From scales to shapes: theory uncertainties with theory nuisance parameters in the Drell-Yan q_T spectrum

based on Tackmann '24 and[WIP] Cridge, GM, Tackmann

Particle Physics Seminar 29.04.25
Vienna, Austria

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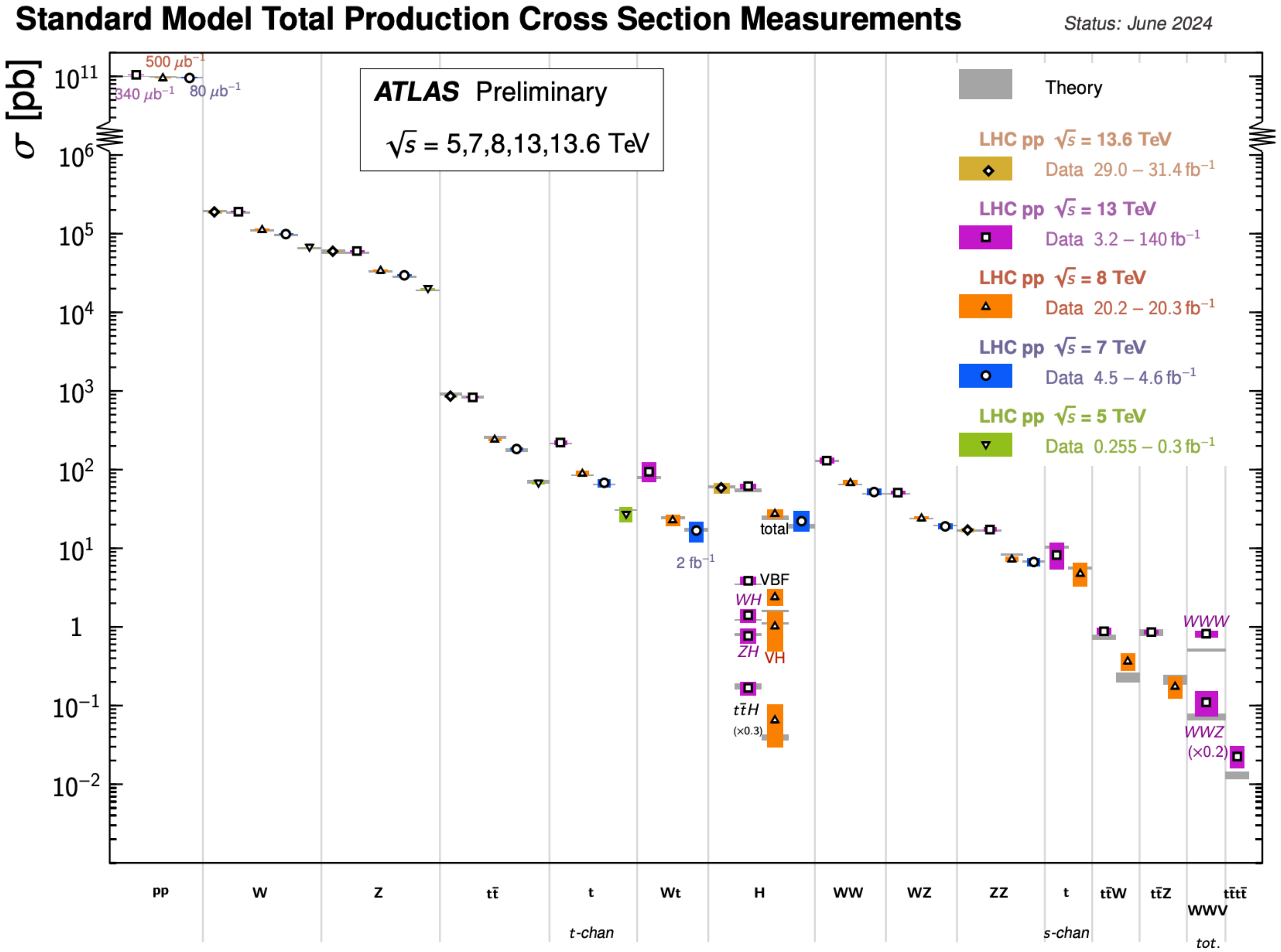
Where are we now?

