

Gauge Invariance and Particles

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Vienna
Austria



NAWI Graz
Natural Sciences

FWF Österreichischer
Wissenschaftsfonds



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Elementary particles are not observables

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- Fröhlich-Morchio-Strocchi mechanism
 - Standard Model
 - Experimental signatures

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- Brout-Englert-Higgs Physics
- Fröhlich-Morchio-Strocchi mechanism
 - Standard Model
 - Experimental signatures
 - Beyond the Standard Model
 - Qualitative changes

Brout-Englert-Higgs Physics - The Standard Model

A toy model

A toy model: Higgs sector of the SM

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- Consider an $SU(2)$ with a fundamental scalar

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- Consider an SU(2) with a fundamental scalar
- Essentially the standard model Higgs

$$L = -\frac{1}{4} W_{\mu\nu}^a W_a^{\mu\nu}$$

$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + gf_{bc}^a W_\mu^b W_\nu^c$$

- W s W_μ^a 

- Coupling g and some numbers f^{abc}



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- **Higgs** h_i 
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- Couplings g, v, λ and some numbers f^{abc} and t_a^{ij}
- Parameters selected for a BEH effect

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- Local SU(2) gauge symmetry

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- Global SU(2) custodial (flavor) symmetry

- Acts as (right-)transformation on the scalar field only

$$W_\mu^a \rightarrow W_\mu^a \qquad h \rightarrow h \Omega$$

Textbook approach

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- Choose a suitable gauge and obtain 'spontaneous gauge symmetry breaking': $SU(2) \rightarrow 1$
- Get masses and degeneracies at tree-level
- Perform perturbation theory

Physical spectrum

Perturbation theory

Mass



0

Physical spectrum

Perturbation theory

Scalar

fixed charge

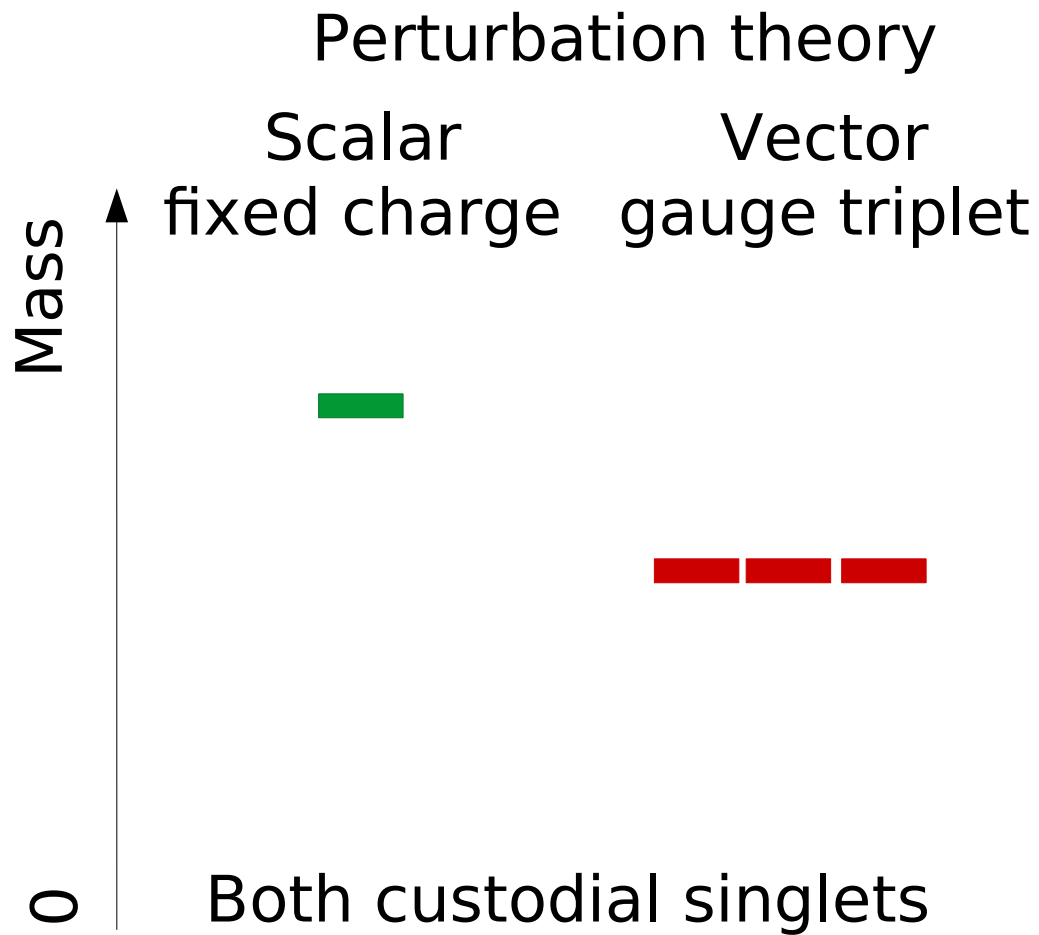
Mass



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Custodial singlet

Physical spectrum



The origin of the problem

[Fröhlich et al.'80,
Banks et al.'79]

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 - And this includes non-perturbative aspects...
 - ...even at weak coupling [Gribov'78, Singer'78, Fujikawa'82]

Physical states

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 - **Cannot** be the elementary particles
 - Non-Abelian nature is relevant

Physical states

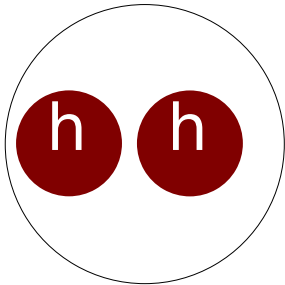
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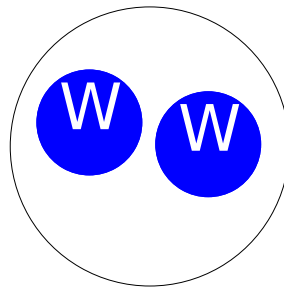
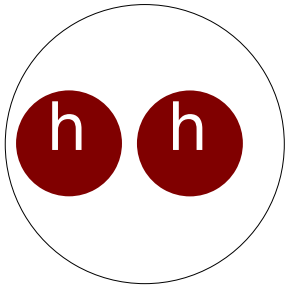
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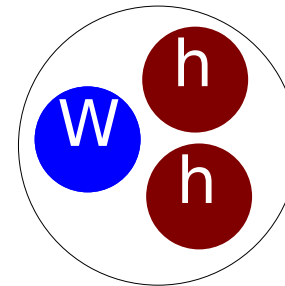
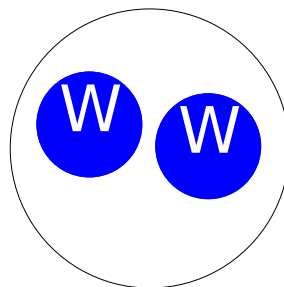
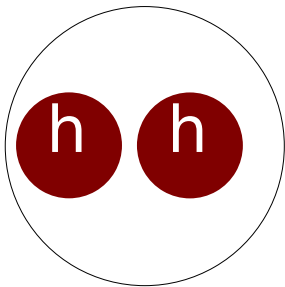
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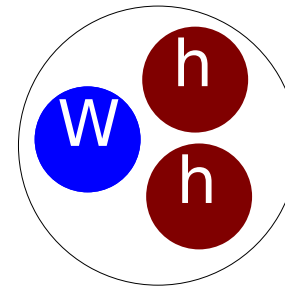
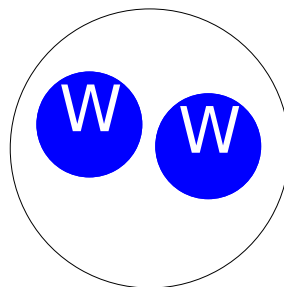
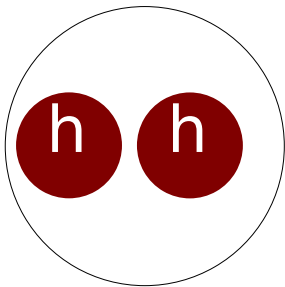
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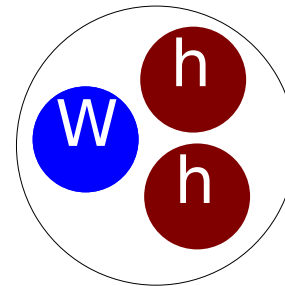
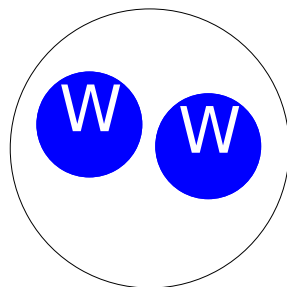
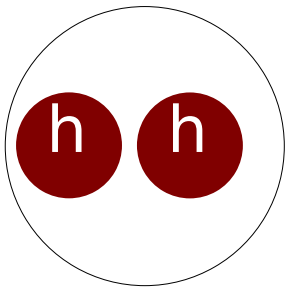


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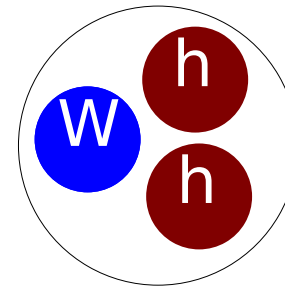
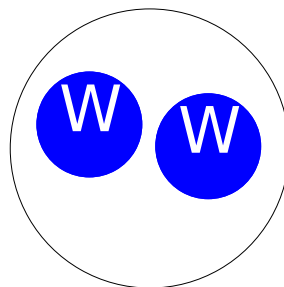
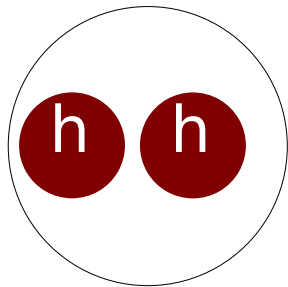


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 - Think QED (hydrogen atom!)

Physical states

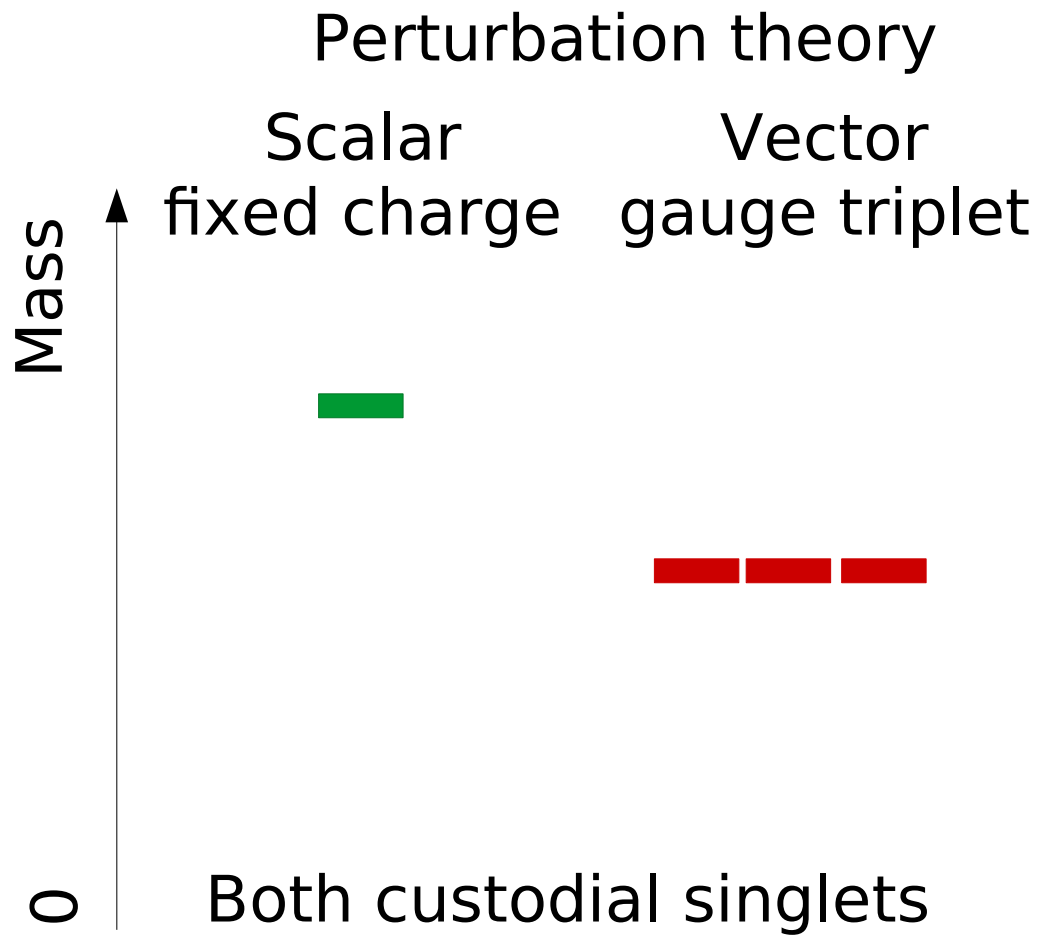
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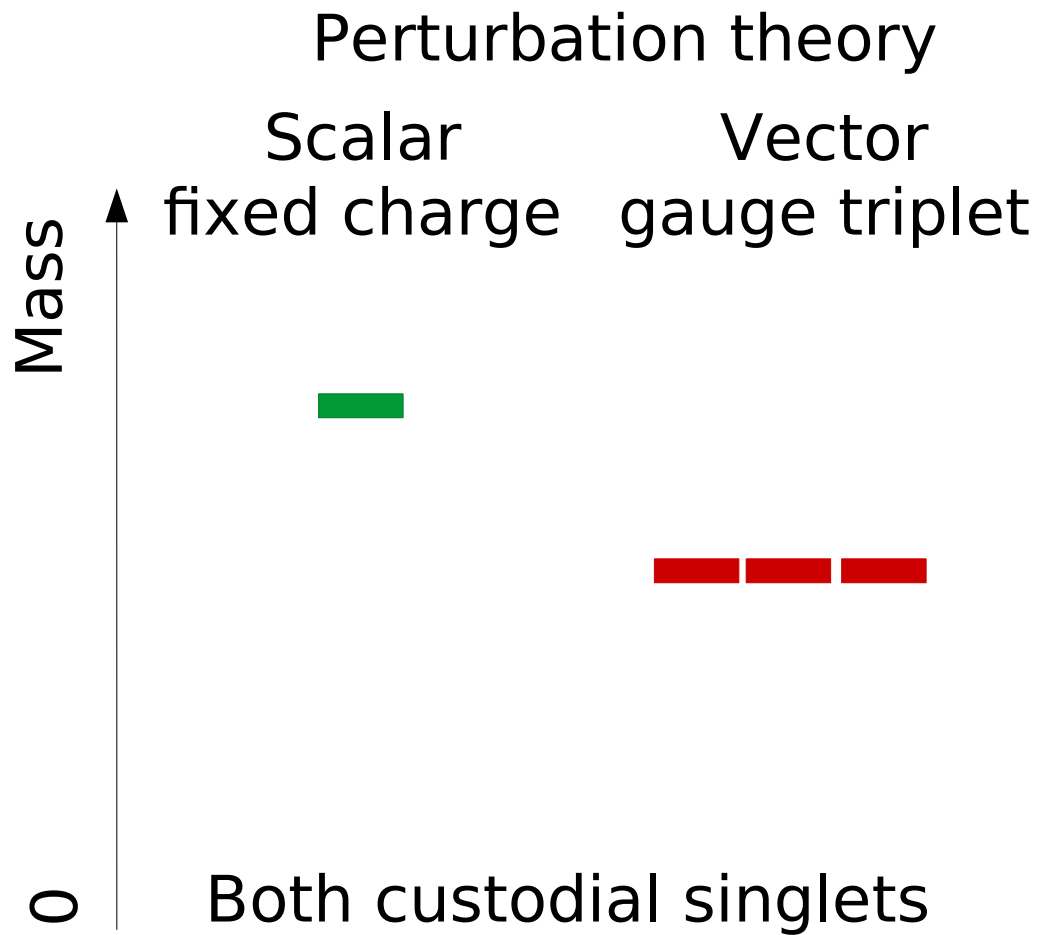
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 - Think QED (hydrogen atom!)
- Can this matter?

Physical spectrum

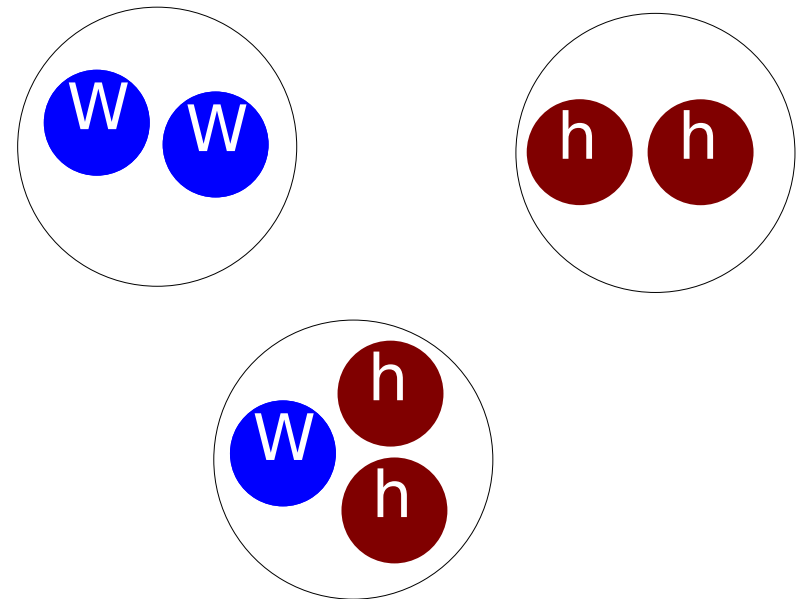


Remember: Experiment tells that somehow the left is correct!

Physical spectrum

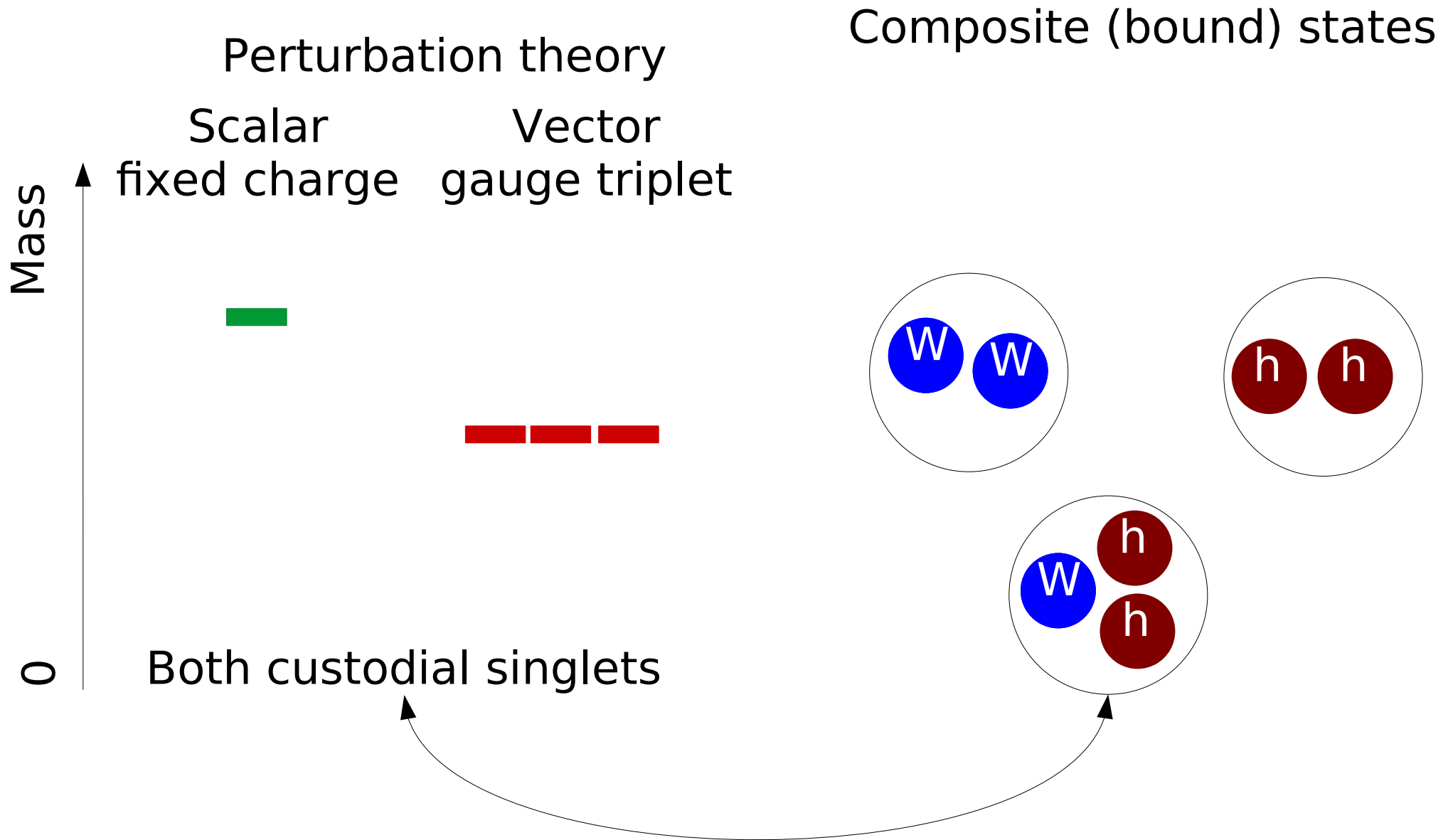


Composite (bound) states



Experiment tells that somehow the left is correct
Theory say the right is correct

Physical spectrum



Experiment tells that somehow the left is correct
Theory say the right is correct
There must exist a relation that both are correct

Physical particles

[Fröhlich et al.'80,'81,
Maas & Törek'16,'18,
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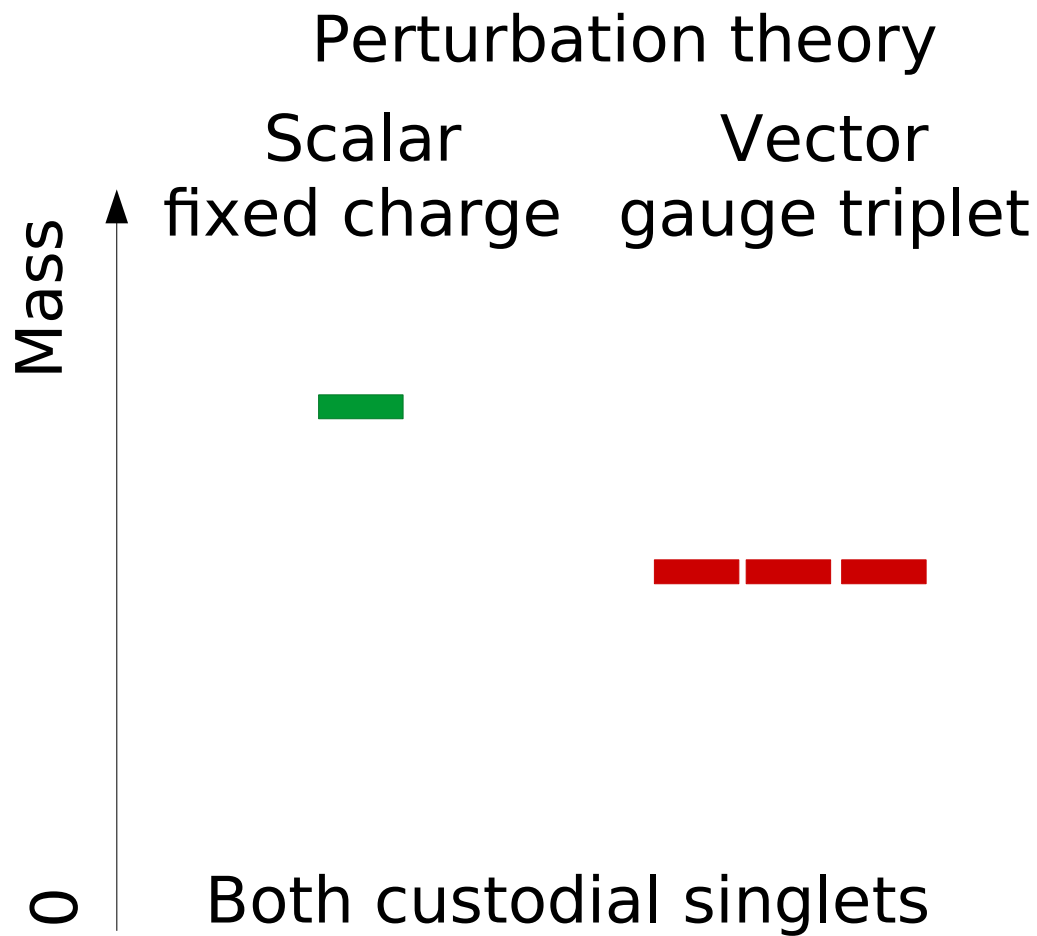
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 - Standard lattice spectroscopy problem
 - Standard methods
 - Smearing, variational analysis, systematic error analysis etc.
 - Very large statistics ($>10^5$ configurations)

Physical spectrum

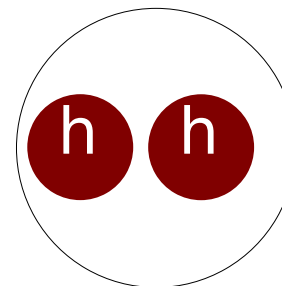
[Maas'12, Maas & Mufti'14]



Gauge-invariant

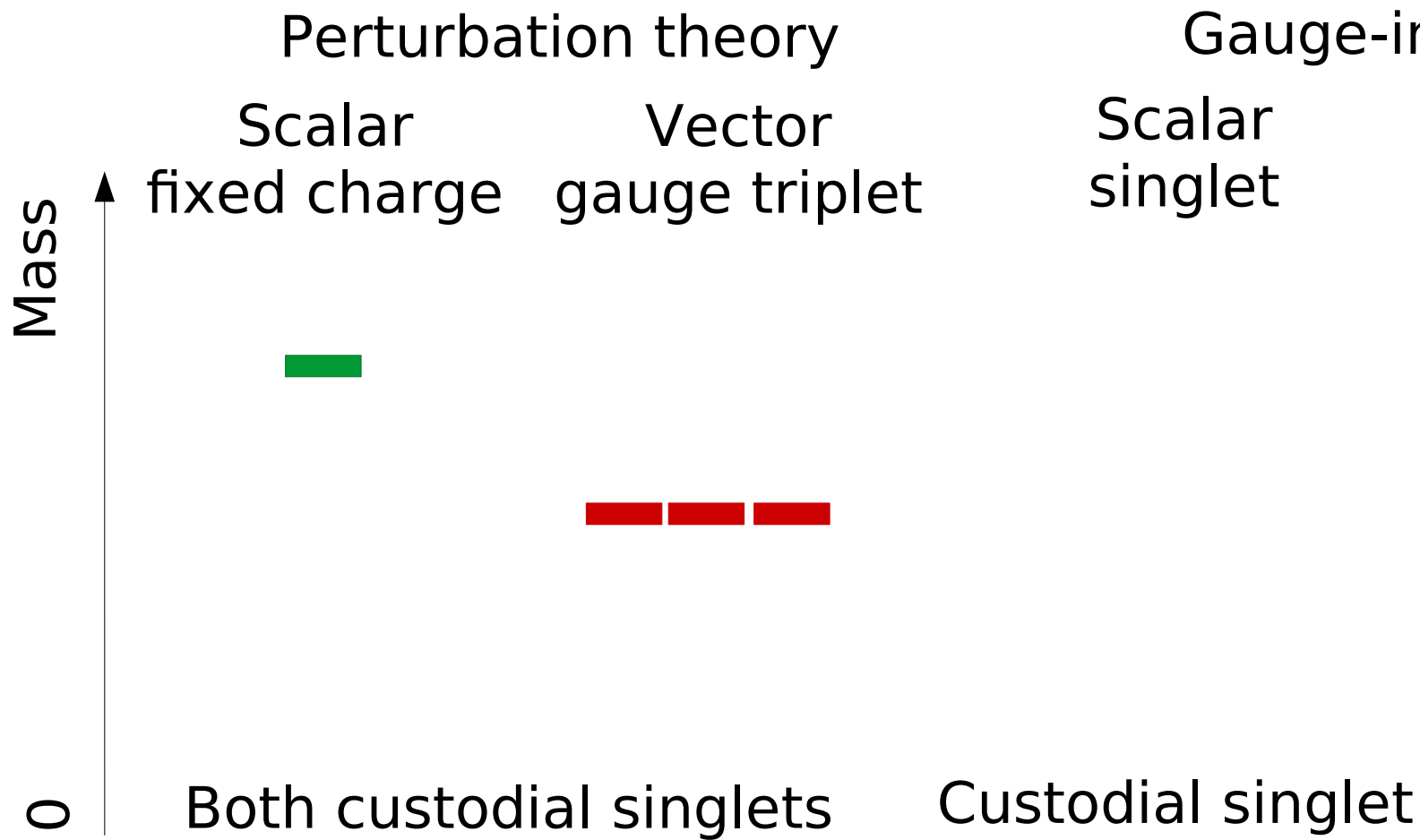
Scalar singlet

$$h(x)^+ h(x)$$

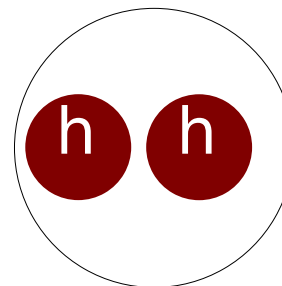


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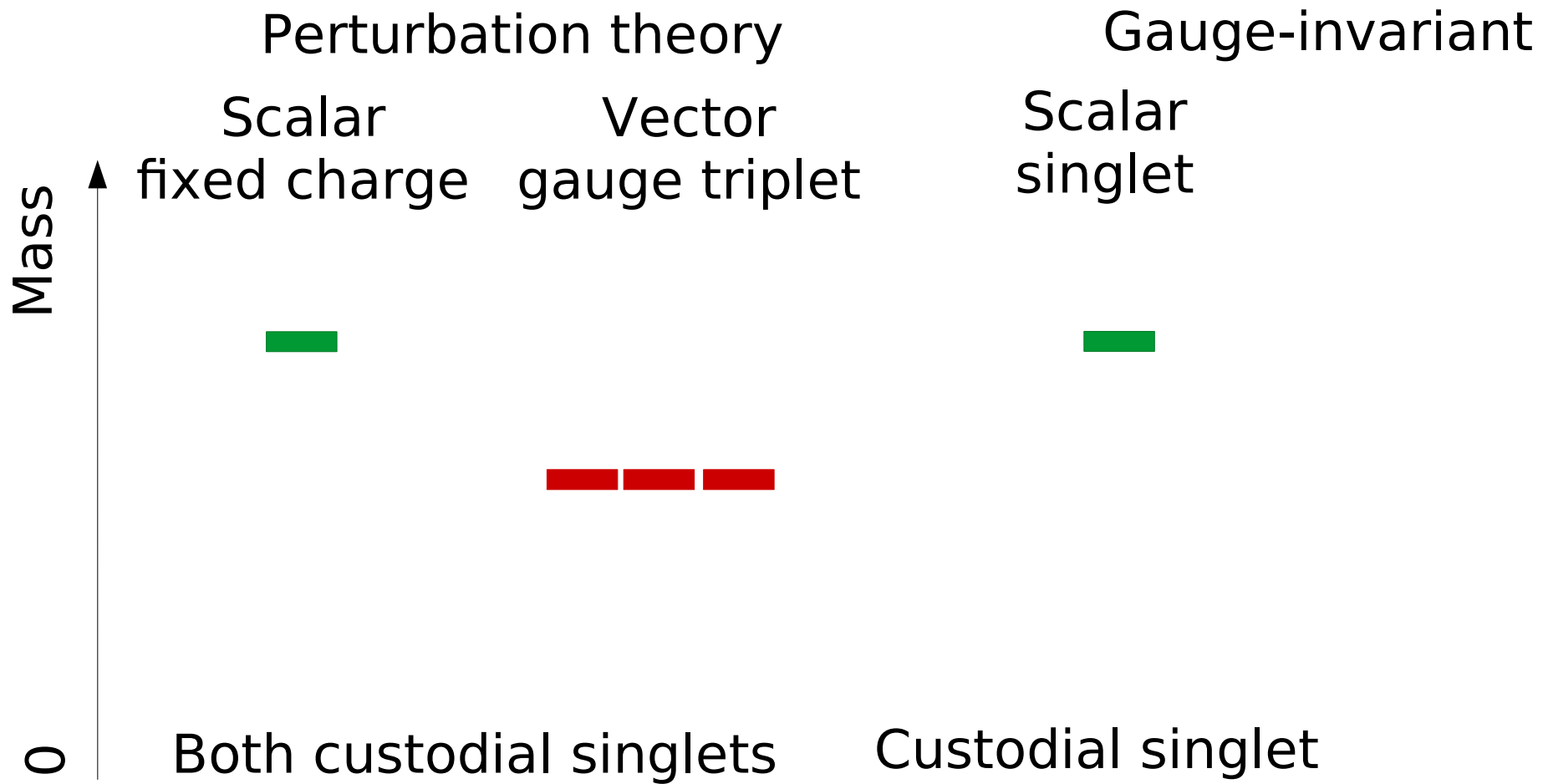


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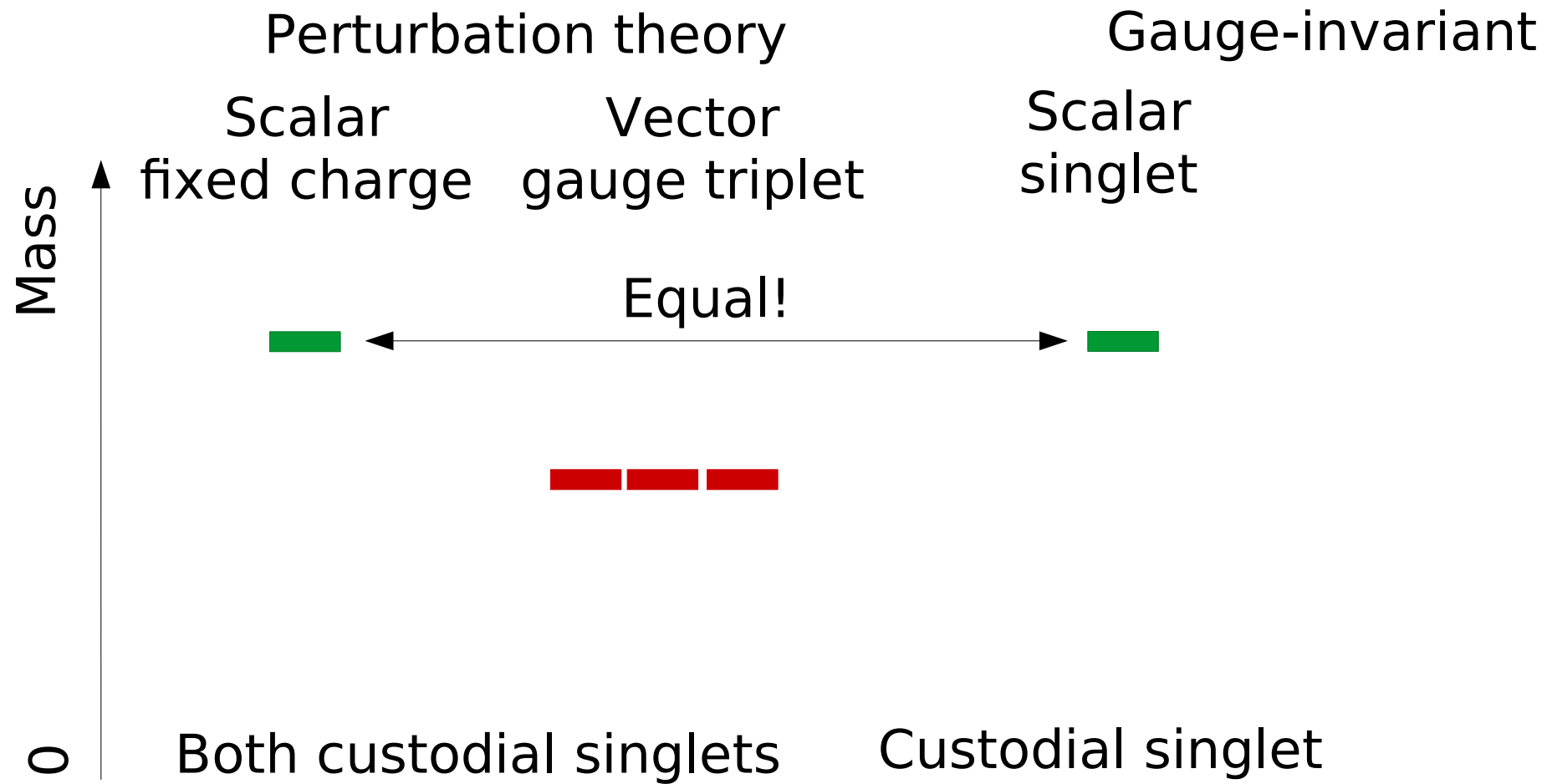
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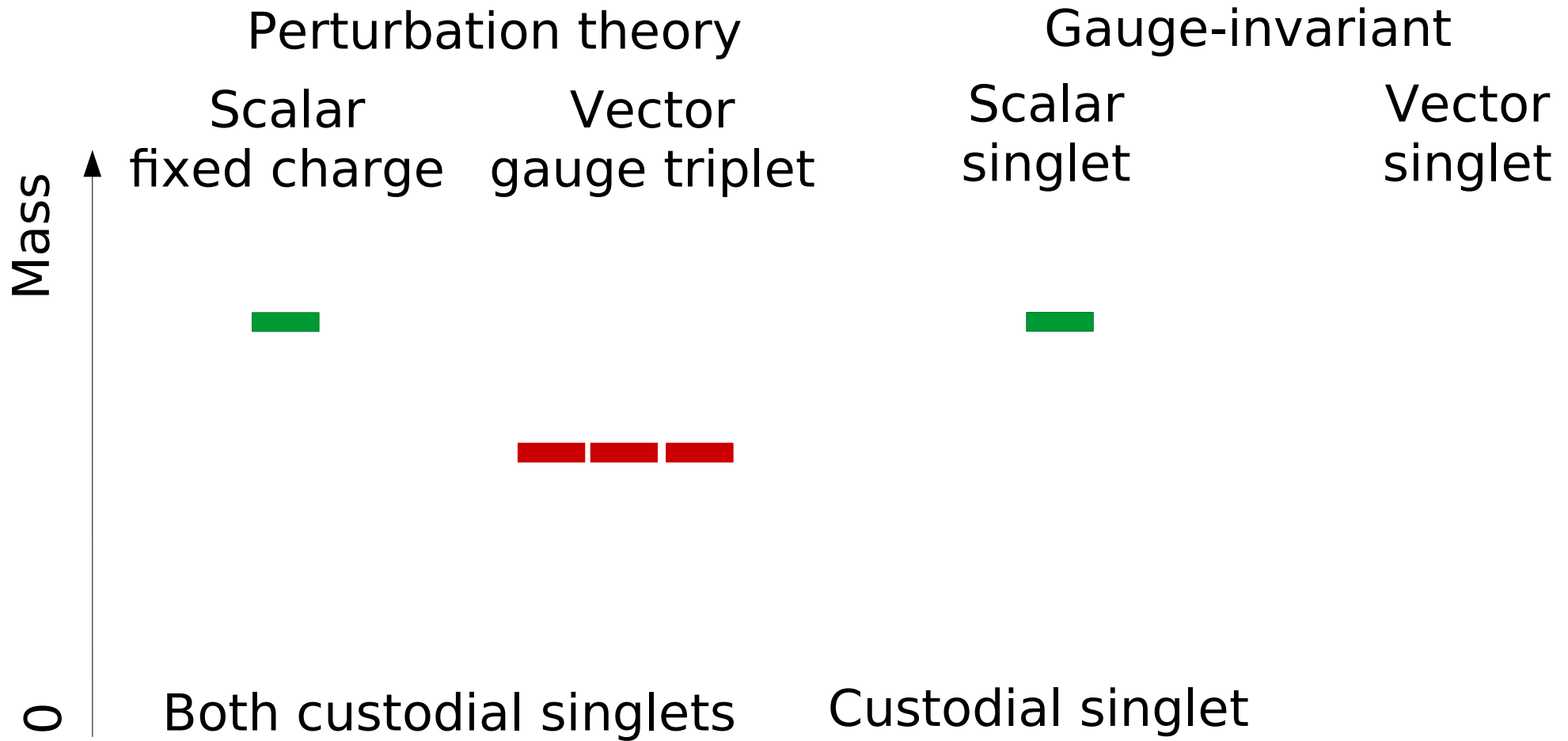
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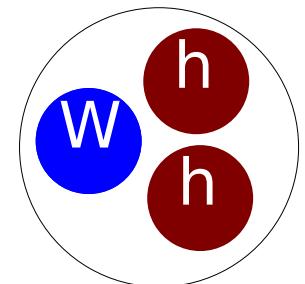


