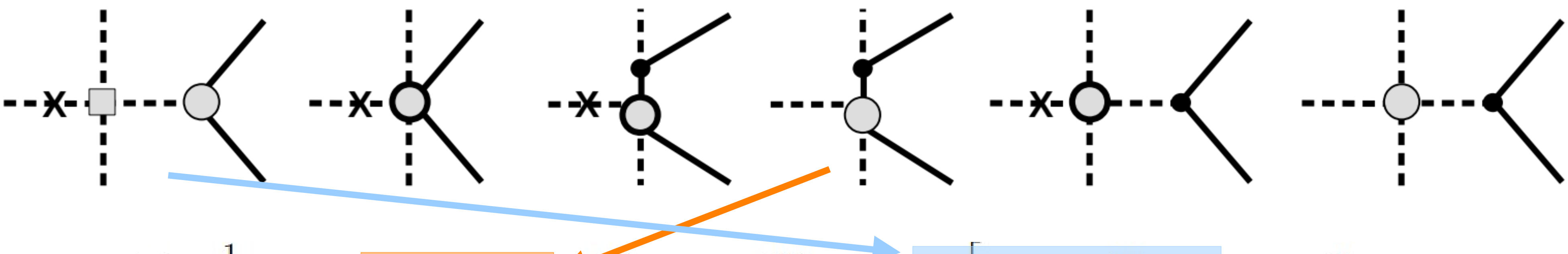


$r'_{\phi e,mn} = -2\mu^2 c_{\phi e,mp}c_{\phi e,pn}$

What about two-fermion operators? [Bakshi, **MC**, Díaz-Carmona, Ren and Vilches; 2409.15408]

New complications: There're much more operators, redundant bosonic matter, on-shell relations crazy, ...

Example: renormalization of $\mathcal{O}_{e\phi} = (\bar{l}\phi e)\phi^\dagger\phi$

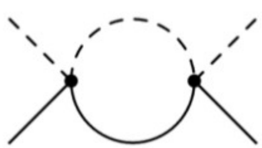


$$(\dot{c}_{e\phi,mn})^{\rm ind} = \frac{1}{2} r'_{\phi D} y^e_{mn} + r'_{\phi e,pn} y^e_{mp} + r'^{(1)}_{\phi l,mp} y^e_{pn} + r'^{(3)}_{\phi l,mp} y^e_{pn} - \mu^2 \Big[-4 c_{\phi\Box} r^{(1)}_{e\phi D,mn} + c_{\phi D} r^{(1)}_{e\phi D,mn}$$

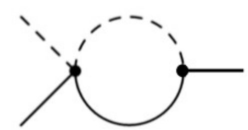
$$\ldots r^{(2)}_{e\phi D,mn} + 2 c_{\phi\Box} r^{(4)}_{e\phi D,mn} - \frac{1}{2} c_{\phi D} r^{(4)}_{e\ldots}$$

$$\ldots{}_{D^2,mn} - r^{(14)}_{le\phi^3 D^2,mn} + \frac{1}{2} r^{(15)}_{le\phi^3 D^2,m\ldots}$$

$$\ldots y^e_{mn} + r^{(11)}_{\phi^4 D^4} y^e_{mn} + \frac{1}{2} r^{(33)}_{l^2\phi^2 D^3,mp}$$



$$r'_{\phi e,mn} = -2\mu^2 c_{\phi e,mp} c_{\phi e,pn}$$



$$r^{(1)}_{e\phi D,mn} = -2 c_{\phi e,pn} y^e_{mn} + 4 c_{le,mprn} y^e_{pr} - 6 c_{ledq,mnpr} y^d_{rp} + 6 c^{(1)}_{lequ,mnpr} y^{u*}_{pr}$$

What about two-fermion operators? [Bakshi, **MC**, Díaz-Carmona, Ren and Vilches; 2409.15408]

New complications: There're much more operators, redundant bosonic matter, on-shell relations crazy, ...

Example: renormalization of $\mathcal{O}_{e\phi} = (\bar{l}\phi e)\phi^\dagger\phi$

$$\dot{c}_{e\phi,mn} = -\mu^2 \Big[48c_{\phi\Box}c_{e\phi,mn} - 12c_{\phi D}c_{e\phi,mn} + 2c_{e\phi,mp}c_{\phi e,pn} - 2c_{e\phi,pn}c^{(1)}_{\phi l,mp} - 6c_{e\phi,pn}c^{(3)}_{\phi l,mp} - 8c_{\phi\Box}c_{\phi e,pn}y^e_{mp} + 2c_{\phi D}c_{\phi e,pn}y^e_{mp} + 8c_{\phi\Box}c^{(1)}_{\phi l,mp}y^e_{pn} - 2c_{\phi D}c^{(1)}_{\phi l,mp}y^e_{pn} + 24c_{\phi\Box}c^{(3)}_{\phi l,mp}y^e_{pn} - 6c_{\phi D}c^{(3)}_{\phi l,mp}y^e_{pn} + 2c_{\phi e,pn}c_{\phi e,qp}y^e_{mq} - 4c_{\phi e,pn}c^{(1)}_{\phi l,mq}y^e_{qp} - 4c_{\phi e,pn}c^{(3)}_{\phi l,mq}y^e_{qp} + 2c^{(1)}_{\phi l,mp}c^{(1)}_{\phi l,pq}y^e_{q,n} + 2c^{(1)}_{\phi l,mp}c^{(3)}_{\phi l,pq}y^e_{qn} + 2c^{(1)}_{\phi l,pq}c^{(3)}_{\phi l,mp}y^e_{qn} + 6c^{(3)}_{\phi l,mp}c^{(3)}_{\phi l,pq}y^e_{qn} + 12c_{e\phi,pq}c_{le,mpqn} - 16c_{\phi\Box}c_{le,mpqn}y^e_{pq} + 4c_{\phi D}c_{le,mpqn}y^e_{pq} + 24c_{\phi\Box}c_{ledq,mnpq}y^d_{qp} - 6c_{\phi D}c_{ledq,mnpq}y^d_{qp} - 24c_{\phi\Box}c^{(1)}_{lequ,mnpq}y^{u*}_{pq} + 6c_{\phi D}c^{(1)}_{lequ,mnpq}y^{u*}_{pq} - 18c_{d\phi,pq}c_{ledq,mnqp} + 18c^{(1)}_{lequ,mnpq}c^*_{u\phi,pq} - 16c^2_{\phi\Box}y^e_{mn} + 10c_{\phi\Box}c_{\phi D}y^e_{mn} - 2c^2_{\phi D}y^e_{mn} \Big] + \cdots ;$$

What about two-fermion operators? [Bakshi, **MC**, Díaz-Carmona, Ren and Vilches; 2409.15408]