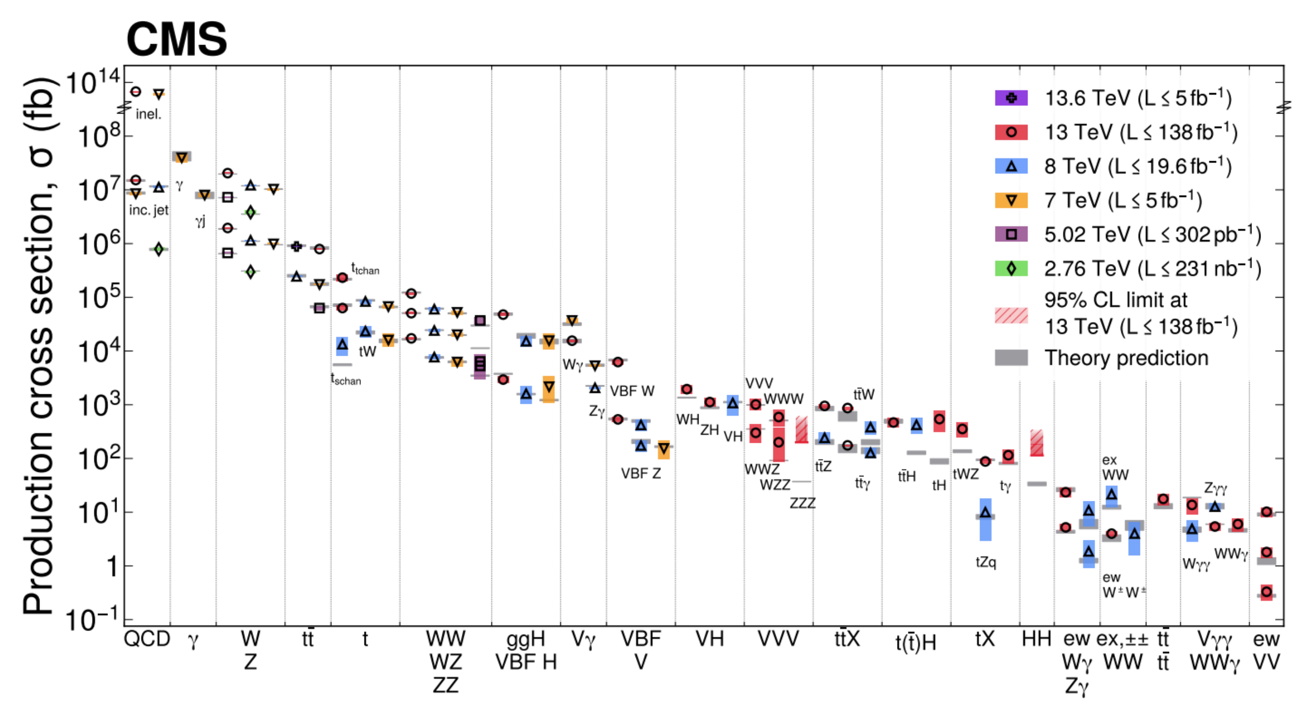
Wonderful measurements of SM processes:



» Good agreement with theory predictions

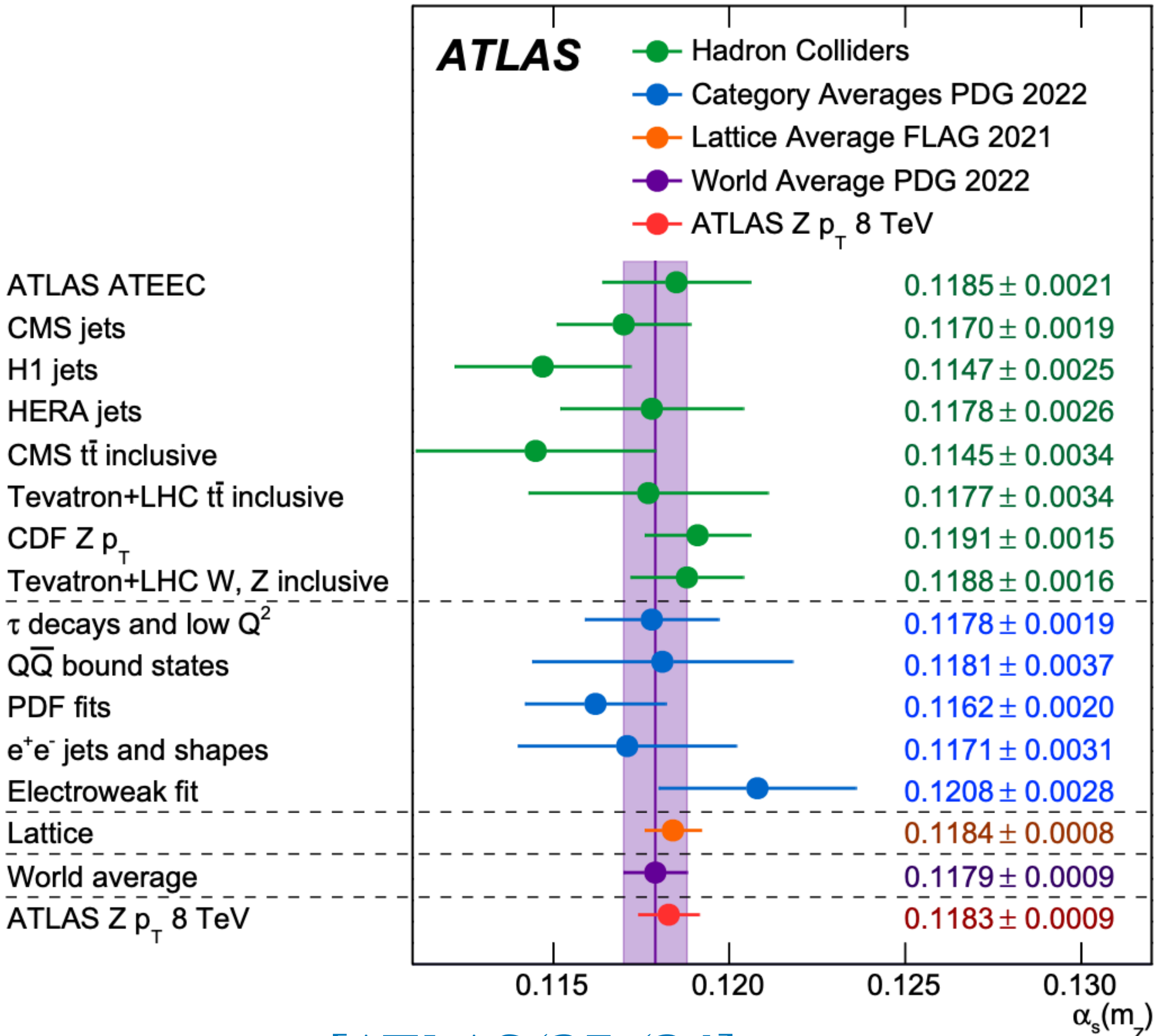
» Rich program for the future
both SM testing and new physics searches