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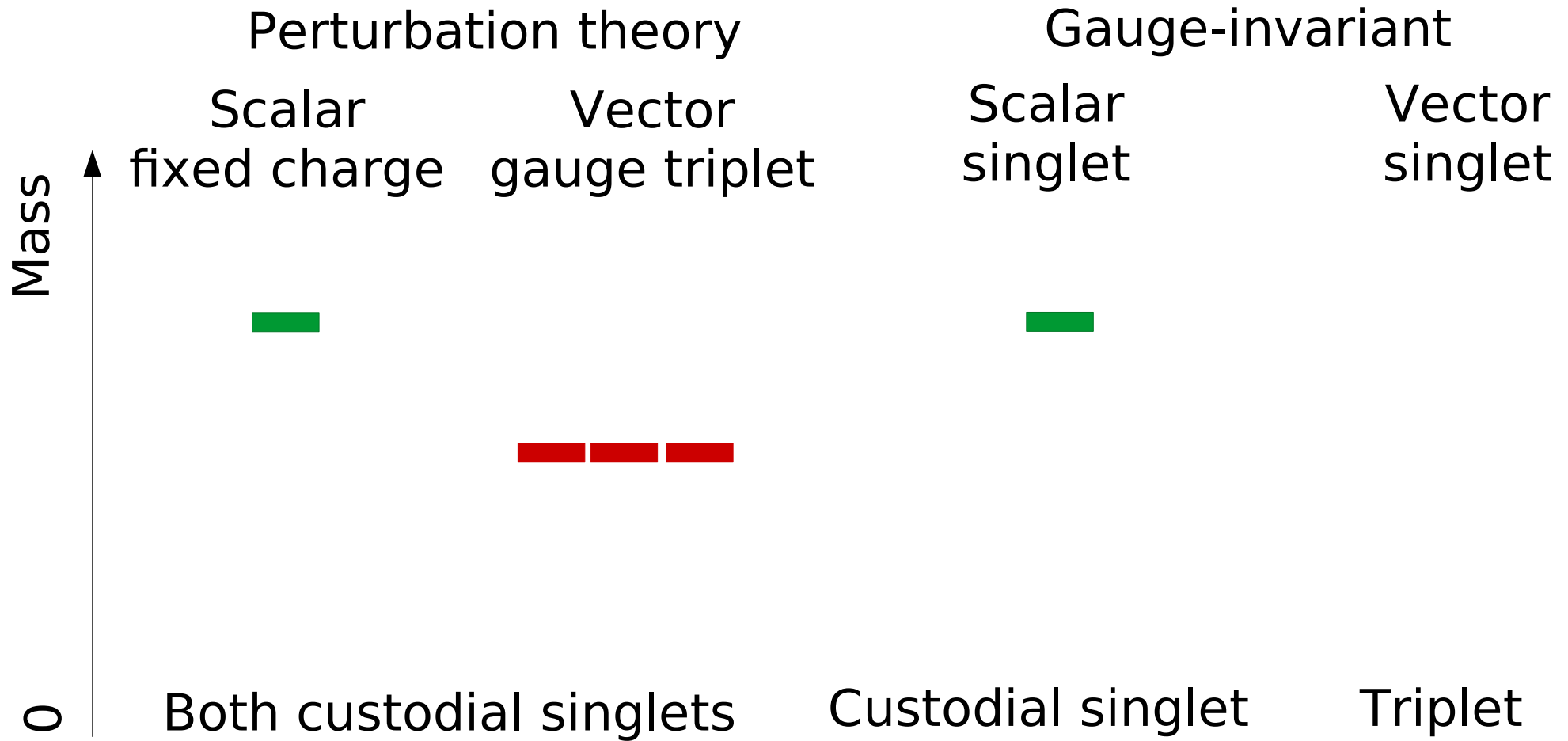


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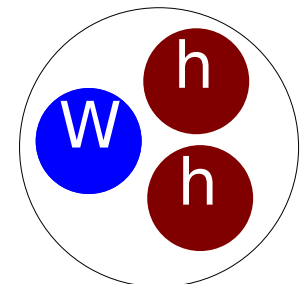


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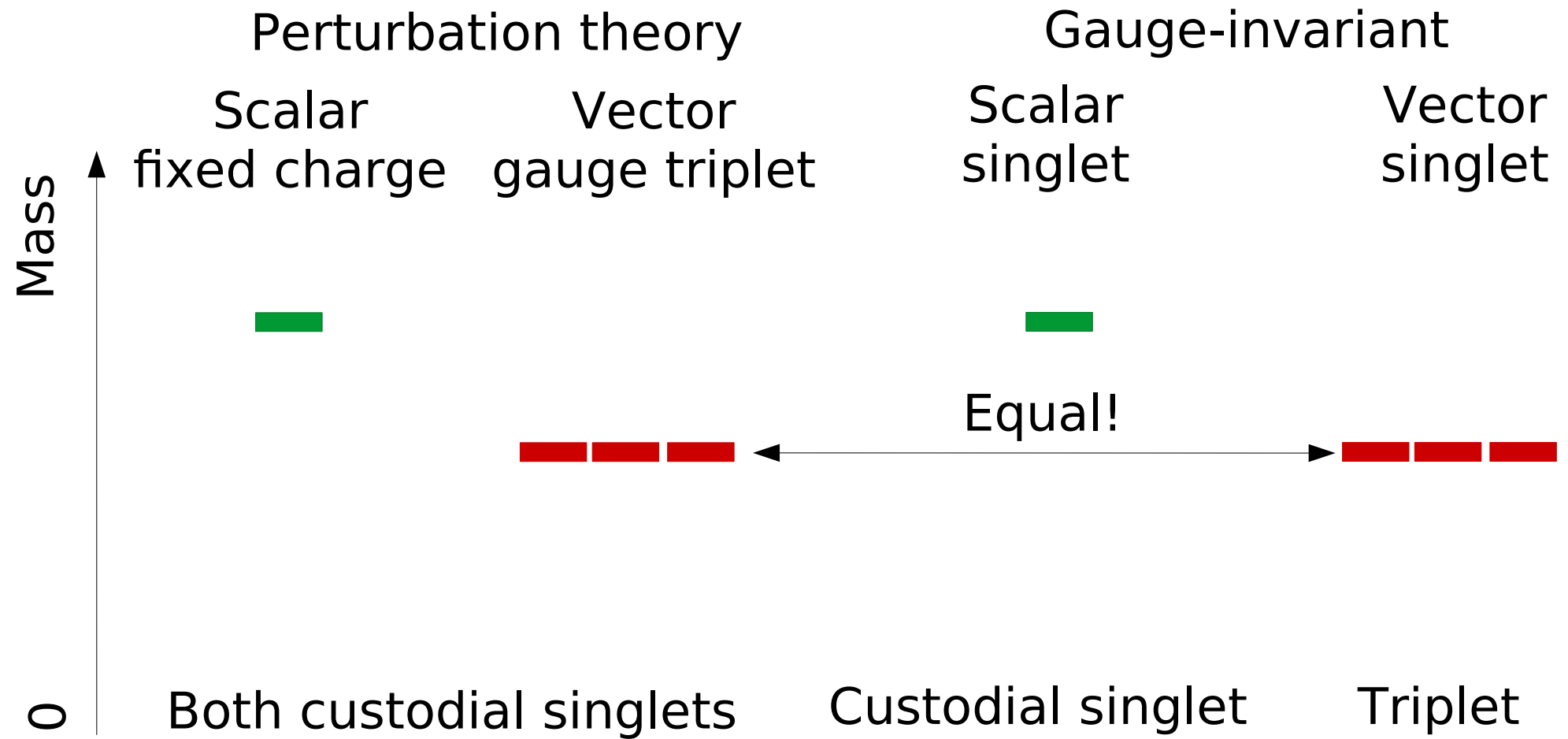


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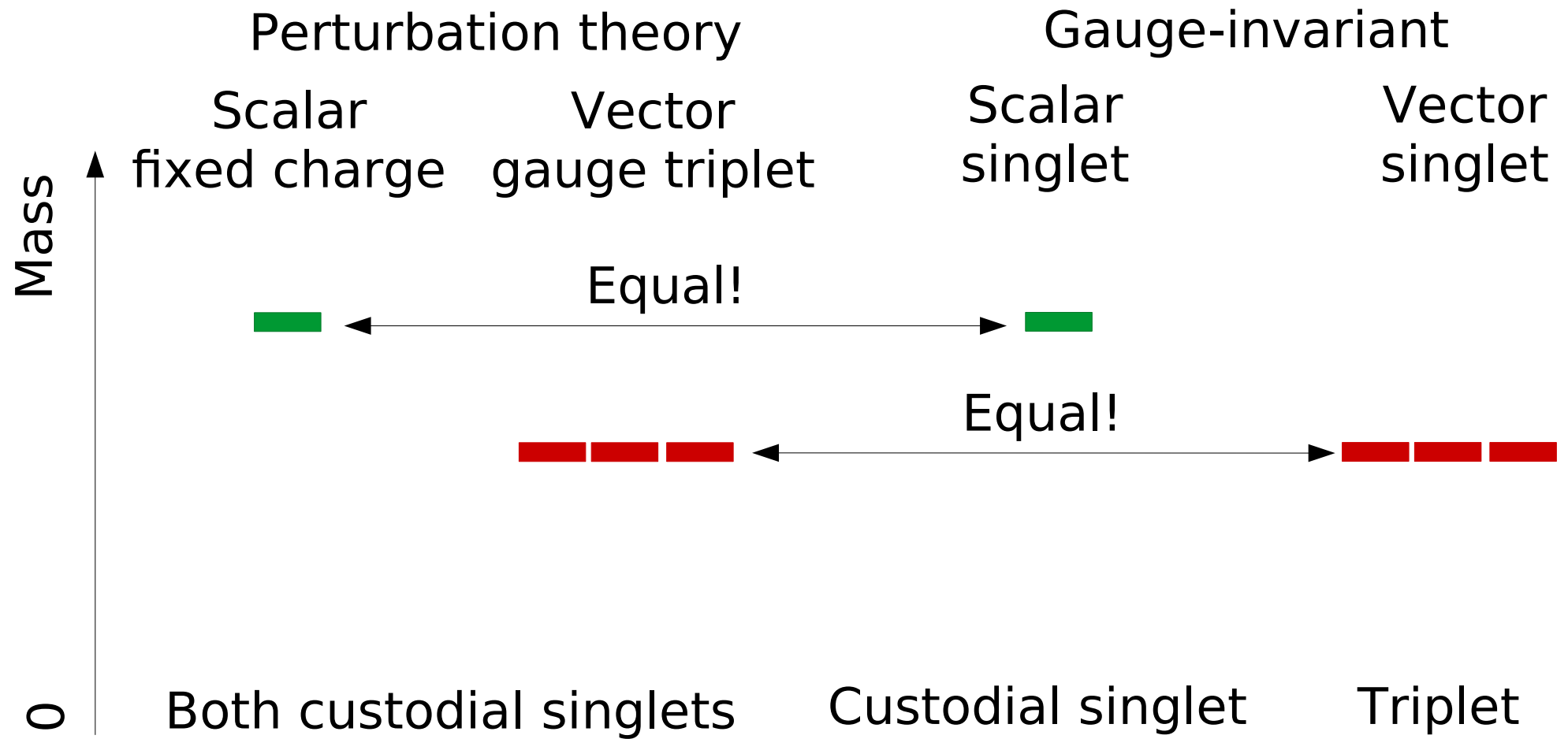
Physical spectrum

[Maas'12, Maas & Mufti'14]



Physical spectrum

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Why?

A microscopic origin

-

Fröhlich-Morchio-Strocchi
mechanism

How to make predictions

[Fröhlich et al.'80,'81,
Maas & Törek'16,'18,
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Maas & Sondenheimer '20]

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 - But coupling is still weak and there is a BEH
 - Perform double expansion [Fröhlich et al.'80, Maas'12]
 - Vacuum expectation value (FMS mechanism)
 - Standard expansion in couplings
 - Together: Augmented perturbation theory

Augmented perturbation theory

[Fröhlich et al.'80,'81
Maas'12,'17]

- 1) Formulate gauge-invariant operator

Augmented perturbation theory

[Fröhlich et al.'80,'81
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0^+ singlet: $\langle (h^\dagger h)(x)(h^\dagger h)(y) \rangle$

Higgs field

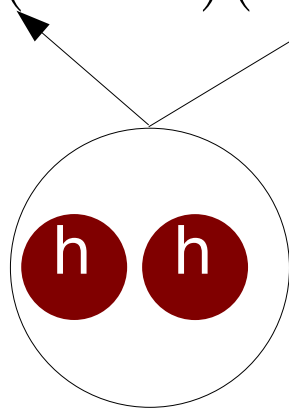


Augmented perturbation theory

[Fröhlich et al.'80,'81
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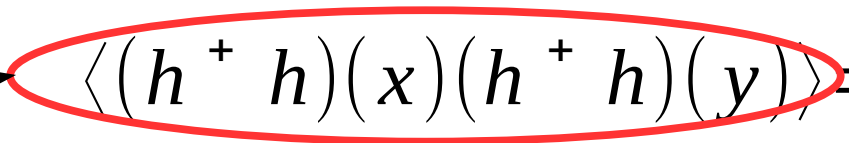
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Trivial two-particle state

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Standard
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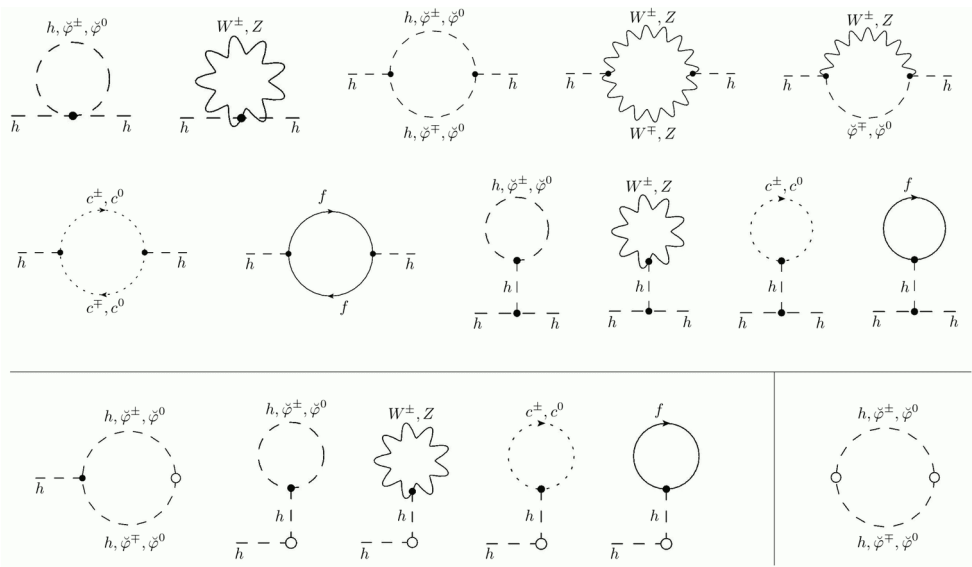
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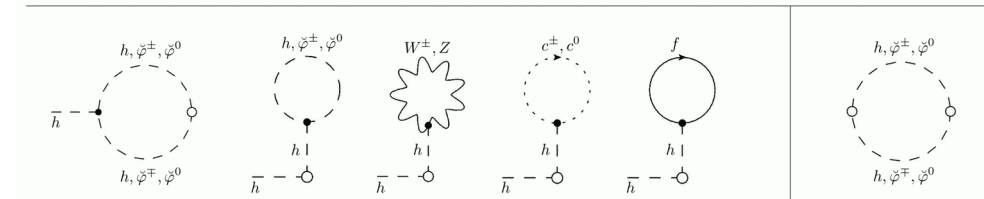
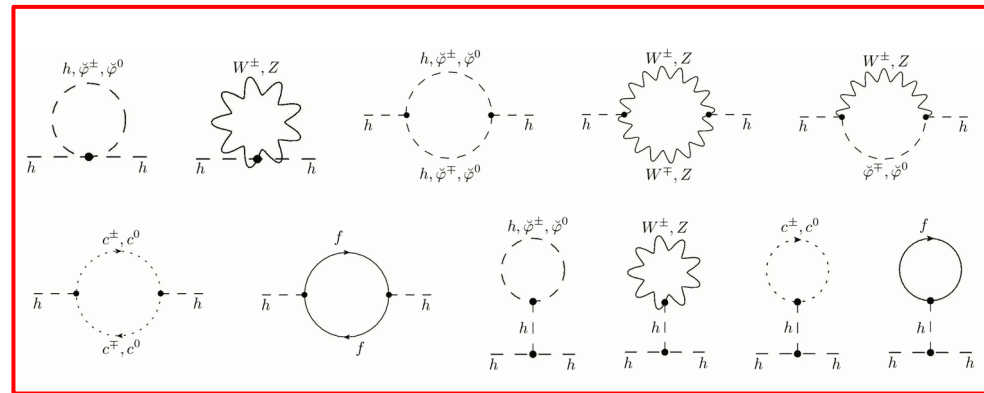
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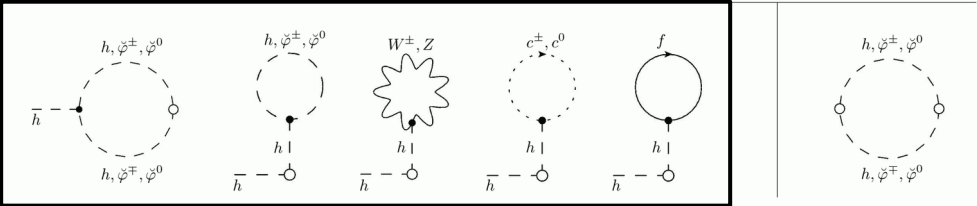
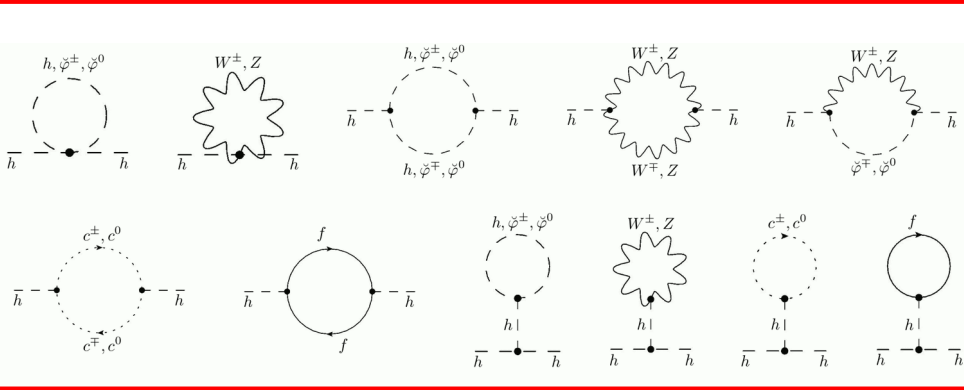
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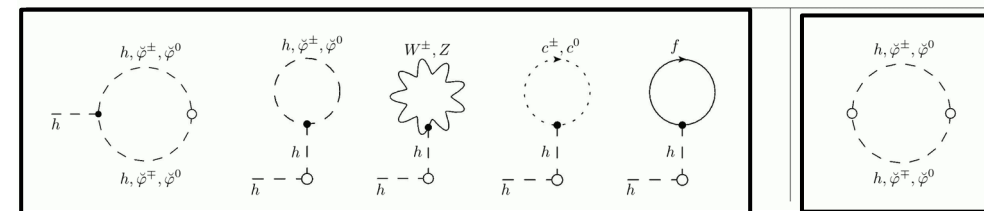
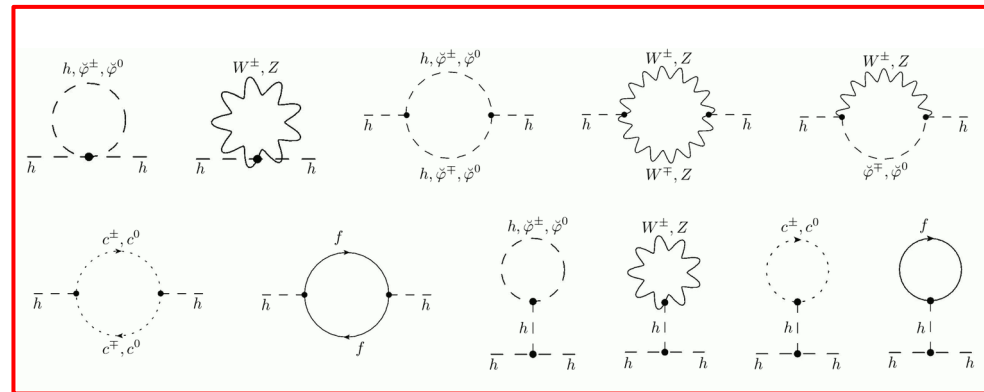
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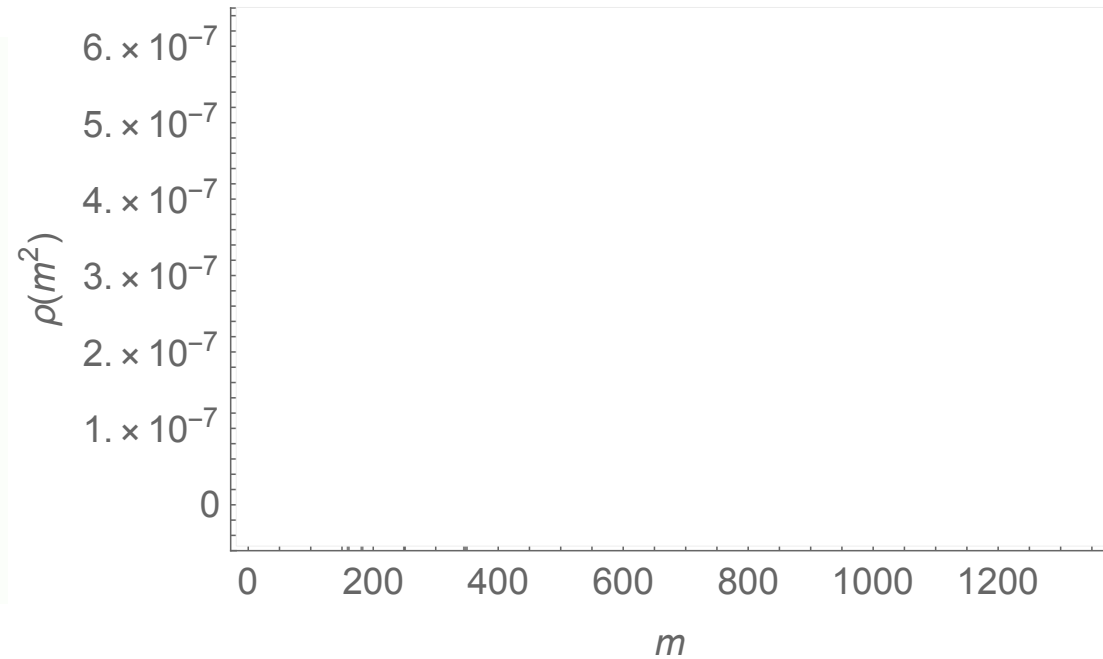
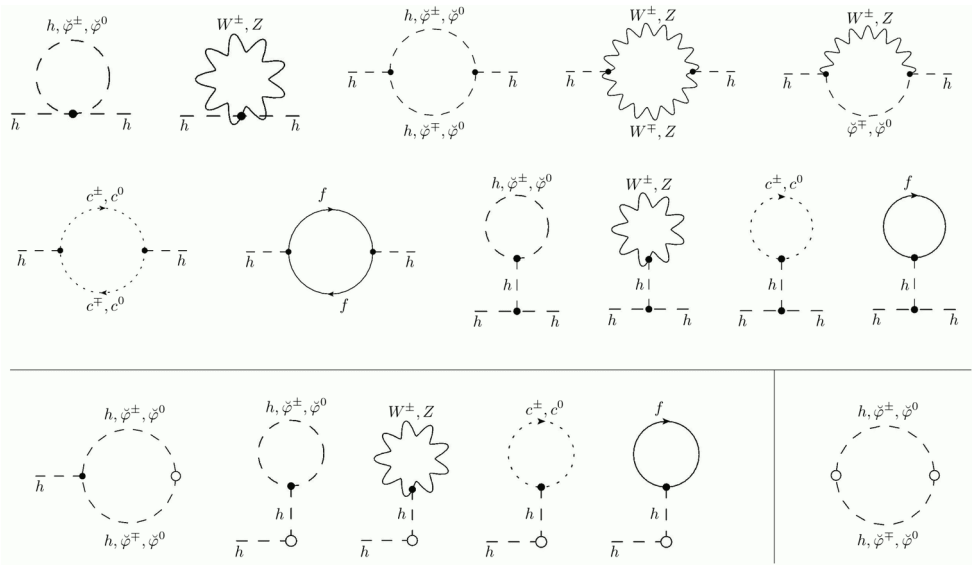
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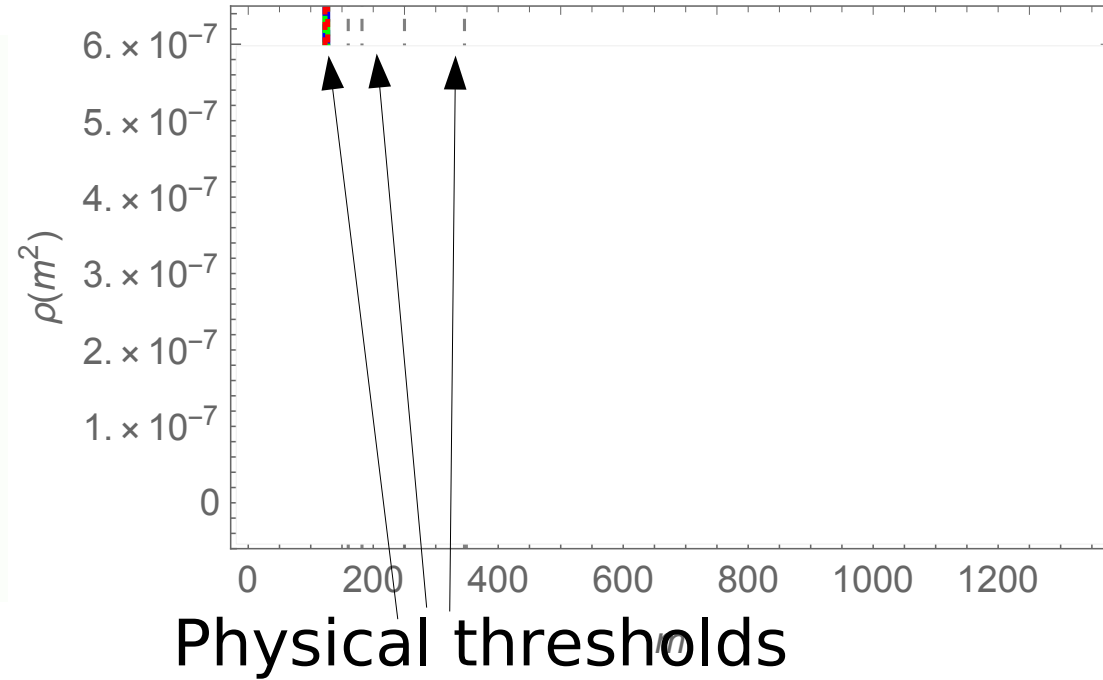
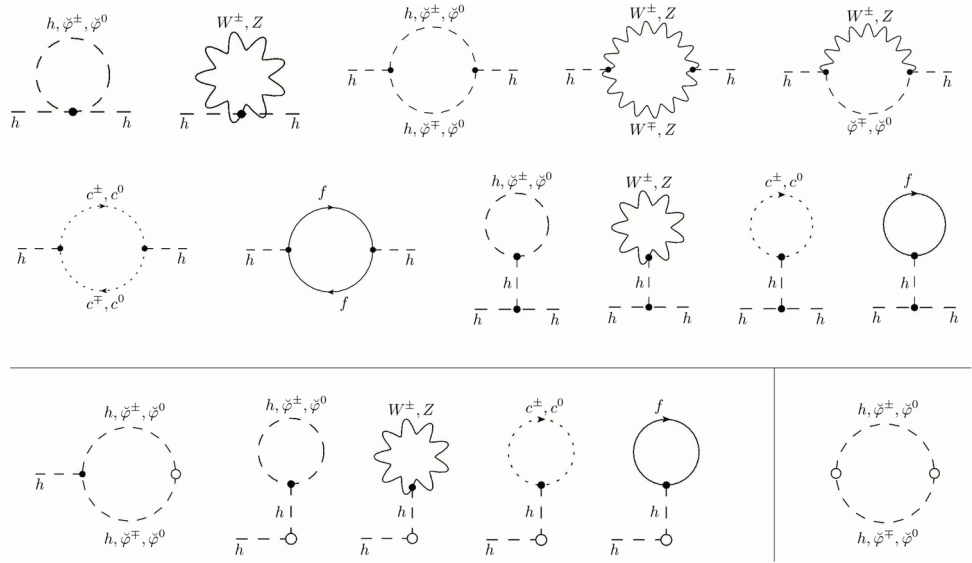
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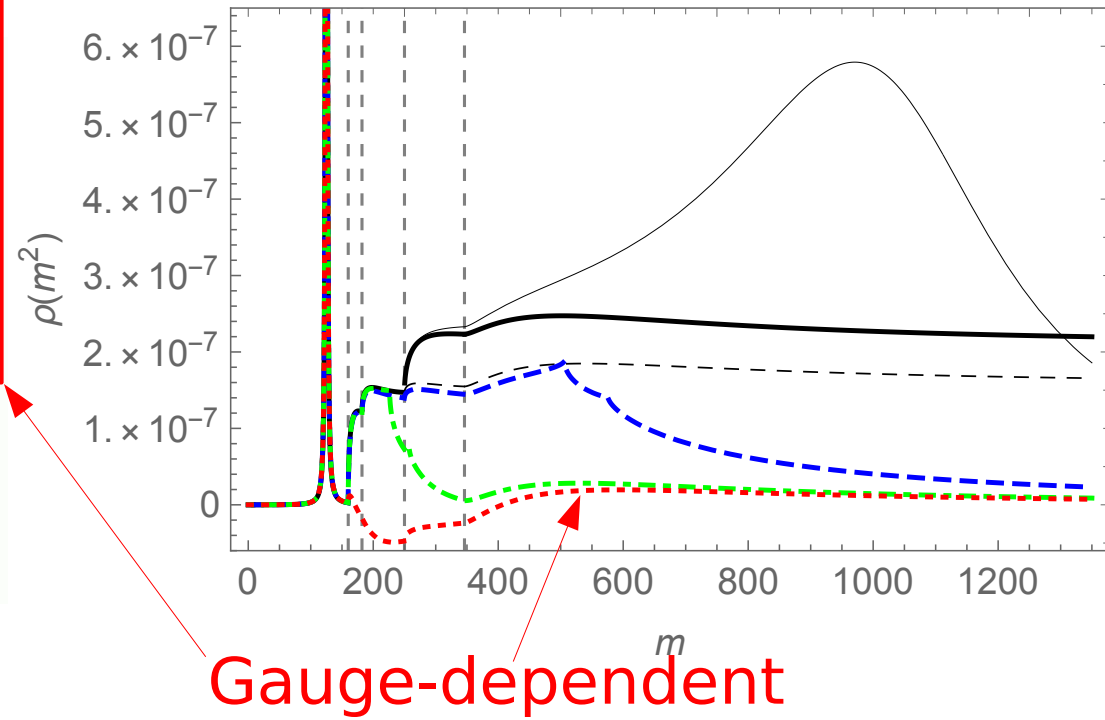
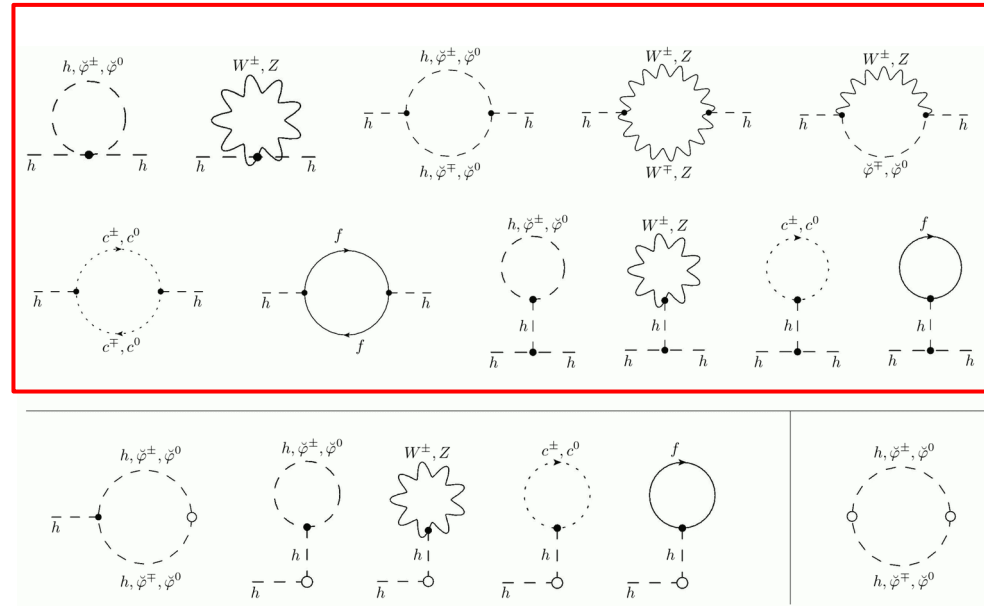
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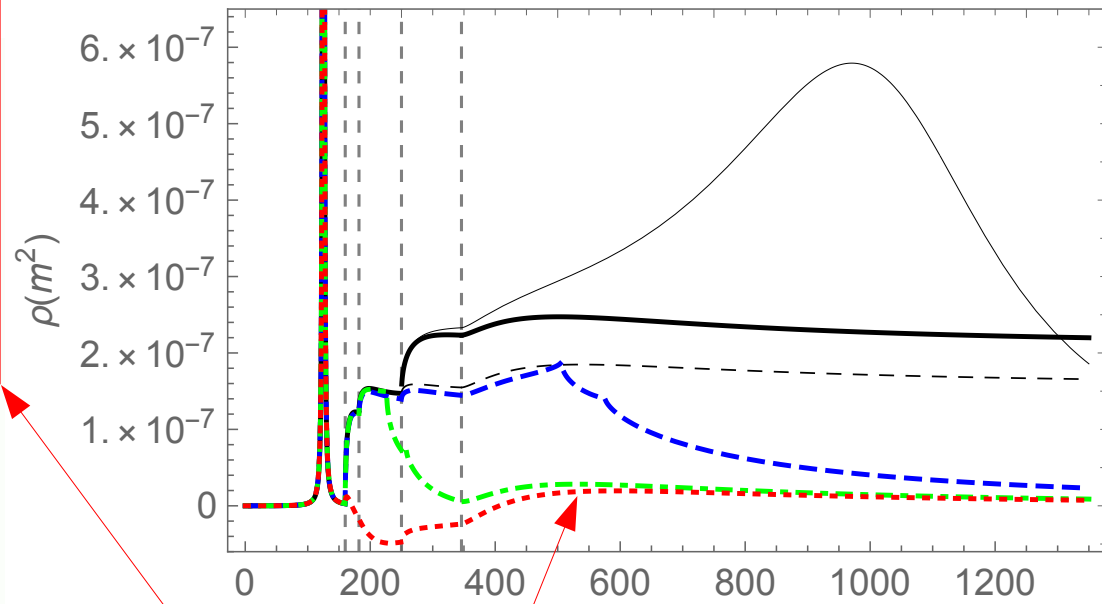