New complications: There're much more operators, redundant bosonic matter, on-shell relations crazy, ...

```
In[3]:= Length[cleH5[a, b] /. redundancies]
Out[3]= 108
In[2]:= cleH5[a, b] /. redundancies
Out[2]= 3 WCH6 alphaReHD1[a, b] - 8 WCHbox λ alphaReHD1[a, b] + 2 WCHD λ alphaReHD1[a, b] + 3/2 WCH6 alphaReHD2[a, b] - 4 WCHbox λ alphaReHD2[a, b] + WCHD λ alphaReHD2[a, b] - 3/2 WCH6 alphaReHD4[a, b] + 4 WCHbox λ alphaReHD4[a, b] - WCHD λ alphaReHD4[a, b] +
  2 λ alphaRleH3D211[a, b] - 2 λ alphaRleH3D214[a, b] + λ alphaRleH3D215[a, b] + 3 i λ alphaRleH3D216[a, b] + 2 λ alphaRleH3D27[a, b] + 2 i λ alphaRleH3D29[a, b] - 3/2 alphaRHDp WCeH[a, b] - i alphaRHDpp WCeH[a, b] - alphaRHep[j1, b] × WCeH[a, j1] +
  i alphaRHepp[j1, b] × WCeH[a, j1] - alphaRHl1p[a, j1] × WCeH[j1, b] - i alphaRHl1pp[a, j1] × WCeH[j1, b] - alphaRHl3p[a, j1] × WCeH[j1, b] - i alphaRHl3pp[a, j1] × WCeH[j1, b] - alphaRH63 YE[a, b] + i alphaRH64 YE[a, b] + 1/4 alphaRBH4D23 g1 YE[a, b] +
  1/4 alphaRWH4D26 g2 YE[a, b] + 1/8 alphaRWH4D27 g2 YE[a, b] + 2 alphaRHDp WCHbox YE[a, b] - 1/4 alphaRHDp WCHD YE[a, b] - 1/2 i alphaRHDpp WCHD YE[a, b] - 1/4 alphaRBDH g1 WCHD YE[a, b] - 1/4 alphaRWDH g2 WCHD YE[a, b] + 2 alphaRH410 λ YE[a, b] +
  2 alphaRH411 λ YE[a, b] - alphaRH412 λ YE[a, b] + 4 alphaRH48 λ YE[a, b] + 1/8 alphaReHD2bar[j1, Gen8] × WCeH[j1, b] × YE[a, Gen8] - 1/8 alphaReHD4bar[j1, Gen8] × WCeH[j1, b] × YE[a, Gen8] - 1/8 alphaReHD2[j1, b] × WCeHbar[j1, Gen8] × YE[a, Gen8] +
  1/8 alphaReHD4[j1, b] × WCeHbar[j1, Gen8] × YE[a, Gen8] - 1/2 i alphaRe2H4D2[j1, b] × YE[a, j1] + alphaRe2H4D3[j1, b] × YE[a, j1] + i alphaRHDpp WCHe[j1, b] × YE[a, j1] + 1/4 alphaReHD2bar[j1, j2] × WCeH[j1, b] × YE[a, j2] +
  1/2 alphaReHD3bar[j1, j2] × WCeH[j1, b] × YE[a, j2] + 1/4 alphaReHD4bar[j1, j2] × WCeH[j1, b] × YE[a, j2] - 2 i alphaRHDpp WCHe[j2, b] × YE[a, j2] + 1/8 alphaReHD2bar[Gen8, j1] × WCeH[a, j1] × YE[Gen8, b] + 3/4 alphaReHD3bar[Gen8, j1] × WCeH[a, j1] × YE[Gen8, b] +
  1/8 alphaReHD4bar[Gen8, j1] × WCeH[a, j1] × YE[Gen8, b] - 1/8 alphaReHD2[a, j1] × WCeHbar[Gen8, j1] × YE[Gen8, b] + 1/4 alphaReHD3[a, j1] × WCeHbar[Gen8, j1] × YE[Gen8, b] - 1/8 alphaReHD4[a, j1] × WCeHbar[Gen8, j1] × YE[Gen8, b] +
  λ alphaRl2H2D333[a, j1] × YE[j1, b] + λ alphaRl2H2D335[a, j1] × YE[j1, b] + 1/2 i alphaRl2H4D5[a, j1] × YE[j1, b] + alphaRl2H4D6[a, j1] × YE[j1, b] + alphaRl2H4D7[a, j1] × YE[j1, b] - i alphaRl2H4D8[a, j1] × YE[j1, b] - i alphaRHDpp WCHl1[a, j1] × YE[j1, b] +
  i alphaRHDpp WCHl3[a, j1] × YE[j1, b] - 1/4 alphaRleH3D213bar[j1, j2] × YE[a, j2] × YE[j1, b] - 3/4 i alphaRleH3D216bar[j1, j2] × YE[a, j2] × YE[j1, b] - 1/2 i alphaRleH3D29bar[j1, j2] × YE[a, j2] × YE[j1, b] + 1/4 alphaReHD2bar[j2, j1] × WCeH[a, j1] × YE[j2, b] -
  1/4 alphaReHD4bar[j2, j1] × WCeH[a, j1] × YE[j2, b] + 2 i alphaRHDpp WCHl1[a, j2] × YE[j2, b] - 1/4 alphaReHD2[a, j2] × WCeH[j1, b] × YEbar[j1, j2] + 1/2 alphaReHD3[a, j2] × WCeH[j1, b] × YEbar[j1, j2] - 1/4 alphaReHD4[a, j2] × WCeH[j1, b] × YEbar[j1, j2] +
  1/4 alphaRleH3D211[j1, b] × YE[a, j2] × YEbar[j1, j2] + 1/8 alphaRleH3D215[j1, b] × YE[a, j2] × YEbar[j1, j2] - 3/8 i alphaRleH3D216[j1, b] × YE[a, j2] × YEbar[j1, j2] - 1/4 alphaRleH3D27[j1, b] × YE[a, j2] × YEbar[j1, j2] -
  1/4 i alphaRleH3D29[j1, b] × YE[a, j2] × YEbar[j1, j2] - 1/2 alphaRl2H2D321[j1, j2] × YE[a, j3] × YE[j2, b] × YEbar[j1, j3] + 1/4 alphaRl2H2D324[j1, j2] × YE[a, j3] × YE[j2, b] × YEbar[j1, j3] + 1/4 alphaRl2H2D325[j1, j2] × YE[a, j3] × YE[j2, b] × YEbar[j1, j3] -
  1/8 alphaRl2H2D333[j1, j2] × YE[a, j3] × YE[j2, b] × YEbar[j1, j3] - 1/8 alphaRl2H2D335[j1, j2] × YE[a, j3] × YE[j2, b] × YEbar[j1, j3] - 1/4 alphaRl2H2D336[j1, j2] × YE[a, j3] × YE[j2, b] × YEbar[j1, j3] - 1/2 alphaRl2H2D338[j1, j2] × YE[a, j3] × YE[j2, b] × YEbar[j1, j3] +
  1/4 alphaRl2H2D37[j1, j2] × YE[a, j3] × YE[j2, b] × YEbar[j1, j3] + 1/4 alphaRl2H2D38[j1, j2] × YE[a, j3] × YE[j2, b] × YEbar[j1, j3] - 1/4 alphaReHD2[j2, b] × WCeH[a, j1] × YEbar[j2, j1] + 1/4 alphaReHD4[j2, b] × WCeH[a, j1] × YEbar[j2, j1] -
  1/4 alphaRleH3D211[a, j1] × YE[j2, b] × YEbar[j2, j1] - 3/4 alphaRleH3D213[a, j1] × YE[j2, b] × YEbar[j2, j1] - 1/8 alphaRleH3D215[a, j1] × YE[j2, b] × YEbar[j2, j1] - 3/8 i alphaRleH3D216[a, j1] × YE[j2, b] × YEbar[j2, j1] + 1/4 alphaRleH3D27[a, j1] × YE[j2, b] × YEbar[j2, j1] -
  1/4 i alphaRleH3D29[a, j1] × YE[j2, b] × YEbar[j2, j1] - 1/2 alphaRe2H2D319[j1, j2] × YE[a, j1] × YE[j3, b] × YEbar[j3, j2] + 1/4 alphaRe2H2D35[j1, j2] × YE[a, j1] × YE[j3, b] × YEbar[j3, j2] + 1/4 alphaRe2H2D36[j1, j2] × YE[a, j1] × YE[j3, b] × YEbar[j3, j2] -
  1/2 alphaRl2H2D321[a, j1] × YE[j1, j2] × YE[j3, b] × YEbar[j3, j2] - 1/4 alphaRl2H2D324[a, j1] × YE[j1, j2] × YE[j3, b] × YEbar[j3, j2] - 1/4 alphaRl2H2D325[a, j1] × YE[j1, j2] × YE[j3, b] × YEbar[j3, j2] + 1/8 alphaRl2H2D333[a, j1] × YE[j1, j2] × YE[j3, b] × YEbar[j3, j2] +
  1/8 alphaRl2H2D335[a, j1] × YE[j1, j2] × YE[j3, b] × YEbar[j3, j2] + 1/4 alphaRl2H2D336[a, j1] × YE[j1, j2] × YE[j3, b] × YEbar[j3, j2] - 1/2 alphaRl2H2D338[a, j1] × YE[j1, j2] × YE[j3, b] × YEbar[j3, j2] - 1/4 alphaRl2H2D37[a, j1] × YE[j1, j2] × YE[j3, b] × YEbar[j3, j2] -
  1/4 alphaRl2H2D38[a, j1] × YE[j1, j2] × YE[j3, b] × YEbar[j3, j2] - 1/2 alphaRe2H2D319[j1, b] × YE[a, j2] × YE[j3, j1] × YEbar[j3, j2] - 1/4 alphaRe2H2D35[j1, b] × YE[a, j2] × YE[j3, j1] × YEbar[j3, j2] - 1/4 alphaRe2H2D36[j1, b] × YE[a, j2] × YE[j3, j1] × YEbar[j3, j2]
```