» LHC not only discovery machine, but also **precision machine**!

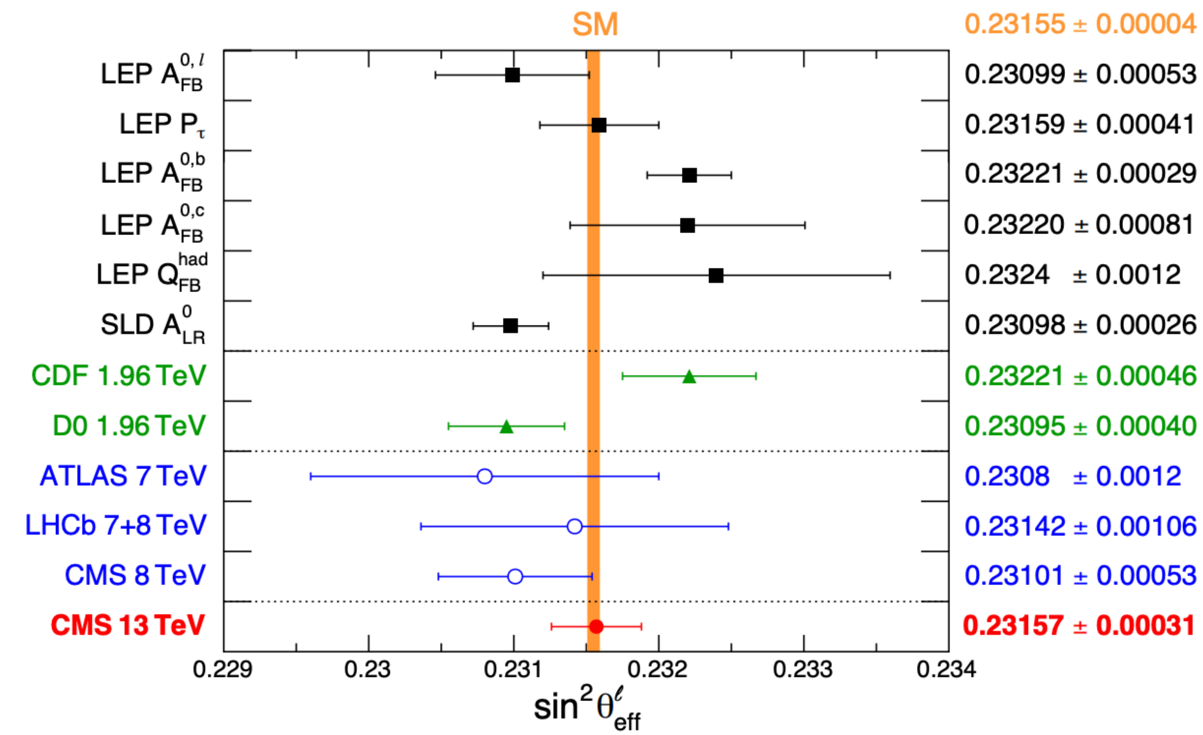


Where are we now?

Drell-Yan production has a special role:



[ATLAS '23, '24]



[CMS '24]