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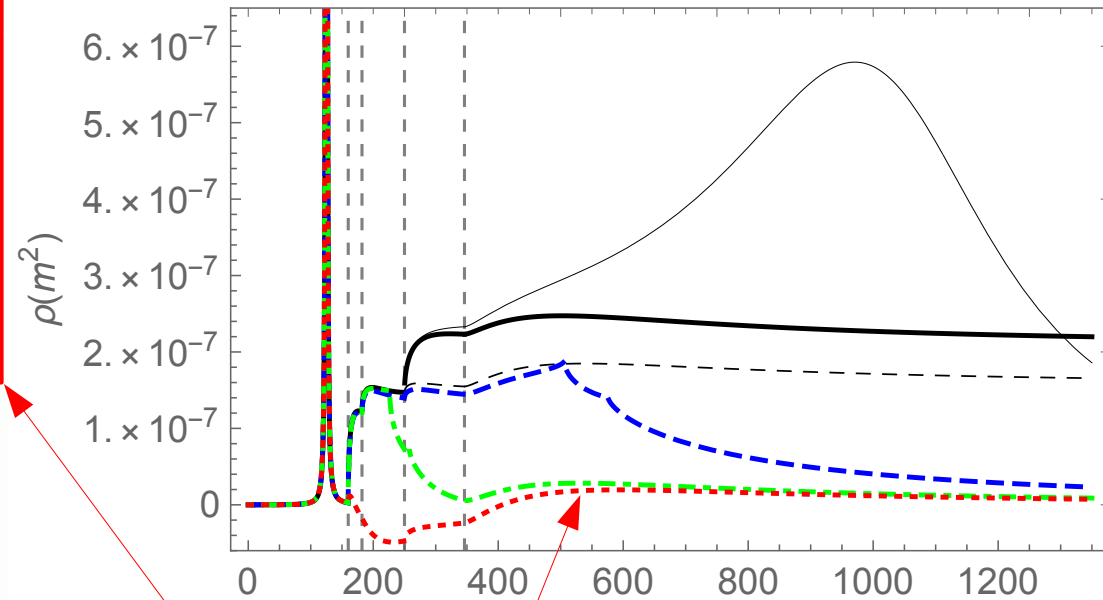
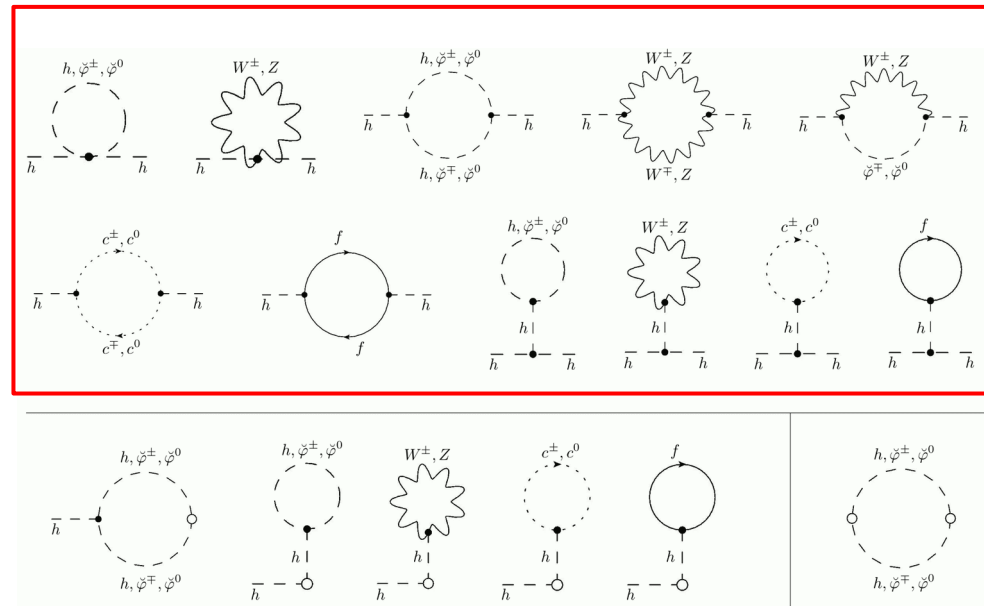
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Gauge-dependent
Unphysical features:
Positivity violation
Additional thresholds

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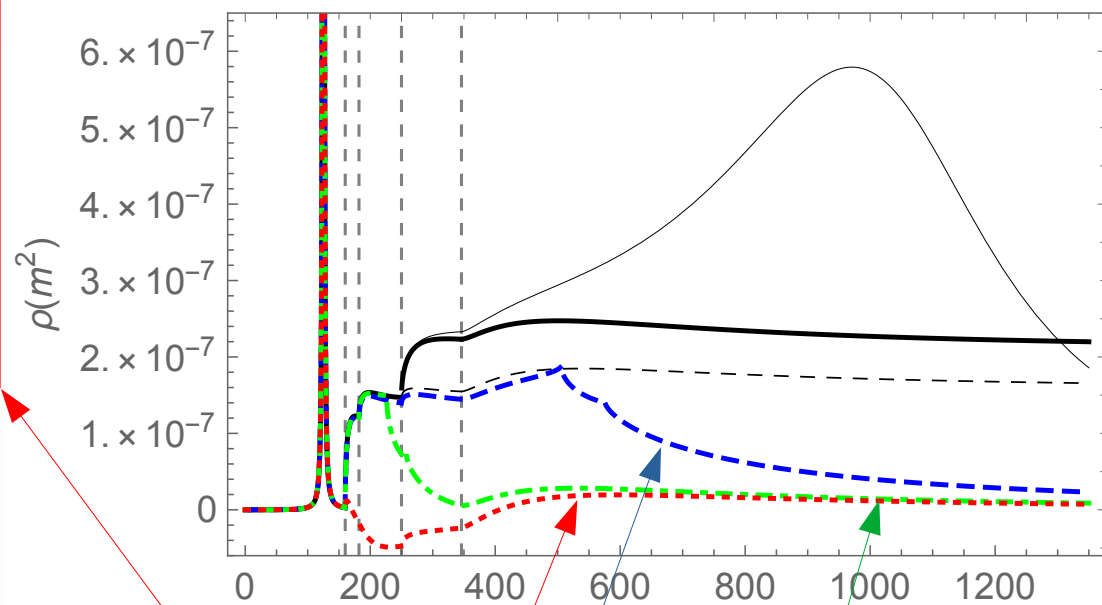
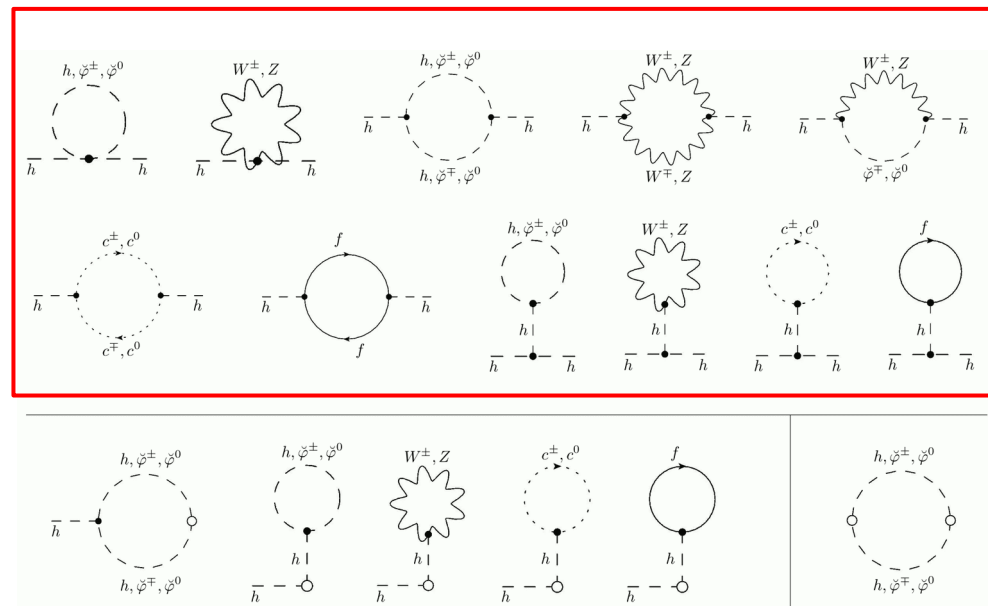


Gauge-dependent
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Not a consequence
of instability: Occurs even
for an asymptotically stable
Higgs in a toy theory

Consequences: The Higgs

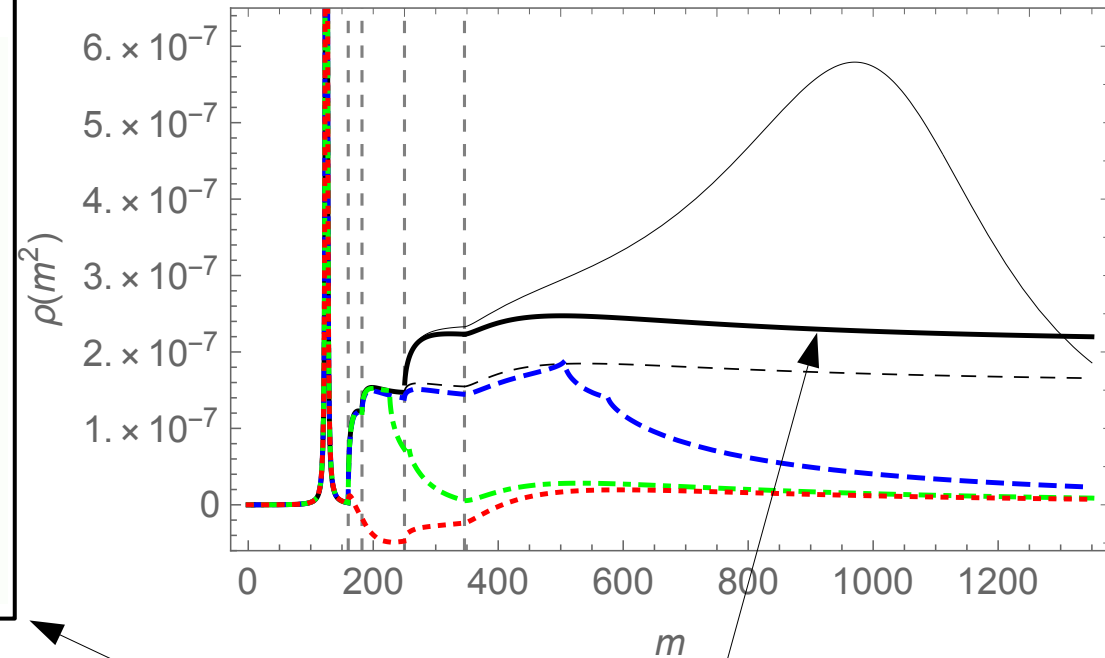
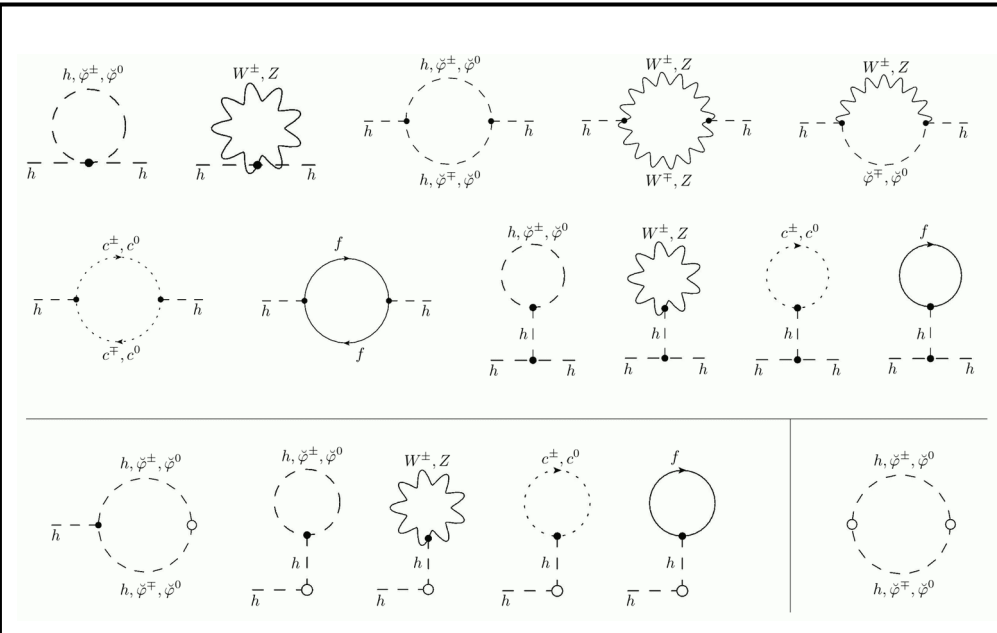
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Gauge-dependent
Other gauge choices

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Physical – same for all gauge choices

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What about the vector?

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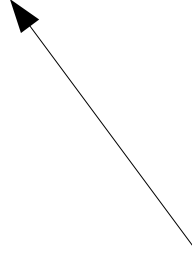
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Matrix from
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c projects custodial
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Exactly one gauge boson
for every physical state

Matrix from
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Phenomenological Implications

-

Can we measure this?

Bound states as extended objects

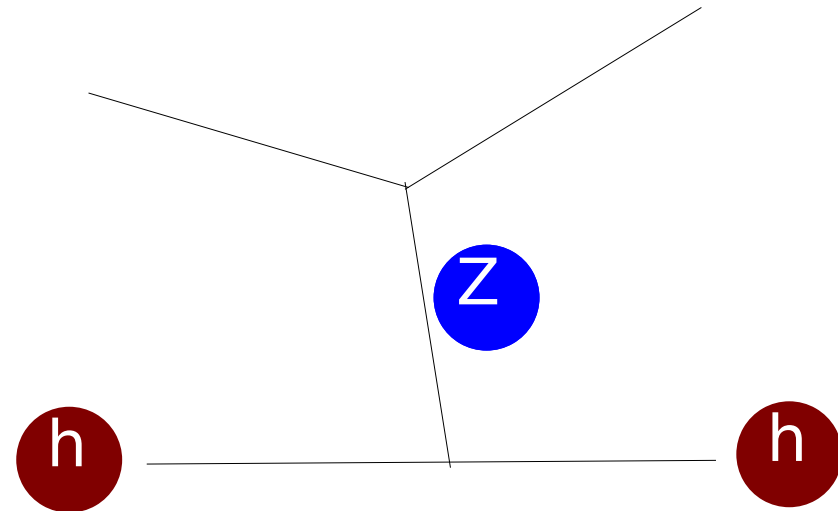
Bound states as extended objects

- Two possibilities to measure extension

Bound states as extended objects

- Two possibilities to measure extension
 - Form factor
 - Difficult
 - Higgs and Z need to be both produced in the same process

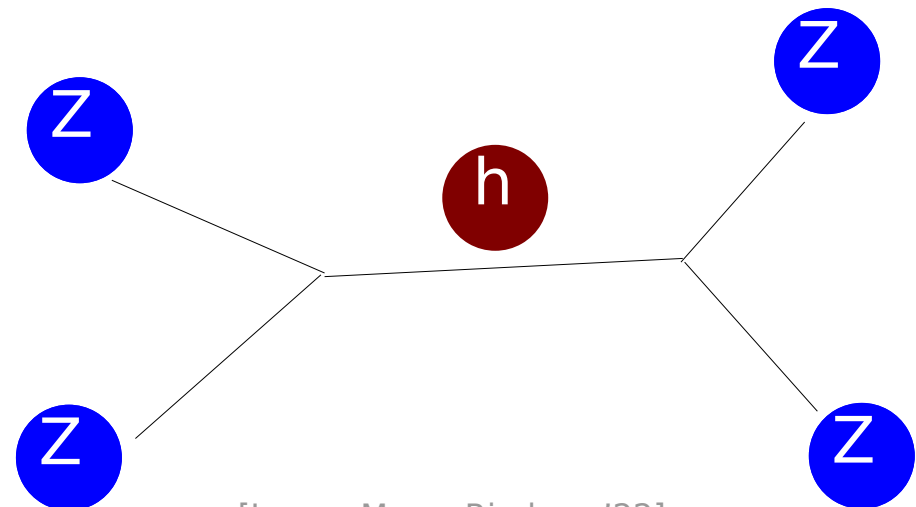
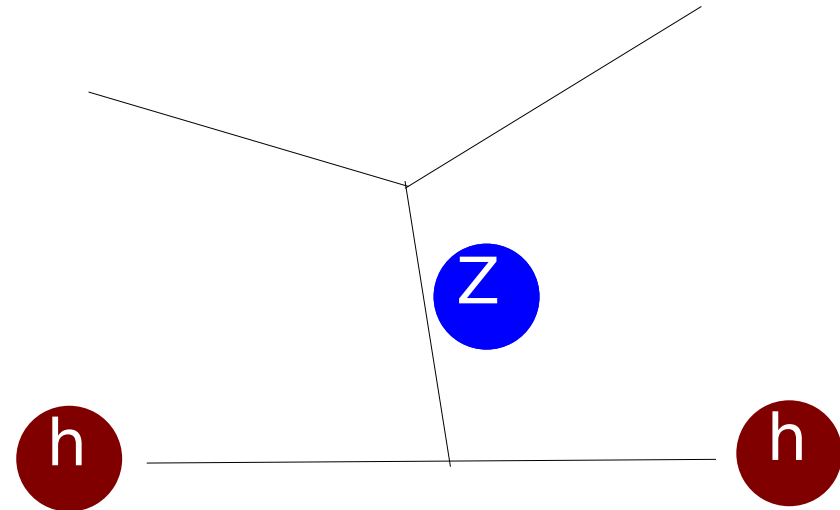
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Bound states as extended objects

- Two possibilities to measure extension
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 - Elastic scattering
 - Standard vector boson scattering process at low energies
 - Use this one

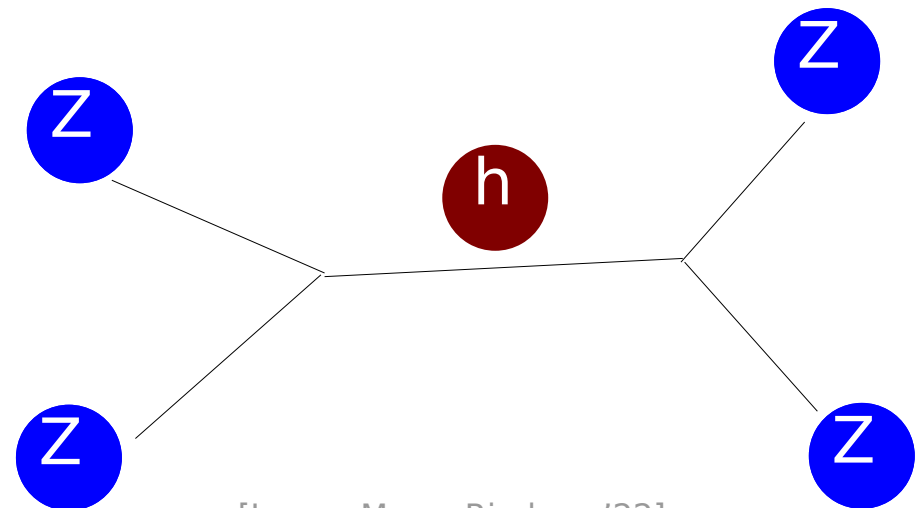
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Radius from elastic scattering in VBS

- Elastic region: $160/180 \text{ GeV} \leq \sqrt{s} \leq 250 \text{ GeV}$
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Cross section

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Matrix element

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Partial wave amplitude $\rightarrow f_J(s)$

Legendre polynomial $\rightarrow P_J(\cos\theta)$

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$$a_0 \xrightarrow{s \rightarrow 4m_W^2} = \tan(\delta_J) / \sqrt{s - 4m_W^2}$$

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$$\mathbf{M}(s, \Omega) = 16\pi \sum_J (2J+1) f_J(s) P_J(\cos \theta)$$

$$f_J(s) = e^{i\delta_J(s)} \sin(\delta_J(s))$$

$$a_0 = \lim_{s \rightarrow 4m_W^2} \tan(\delta_J) / \sqrt{s - 4m_W^2}$$

Scattering length ~ "size" Phase shift

Radius from elastic scattering in VBS

- Elastic region: $160/180 \text{ GeV} \leq \sqrt{s} \leq 250 \text{ GeV}$
 - s is the CMS energy in the initial/final ZZ/WW system
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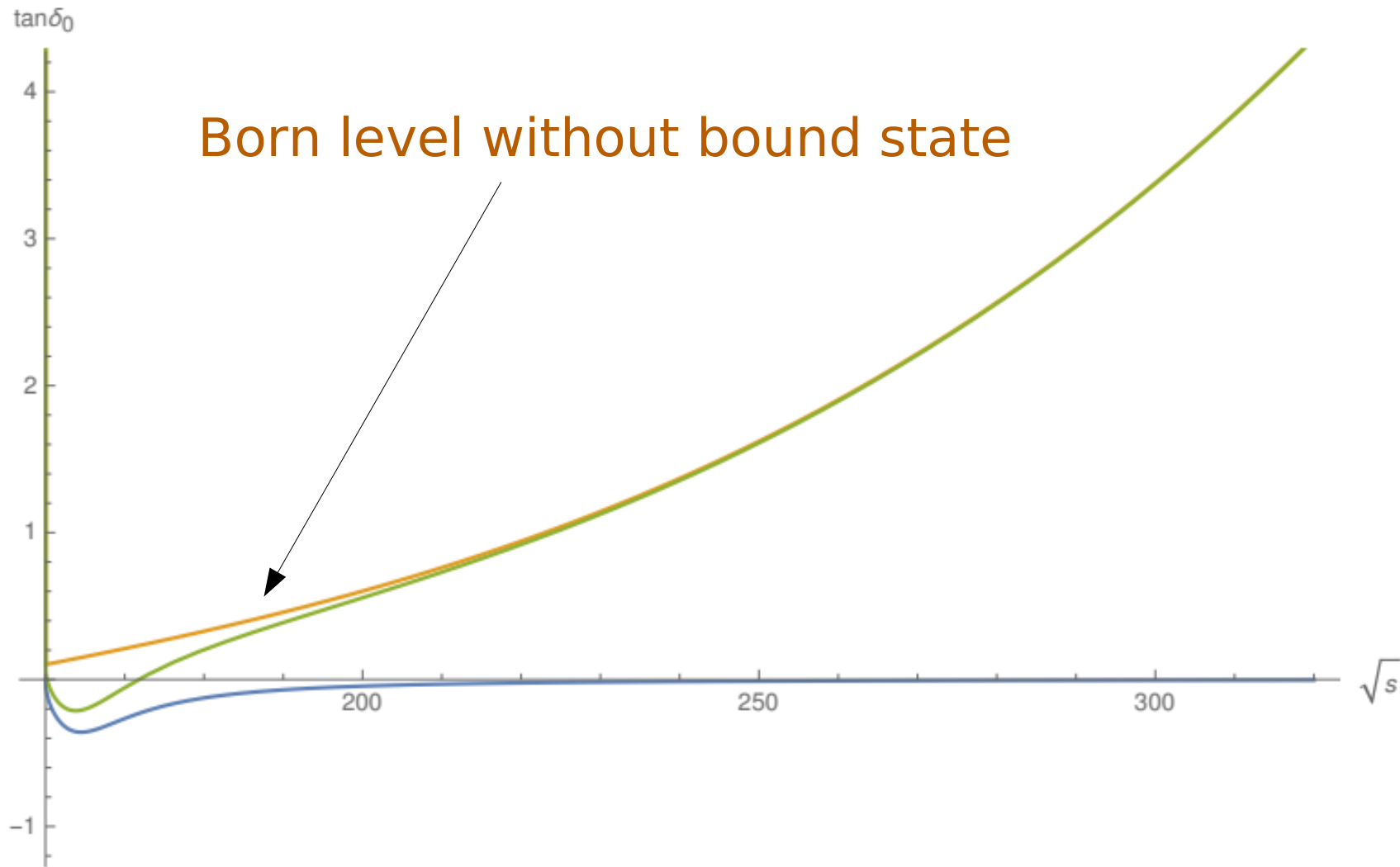
Phase shift

→ Lattice Lüscher analysis

Impact of a finite size of the Higgs

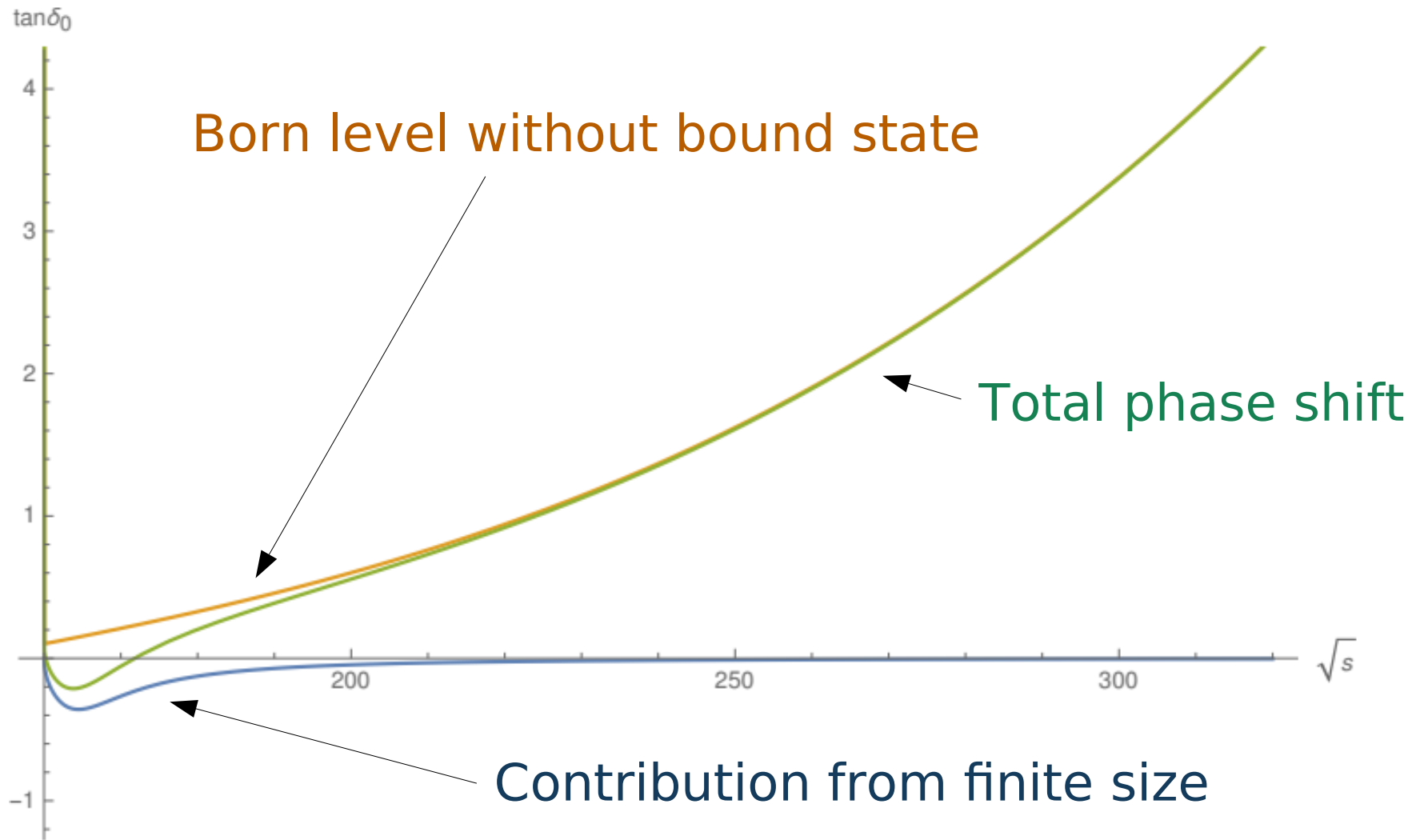
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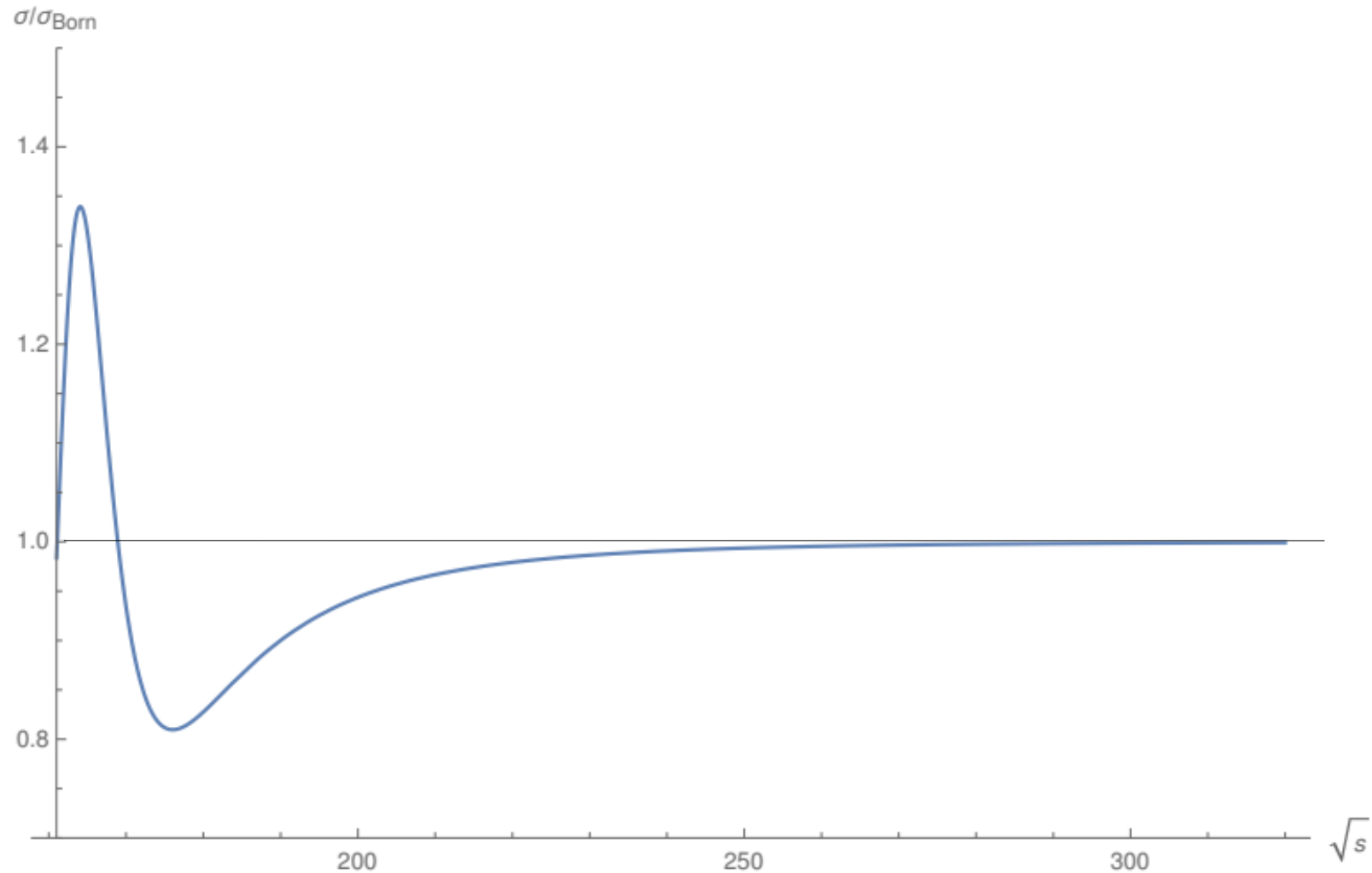
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Impact on the radius of the Higgs

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 - Parameters slightly different
 - Higgs 145 GeV and weak coupling larger