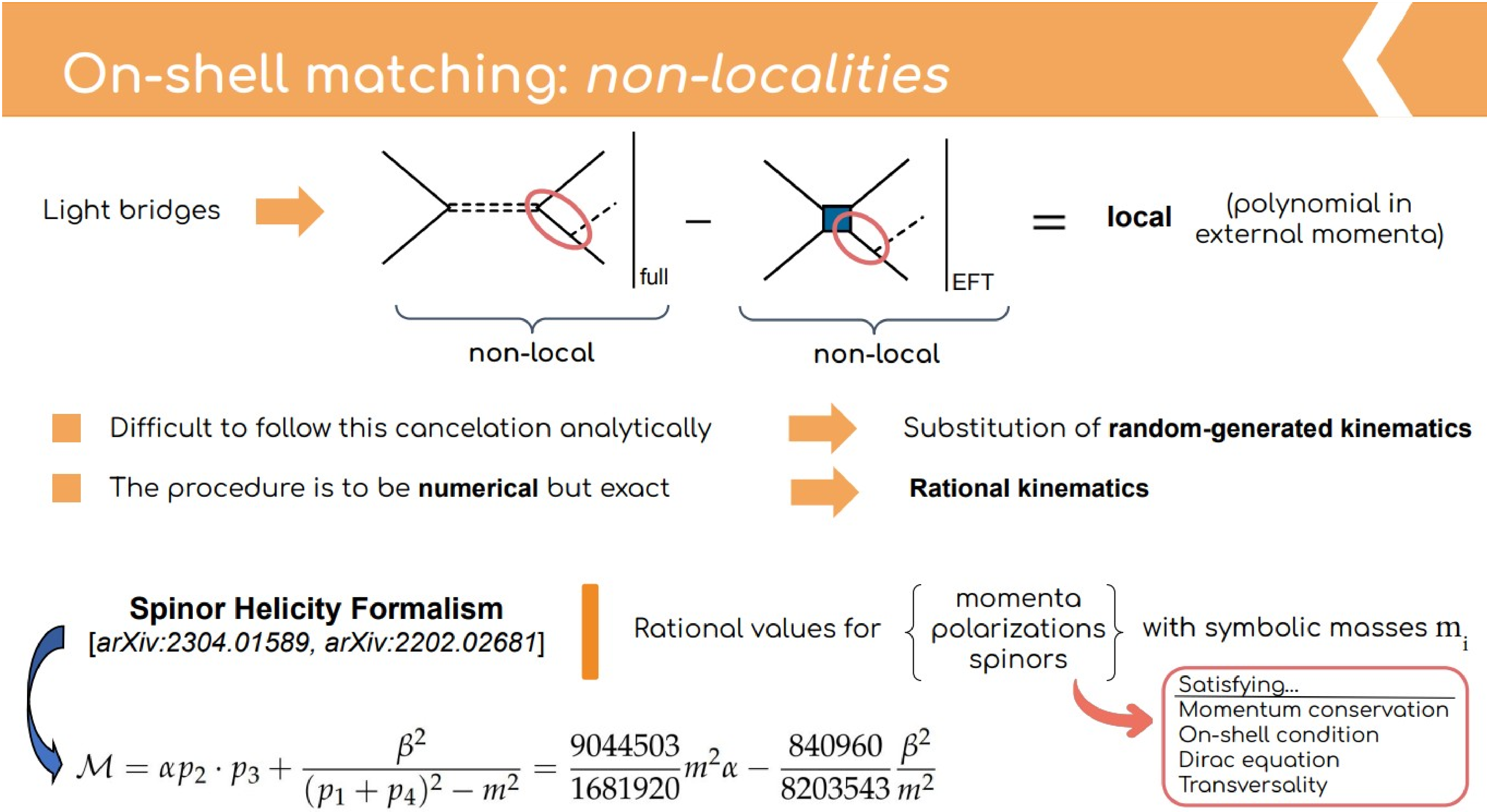
What about two-fermion operators? [Bakshi, **MC**, Díaz-Carmona, Ren and Vilches; 2409.15408]

New complications: There're much more operators, redundant bosonic matter, on-shell relations crazy, ...

How to address this? On-shell matching at tree level!

Main point:

amplitudes computed with and without redundant operators must match; this can be turn into a linear system of equations whose solutions are how physical operators are shifted in terms of redundant ones [MC, López-Miras, Santiago, Vilches 2411.12798]



Taken from Javier López-Miras, SMEFT-Tools 2025

What about two-fermion operators? [Bakshi, **MC**, Díaz-Carmona, Ren and Vilches; 2409.15408]

$\psi^2\phi^2 D^3$	$c_{\phi^4 D^2}$	c_{ϕ^6}	$c_{\psi^2\phi^2 D}$	$c_{\psi^2\phi^3}$	c_{ψ^4}
$c_{\phi^4 D^2}$	0	0	$\dots$	0	0
c_{ϕ^6}		0	0	0	0
$c_{\psi^2\phi^2 D}$			$\dots$	0	$\dots$
$c_{\psi^2\phi^3}$				0	0
c_{ψ^4}					0

$X\psi^2\phi^2 D$	$c_{\phi^4 D^2}$	c_{ϕ^6}	$c_{\psi^2\phi^2 D}$	$c_{\psi^2\phi^3}$	c_{ψ^4}
$c_{\phi^4 D^2}$	0	0	g	0	0
c_{ϕ^6}		0	0	0	0
$c_{\psi^2\phi^2 D}$			g	0	g
$c_{\psi^2\phi^3}$				0	0
c_{ψ^4}					0

$\psi^2\phi^3 D^2$	$c_{\phi^4 D^2}$	c_{ϕ^6}	$c_{\psi^2\phi^2 D}$	$c_{\psi^2\phi^3}$	c_{ψ^4}
$c_{\phi^4 D^2}$	y	0	y	$\dots$	y
c_{ϕ^6}		0	0	0	0
$c_{\psi^2\phi^2 D}$			y	$\dots$	y
$c_{\psi^2\phi^3}$				0	$\dots$
c_{ψ^4}					0

$X\psi^2\phi^3$	$c_{\phi^4 D^2}$	c_{ϕ^6}	$c_{\psi^2\phi^2 D}$	$c_{\psi^2\phi^3}$	c_{ψ^4}
$c_{\phi^4 D^2}$	0	0	gy	0	0
c_{ϕ^6}		0	0	0	0
$c_{\psi^2\phi^2 D}$			gy	g	0
$c_{\psi^2\phi^3}$				0	0
c_{ψ^4}					0

What about two-fermion operators? [Bakshi, **MC**, Díaz-Carmona, Ren and Vilches; 2409.15408]

Some cross-checks:

- Positivity bounds
- Manohar et al work from geometry
- Angelis and Huber work using amplitudes
- ...

Further work:

- Impact on electroweak observables, ...
- Breaking of universality within the SMEFT [Wells, Zhang '15]

Why to worry about a theory with several thousands of free parameters?

1. The SMEFT is the low-energy limit of countless theories

2. Only few operators contribute at leading order to a given observable

3. Pushing calculation to the limit: new methods, new tools, ...

4. New theoretical insights: positivity bounds, evanescent operators, anomalies...

Outlook

The SMEFT RGEs are instrumental for testing the theory against all experimental data (obtained at very different scales)

As of now, almost all RGEs ensuing from loops of tree-level operators have been calculated (and partially cross-checked)

All results can be accessed at:

github.com/SMEFTDimension8-RGEs

Thank you!