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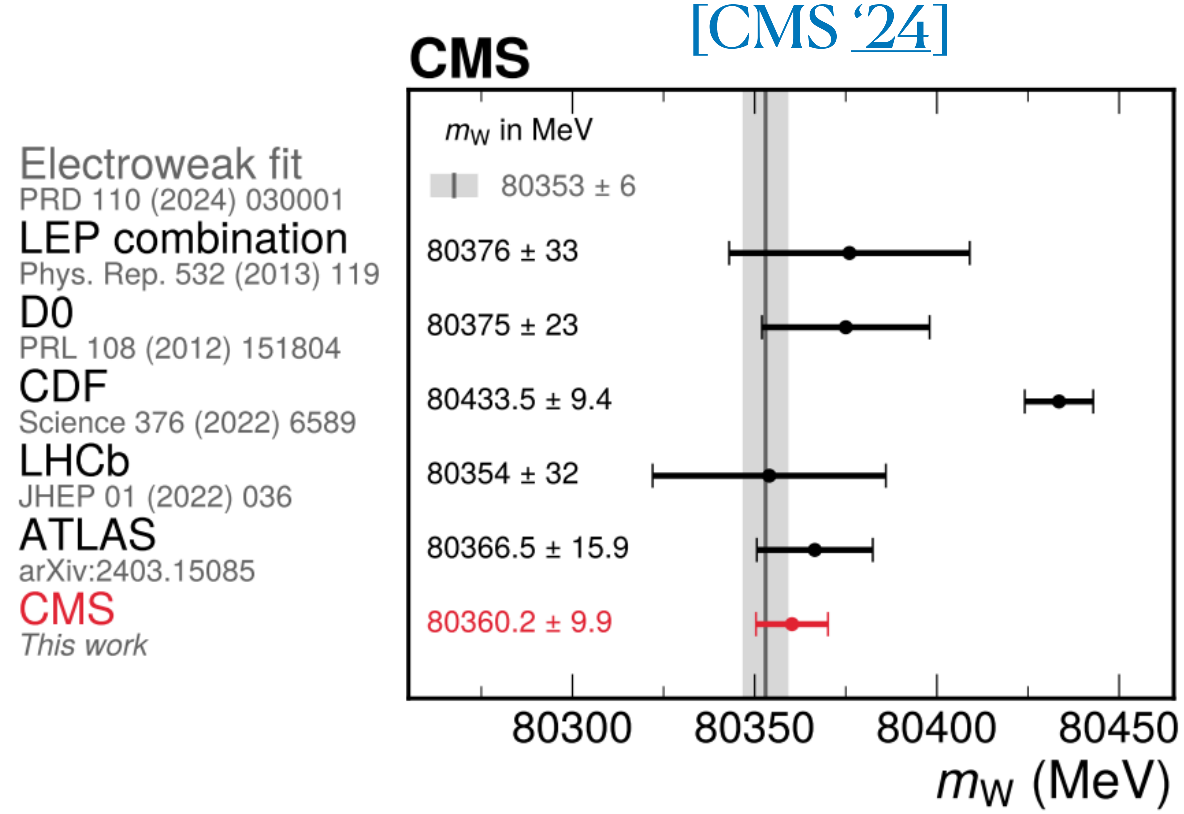

Color singlet q_T spectrum crucial observable

Drell-Yan q_T spectrum

- Wide-ranging applications, many precise measurements:

  ATLAS '20, ATLAS '24, CMS '17, CMS '19, LHCb '16, ...

  - determination of the strong coupling α_s
  - W mass measurement
  - weak mixing angle
  - determination of PDFs at full N^{3}LO
  - Higgs Yukawa couplings constraining to light quarks

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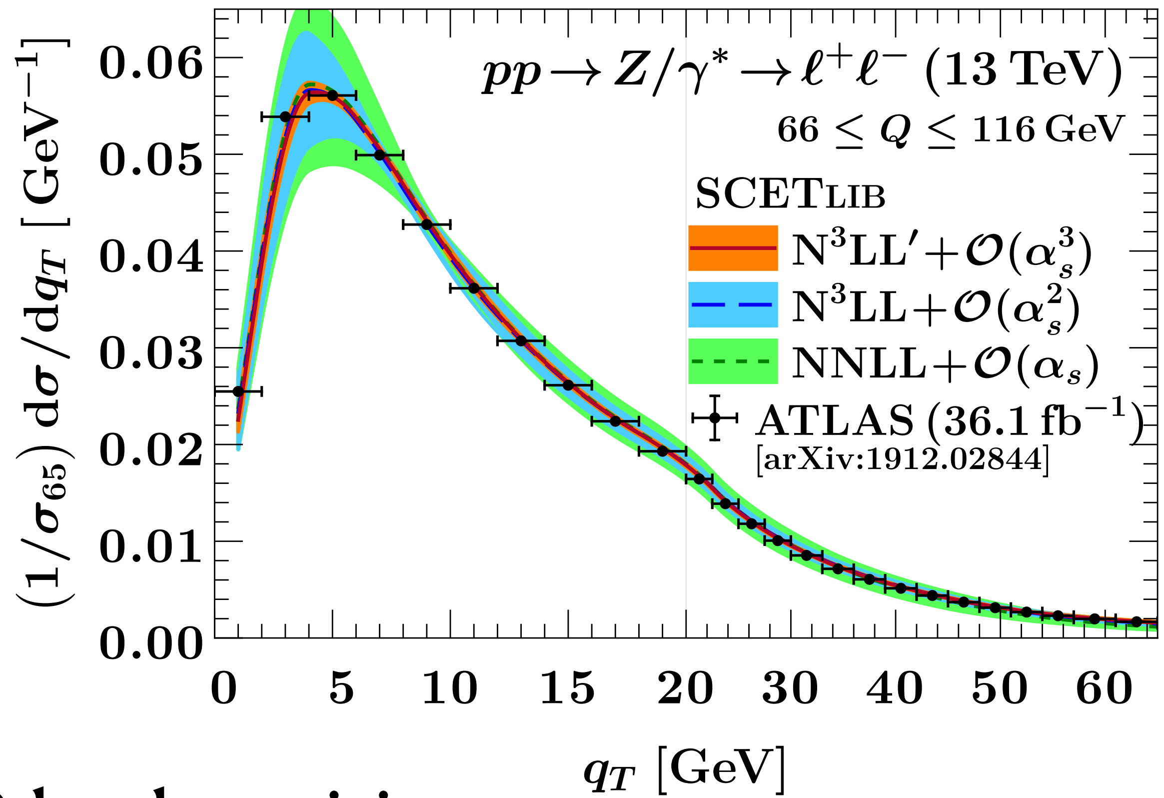