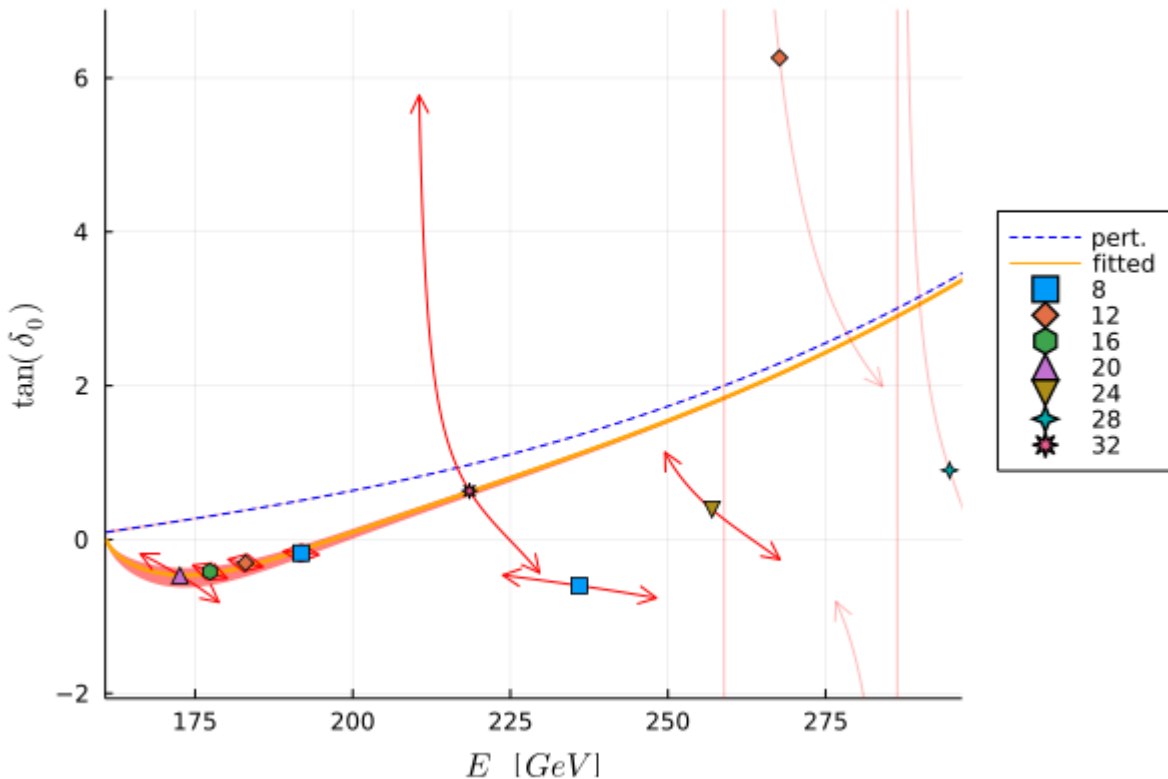
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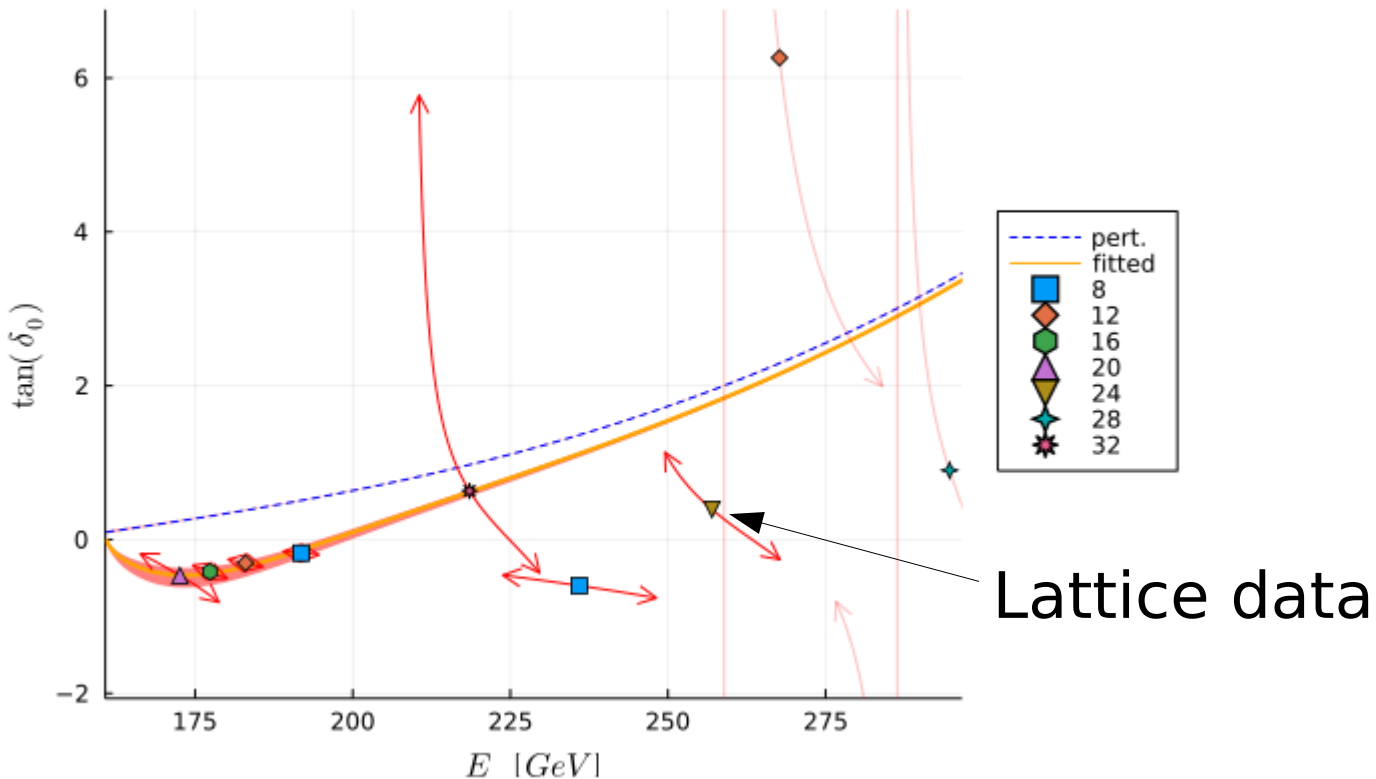
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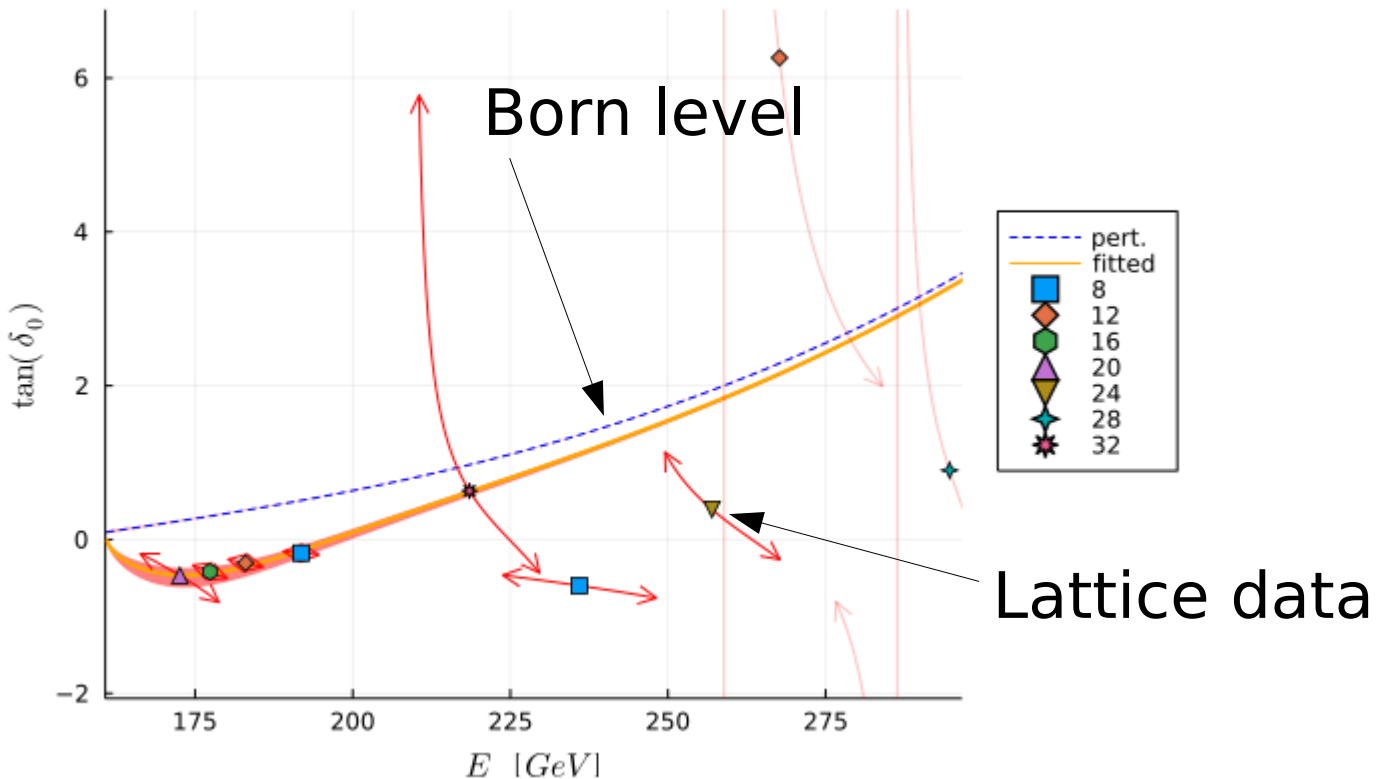
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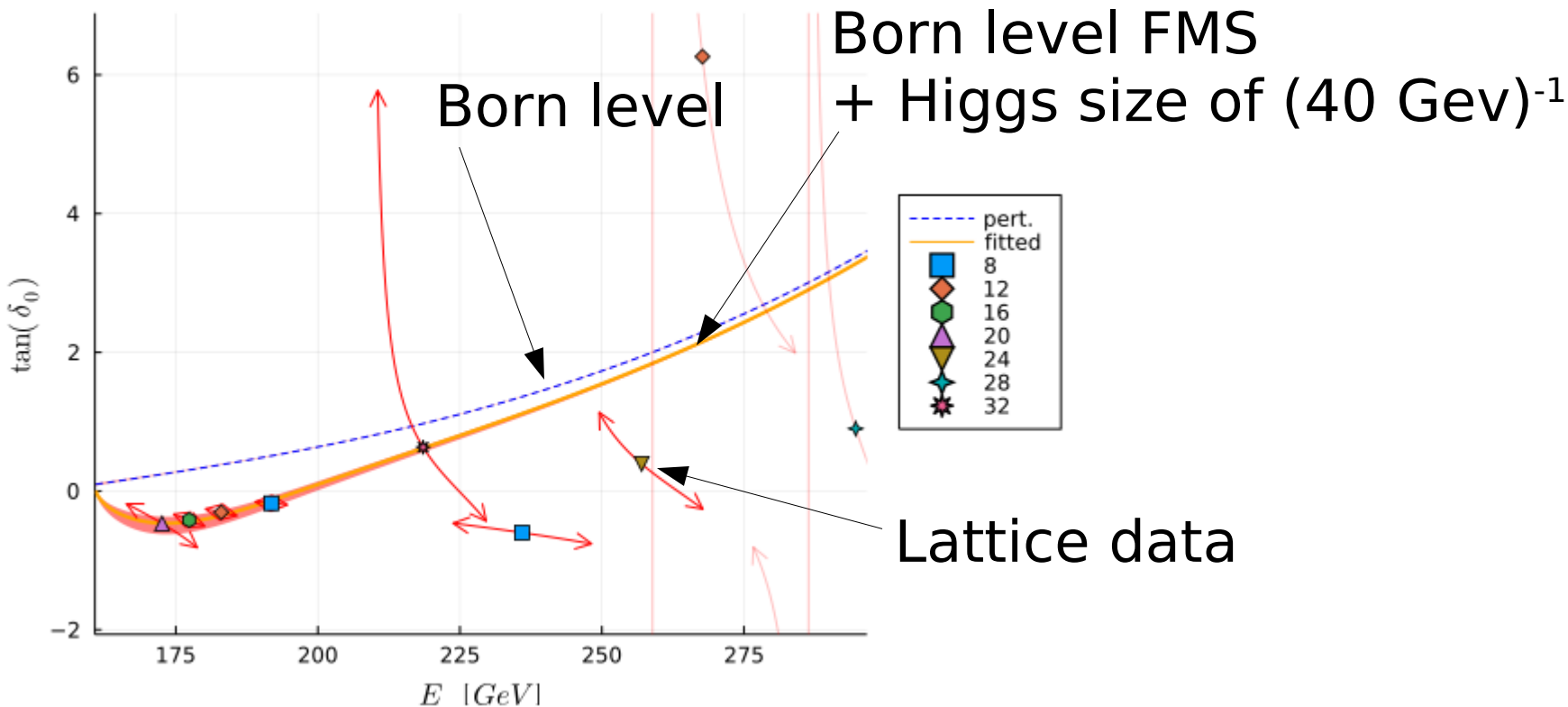
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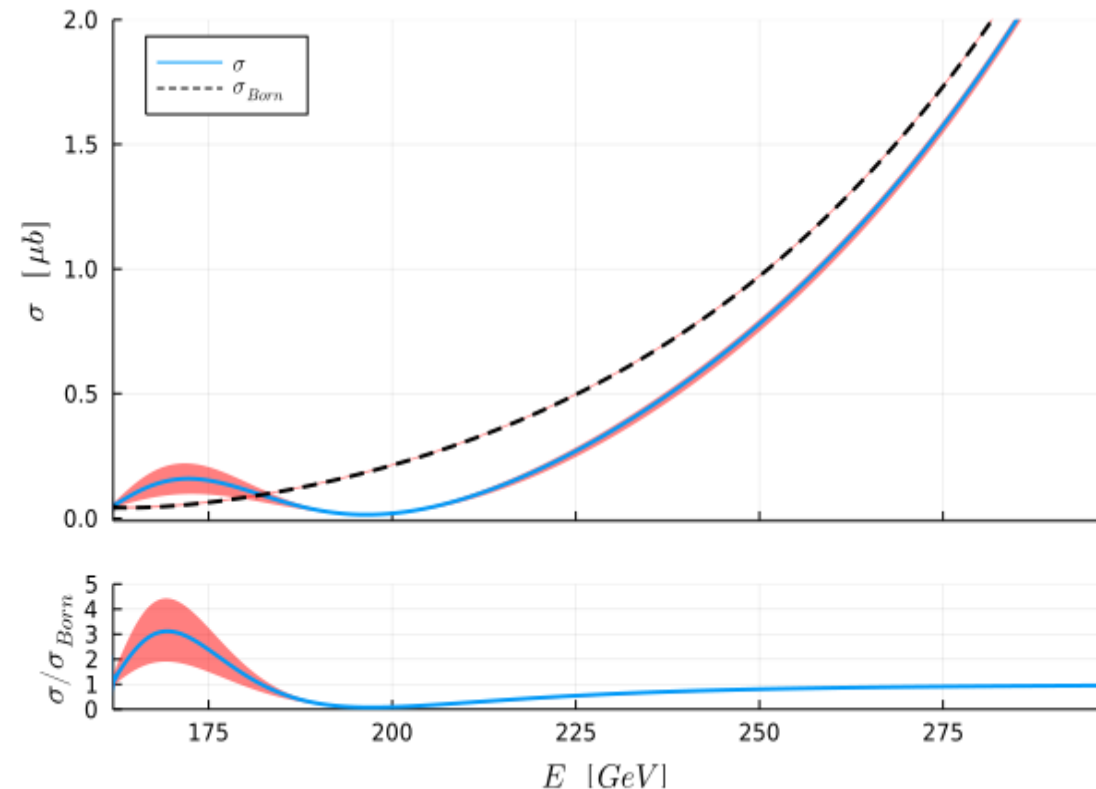
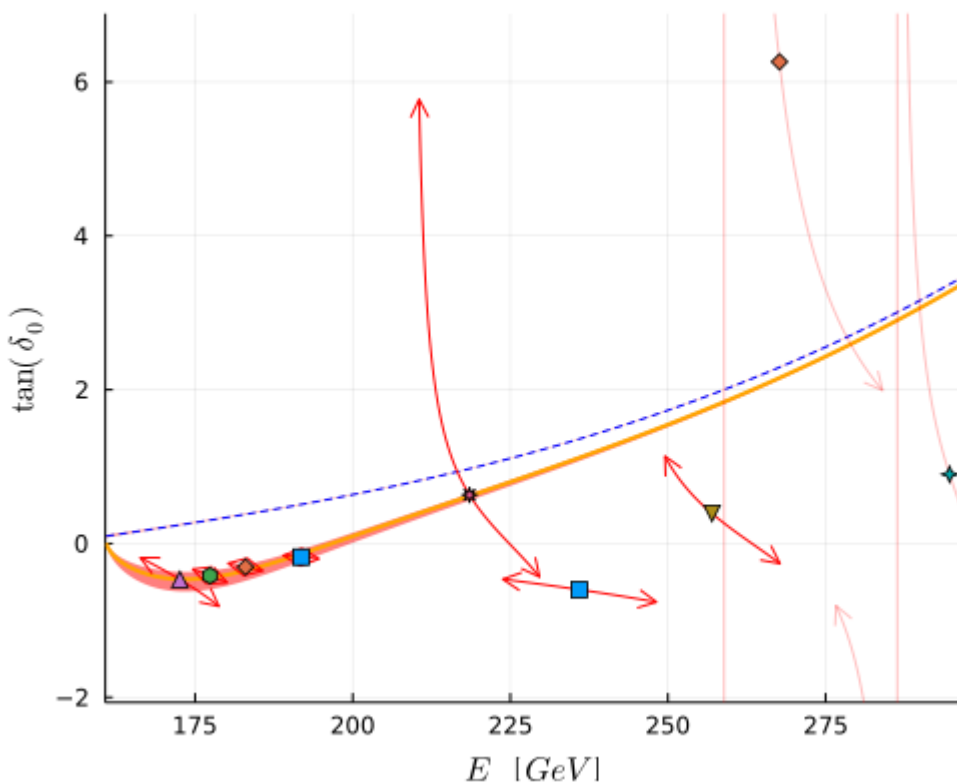
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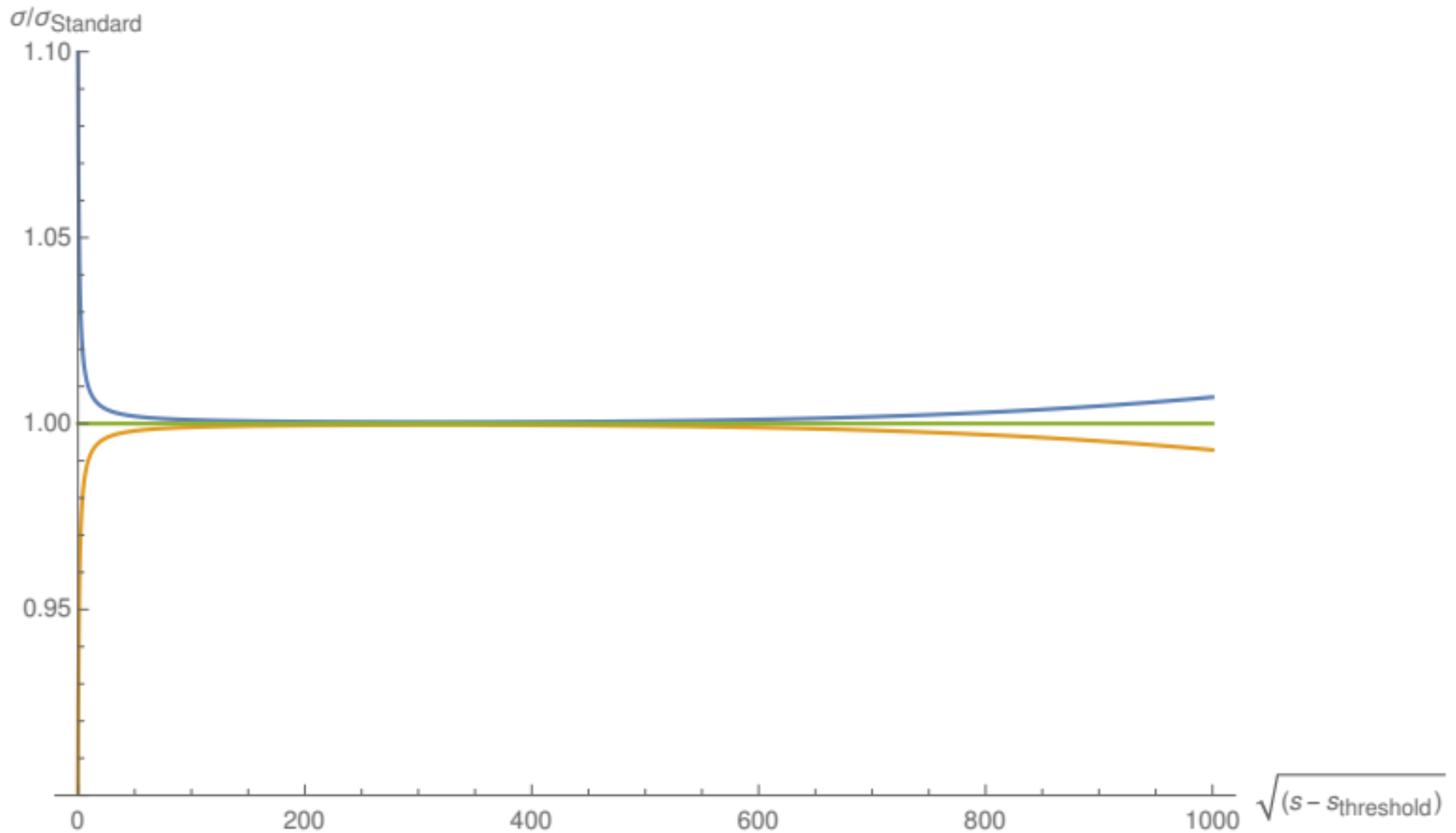


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Generic behavior

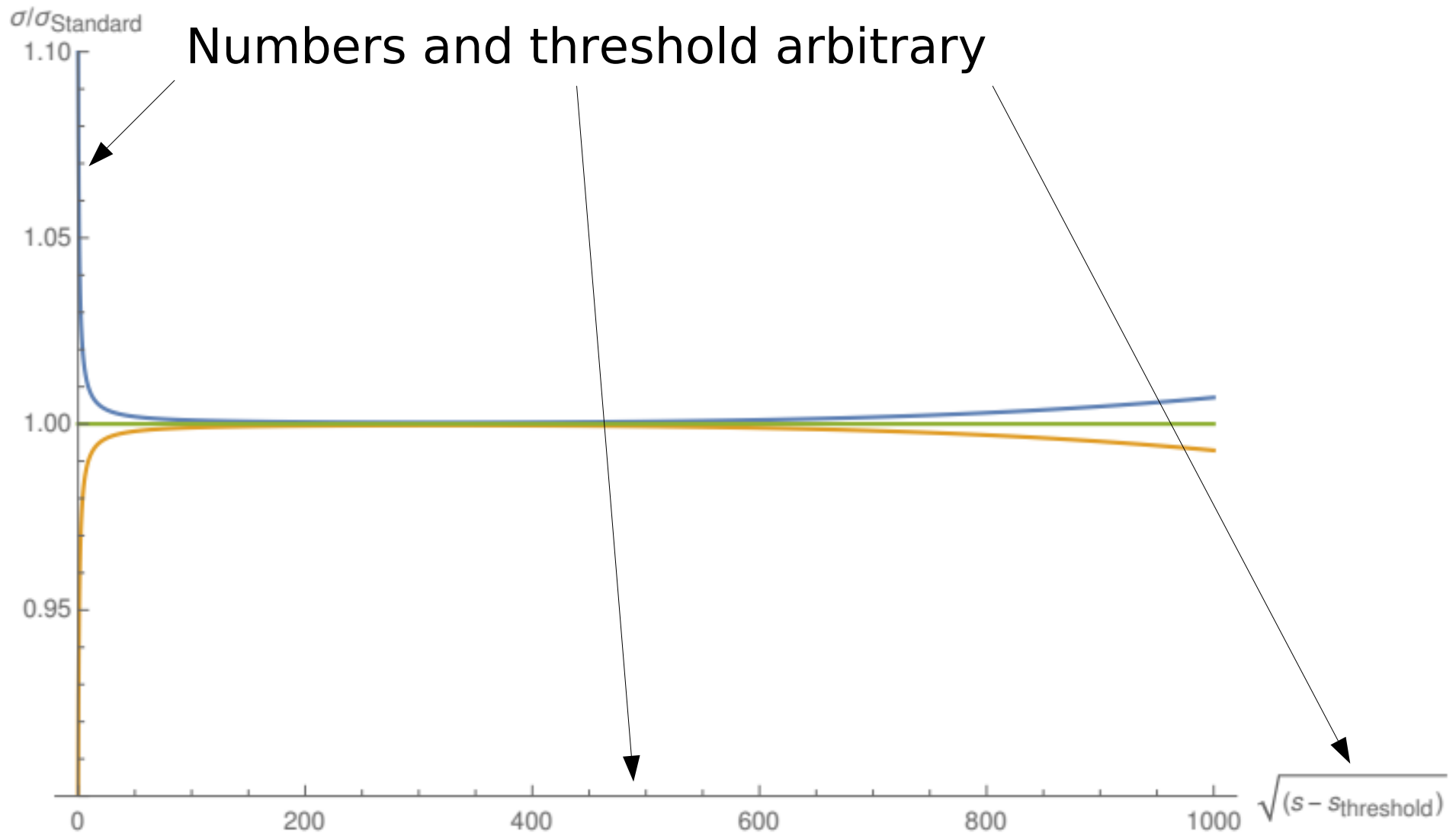
Has been done for several observables

Generic behavior: DIS-like



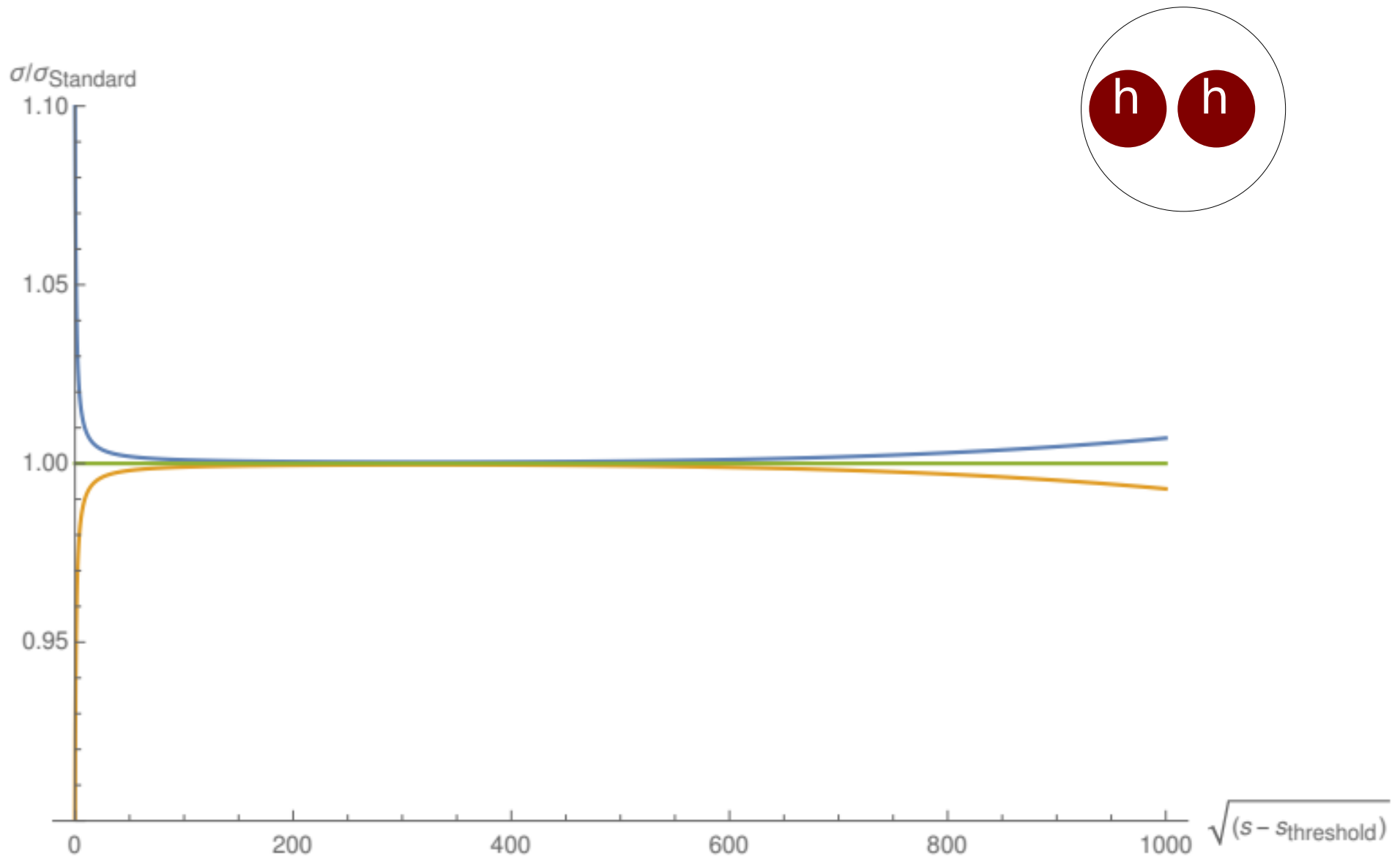
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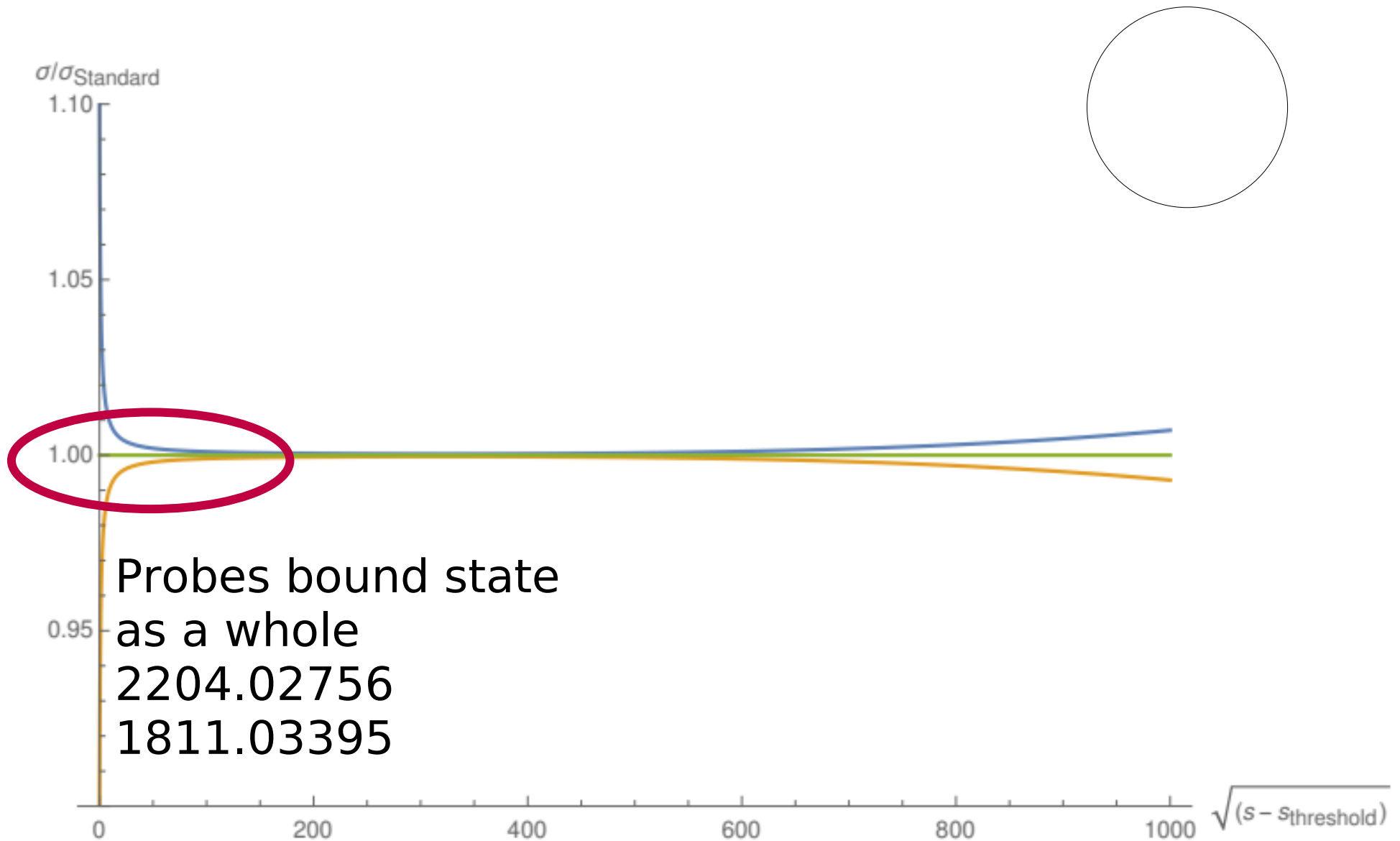
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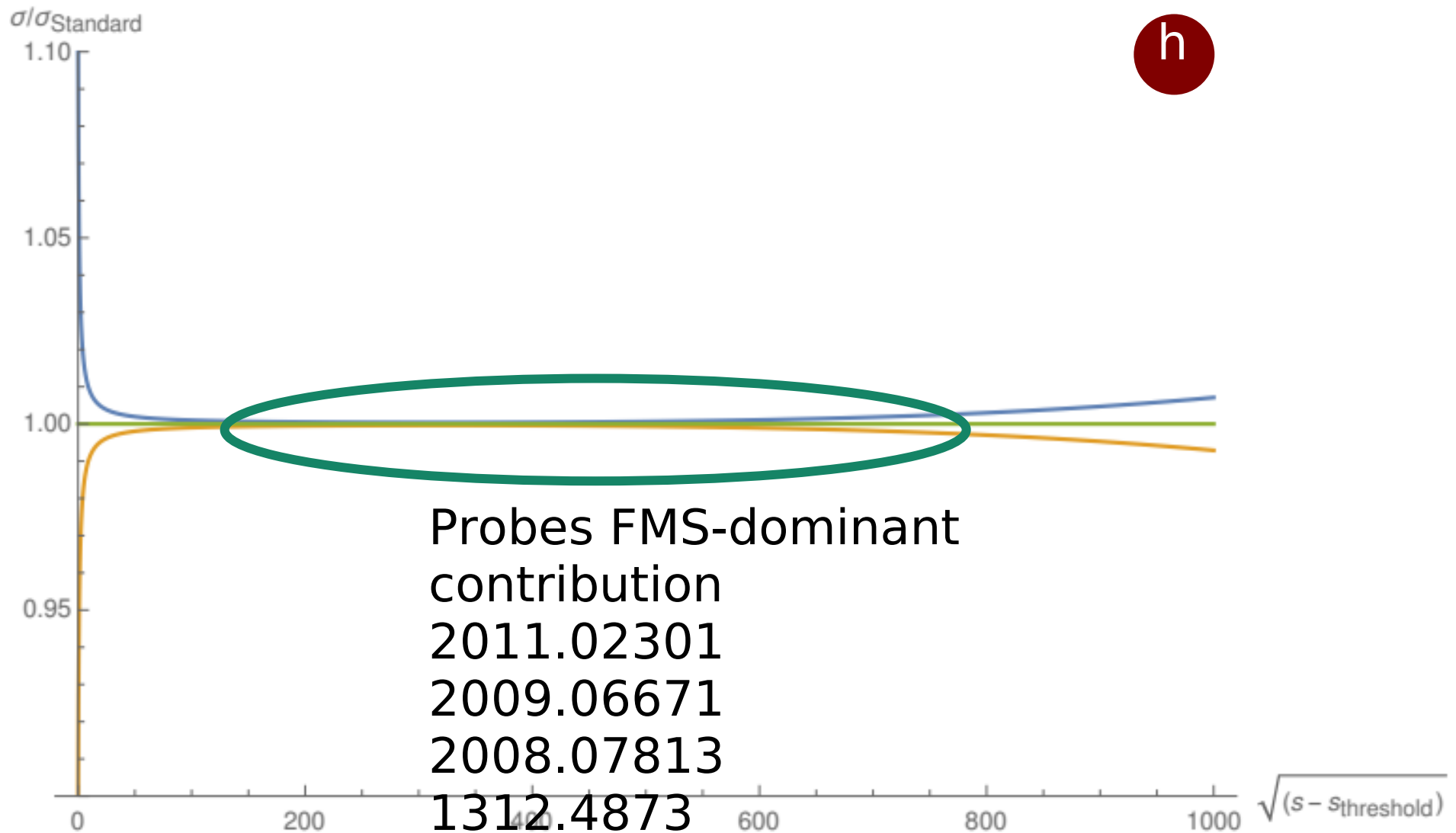
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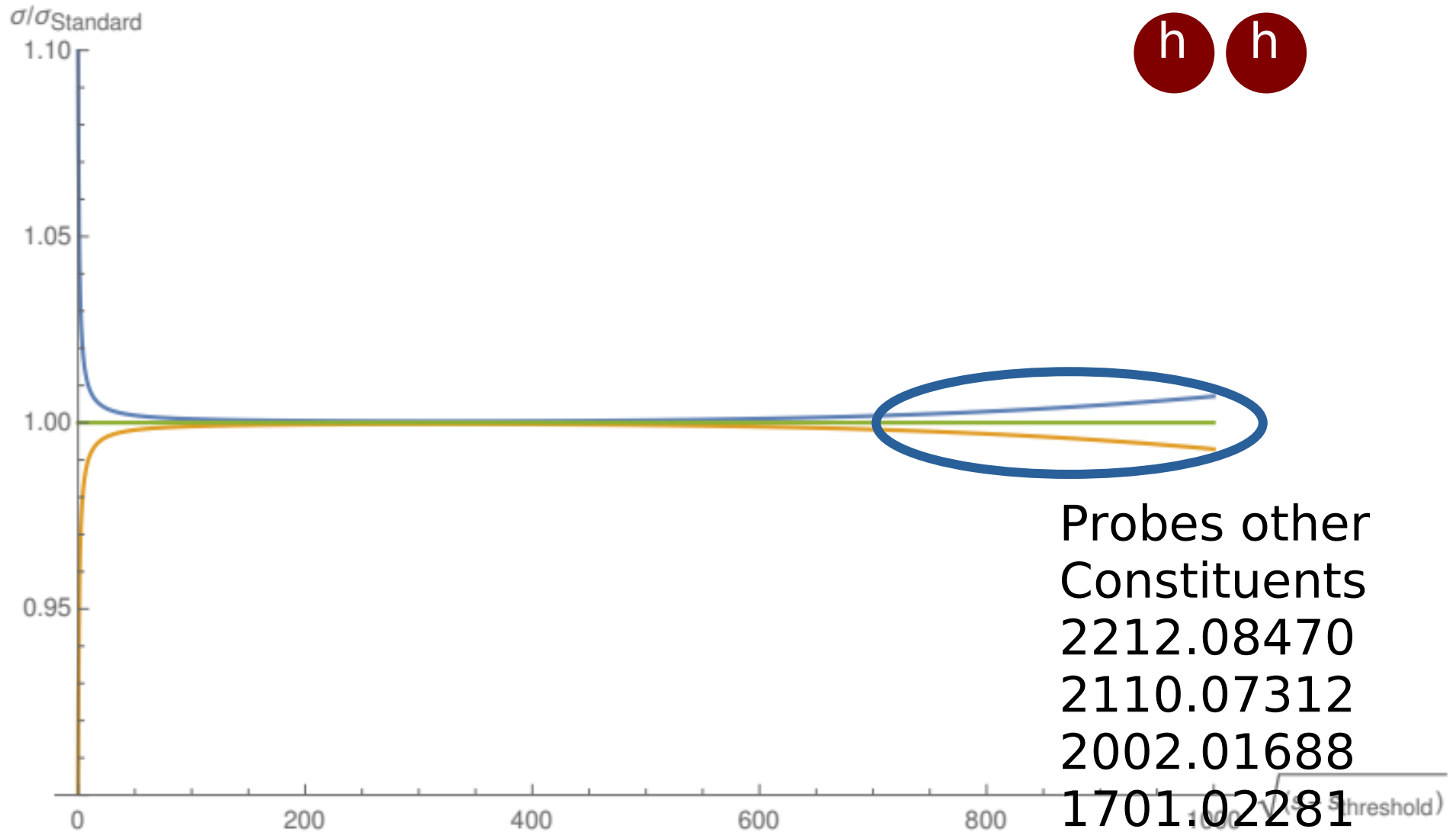
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Phenomenological
Implications

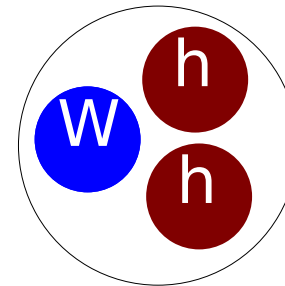
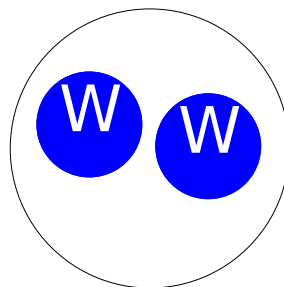
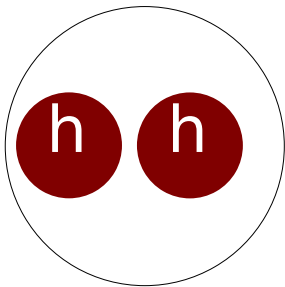
-

Adding matter

Physical states

[Fröhlich et al.'80,
Banks et al.'79]

- Need physical, gauge-invariant particles
 - **Cannot** be the elementary particles
 - Non-Abelian nature is relevant
- Need more than one particle: Composite particles
 - Higgs-Higgs, W-W, Higgs-Higgs-W etc.