Backup

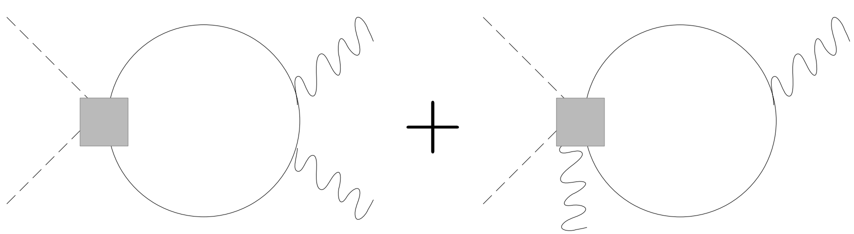
It is obvious that there are zeros in mixing of specific operators of different classes

It is not so clear how to anticipate them, not even with amplitude methods

$$\mathcal{O}^{(1)}_{e^2\phi^2 D^3} = i(\bar{e}\gamma^\mu D^\nu e)(D_{(\mu}D_{\nu)}\phi^\dagger\phi) + \text{h.c.}$$

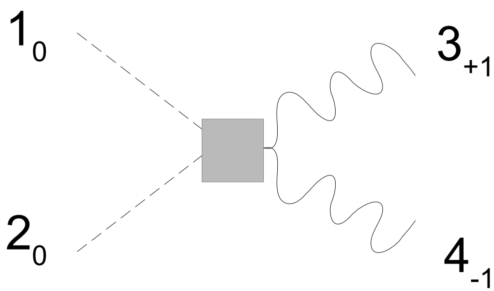
$$\mathcal{O}^{(1)}_{B^2\phi^2 D^2} = (D^\mu\phi^\dagger D^\nu\phi)B_{\mu\rho}B_\nu^\rho$$

$$\mathcal{O}^{(2)}_{e^2\phi^2 D^3} = i(\bar{e}\gamma^\mu D^\nu e)(\phi^\dagger D_{(\mu}D_{\nu)}\phi) + \text{h.c.}$$

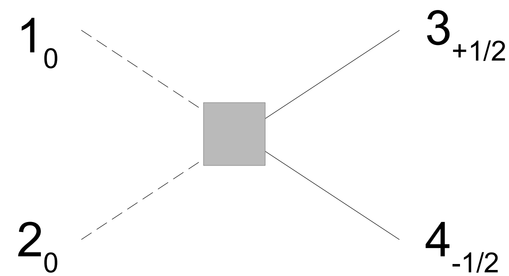


$+\cdots$

$$\underbrace{\mathcal{O}^{(1)}_{e^2\phi^2 D^3} - \mathcal{O}^{(2)}_{e^2\phi^2 D^3}}_{\widetilde{\mathcal{O}}^{(1)}_{e^2\phi^2 D^3}} \not\rightarrow \mathcal{O}^{(1)}_{B^2\phi^2 D^2}$$

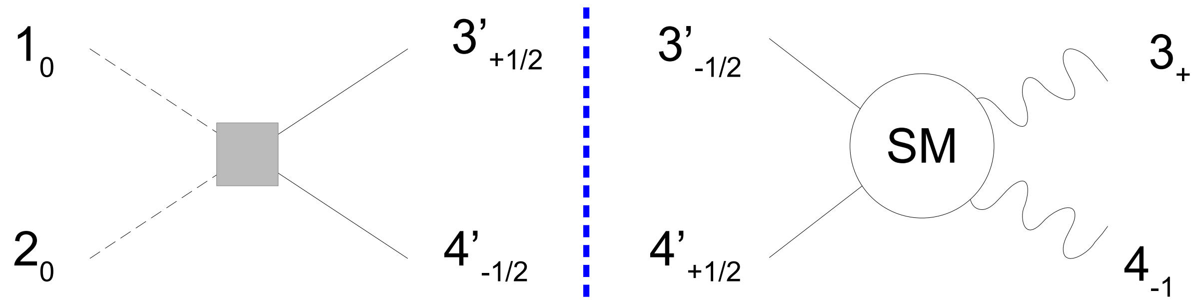


$$= \langle 41\rangle^2[31]^2$$



$$= \langle 43\rangle\langle 41\rangle[43][31]$$

$$\gamma_{\tilde{c}^{(1)}_{e^2\phi^2D^3}\to c^{(1)}_{B^2\phi^2D^2}} \propto$$

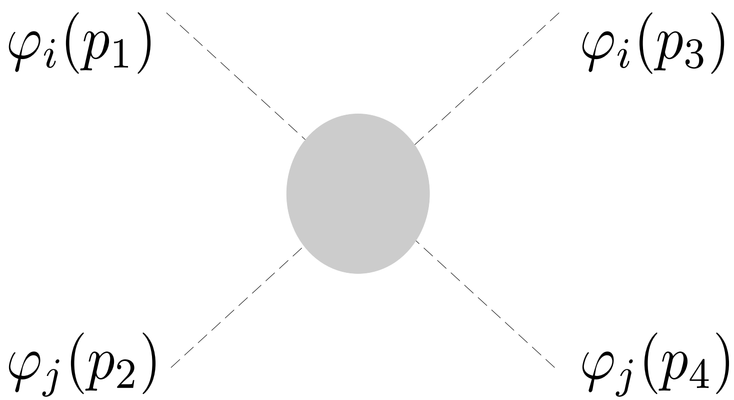


$$= \int d\text{LIPS}\langle 4'3'\rangle\langle 4'1\rangle[4'3'][3'1]\frac{\langle 3'4\rangle^2}{\langle 3'3\rangle\langle 34'\rangle}$$

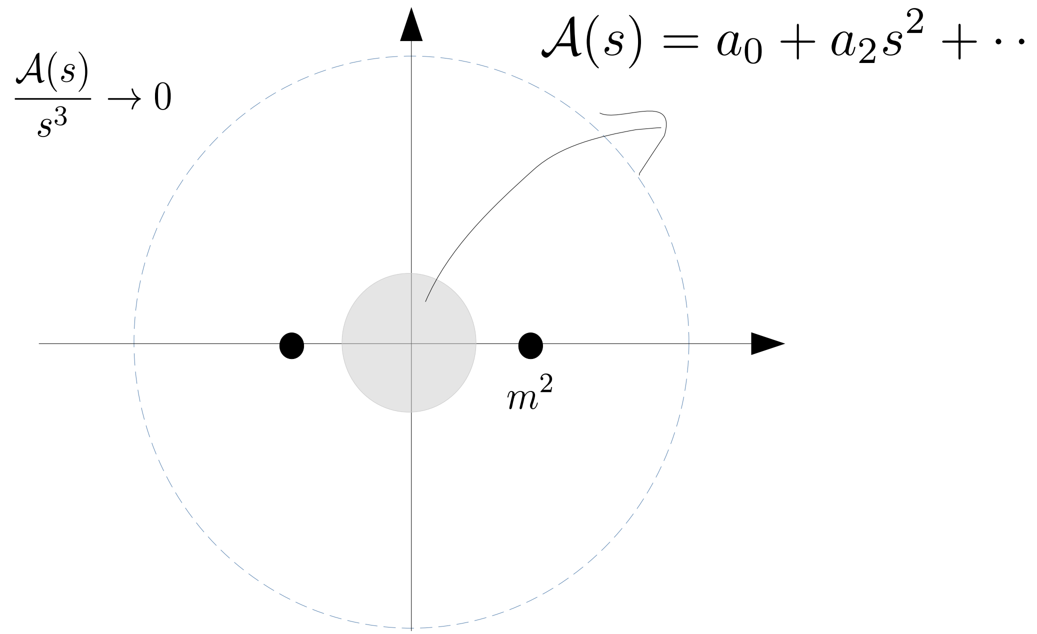
0

$$= \int_0^{2\pi} d\phi \int_0^{\pi/2} d\theta s_\theta c_\theta \left[\#_1 e^{i\phi} + \#_2 e^{2i\phi} + \cdots\right]$$

A different perspective: certain operators are constrained by positivity, from unitarity+locality [Adams, Arkani-Hamed, Nicolis, Rattazzi ‘06]



$\mathcal{A}(s) \equiv \mathcal{A}(s, t=0)$



$$0 = \sum \text{res} \frac{\mathcal{A}(s)}{s^3} = a_2 - \frac{1}{\pi} \int s \frac{\sigma(s)}{(m^2)^3} \Rightarrow a_2 > 0$$

A different perspective: certain operators are constrained by positivity, from unitarity+locality [Adams, Arkani-Hamed, Nicolis, Rattazzi '06]