- Many theory requirements to reach $\mathcal{O}(1\%)$ level precision:
  - resummation $\mathcal{O}\left(\log^{2n}(q_T/m_Z)\right) \longrightarrow$ N^{3}LL$'$/ approx N^{4}LL
  - perturbative corrections $\mathcal{O}\left(q_T^2/Q^2\right)$
  - nonperturbative modeling $\mathcal{O}\left(\Lambda_{\mathrm{NP}}^2/q_T^2\right)$
  - quark mass and EW corrections $\mathcal{O}\left(m_q^2/q_T^2\right)$, $(\alpha_{\mathrm{em}} \sim \alpha_S^2)$
  - PDFs

Billis, Michel, Tackmann '25,
Moos, Scimeni, Vladimirov, Zurita '24,
Camarda, Cieri, Ferrera '23,
...

Theory uncertainties

- Every theory prediction needs its theory uncertainty:

$$\Delta_{\text{theo}} \gg \Delta_{\text{exp}}$$

"quite embarrassing"

Major source of uncertainty: missing higher orders (MHO)

Usually determined through ‣ scale variations

$\longrightarrow$ scale variations really easy to implement and use, but with many limitations

Other approaches scale variations based
- scale variation with bayesian approach
- series acceleration
- using reference processes

Meaningful theory uncertainty:

- must reflect our degree of knowledge (or ignorance)
- provide correct **correlations** for different predictions
- have a statistical meaning needed for the interpretation of experimental measurements

 [nice overview of exp-theo status for DY SM@LHC25 Buonocore's talk]

Scale variations approach

Consider a series expansion in a small parameter α:

$$f(\alpha) = f_0 + \alpha f_1 + \alpha^2 f_2 + \mathcal{O}(\alpha^3)$$

$$\text{LO}: \; f(\alpha) = \hat{f}_0 \pm \Delta f$$

$$\text{NLO}: \; f(\alpha) = \hat{f}_0 + \alpha \hat{f}_1 \pm \Delta f$$

Δf is due to the series of the unknown true values $\hat{f}_n$ ⟶ missing higher orders (MHOs)

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Make now a variable transformation

$$\tilde{\alpha}(\alpha) = \alpha[1 + b_0\alpha + b_1\alpha^2 + \mathcal{O}(\alpha^3)]$$

and do again the expansion in $\tilde{\alpha}(\alpha)$

$$\tilde{f}(\tilde{\alpha}) = \tilde{f}_0 + \tilde{f}_1\tilde{\alpha} + \tilde{f}_2\tilde{\alpha}^2 + \mathcal{O}(\tilde{\alpha}^3)$$

$$\text{LO}: \quad \tilde{f}(\tilde{\alpha}) = \tilde{f}_0 = \hat{f}_0$$

$$\text{NLO}: \quad \tilde{f}(\tilde{\alpha}) = \tilde{f}_0 + \tilde{\alpha}\tilde{f}_1 = \hat{f}_0 + \hat{f}_1\alpha + b_0\hat{f}_1\alpha^2 + b_1\hat{f}_1\alpha^3 + \dots$$

Scale variations approach

To estimate the uncertainty, take the difference between the two "schemes"

$$\text{LO}: \quad \Delta f(\alpha) = 0$$

$$\text{NLO}: \quad \Delta f(\alpha) = b_0 \hat{f}_1 \alpha^2 + b_1 \hat{f}_1 \alpha^3 + \mathcal{O}(\alpha^4)$$

Estimating the MHOs uncertainty by approximating them by some linear combination of known lower-order terms [$f_2 \approx b_0 \hat{f}_1$]

- ✓ $\Delta f(\alpha)$ is genuinely of higher order

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Estimating the MHOs uncertainty by approximating them by some linear combination of known lower-order terms [$f_2 \approx b_0 \hat{f}_1$]

- ✓ $\Delta f(\alpha)$ is genuinely of higher order
- ✗ nothing guarantees this is any good
- ✗ f_{n+1} generally more complex internal structure than $f_{\leq n}$
- ✗ b_0 (and b_n) are just arbitrary constants and usually the same for any f
- ✗ μ or b_0 are not actual physical parameter with a true value
- ✗ <u>correlation and shape uncertainties?</u>

$\alpha \equiv \alpha_s(\mu_0)$ $\tilde{\alpha} \equiv \alpha_s(\mu)$

$$b_0 = \frac{\beta_0}{2\pi} \ln \frac{\mu}{\mu_0}$$

why vary μ by 2?