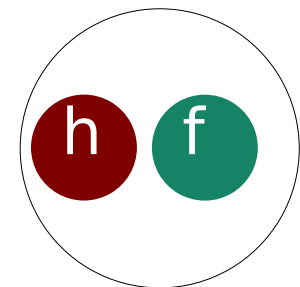
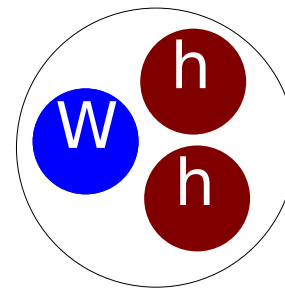
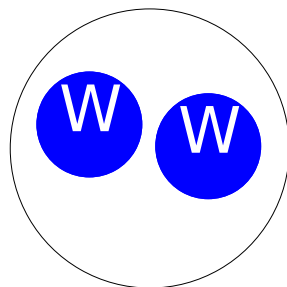
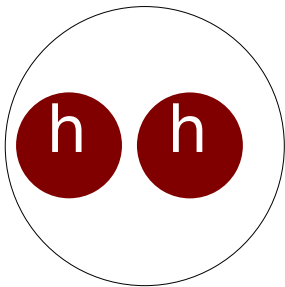


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Flavor on the lattice

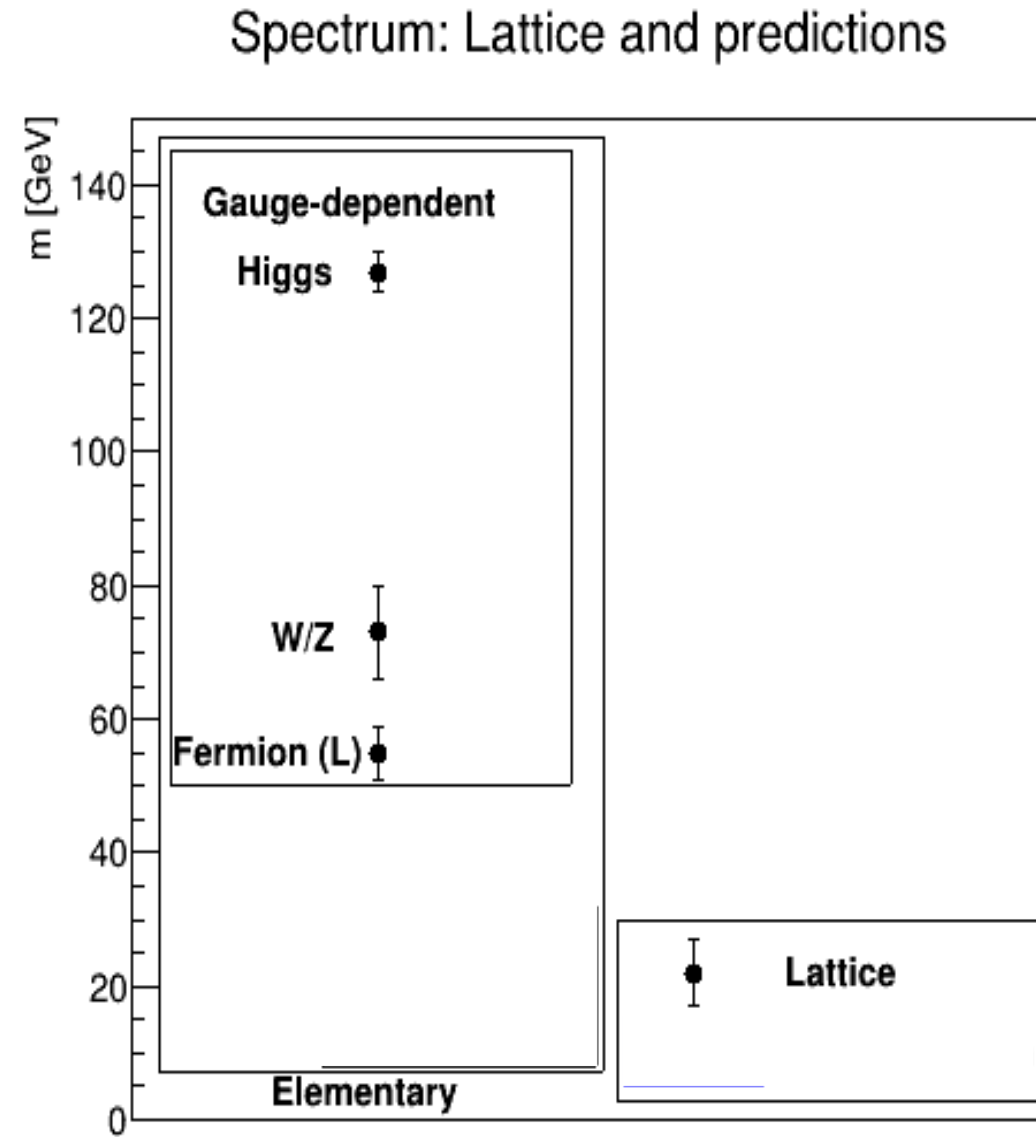
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 - One generation
 - Degenerate leptons and neutrinos
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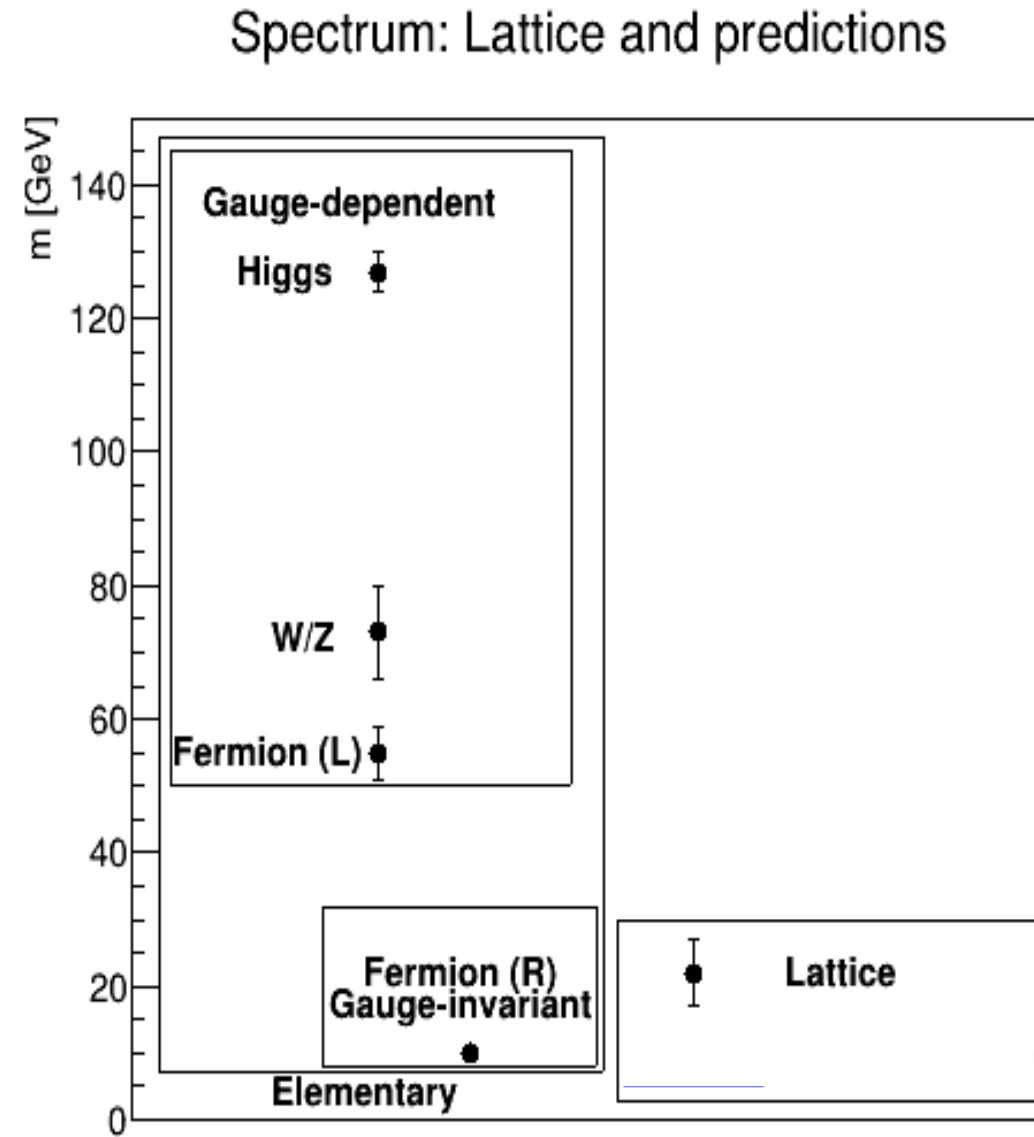
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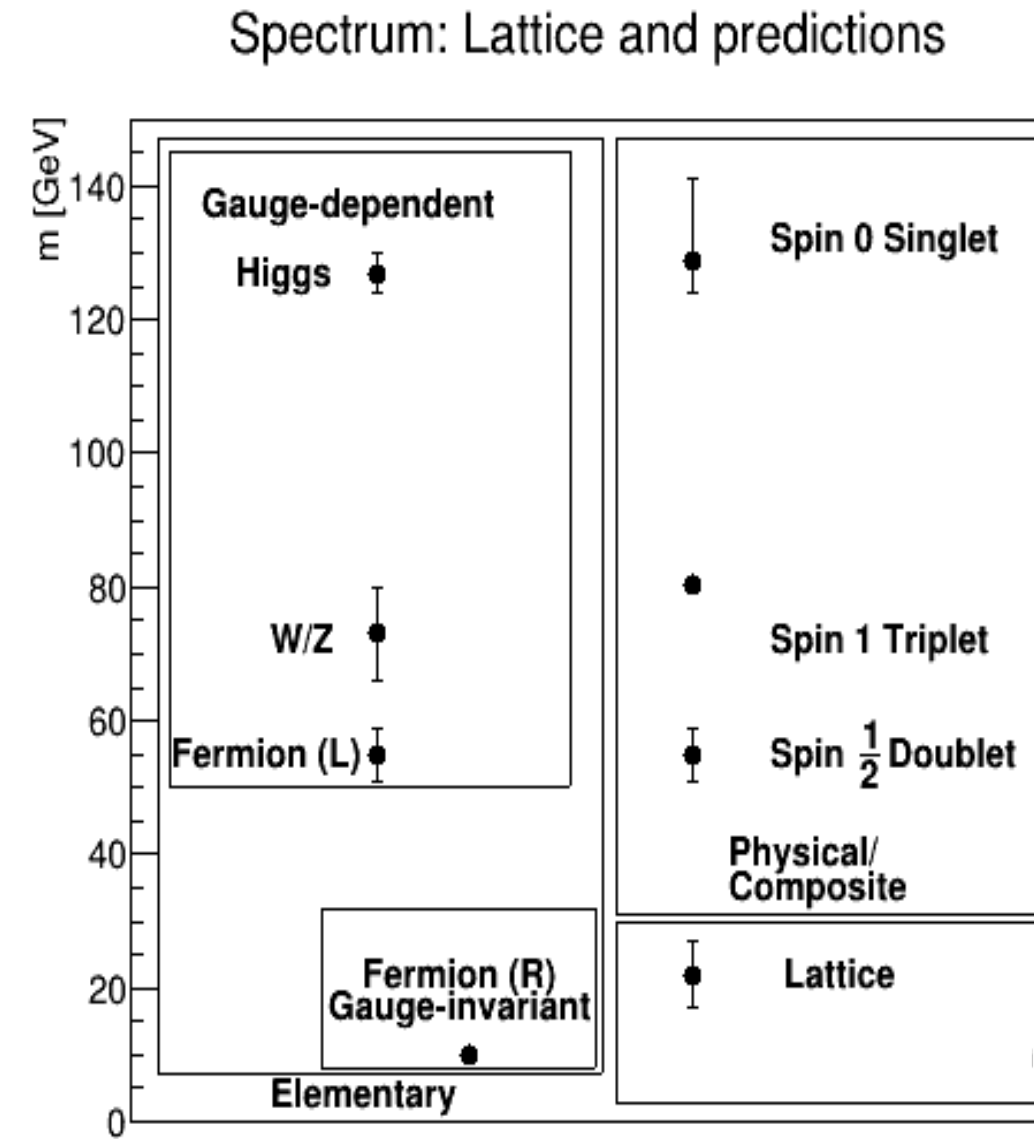
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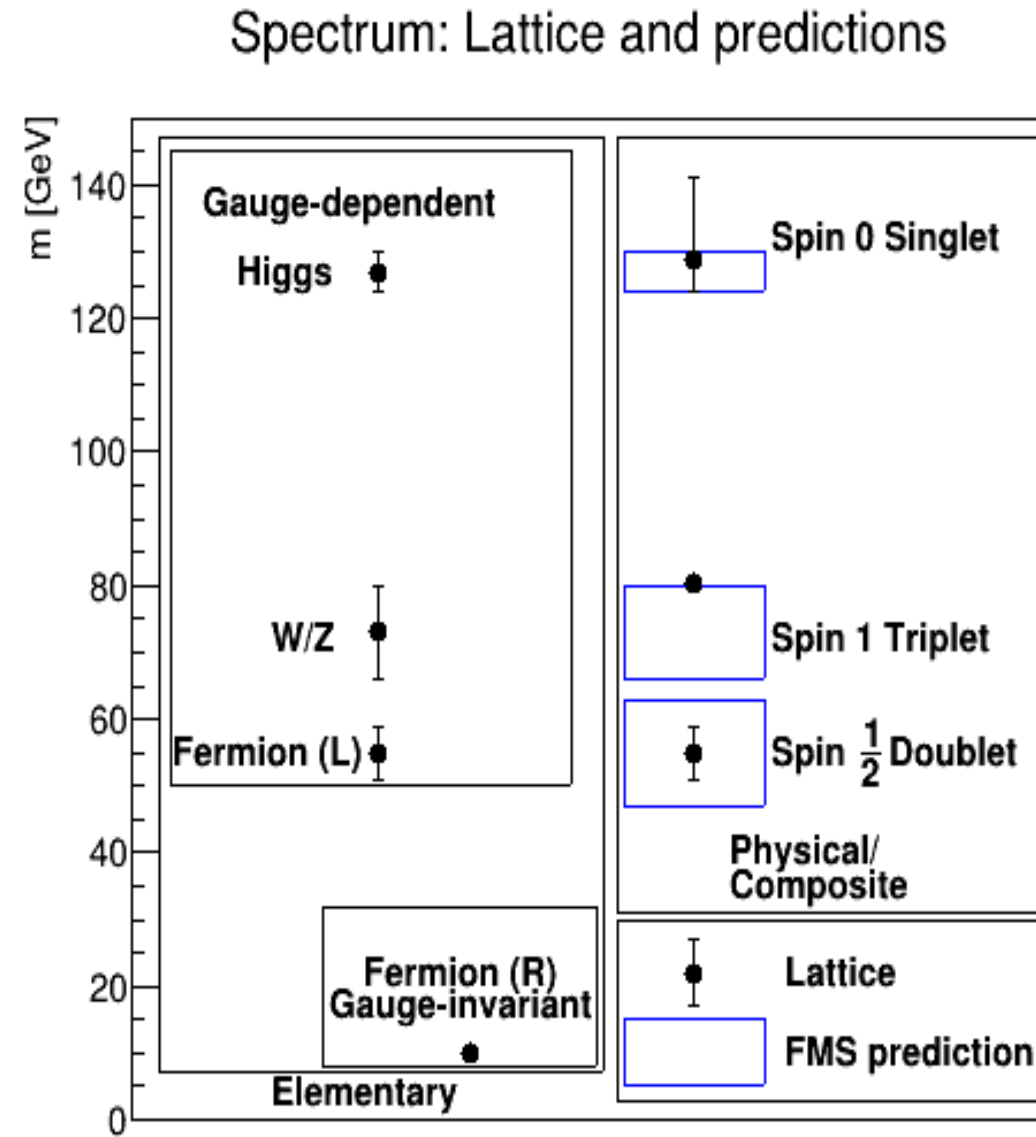
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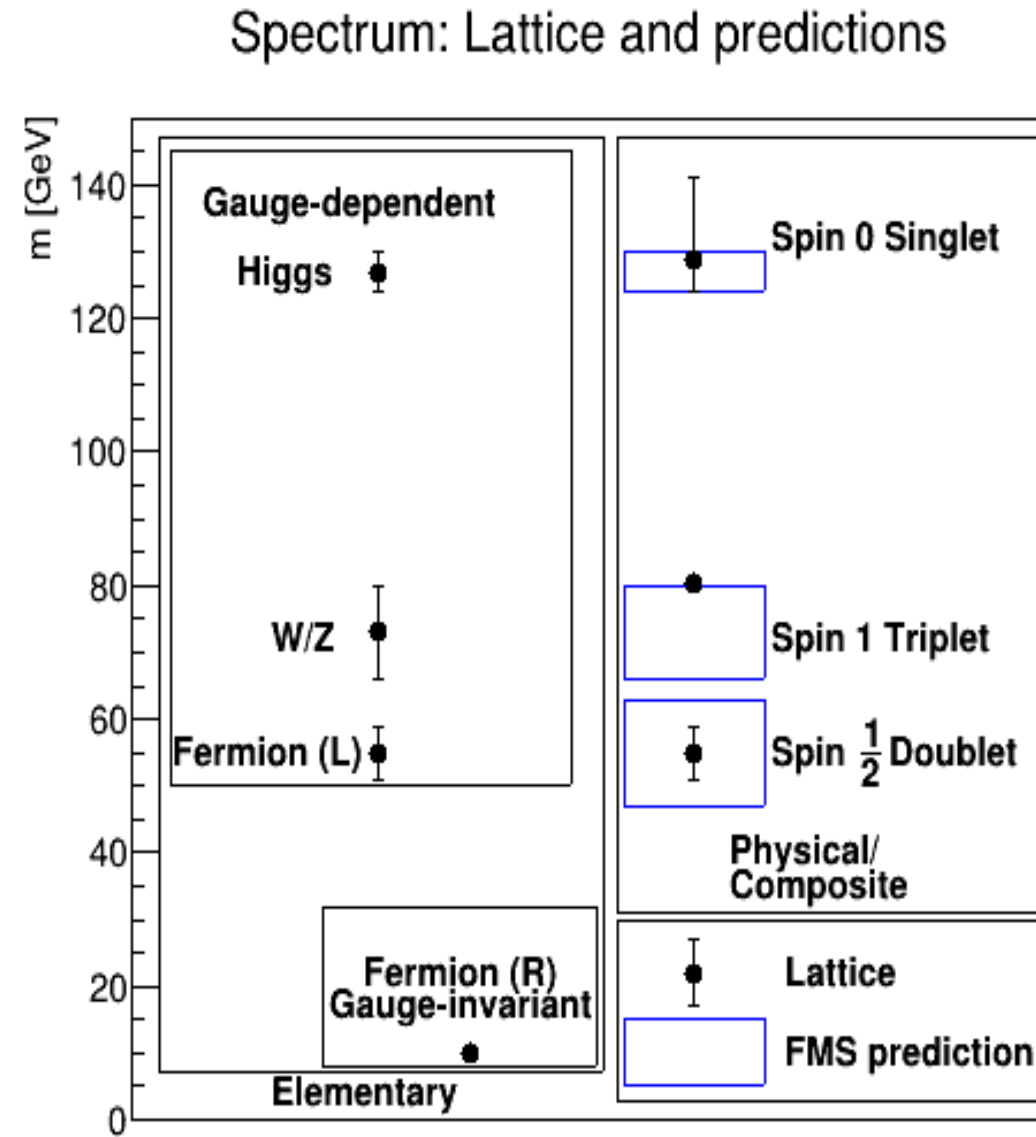
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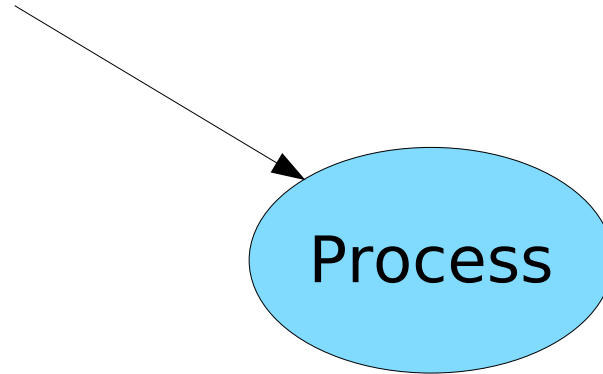
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- Supports FMS prediction – grant for unquenching '24-'28



Scattering

[Maas et al.'17
Maas & Reiner '22
Maas, Plätzer et al.' unpublished]

Incoming (asymptotic) particle
Standard LSZ: Elementary particle

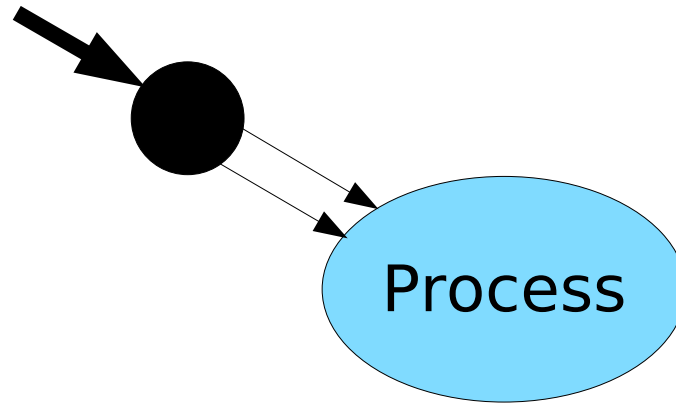


$$\langle f(p) \dots \rangle$$

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Gauge-invariant LSZ: Bound state

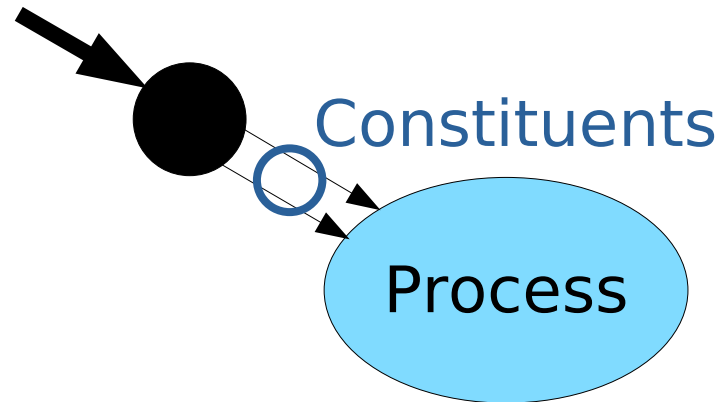


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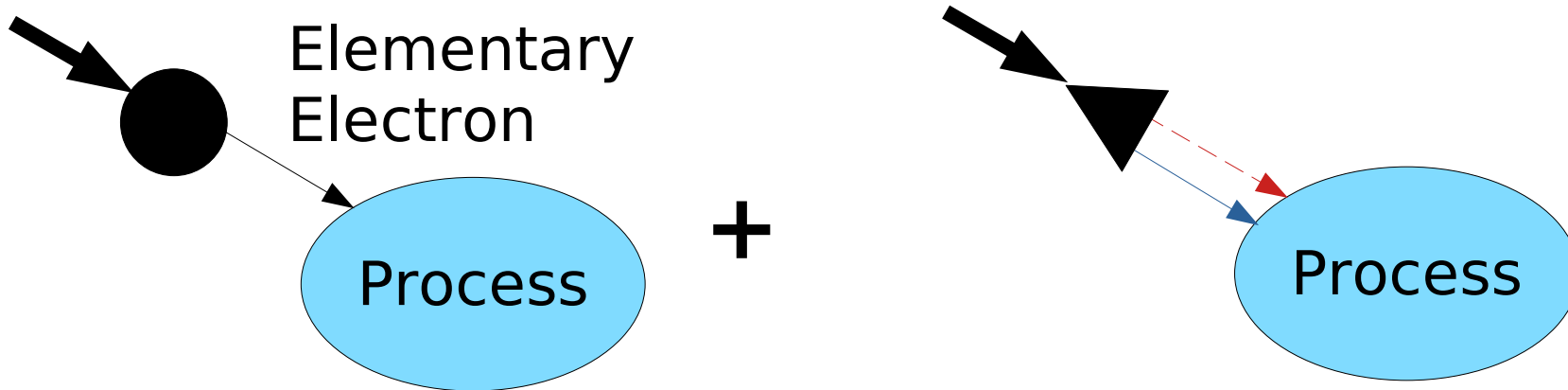


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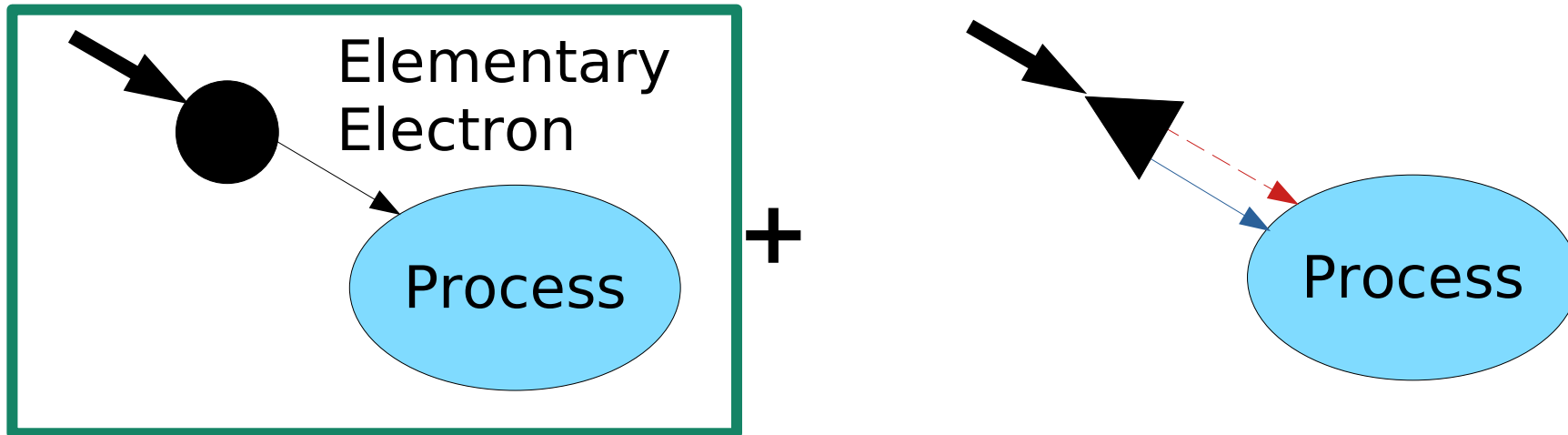


$$v \langle f(p) \dots \rangle + \int dq \Gamma(P, q) D_f(p - q) D_h(q) \langle h(q) f(P - q) \dots \rangle$$

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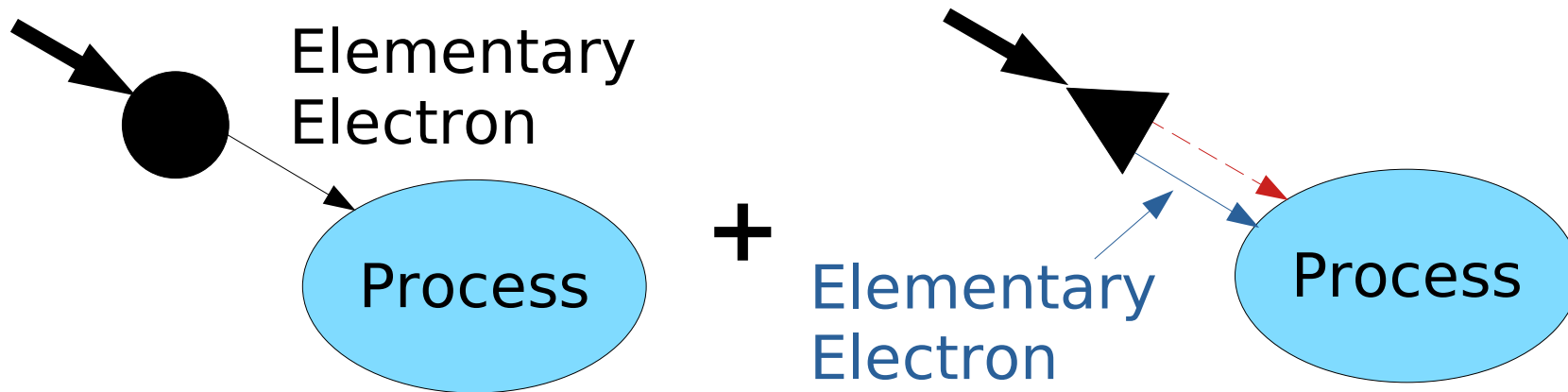
Standard perturbation theory

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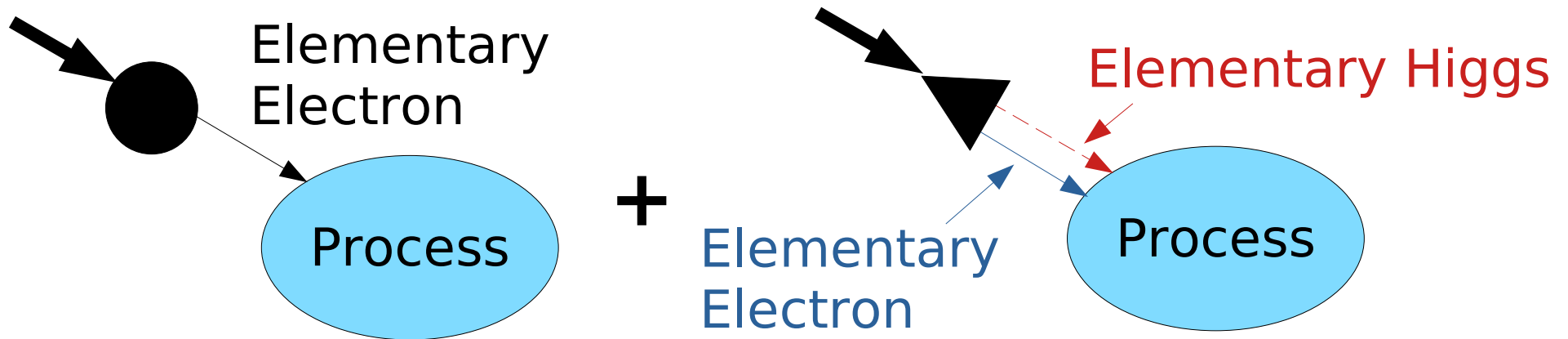


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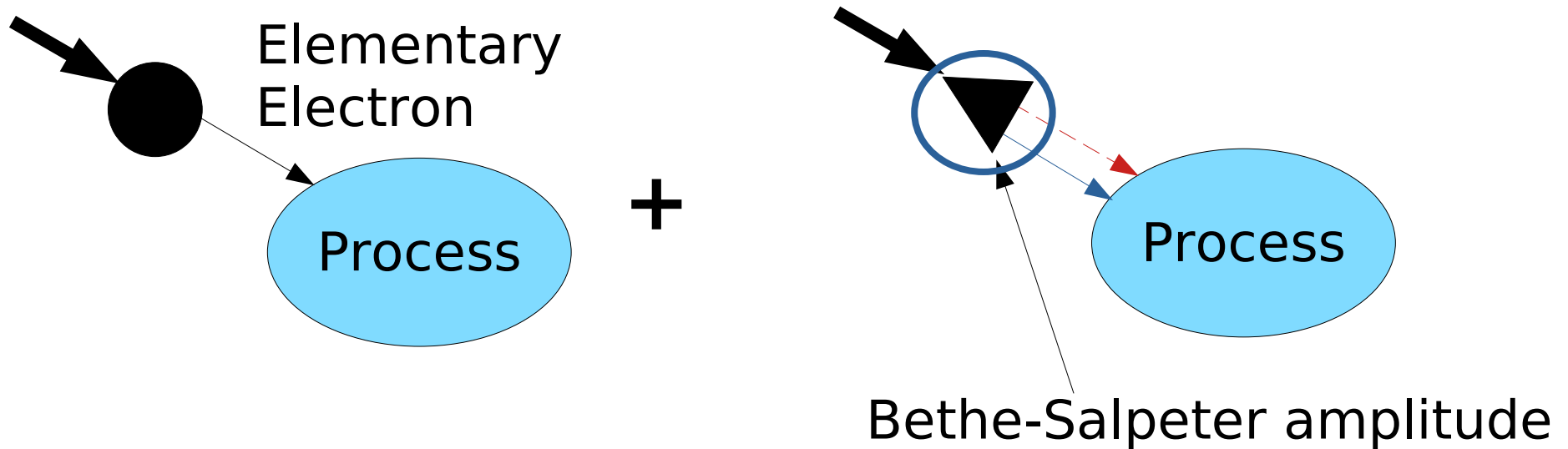


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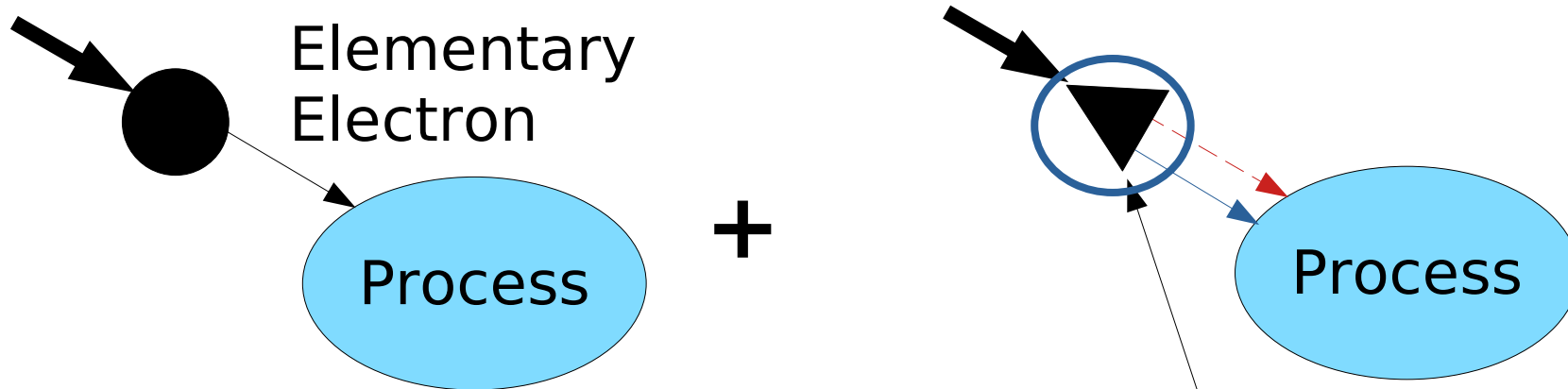


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Scattering

[Maas et al.'17
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Maas, Plätzer et al.'24
Maas, Plätzer et al. unpublished]

Incoming (asymptotic) particle
FMS LSZ: Elementary and fluctuations



Bethe-Salpeter amplitude:
Calculate in augmented PT

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Bethe-Salpeter Amplitude

[Maas, Plätzer et al. unpublished
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Bethe-Salpeter Amplitude