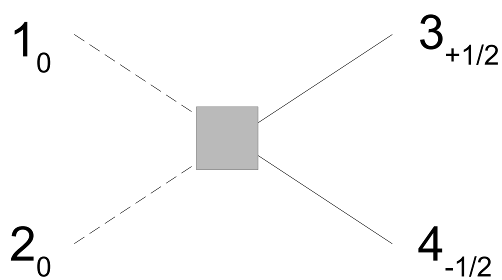
$$c^{(1)}_{B^2\phi^2 D^2} \leq 0$$

A different perspective: certain operators are constrained by positivity, from unitarity+locality [Adams, Arkani-Hamed, Nicolis, Rattazzi '06]

$$c^{(1)}_{B^2\phi^2D^2} \leq 0$$

But some others are not:

$$\widetilde{\mathcal{O}}^{(1)}_{e^2\phi^2D^3} =$$



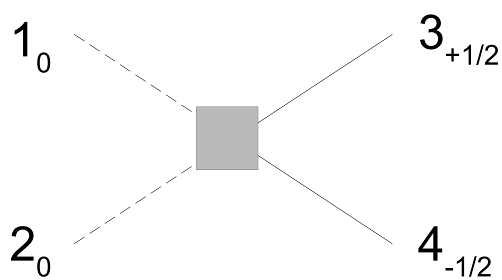
$$= \langle 43\rangle\langle 41\rangle[43][31]$$

A different perspective: certain operators are constrained by positivity, from unitarity+locality [Adams, Arkani-Hamed, Nicolis, Rattazzi '06]

$$c^{(1)}_{B^2\phi^2D^2} \leq 0$$

But some others are not:

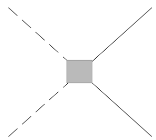
$$\widetilde{\mathcal{O}}^{(1)}_{e^2\phi^2D^3} =$$

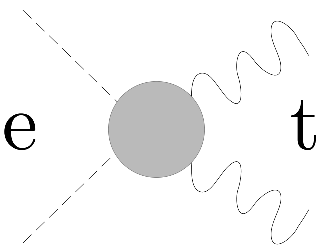


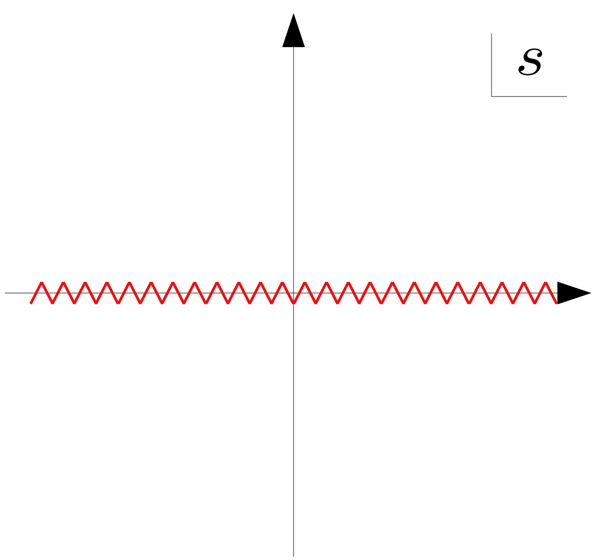
$$= \langle 43\rangle\langle 41\rangle[43][31]$$

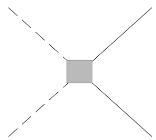
"Therefore",

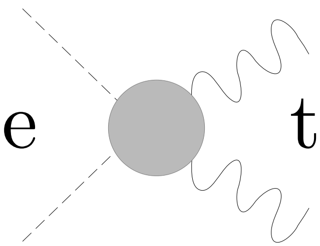
$$\dot{c}^{(1)}_{B^2\phi^2D^2} = \#_1 \tilde{c}^{(1)}_{e^2\phi^2D^3} + \cdots \Rightarrow \#_1 = 0$$

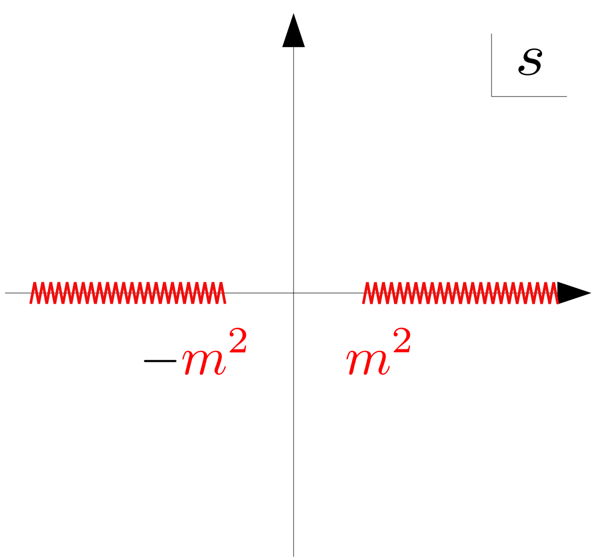
1. For any $(c^{(1)}_{e^2\phi^2 D^3}, c^{(2)}_{e^2\phi^2 D^3})$ compatible with the positivity bounds ($c^{(1)}_{e^2\phi^2 D^3} + c^{(2)}_{e^2\phi^2 D^3} \leq 0$) , there exists UV such that only $(c^{(1)}_{e^2\phi^2 D^3}, c^{(2)}_{e^2\phi^2 D^3})$ (and lower-dimensional  ones) at tree level.

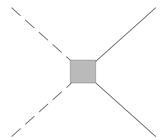
2. Within any such UV, compute  to order $O(g^2)$

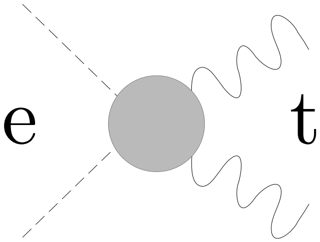


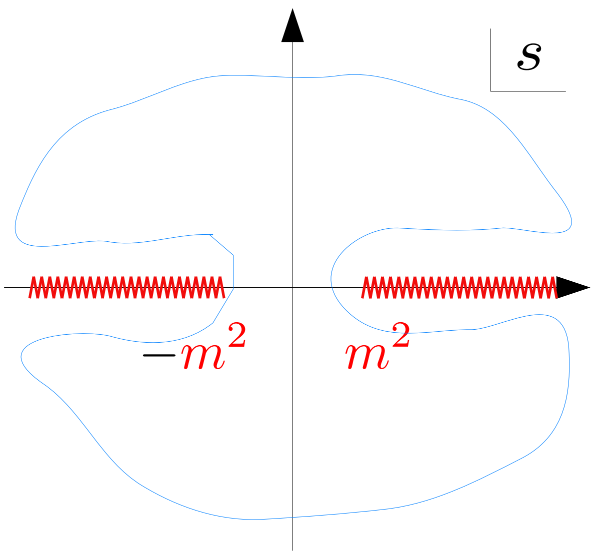
1. For any $(c^{(1)}_{e^2\phi^2 D^3}, c^{(2)}_{e^2\phi^2 D^3})$ compatible with the positivity bounds ($c^{(1)}_{e^2\phi^2 D^3} + c^{(2)}_{e^2\phi^2 D^3} \leq 0$) , there exists UV such that only $(c^{(1)}_{e^2\phi^2 D^3}, c^{(2)}_{e^2\phi^2 D^3})$ (and lower-dimensional  ones) at tree level.

2. Within any such UV, compute  to order $O(g^2)$

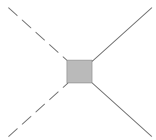


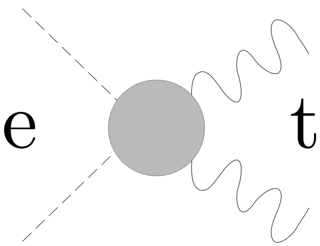
1. For any $(c^{(1)}_{e^2\phi^2 D^3}, c^{(2)}_{e^2\phi^2 D^3})$ compatible with the positivity bounds ($c^{(1)}_{e^2\phi^2 D^3} + c^{(2)}_{e^2\phi^2 D^3} \leq 0$) , there exists UV such that only $(c^{(1)}_{e^2\phi^2 D^3}, c^{(2)}_{e^2\phi^2 D^3})$ (and lower-dimensional  ones) at tree level.