Best can be done is to *assume* some theoretically motivated but *ad hoc* correlation.

Correlations are needed to correctly propagate the uncertainty

Correlation examples

Extract g from the measure of the period of a pendulum: $T = 2\pi\sqrt{\frac{L}{g}} \rightarrow g = 4\pi^2 \frac{L^2}{T^2}$

1 Using two different pendulums with two different lengths L_1, L_2:

- both lengths will have their uncertainty $\longrightarrow$ $g_{1,2}$ will not have a correlated uncertainty

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2 Using the same pendulum with length $L = 1.00 \pm 0.01$ m

$T_1 = 2.02 \pm 0.02\ s$ $\quad$ $T_2 = 2.06 \pm 0.03\ s$

$g_1 = 9.68 \pm 0.19 \pm 0.19\ \ m/s^2$ $\quad$ $g_2 = 9.30 \pm 0.19 \pm 0.27\ \ m/s^2$

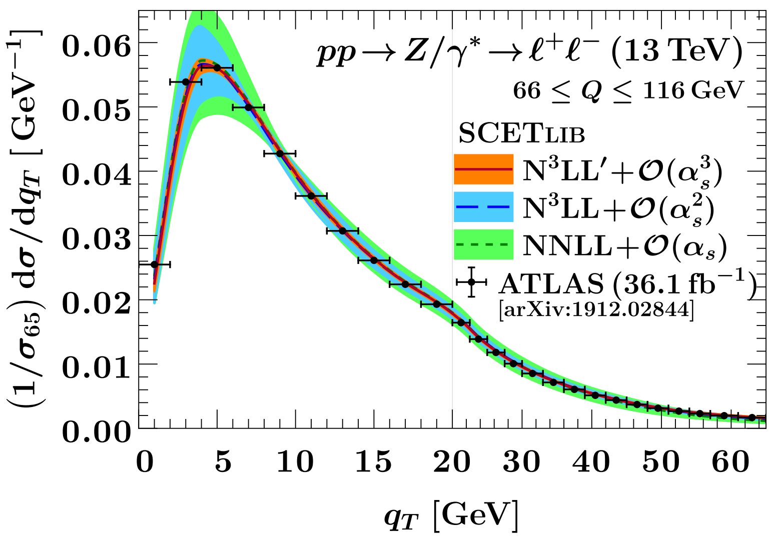
uncertainties on g_1 and g_2 are correlated! ⟶ uncertainty on L is inducing a correlation!

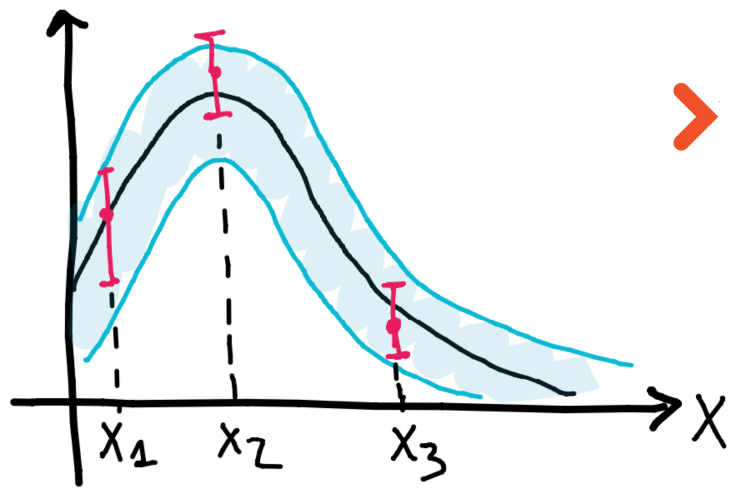
To have a final estimate of g, take into account such correlation!

(weighted average of $g_{1,2}$, or more involved procedure as fits..)

Correlation and scale variations

Taking a differential spectrum, each bin as separate predictions and separate measurements





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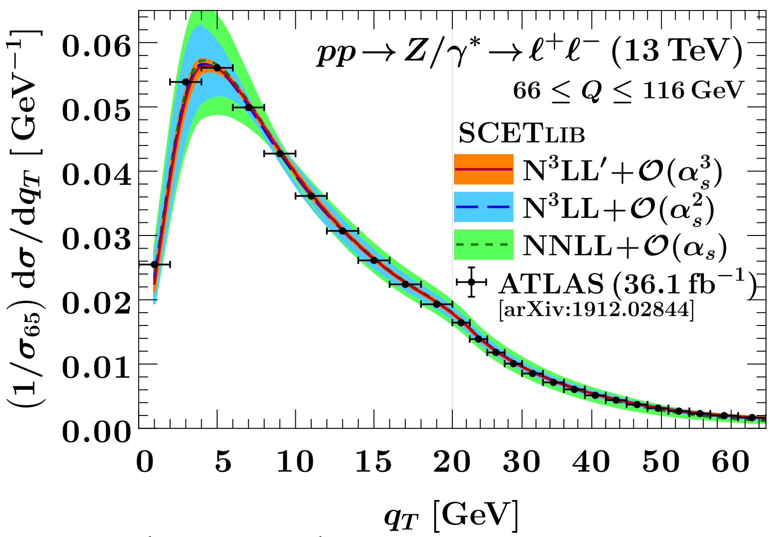