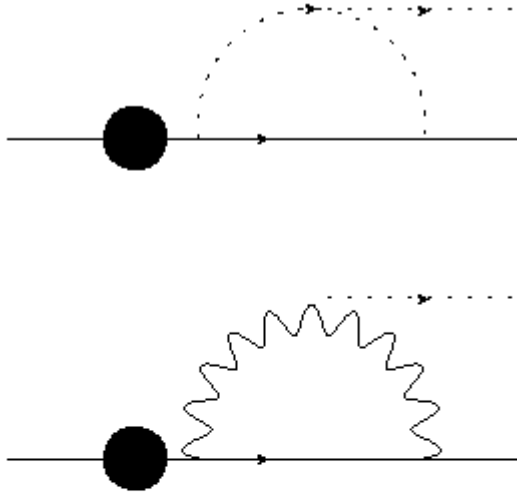
[Maas, Plätzer et al. unpublished
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Calculable itself in augmented perturbation theory

Bethe-Salpeter Amplitude

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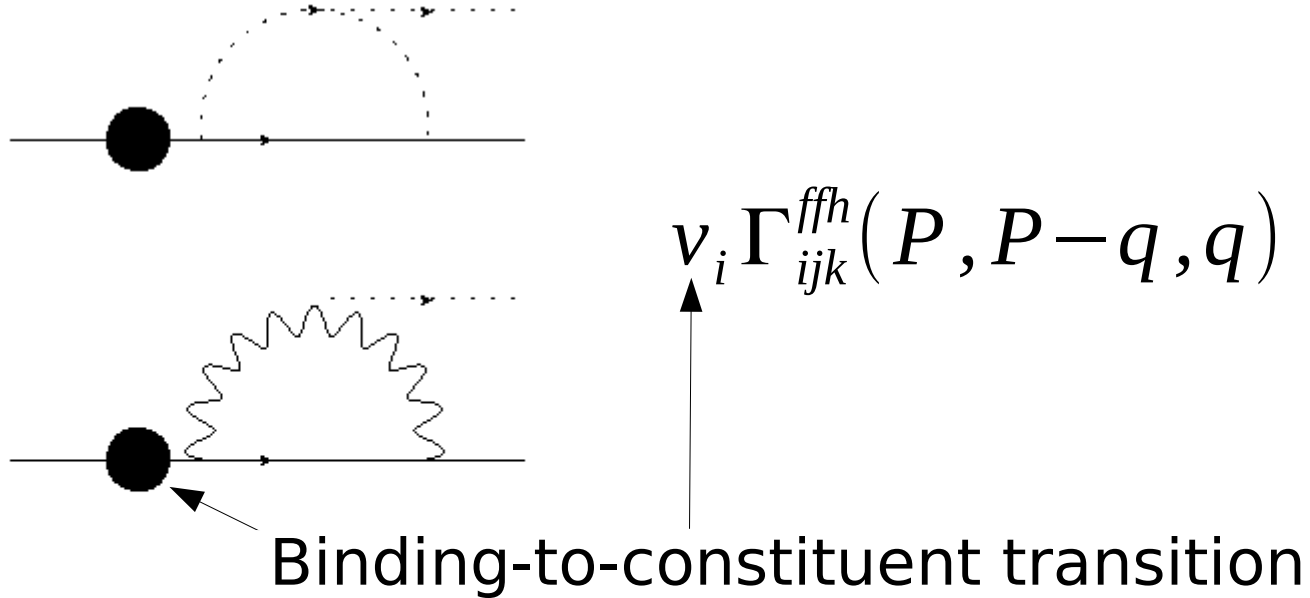


$$v_i \Gamma_{ijk}^{ffh}(P, P-q, q)$$

Bethe-Salpeter Amplitude

[Maas, Plätzer et al. unpublished
Maas, Plätzer et al.'24]

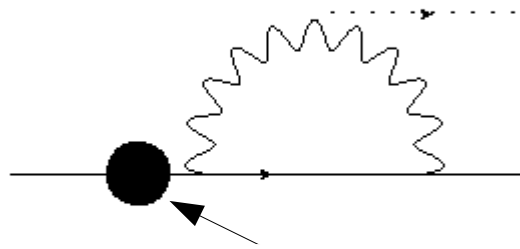
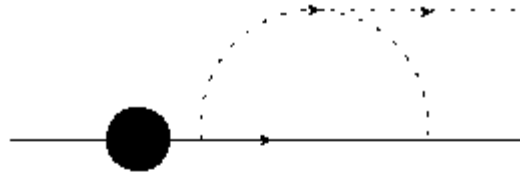
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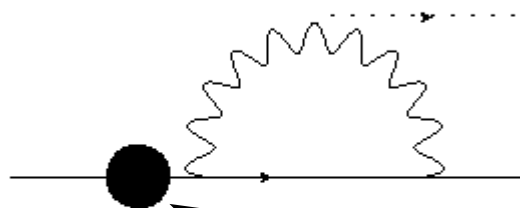
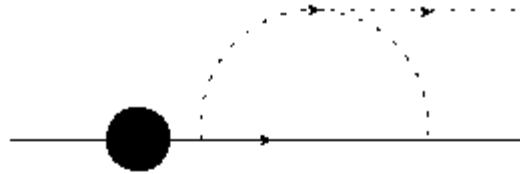
Binding-to-constituent transition

Reweights
standard
diagrams
@N(N)LO

Bethe-Salpeter Amplitude

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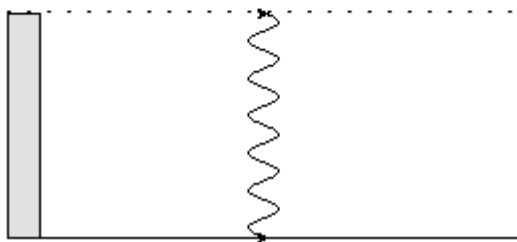
Calculable itself in augmented perturbation theory



Binding-to-constituent transition

$$v_i \Gamma_{ijk}^{ffh}(P, P-q, q)$$

Reweights
standard
diagrams
@N(N)LO

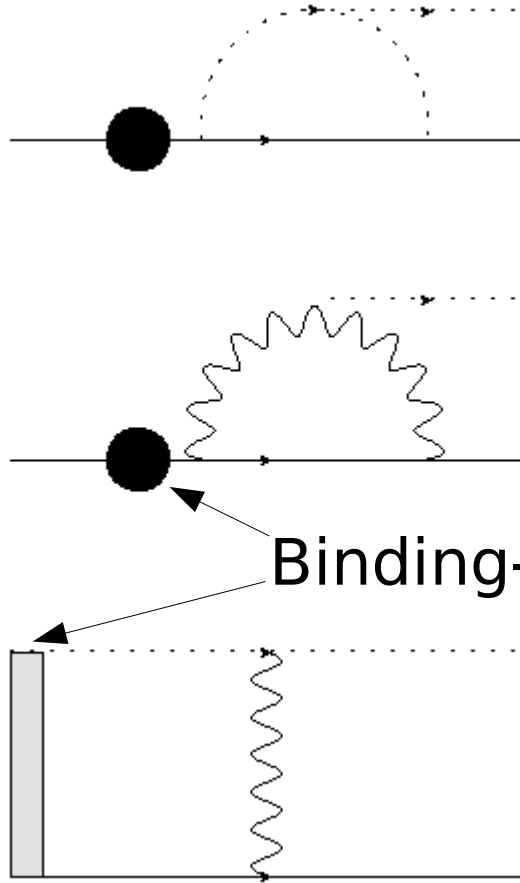


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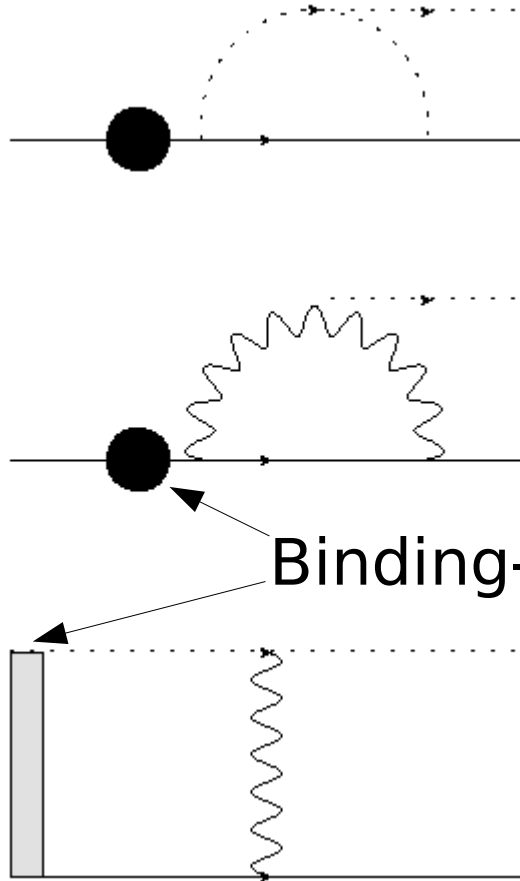
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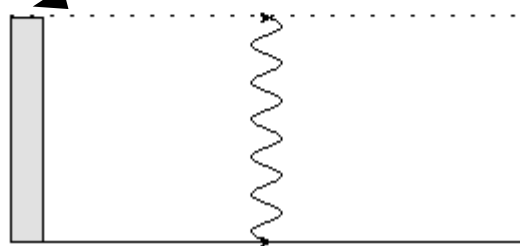
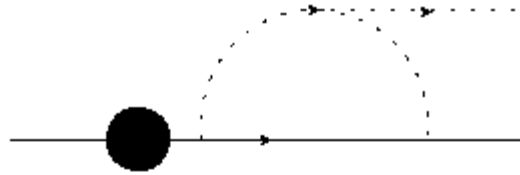
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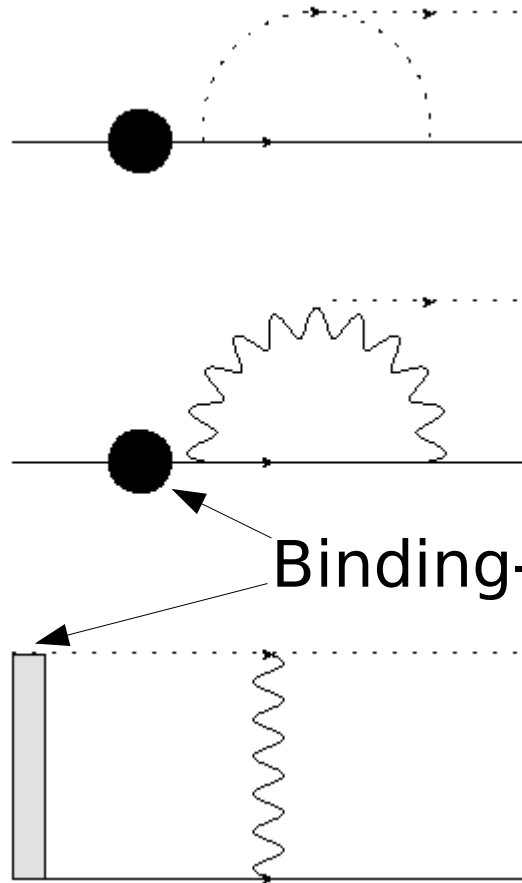
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Neither are Yukawa suppressed

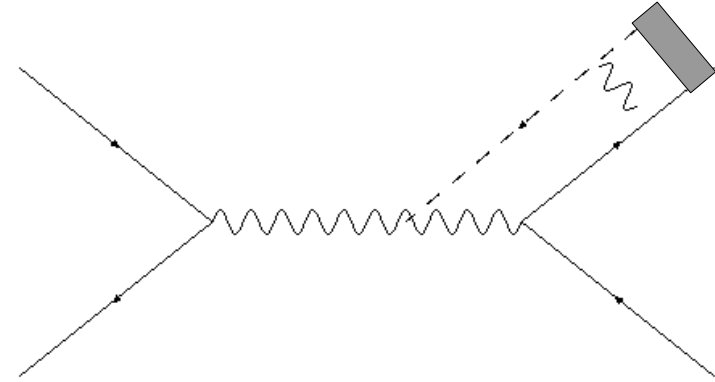
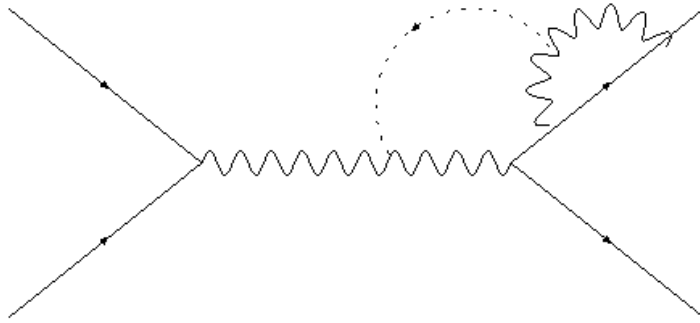
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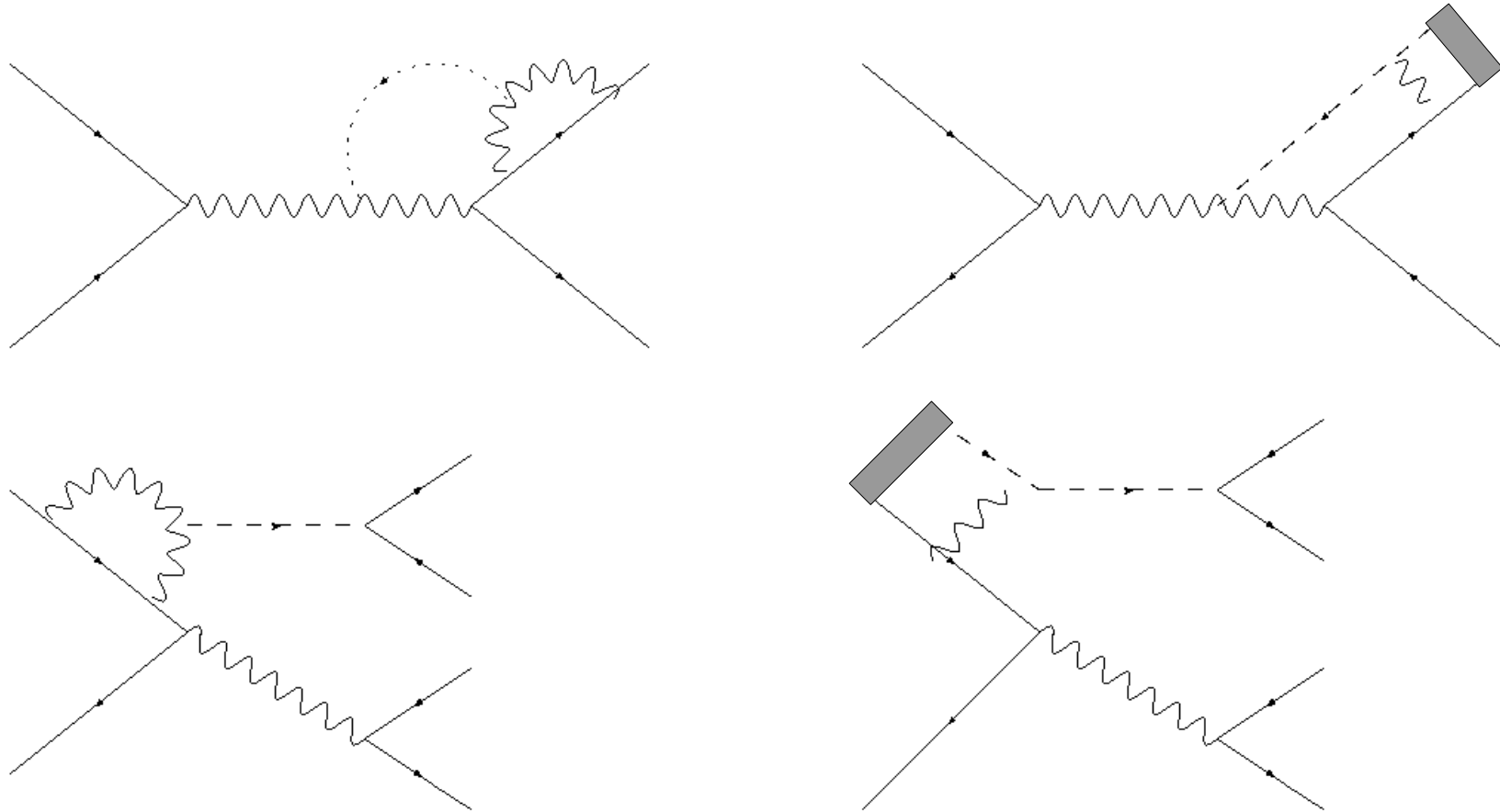
Process $ff \rightarrow ff$: 2/1-loop (in g_{weak}) suppressed contribution



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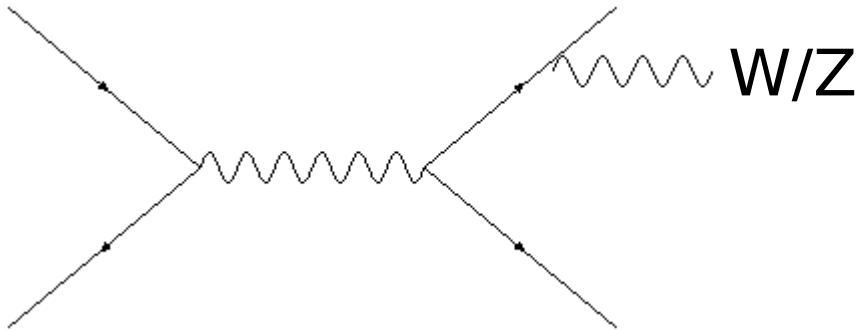
Resummation effects at $s \gg M_Z^2$

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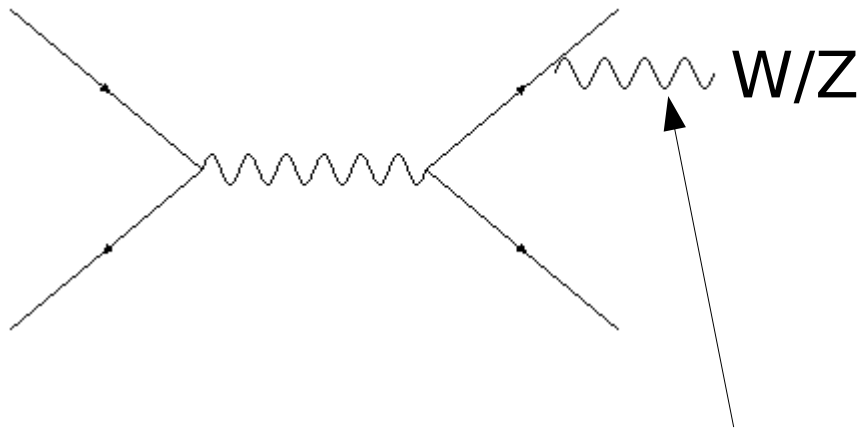
Standard perturbation theory



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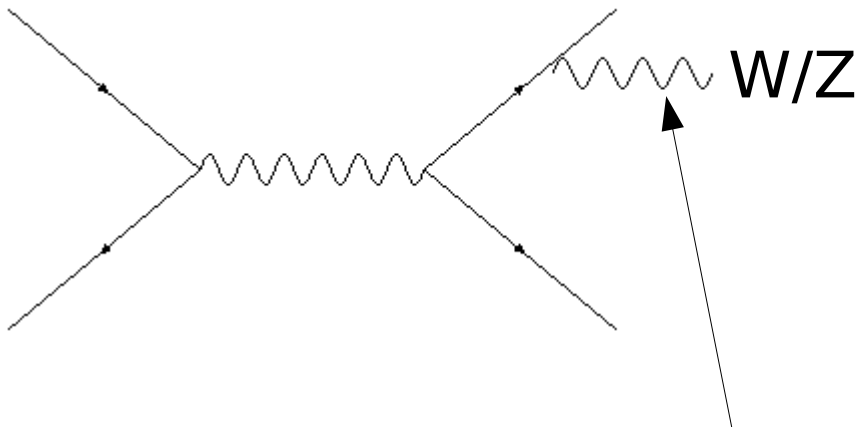


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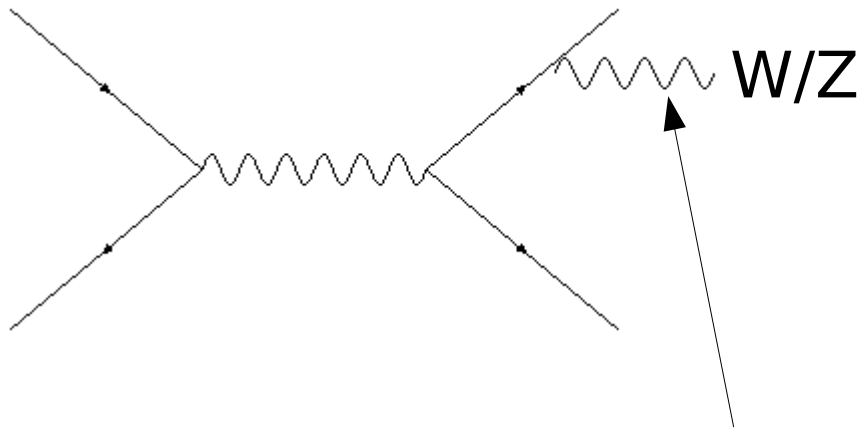


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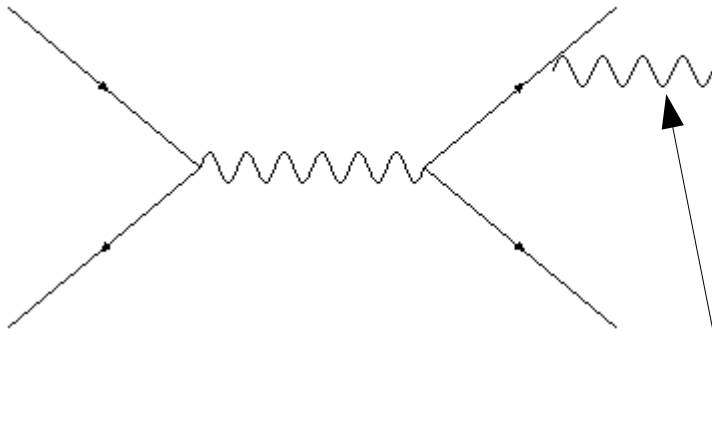
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at 1 TeV of the same
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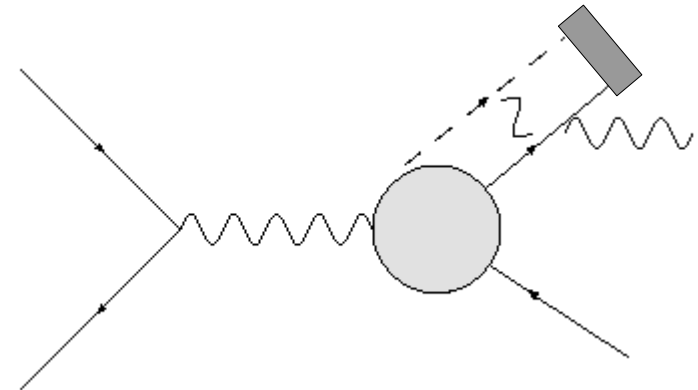
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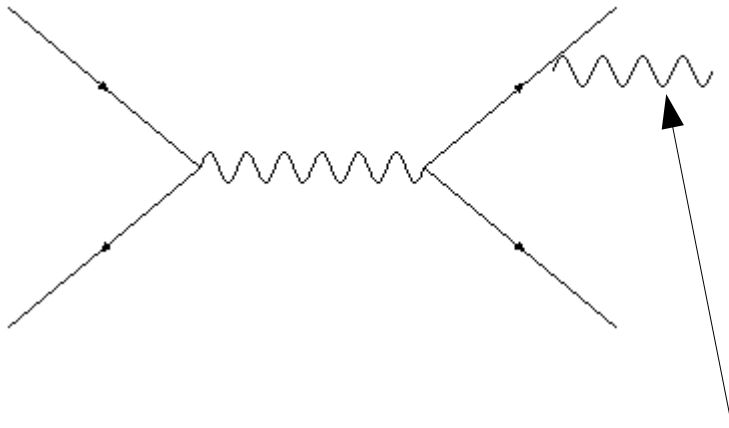
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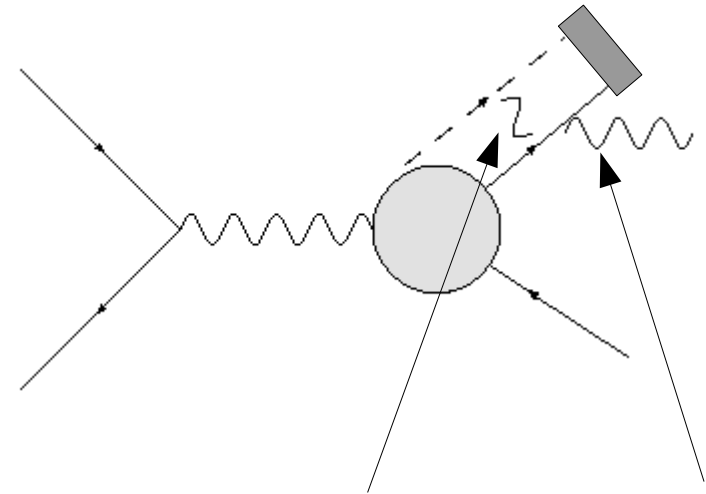


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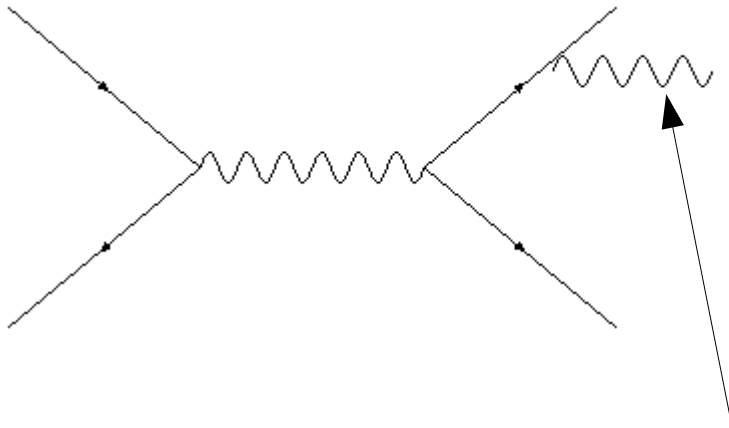
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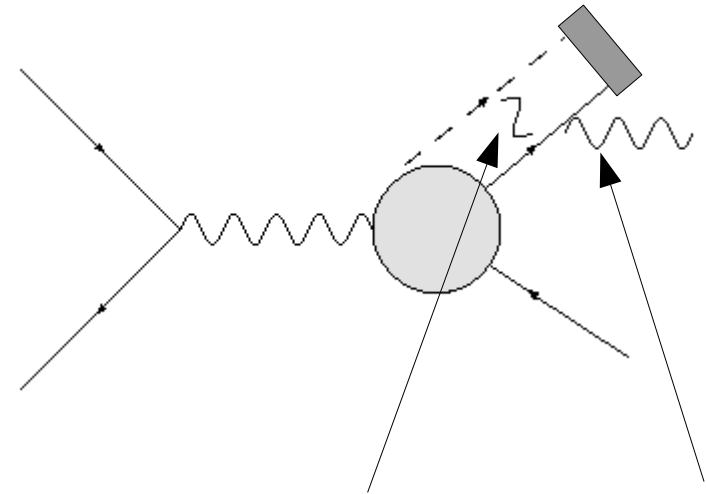


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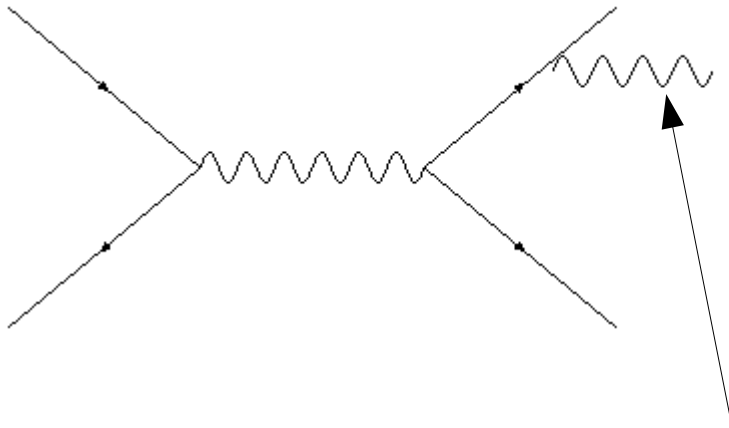
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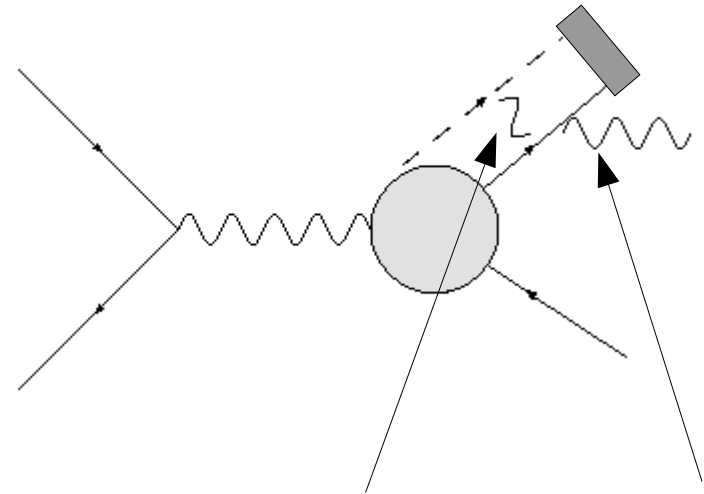


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Cancel this effect:
Less weak jets

New physics

-

Qualitative changes

Beyond the standard model

[Maas'15
Maas, Sondenheimer, Törek'17]

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- Global U(1) custodial (flavor) symmetry

- Acts as (right-)transformation on the scalar field only

$$W_\mu^a \rightarrow W_\mu^a \qquad h \rightarrow \exp(ia) h$$

Spectrum

Gauge-dependent
Vector

Mass
↑
0



'SU(3) → SU(2)'

Spectrum

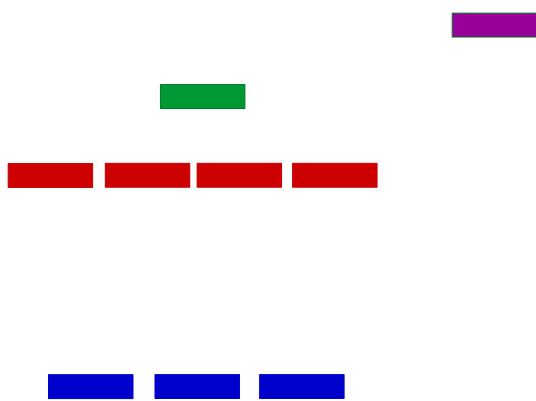
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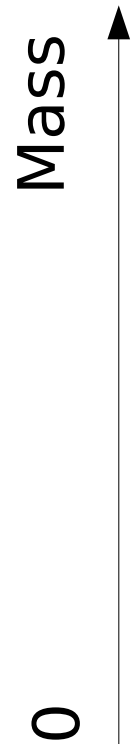


Spectrum

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Confirmed in gauge-fixed
lattice calculations [Maas et al.'16]

Spectrum

[Maas & Törek'16,'18
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Gauge-dependent

Gauge-invariant

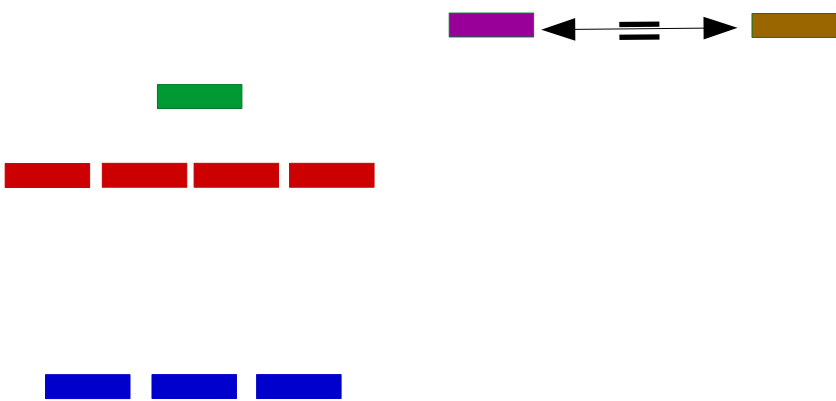
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Scalar

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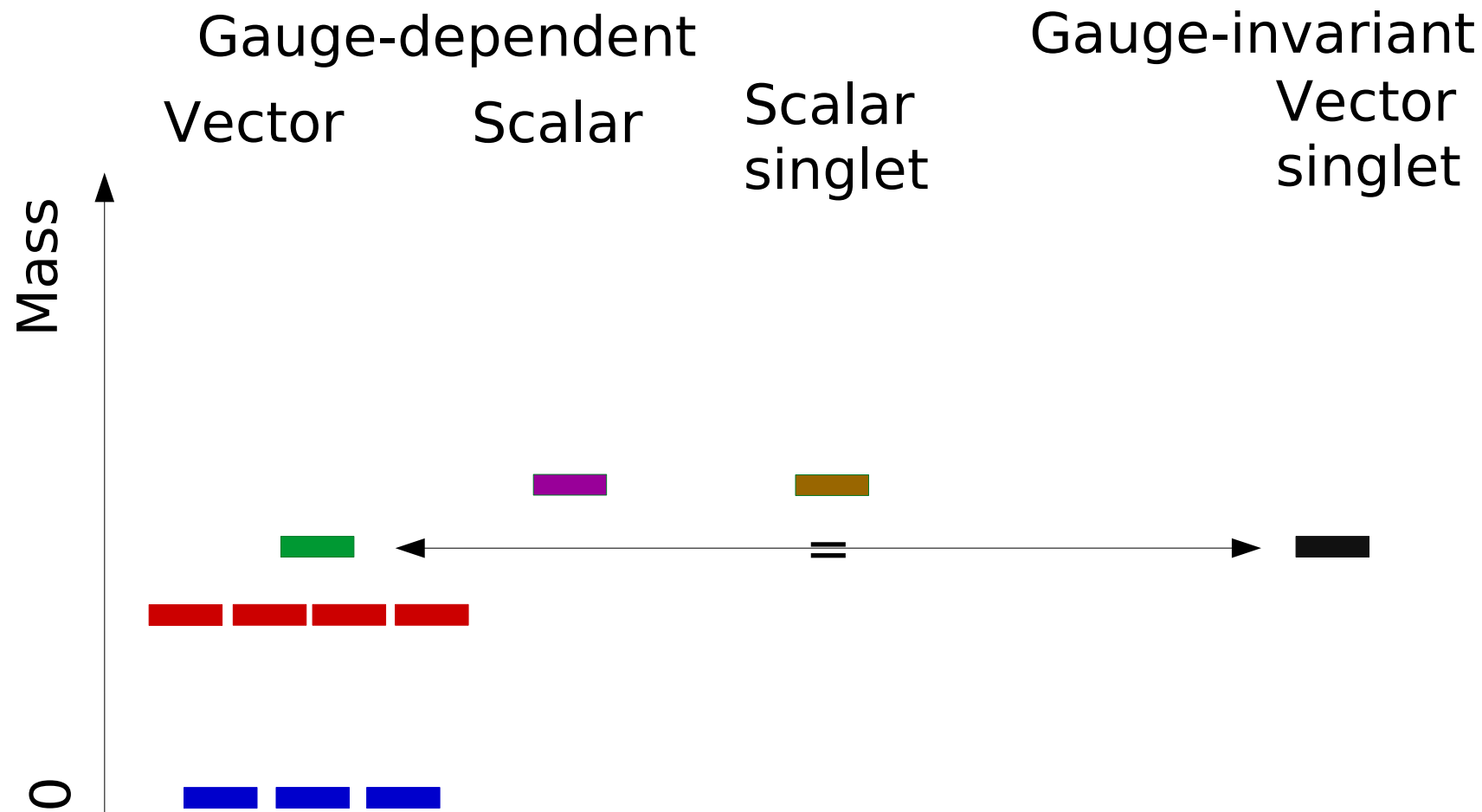
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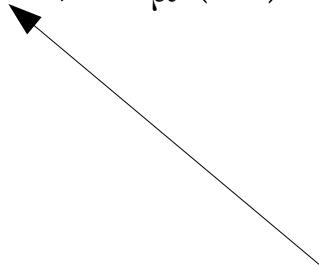
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Only one state remains in the spectrum
at mass of gauge boson 8 (heavy singlet)

Matrix from
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A different kind of states

- Group theory forced same gauge multiplets and custodial multiples for $SU(2)$
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- Now: States without elementary analouge
 - Gauge-invariant states from 3 Higgs fields
 - Baryon analogue – $U(1)$ acts as baryon number
 - Lightest must exist and be absolutely stable