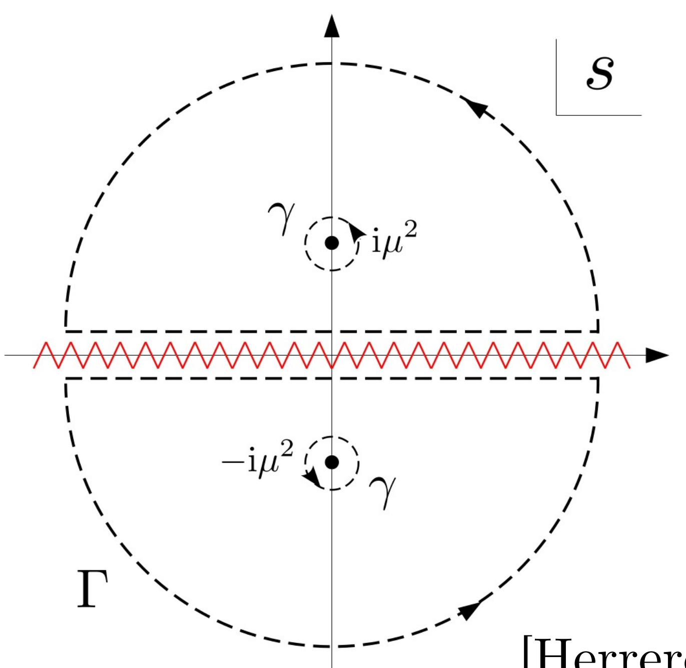
2. Within any such UV, compute  to order $O(g^2)$



See Minyuan's talk

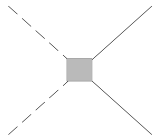
1. For any $(c^{(1)}_{e^2\phi^2D^3}, c^{(2)}_{e^2\phi^2D^3})$ compatible with the positivity bounds ($c^{(1)}_{e^2\phi^2D^3} + c^{(2)}_{e^2\phi^2D^3} \leq 0$) , there exists UV such that only $(c^{(1)}_{e^2\phi^2D^3}, c^{(2)}_{e^2\phi^2D^3})$ (and lower-dimensional  ones) at tree level.

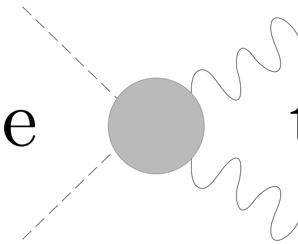
2. Within any such UV, compute  to order $O(g^2)$

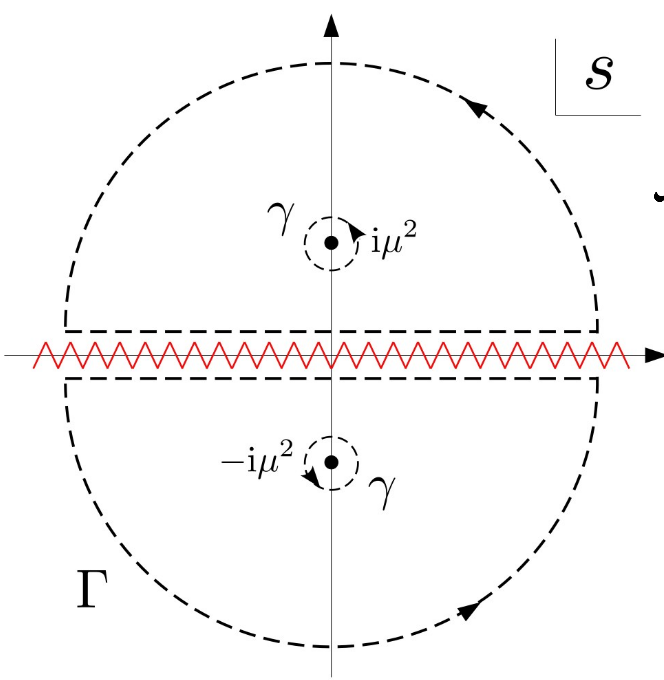


$$\Sigma(\mu) \equiv \frac{1}{2\pi i}\int_\gamma \frac{\mathcal{A}(s)s^3}{(s^2+\mu^4)^3} \geq 0$$

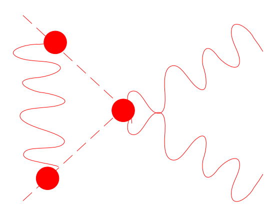
[Herrero-Valea et al ‘20]

1. For any $(c^{(1)}_{e^2\phi^2D^3}, c^{(2)}_{e^2\phi^2D^3})$ compatible with the positivity bounds ($c^{(1)}_{e^2\phi^2D^3} + c^{(2)}_{e^2\phi^2D^3} \leq 0$), there exists UV such that only $(c^{(1)}_{e^2\phi^2D^3}, c^{(2)}_{e^2\phi^2D^3})$ (and lower-dimensional  ones) at tree level.

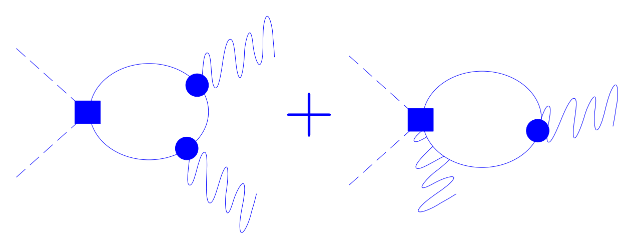
2. Within any such UV, compute  to order $O(g^2)$



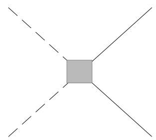
$$\mathcal{A}(s) \sim -(\beta_4 + \beta_8 s^2 + \beta_{12} s^4 + \cdots) \log \frac{s}{\Lambda^2}$$

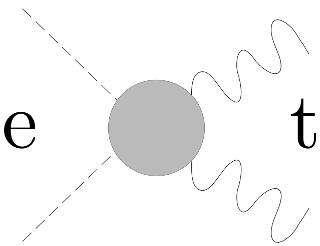


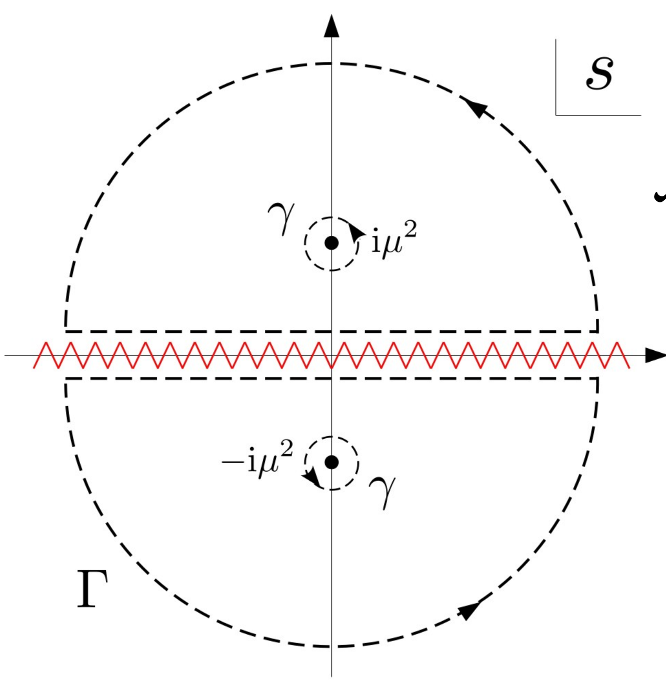
$+ \cdots$



$+ \cdots$

1. For any $(c^{(1)}_{e^2\phi^2 D^3}, c^{(2)}_{e^2\phi^2 D^3})$ compatible with the positivity bounds ($c^{(1)}_{e^2\phi^2 D^3} + c^{(2)}_{e^2\phi^2 D^3} \leq 0$) , there exists UV such that only $(c^{(1)}_{e^2\phi^2 D^3}, c^{(2)}_{e^2\phi^2 D^3})$ (and lower-dimensional  ones) at tree level.