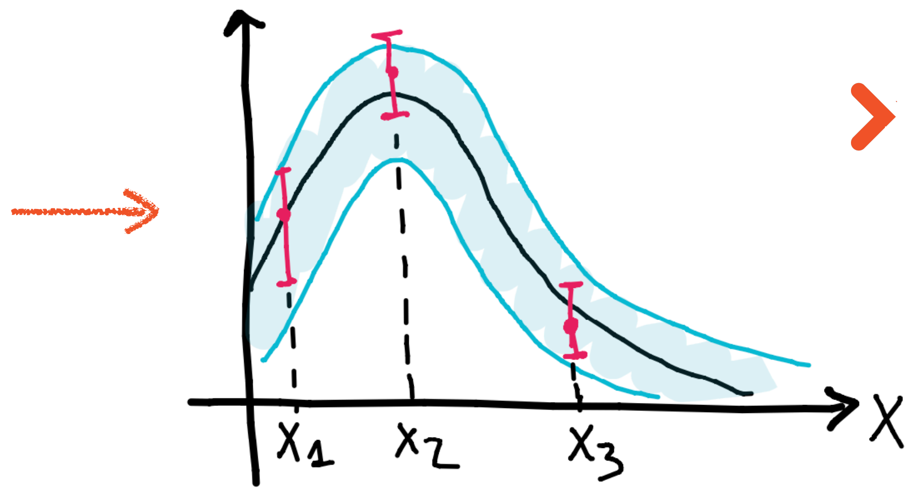

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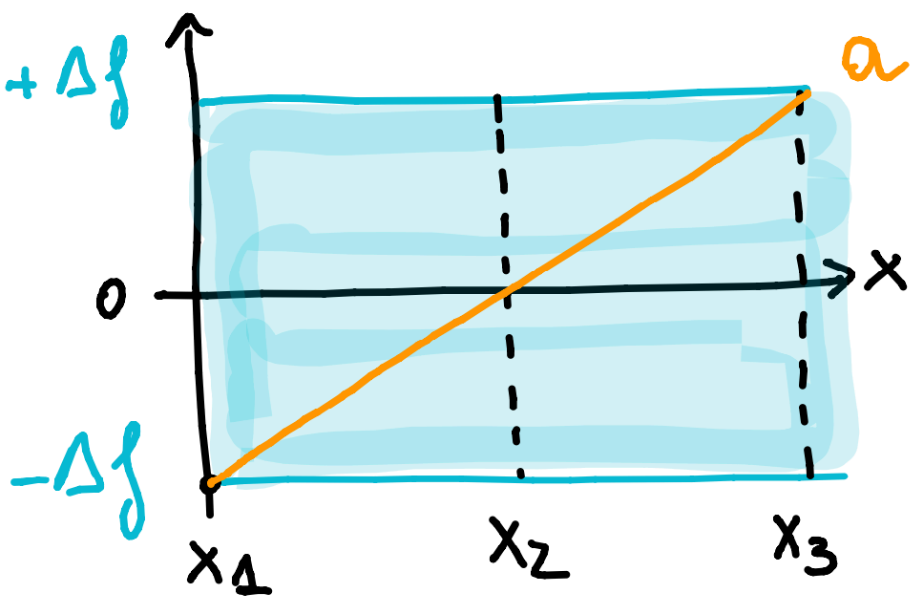
Let's be realistic: uncertainty band given by scale variations. What about its **shape**?



	ρ_{12}	ρ_{13}	ρ_{23}
a	0	−1	0

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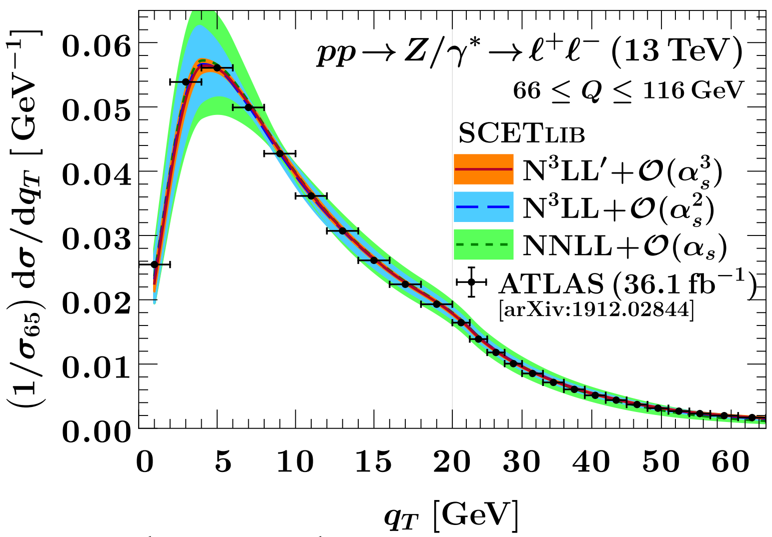