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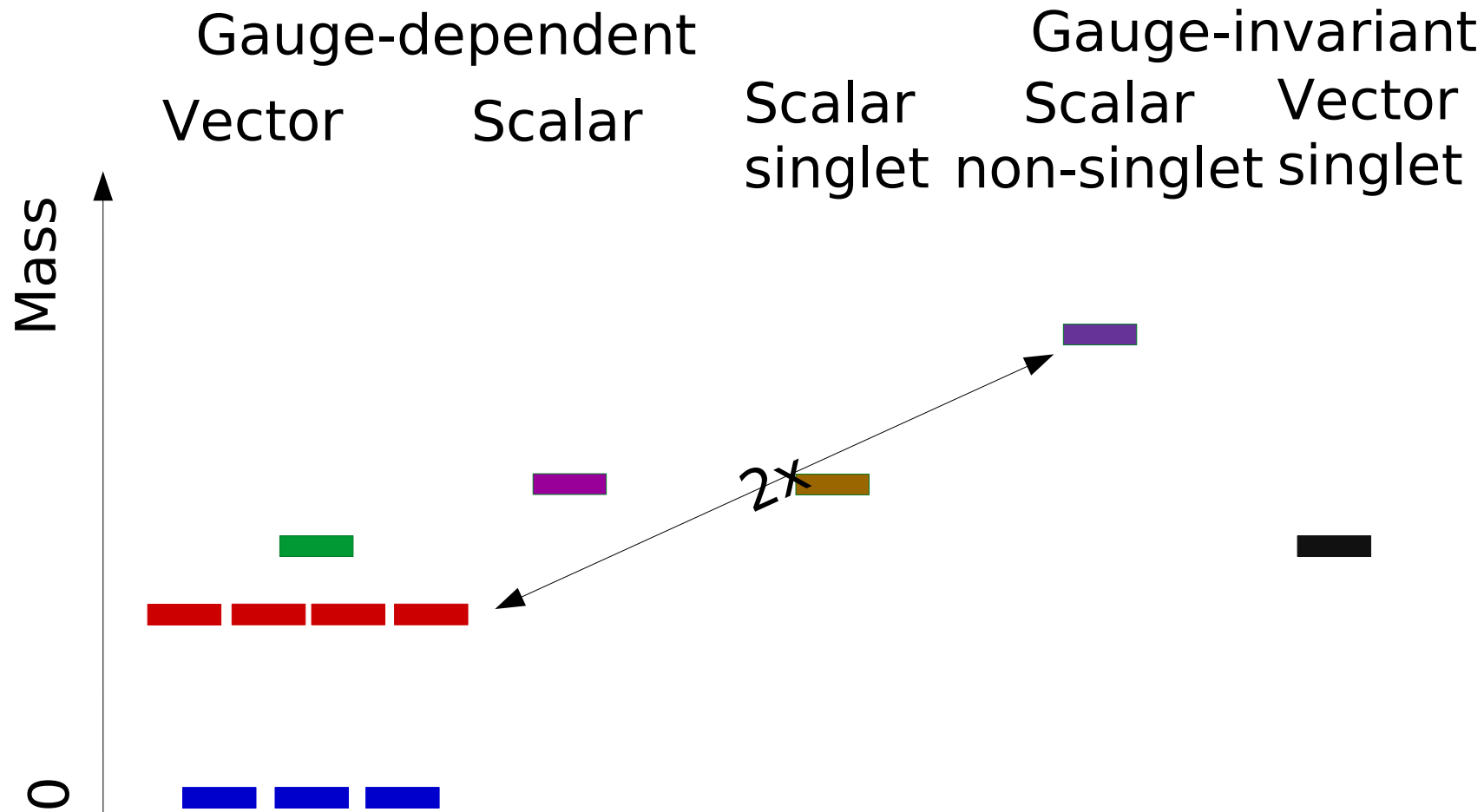
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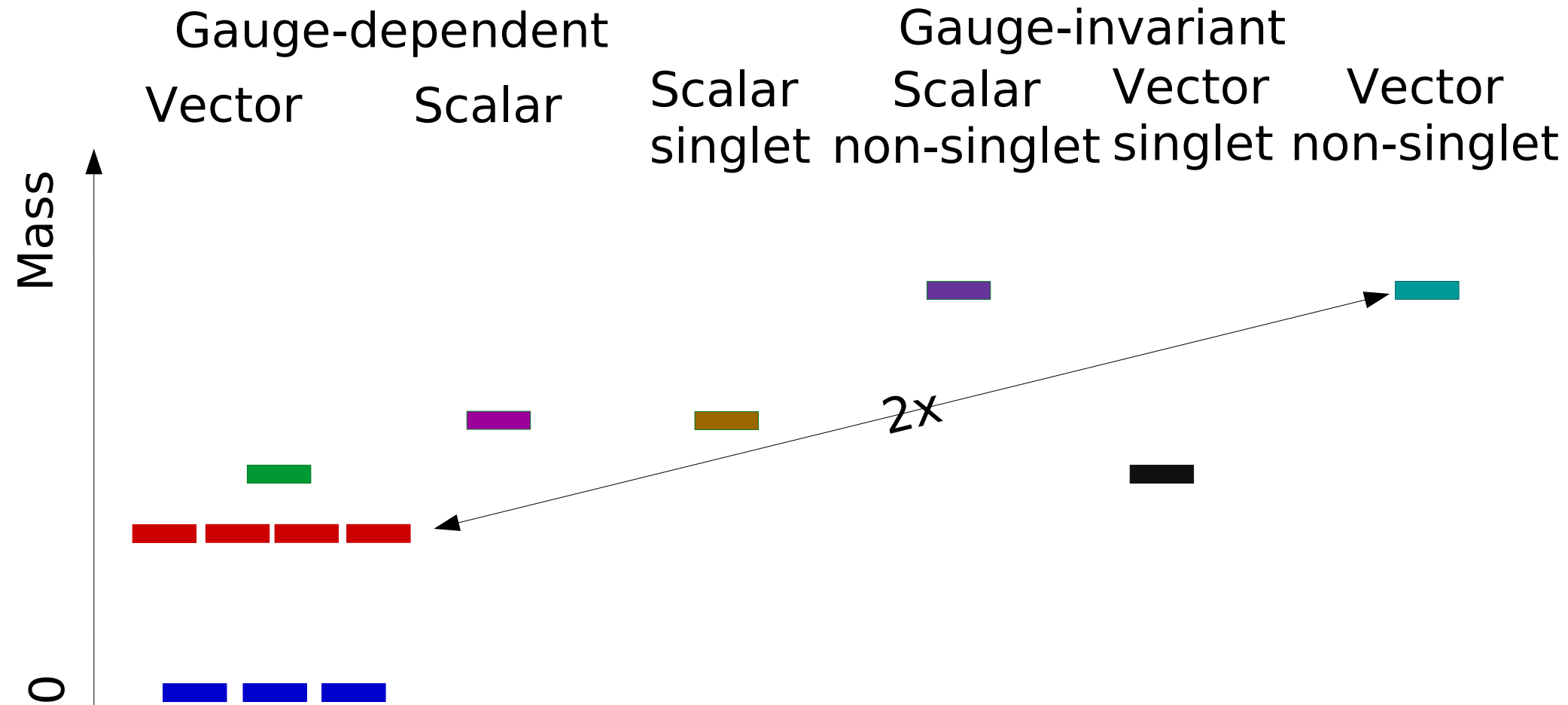
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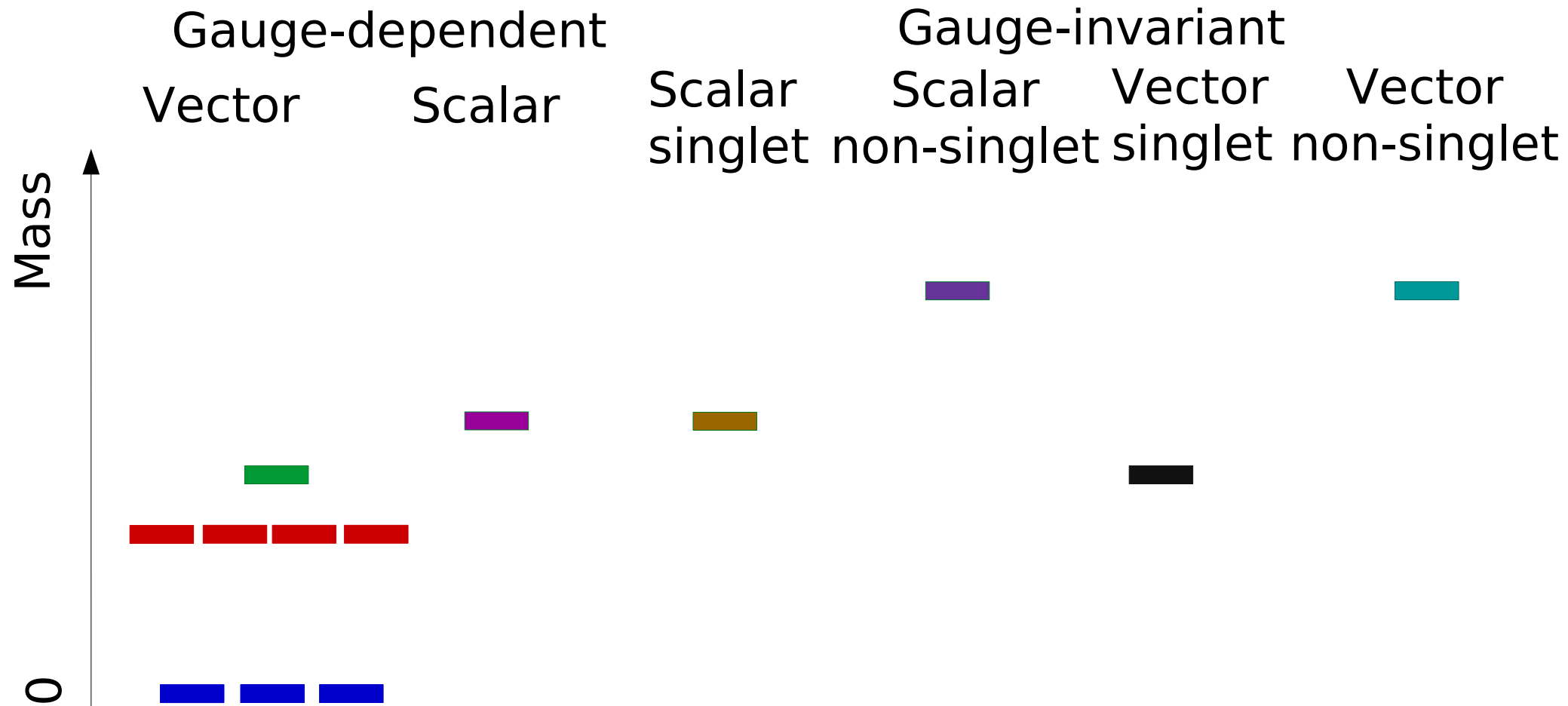
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- Qualitatively different spectrum
- No mass gap!

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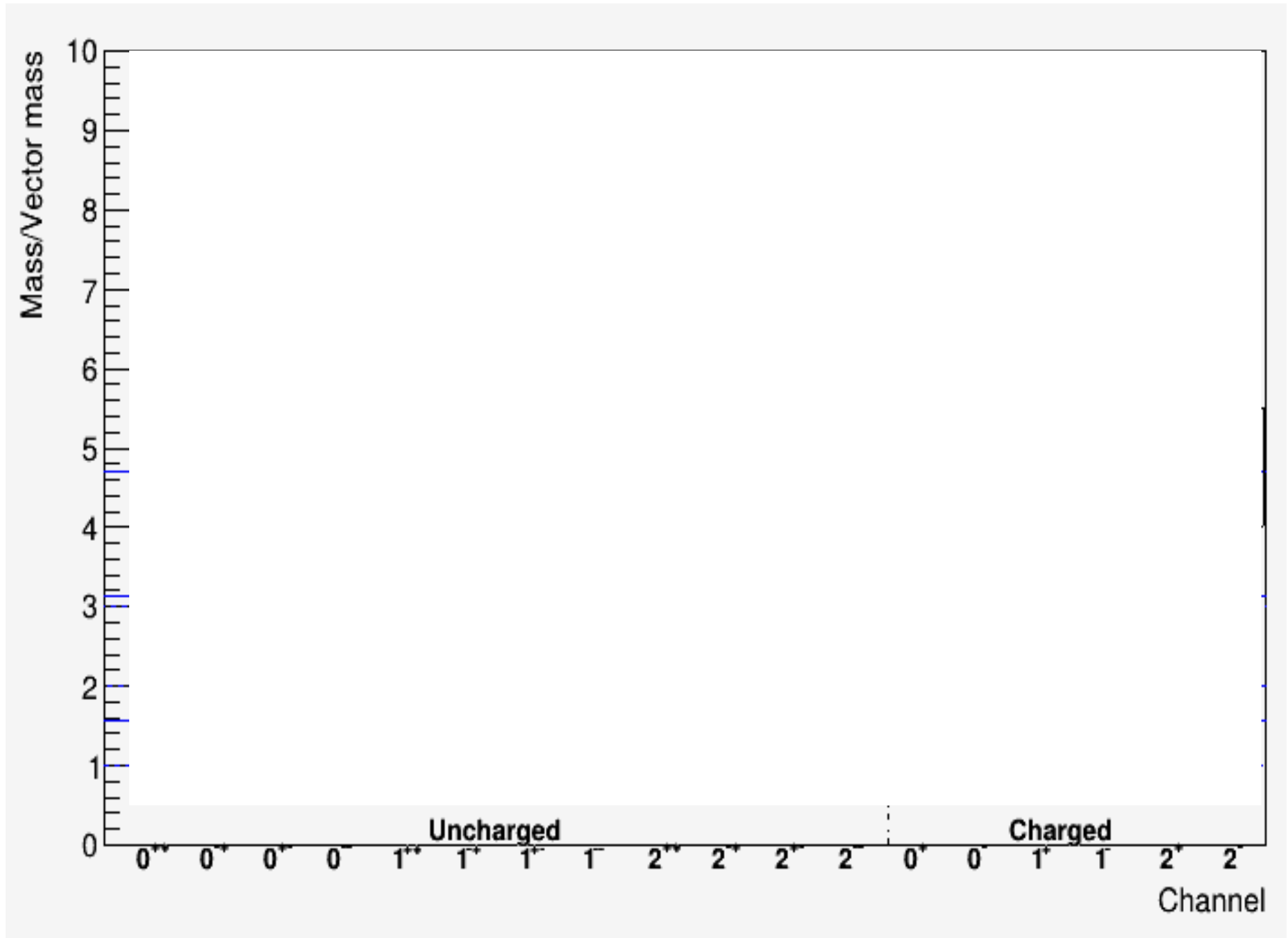
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 - All channels: $J < 3$
 - Aim: Ground state for each channel
 - Characterization through scattering states

Typical spectrum

PRELIMINARY

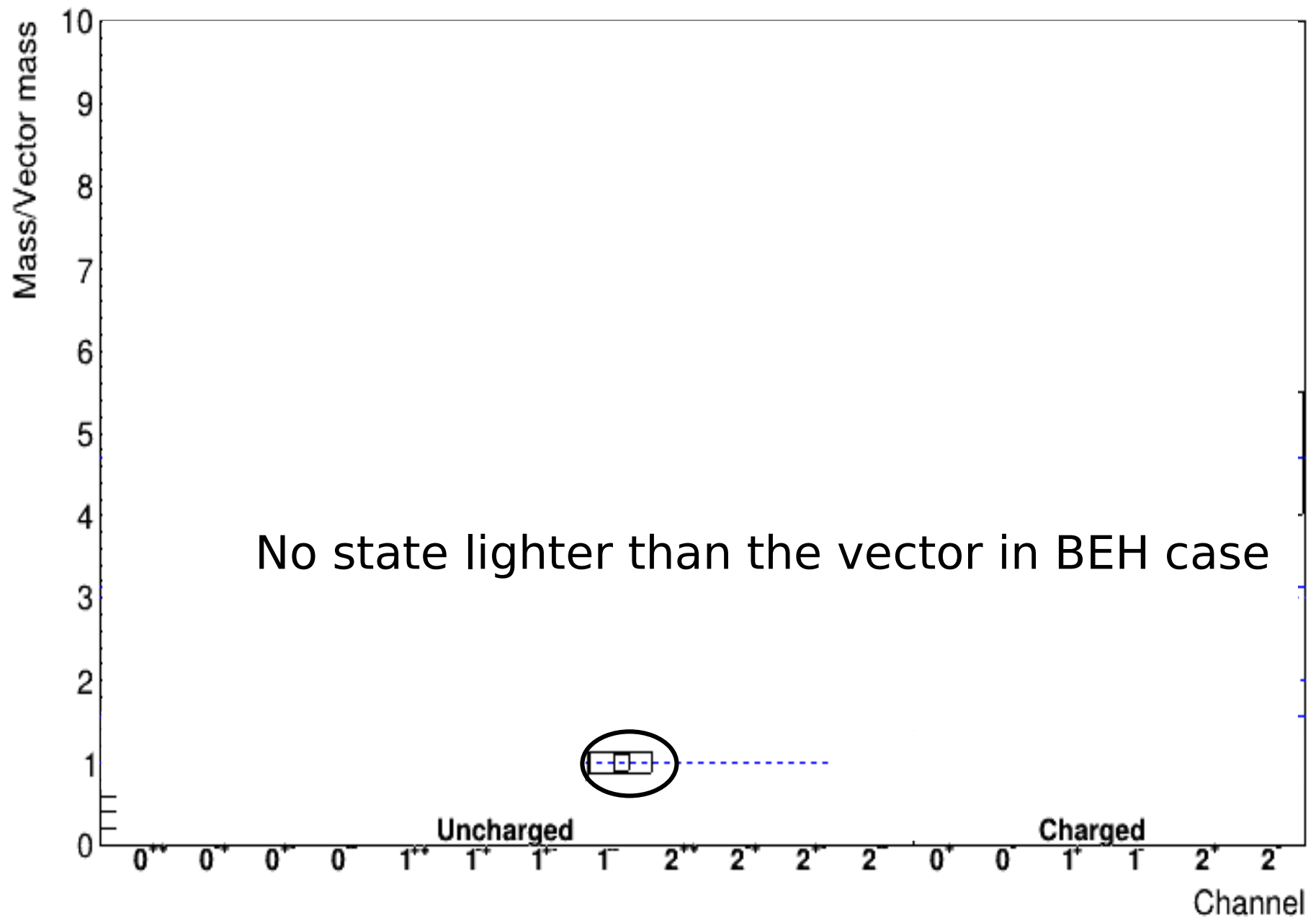
[Dobson et al.'25]