2. Within any such UV, compute  to order $O(g^2)$



$$\mathcal{A}(s) \sim -(\beta_4 + \beta_8 s^2 + \beta_{12} s^4 + \cdots) \log \frac{s}{\Lambda^2}$$

$$\Sigma(\mu) = -\beta_8 + \beta_{12}\mu^4 + \cdots$$

$$\Rightarrow \lim_{\mu \to 0} \Sigma(\mu) = -\beta_8 \geq 0$$

So $\beta_8 \le 0$ in any of the aforementioned UV, and therefore for all values of $(c^{(1)}_{e^2\phi^2 D^3}, c^{(2)}_{e^2\phi^2 D^3})$ compatible with $c^{(1)}_{e^2\phi^2 D^3} + c^{(2)}_{e^2\phi^2 D^3} \le 0$

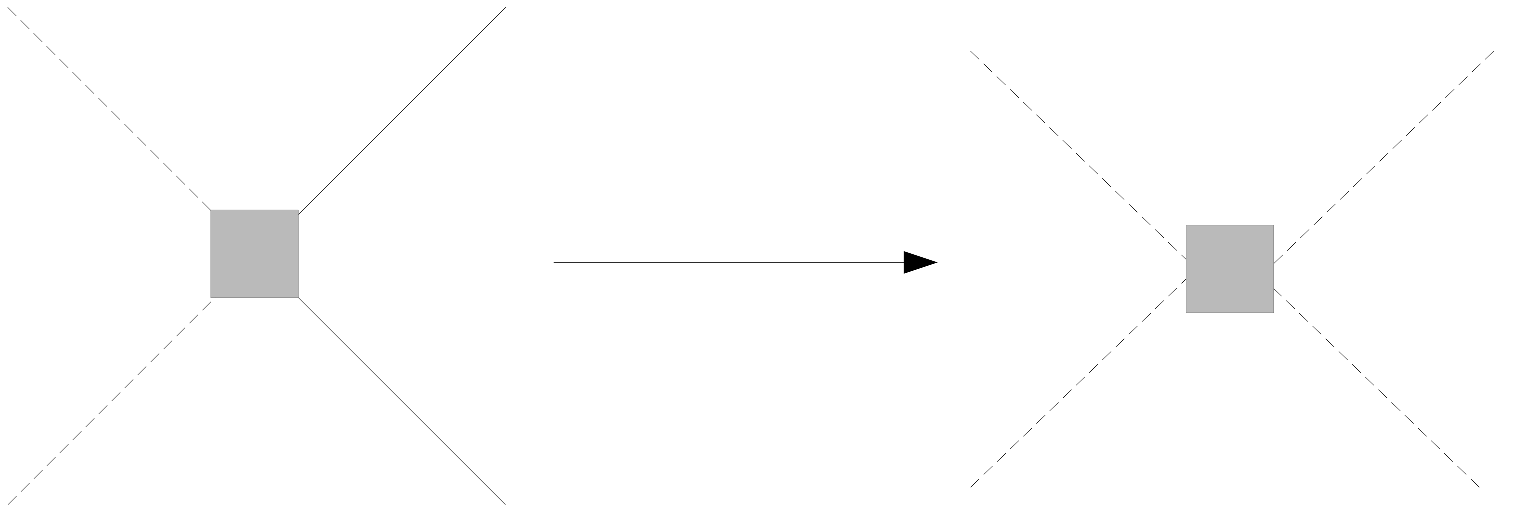
3. The beta function is linear in the Wilson coefficients:

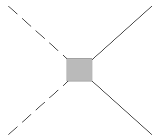
$$\beta_8 = \alpha(c^{(1)}_{e^2\phi^2 D^3} + c^{(2)}_{e^2\phi^2 D^3}) , \quad \alpha \ge 0$$

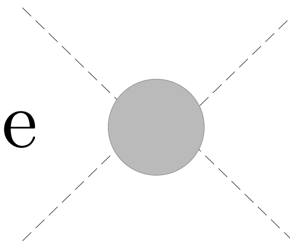
Therefore,

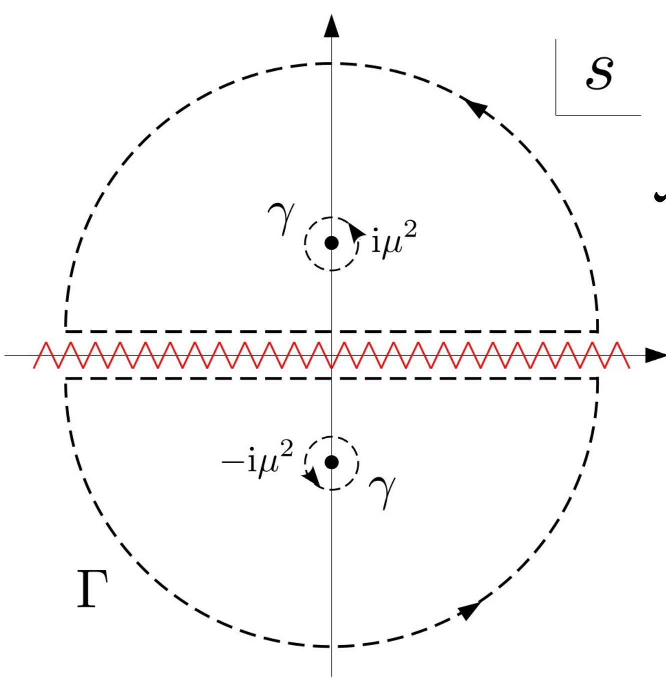
$$\underbrace{\mathcal{O}^{(1)}_{e^2\phi^2 D^3} - \mathcal{O}^{(2)}_{e^2\phi^2 D^3}}_{\widetilde{\mathcal{O}}^{(1)}_{e^2\phi^2 D^3}} \not\longrightarrow \mathcal{O}^{(1)}_{B^2\phi^2 D^2}$$

How do things change if we consider instead...?

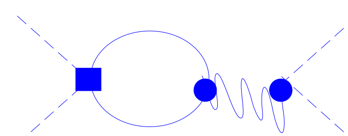


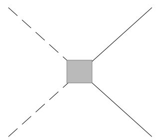
1. For any $(c^{(1)}_{e^2\phi^2 D^3}, c^{(2)}_{e^2\phi^2 D^3})$ compatible with the positivity bounds ($c^{(1)}_{e^2\phi^2 D^3} + c^{(2)}_{e^2\phi^2 D^3} \leq 0$) , there exists UV such that only $(c^{(1)}_{e^2\phi^2 D^3}, c^{(2)}_{e^2\phi^2 D^3})$ (and lower-dimensional  ones) at tree level.

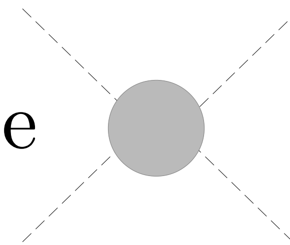
2. Within any such UV, compute  to order $\mathrm{O}(g^2)$

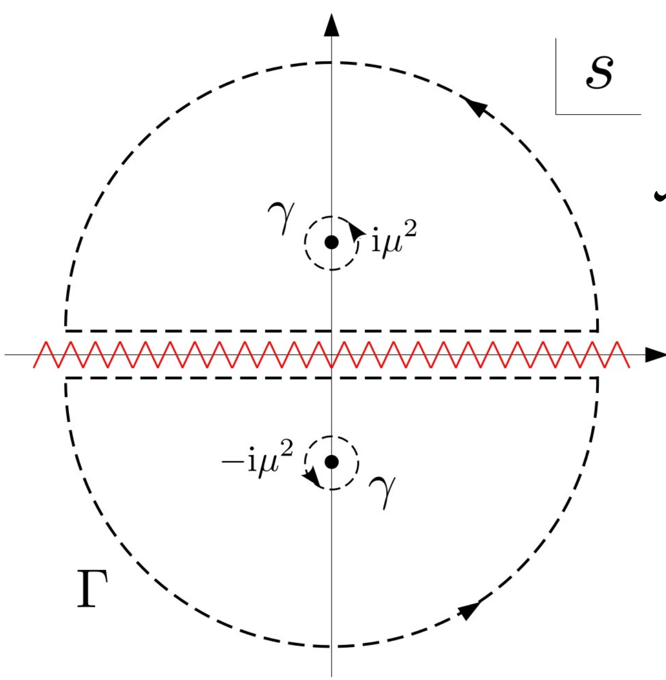


$$\mathcal{A}(s) \sim -(\beta_4 + \beta_8 s^2 + \beta_{12} s^4 + \cdots) \log \frac{s}{\Lambda^2}$$

$\cdots$  $+ \cdots$

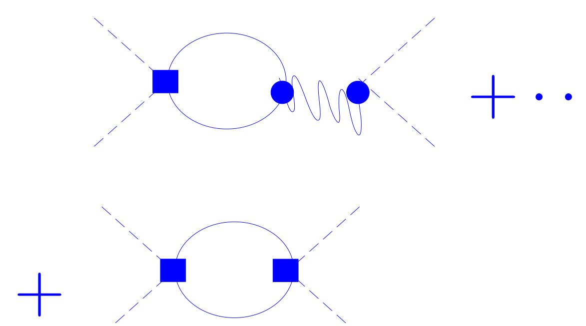
1. For any $(c^{(1)}_{e^2\phi^2 D^3}, c^{(2)}_{e^2\phi^2 D^3})$ compatible with the positivity bounds ($c^{(1)}_{e^2\phi^2 D^3} + c^{(2)}_{e^2\phi^2 D^3} \leq 0$) , there exists UV such that only $(c^{(1)}_{e^2\phi^2 D^3}, c^{(2)}_{e^2\phi^2 D^3})$ (and lower-dimensional  ones) at tree level.

2. Within any such UV, compute  to order $\mathrm{O}(g^2)$



$$\mathcal{A}(s) \sim -\left(\beta_4 + \beta_8 s^2 + \beta_{12} s^4 + \cdots\right) \log \frac{s}{\Lambda^2}$$

...



The dim-6 squared contributions fulfill positivity:

$$c_{\phi^4}^{(2)} \geq 0\,, \quad c_{\phi^4}^{(1)} + c_{\phi^4}^{(2)} \geq 0\,, \quad c_{\phi^4}^{(1)} + c_{\phi^4}^{(2)} + c_{\phi^4}^{(3)} \geq 0$$

The dim-6 squared contributions fulfill positivity:

$$c^{(2)}_{\phi^4} \geq 0\,, \qquad c^{(1)}_{\phi^4} + c^{(2)}_{\phi^4} \geq 0\,, \qquad c^{(1)}_{\phi^4} + c^{(2)}_{\phi^4} + c^{(3)}_{\phi^4} \geq 0$$

$$16\pi^2 \beta^{(1)}_{H^4D^4} = \frac{8}{3}\Bigg[-2(c^{(1)}_{H^4D^2})^2 - \frac{11}{8}(c^{(2)}_{H^4D^2})^2 + 4c^{(1)}_{H^4D^2}c^{(2)}_{H^4D^2} \\ \underline{+3c^2_{Hd}}\ \underline{\underline{+c^2_{He}}}\ \underline{\underline{\underline{+2(c^{(1)}_{Hl})^2}}} - 2(c^{(3)}_{Hl})^2 \underline{\underline{\underline{\underline{+6(c^{(1)}_{Hq})^2}}}} - 6(c^{(3)}_{Hq})^2 + \underline{\underline{\underline{\underline{3c^2_{Hu}}}}} - 3c^2_{Hud}\Bigg]\,,$$

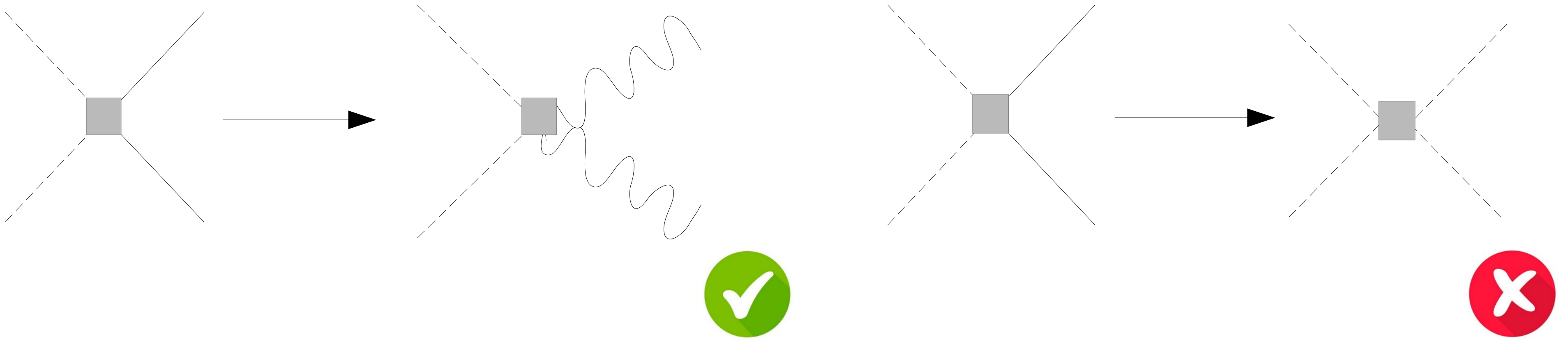
$$16\pi^2 \beta^{(2)}_{H^4D^4} = \frac{8}{3}\Bigg[-2(c^{(1)}_{H^4D^2})^2 - \frac{5}{8}(c^{(2)}_{H^4D^2})^2 - 2c^{(1)}_{H^4D^2}c^{(2)}_{H^4D^2} \\ \underline{-3c^2_{Hd}}\ \underline{\underline{-c^2_{He}}}\ \underline{\underline{\underline{-2(c^{(1)}_{Hl})^2}}} - 2(c^{(3)}_{Hl})^2 \underline{\underline{\underline{\underline{-6(c^{(1)}_{Hq})^2}}}} - 6(c^{(3)}_{Hq})^2 \underline{\underline{\underline{\underline{-3c^2_{Hu}}}}}\Bigg]\,,$$

$$16\pi^2 \beta^{(3)}_{H^4D^4} = \frac{8}{3}\Bigg[-5(c^{(1)}_{H^4D^2})^2 + \frac{7}{8}(c^{(2)}_{H^4D^2})^2 - 2c^{(1)}_{H^4D^2}c^{(2)}_{H^4D^2} + 4(c^{(3)}_{Hl})^2 + 12(c^{(3)}_{Hq})^2 + 3c^2_{Hud}\Bigg]$$

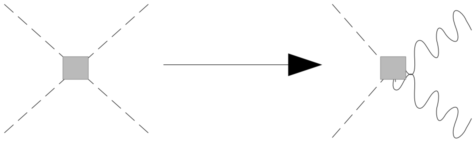
Resorting to the UV to understand the IR is only a trick. In general:

(1) Some tree-level O_i obey $c_i \geq 0$

(2) If O_i involves fields not present in O_j and c_j not constrained by positivity, then $\gamma_{ij} = 0$



Other aspects of anomalous dimensions: signs and inequalities

Let us consider the mixing 

Positivity bounds:

$$c^{(2)}_{\phi^4} \geq 0\,, \quad c^{(1)}_{\phi^4} + c^{(2)}_{\phi^4} \geq 0\,, \quad c^{(1)}_{\phi^4} + c^{(2)}_{\phi^4} + c^{(3)}_{\phi^4} \geq 0$$

$$\dot{c}^{(1)}_{B^2\phi^2 D^2} \geq 0$$

From where we obtain:

$$\begin{aligned}\dot{c}^{(1)}_{B^2\phi^2 D^2} &= \alpha(c^{(1)}_{\phi^4} + c^{(2)}_{\phi^4} + c^{(3)}_{\phi^4}) + \beta(c^{(1)}_{\phi^4} + c^{(2)}_{\phi^4}) + \gamma c^{(2)}_{\phi^4} + \cdots \\ &= (\alpha+\beta)c^{(1)}_{\phi^4} + (\alpha+\beta+\gamma)c^{(2)}_{\phi^4} + \alpha c^{(3)}_{\phi^4} + \cdots\,,\end{aligned}$$

Other aspects of anomalous dimensions: signs and inequalities

$$\dot{c}^{(1)}_{B^2\phi^2D^2} = \alpha(c^{(1)}_{\phi^4} + c^{(2)}_{\phi^4} + c^{(3)}_{\phi^4}) + \beta(c^{(1)}_{\phi^4} + c^{(2)}_{\phi^4}) + \gamma c^{(2)}_{\phi^4} + \cdots$$
$$= (\alpha+\beta)c^{(1)}_{\phi^4} + (\alpha+\beta+\gamma)c^{(2)}_{\phi^4} + \alpha c^{(3)}_{\phi^4} + \cdots\,,$$

1. The anomalous dimensions are positive

	$c^{(1)}_{\phi^4D^4}$	$c^{(2)}_{\phi^4D^4}$	$c^{(3)}_{\phi^4D^4}$
$c^{(1)}_{B^2\phi^2D^2}$	+	+	+

2. They fulfill

$$\gamma_{c^{(1)}_{B^2\phi^2D^2},\,c^{(2)}_{\phi^4D^4}} \geq \gamma_{c^{(1)}_{B^2\phi^2D^2},\,c^{(1)}_{\phi^4D^4}} \geq \gamma_{c^{(1)}_{B^2\phi^2D^2},\,c^{(3)}_{\phi^4D^4}}$$