Typical spectrum

PRELIMINARY

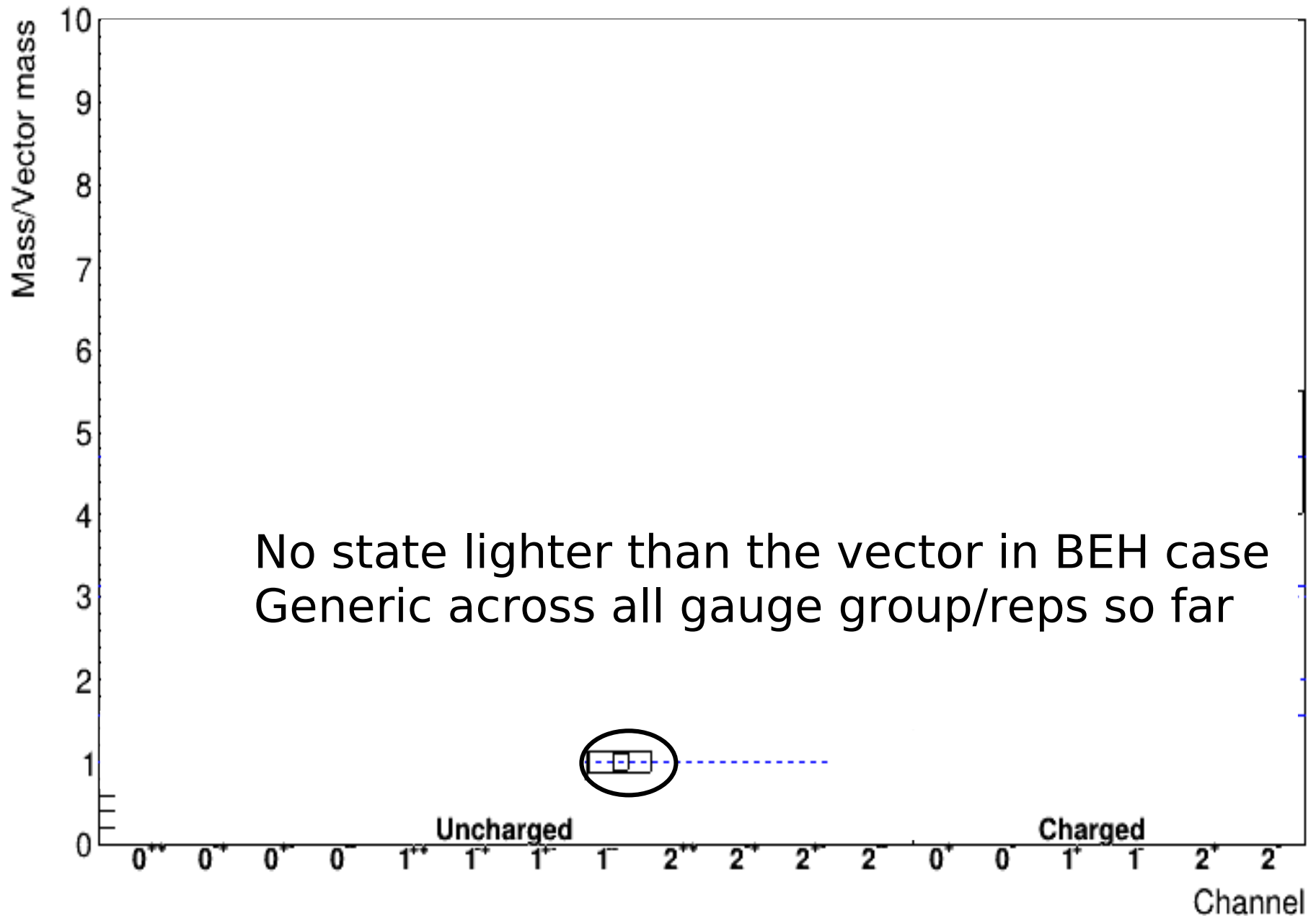
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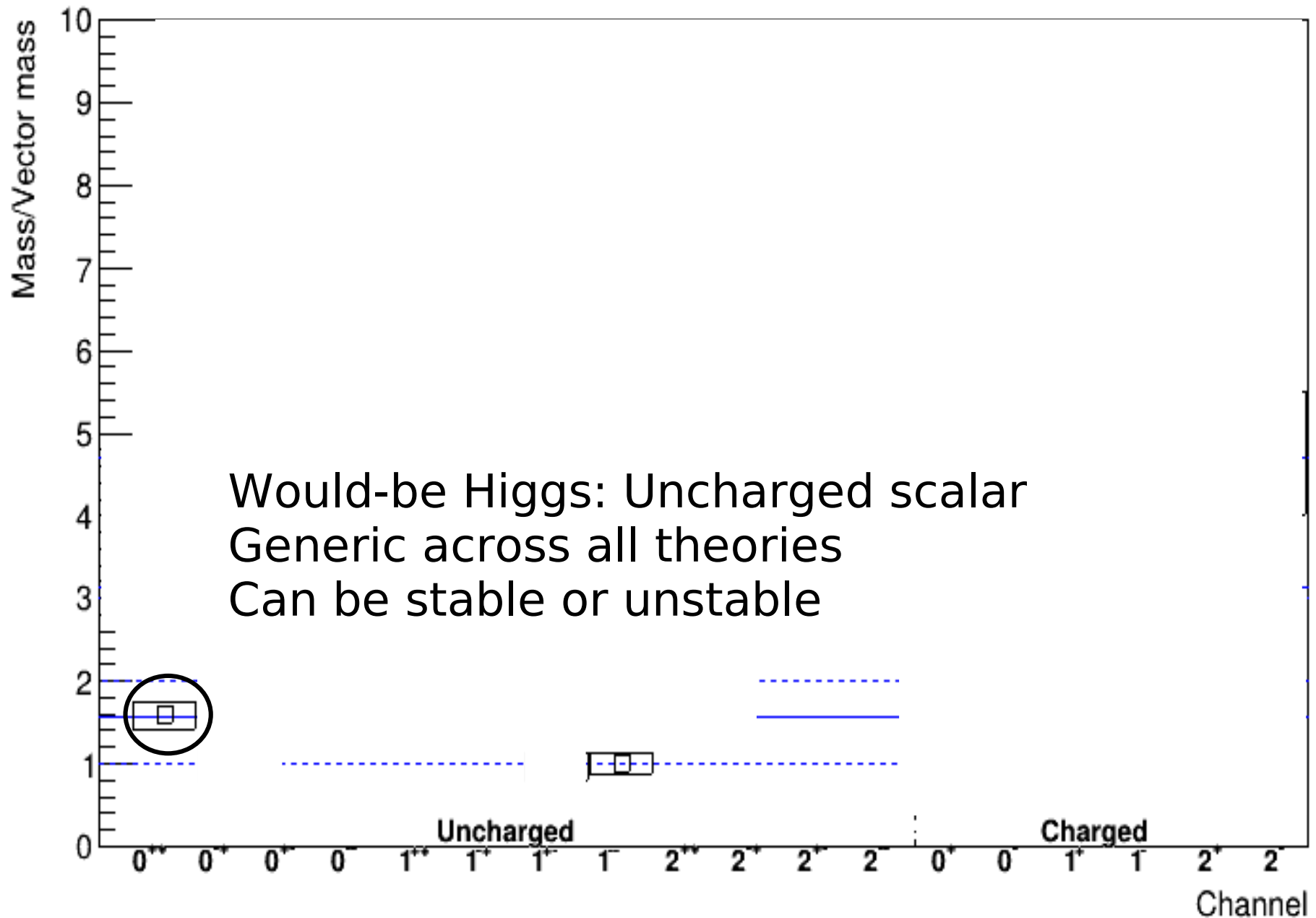
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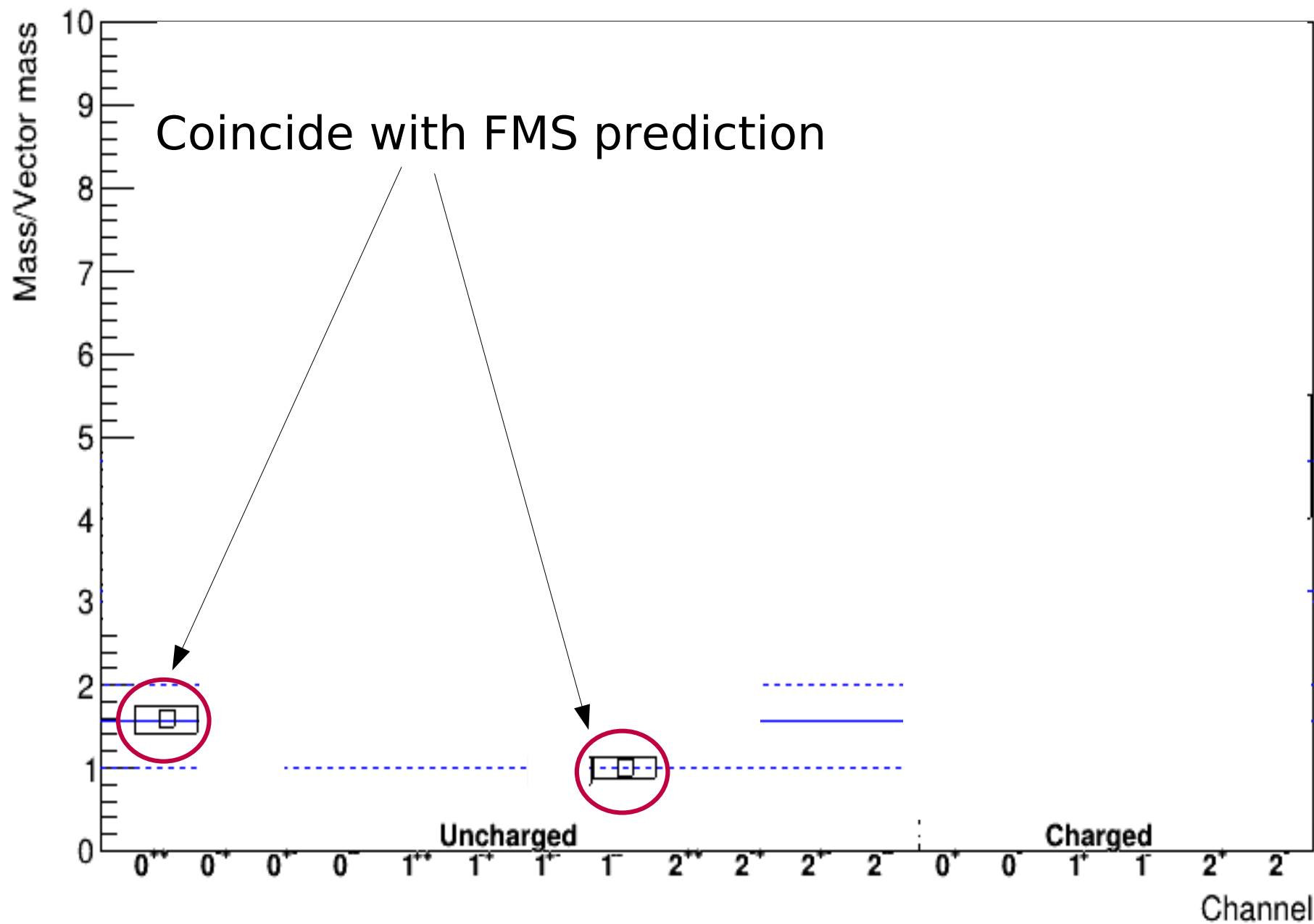
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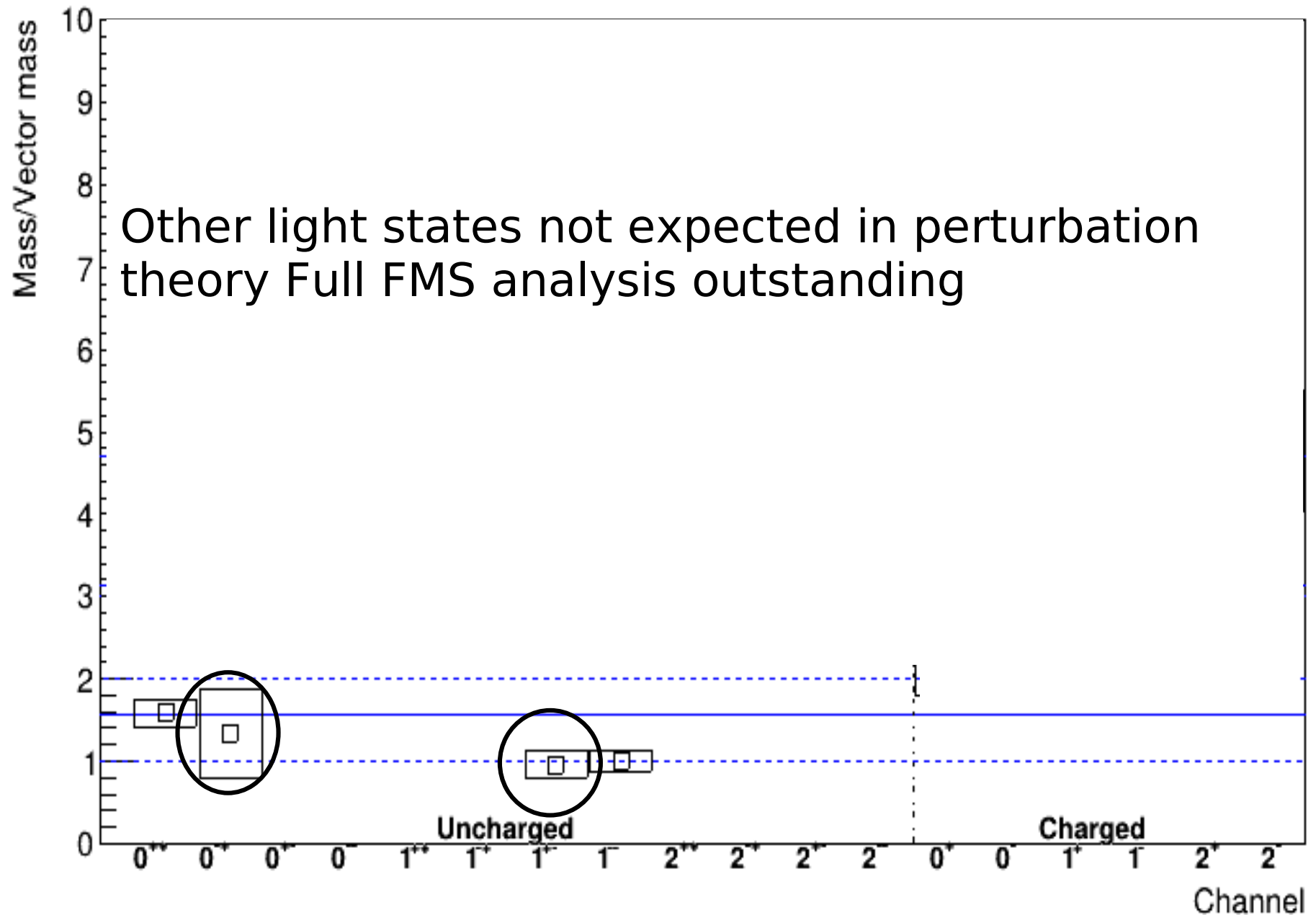
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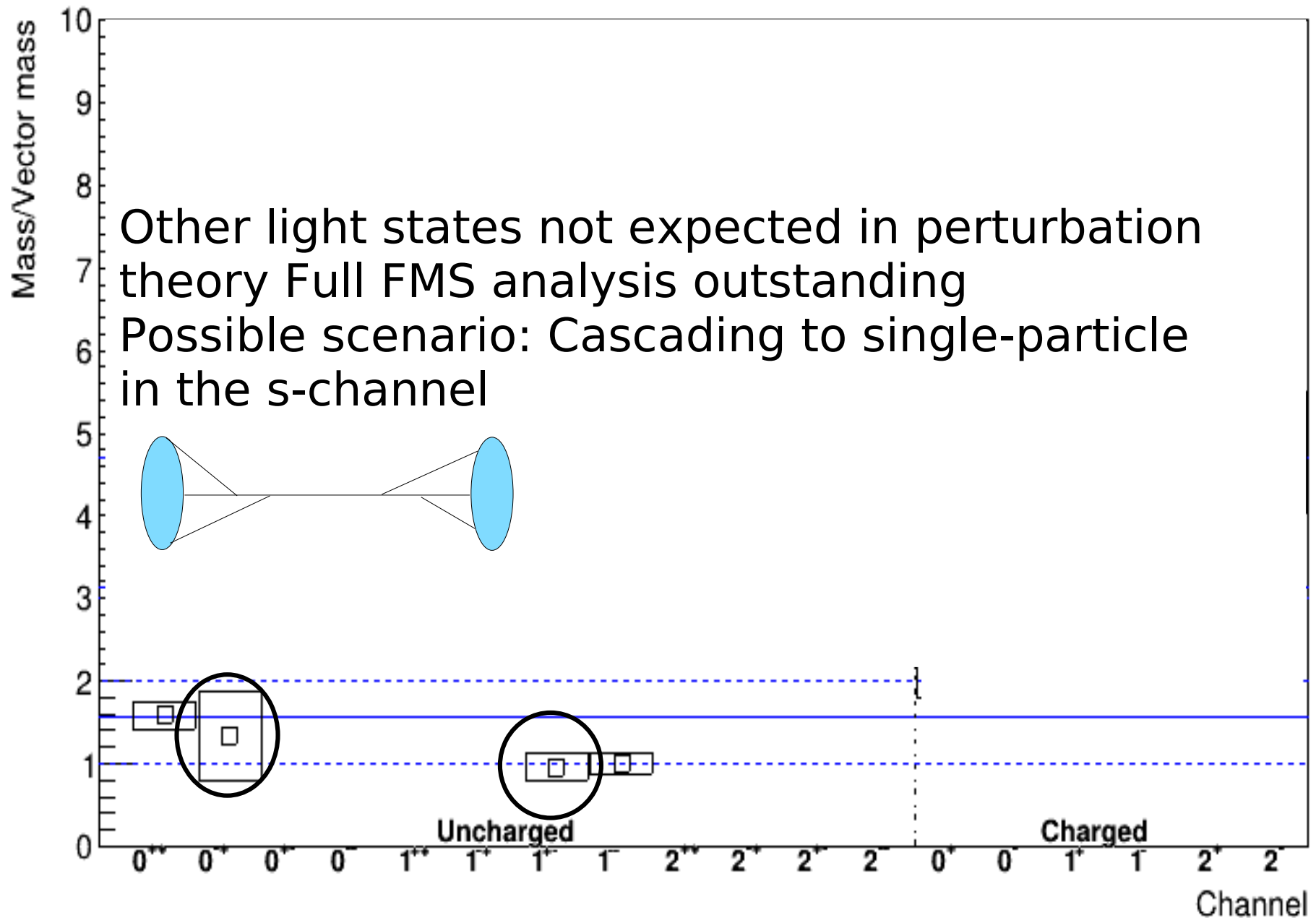
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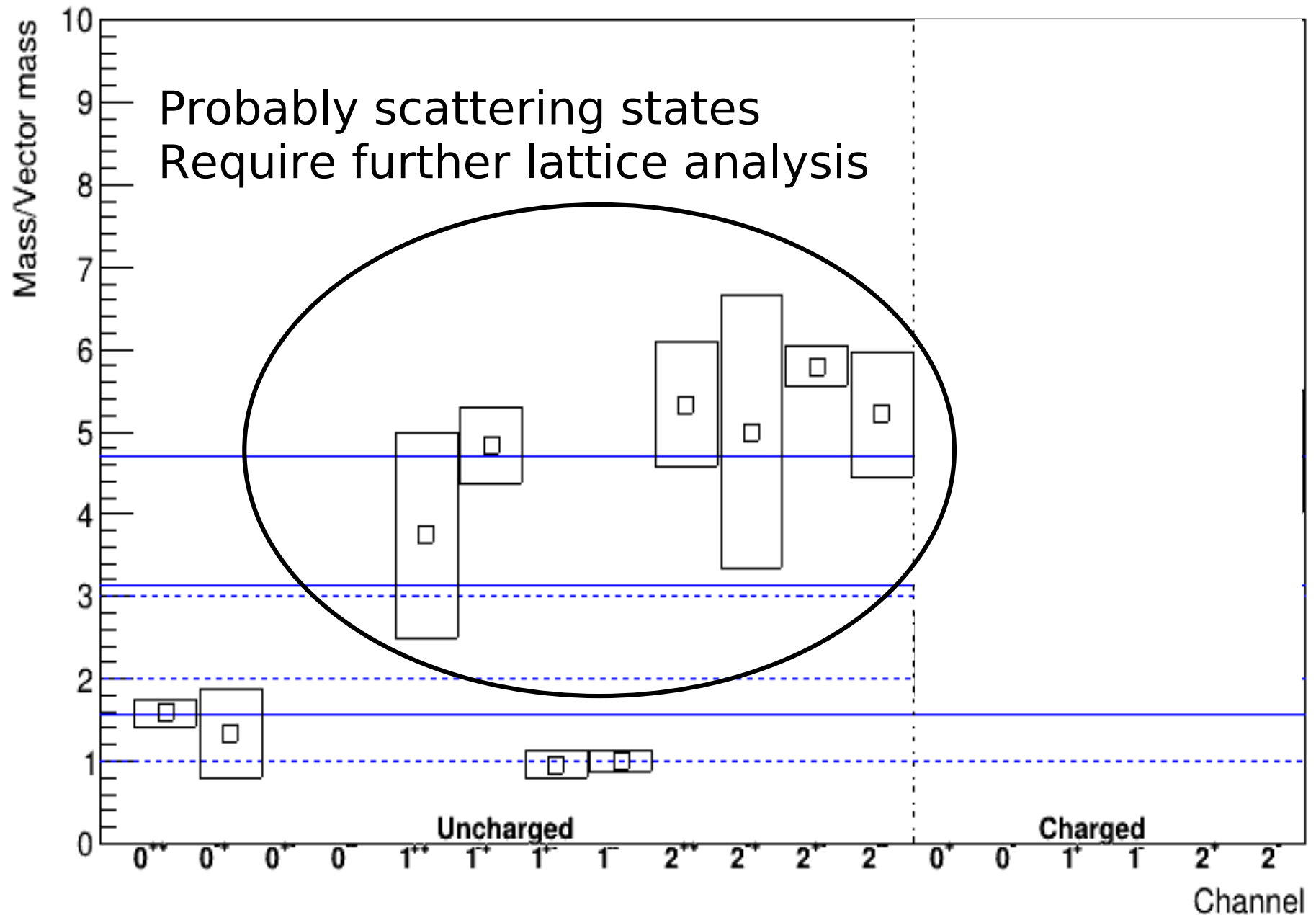
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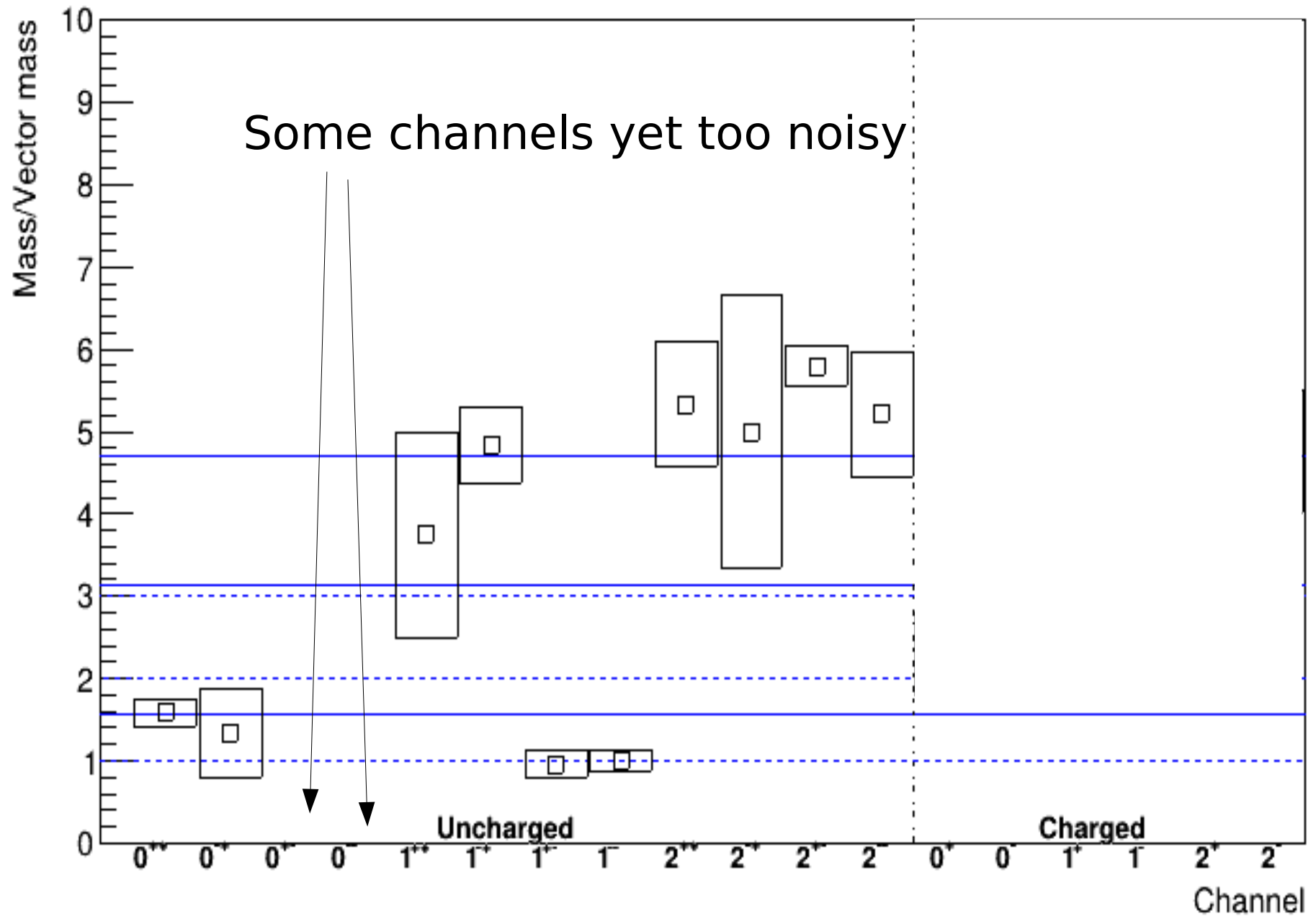
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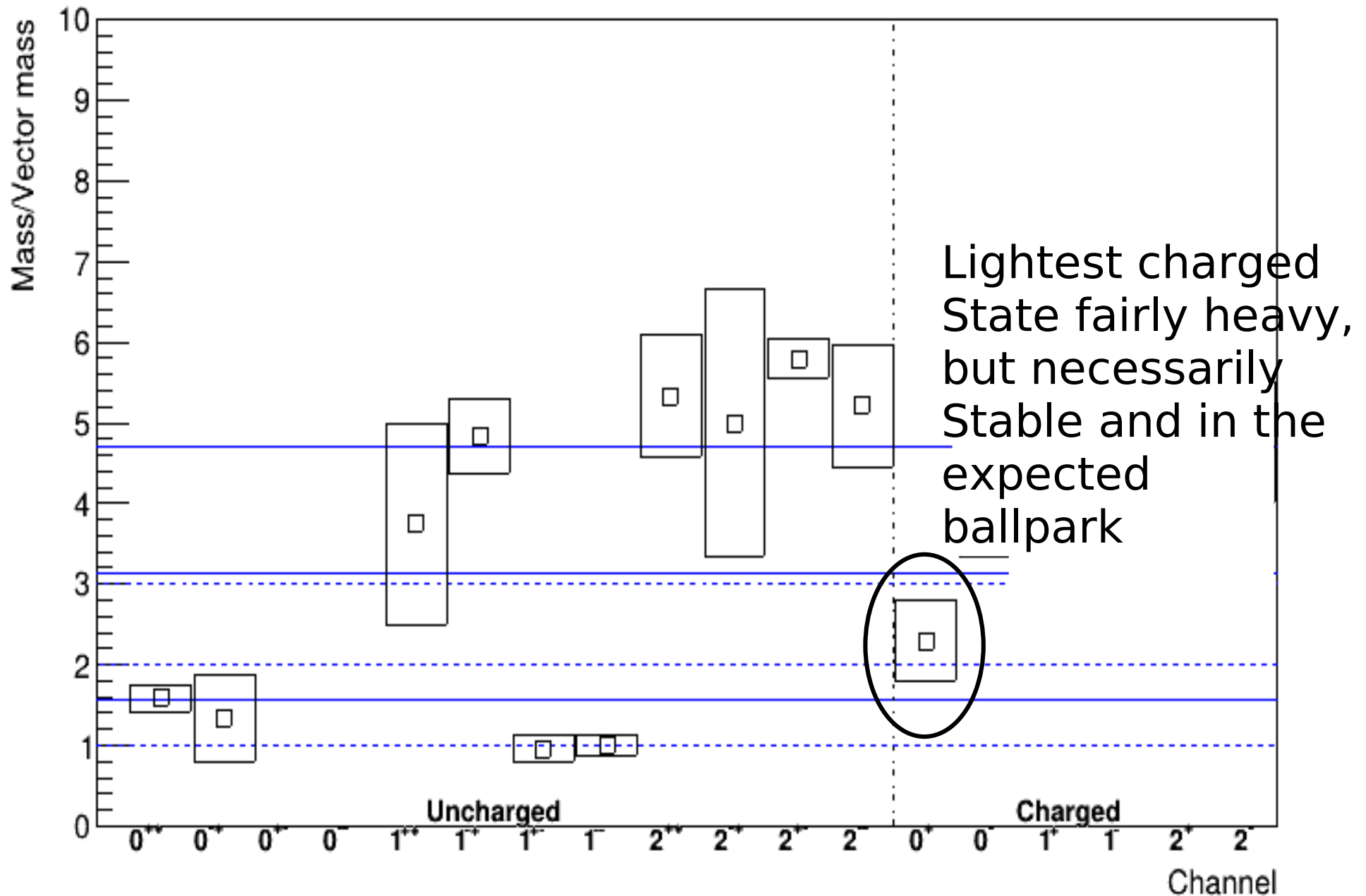
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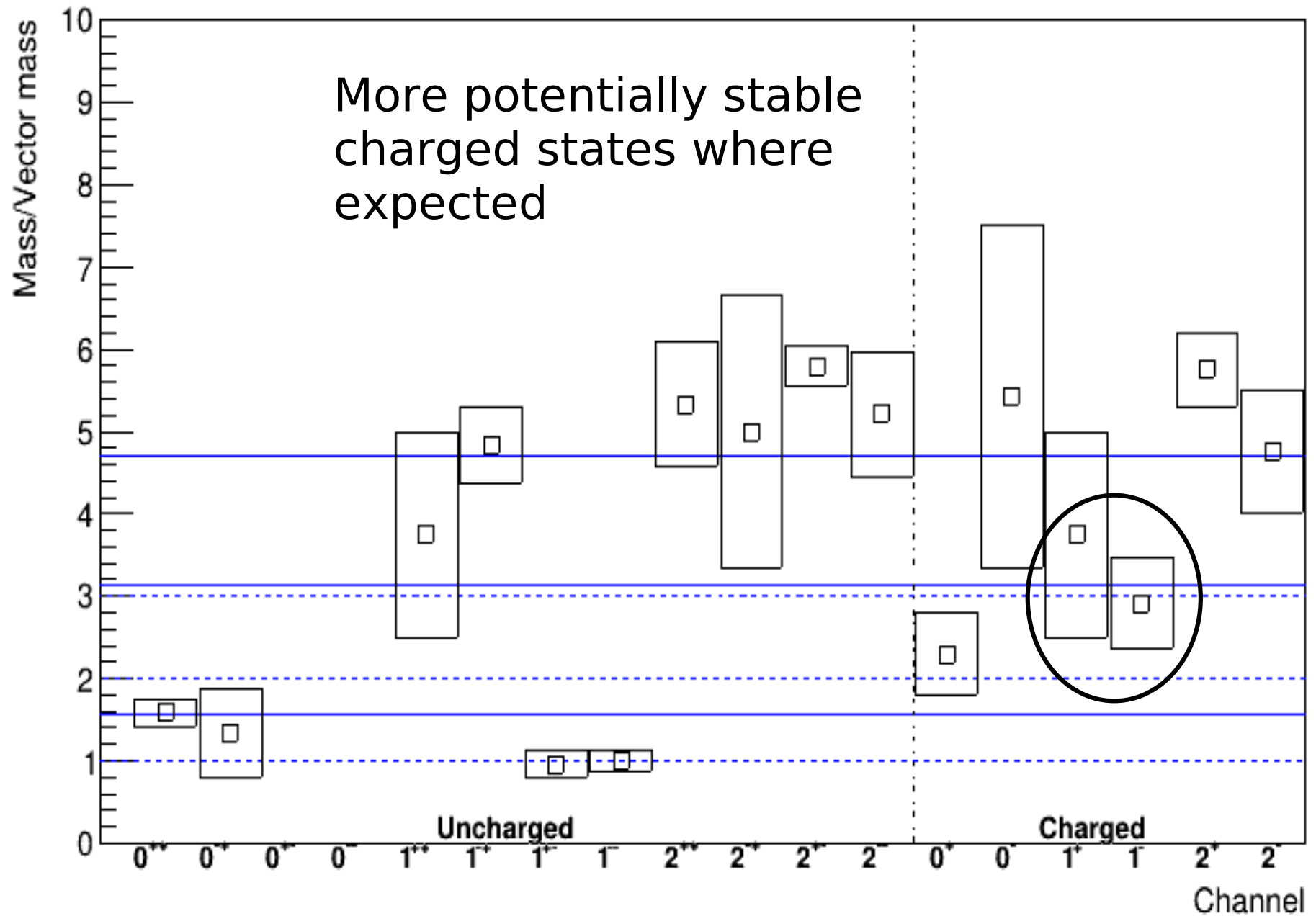
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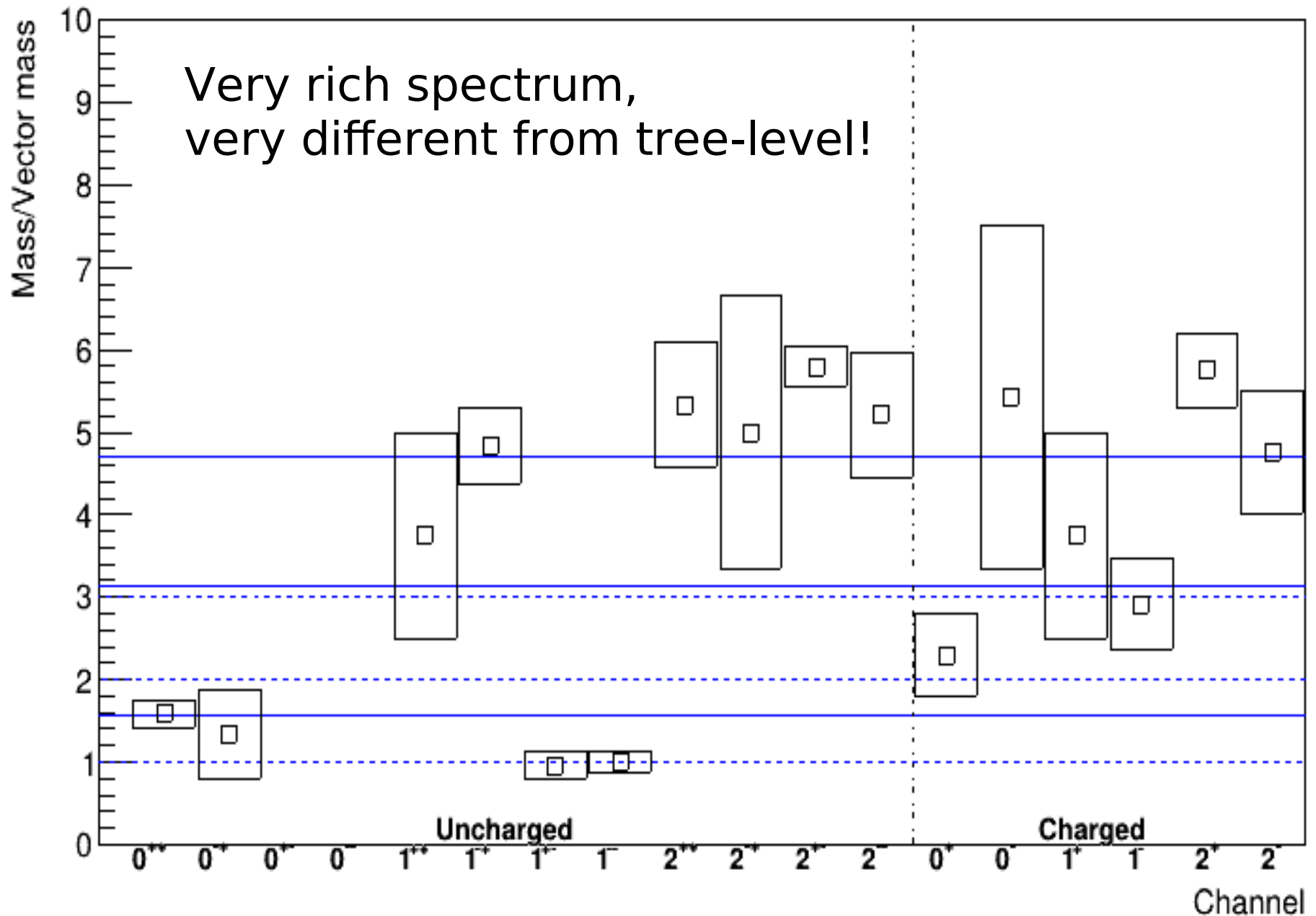
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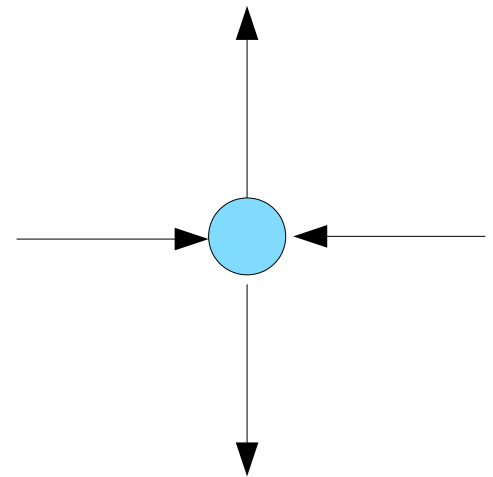
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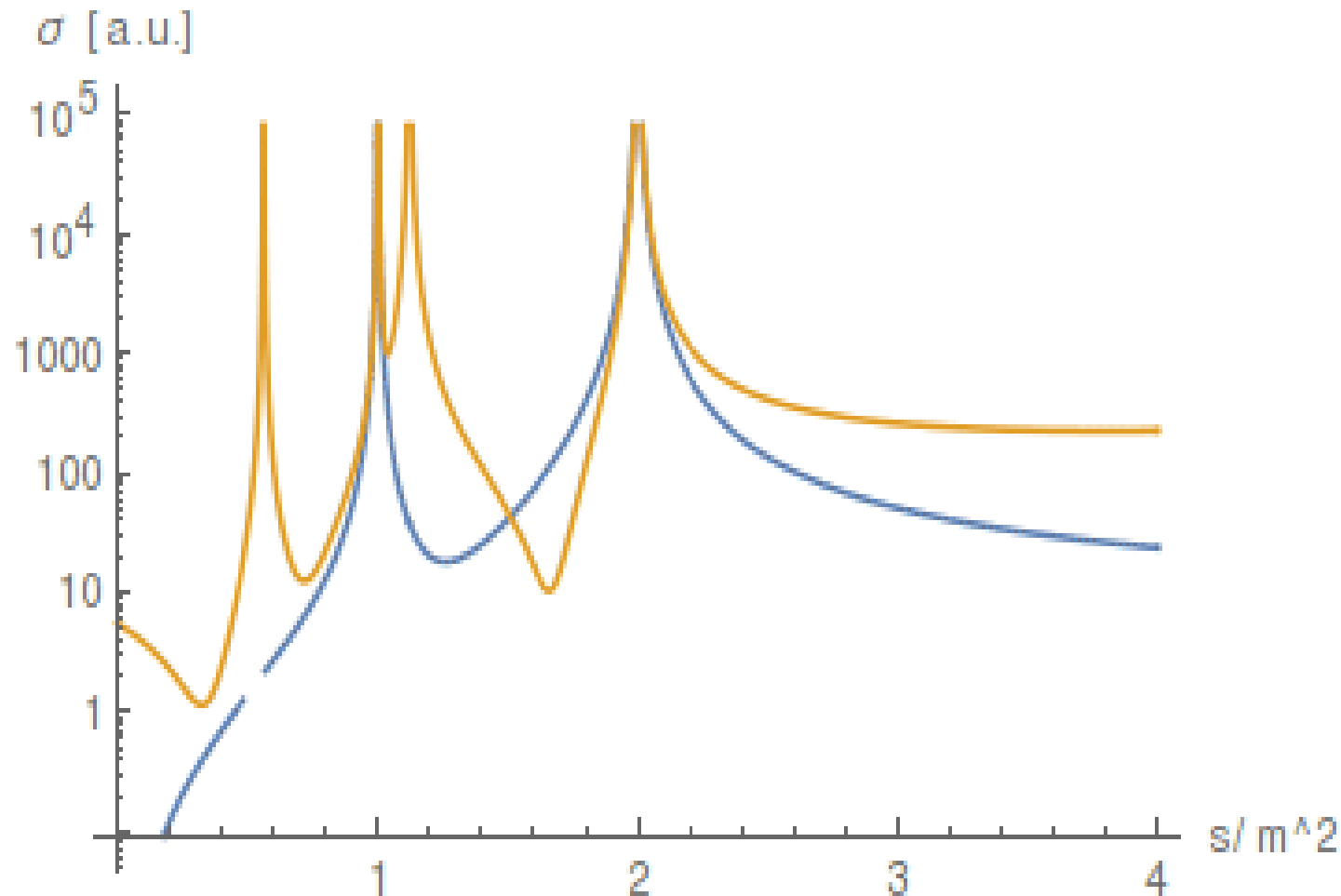
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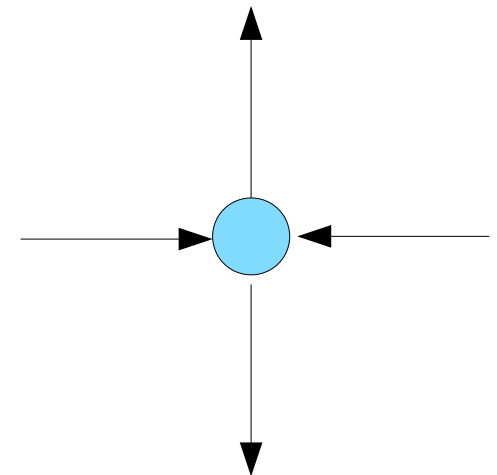


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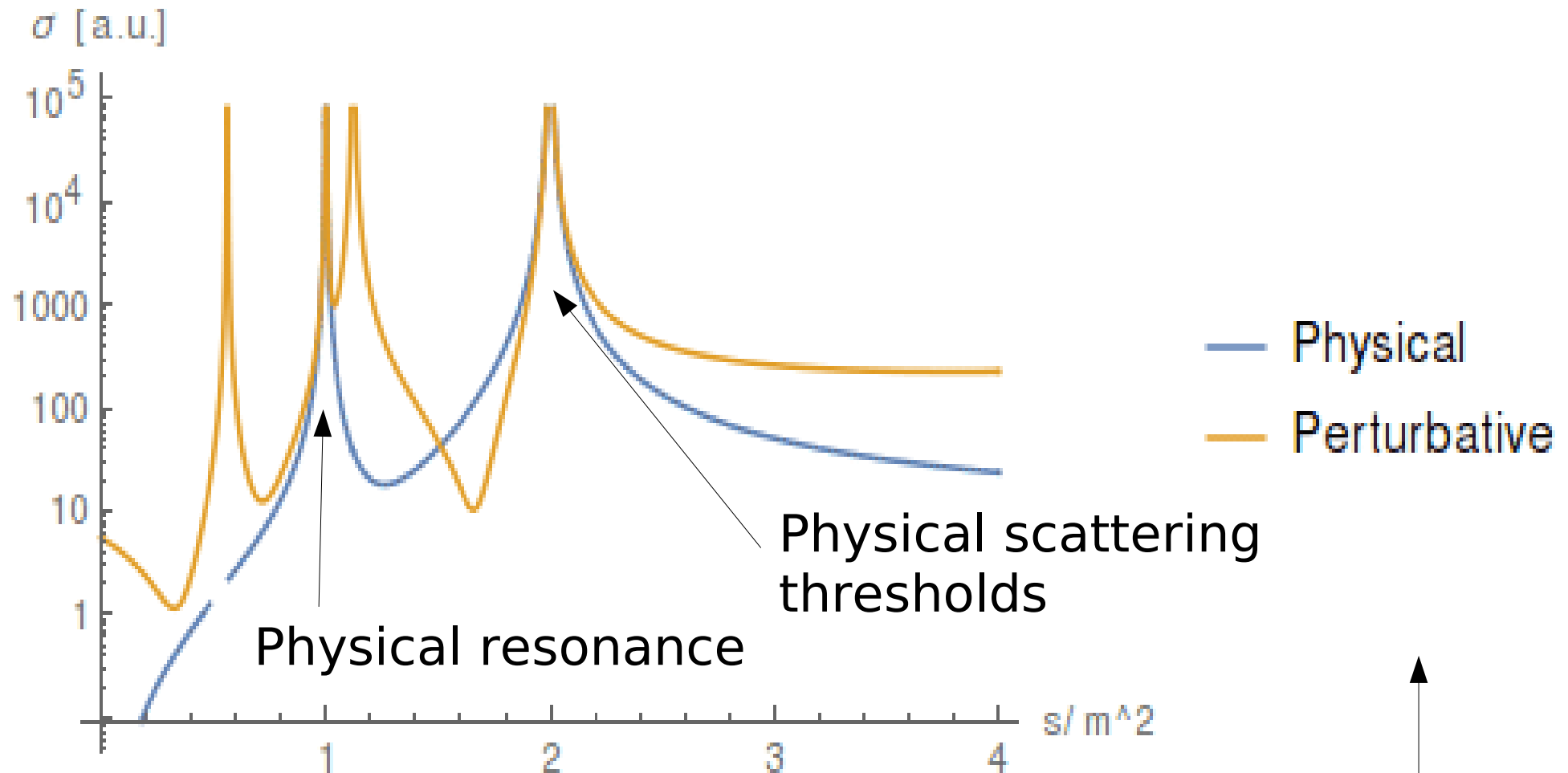


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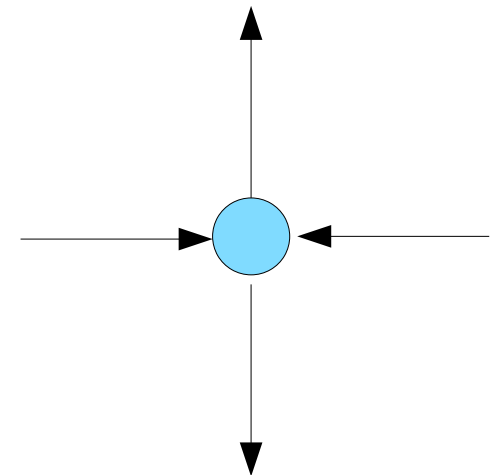


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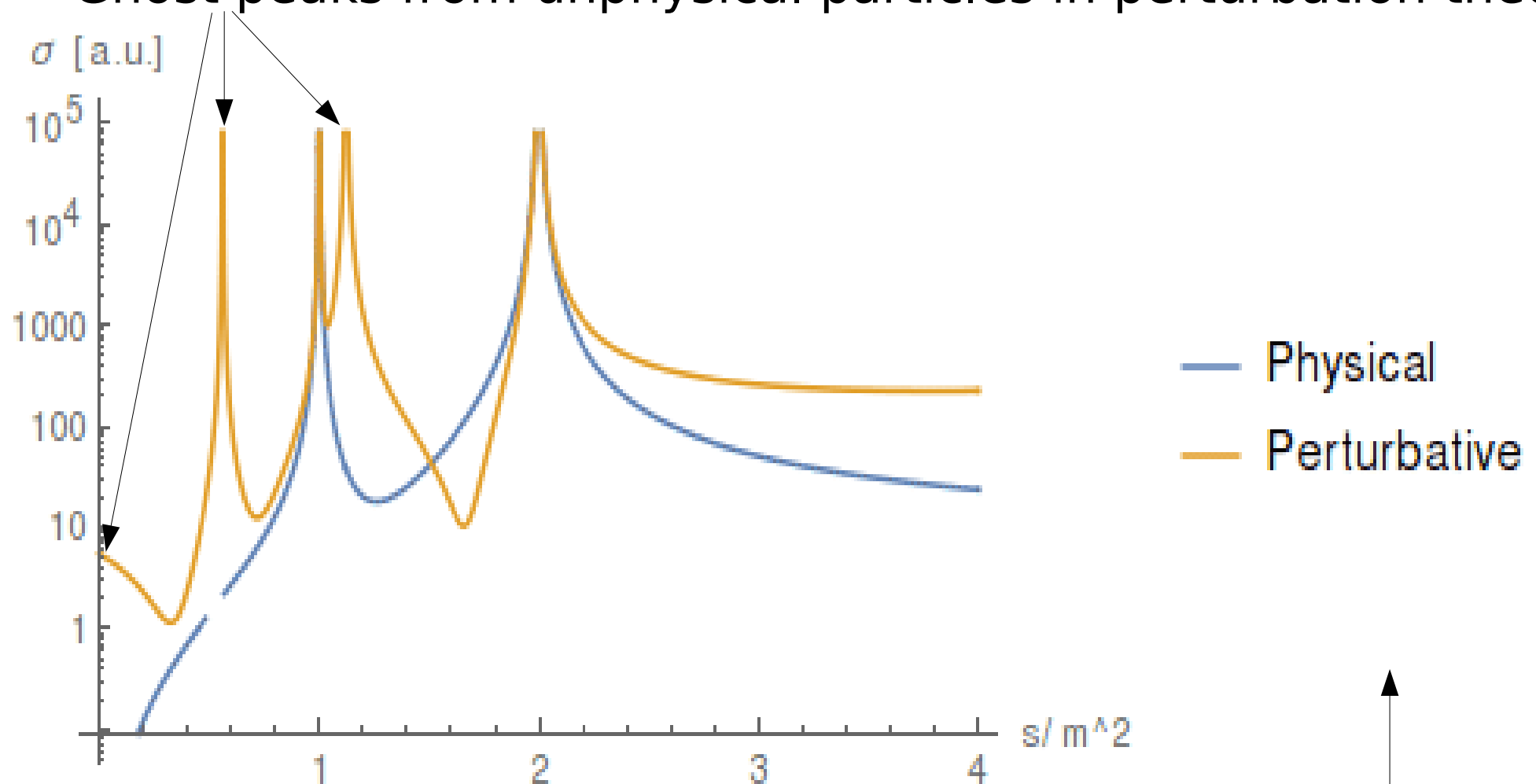
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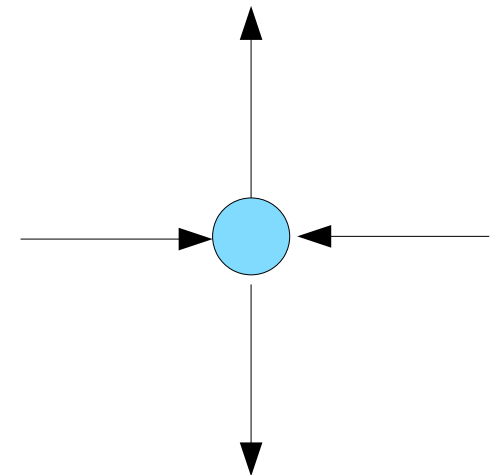
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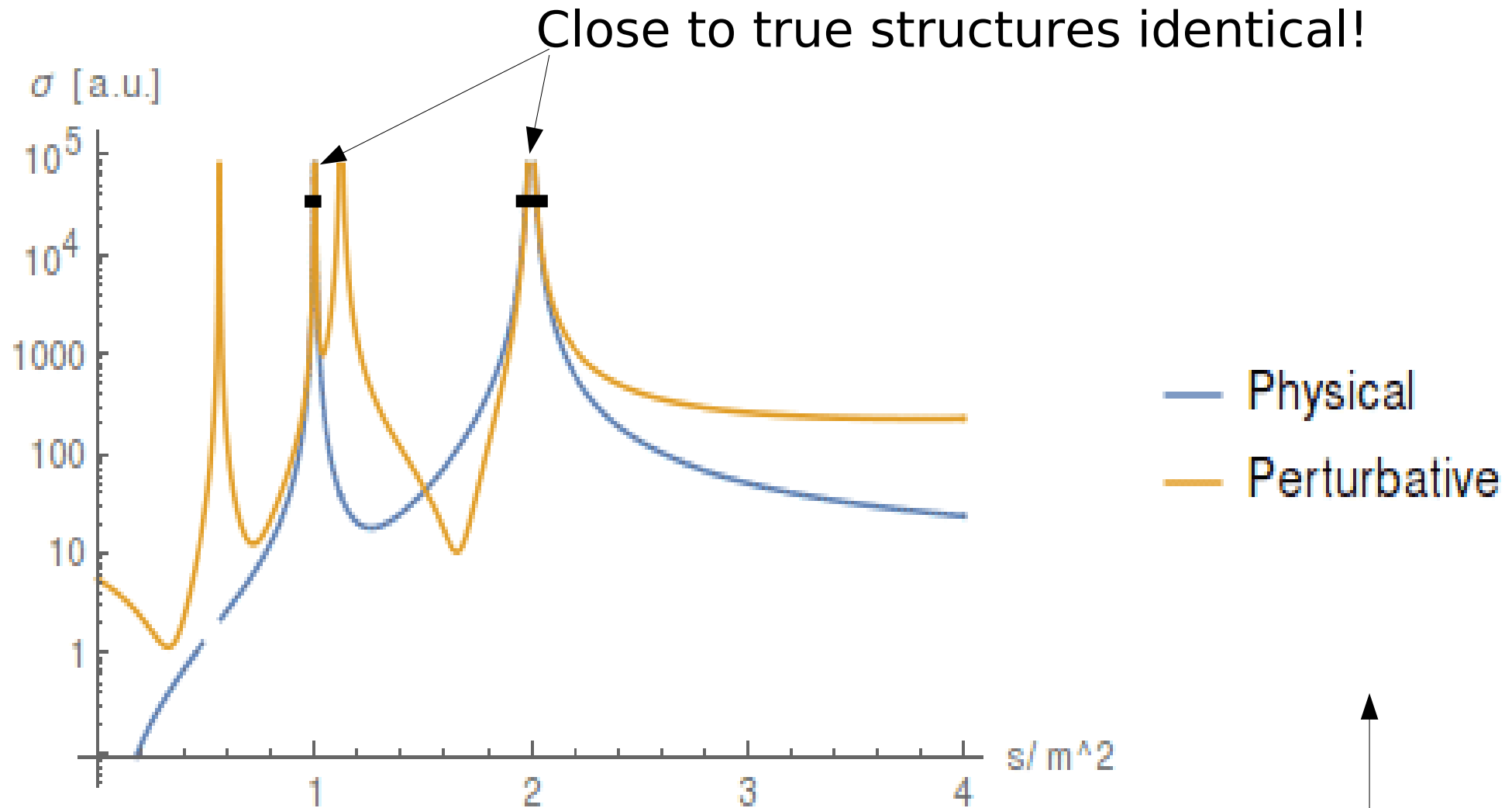


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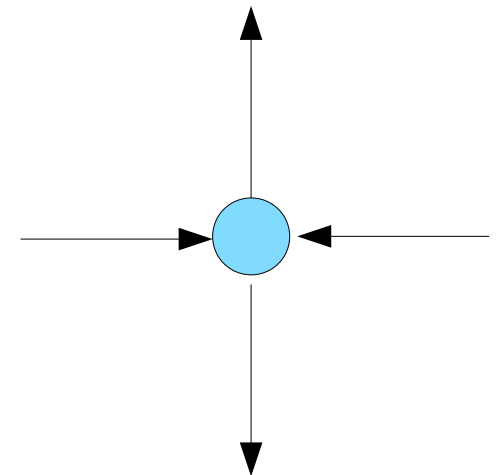


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[Maas'19,
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 - Hints for such states seen in CDT [Maas, Plätzer, Pressler'25]

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