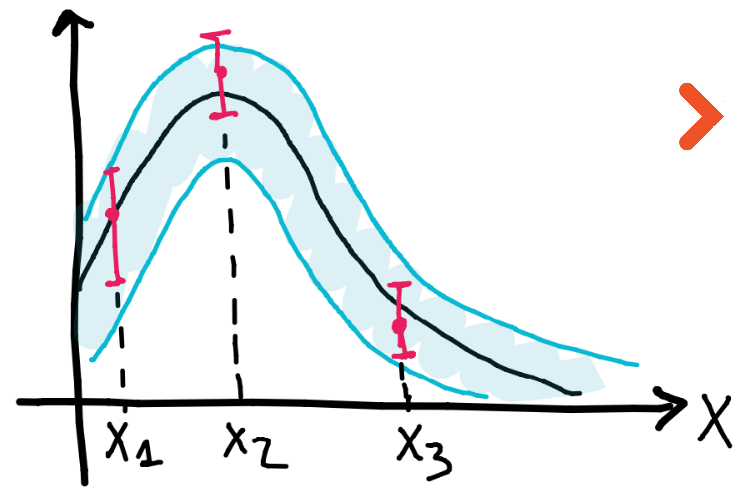

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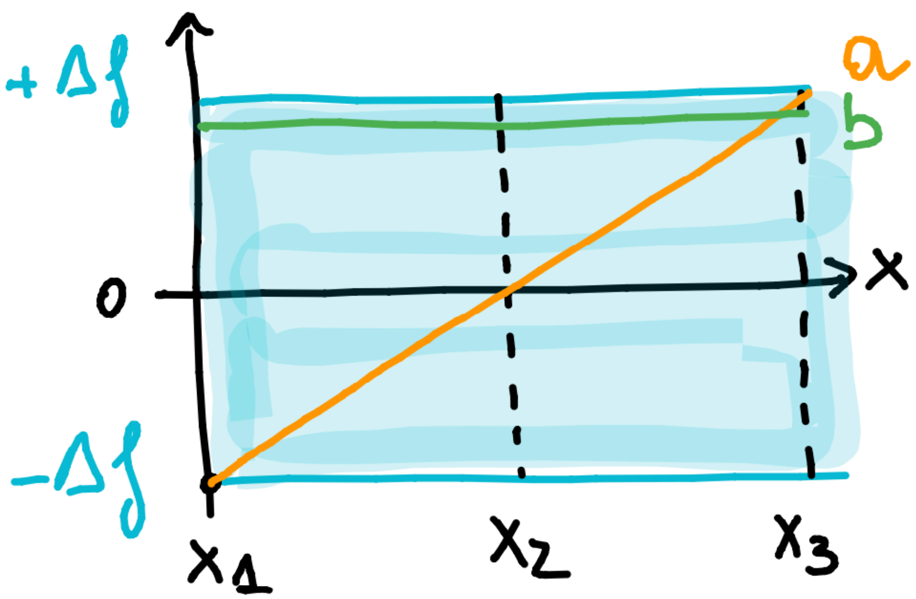

	ρ_{12}	ρ_{13}	ρ_{23}
a	0	−1	0
b	1	1	1

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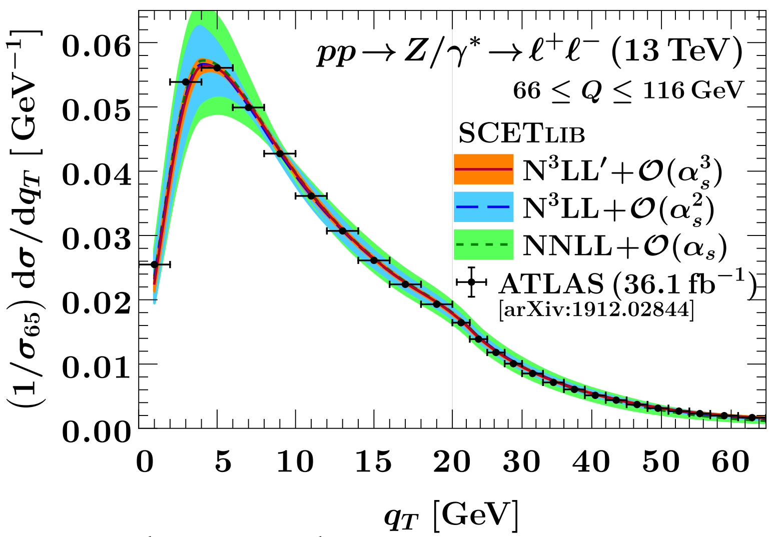

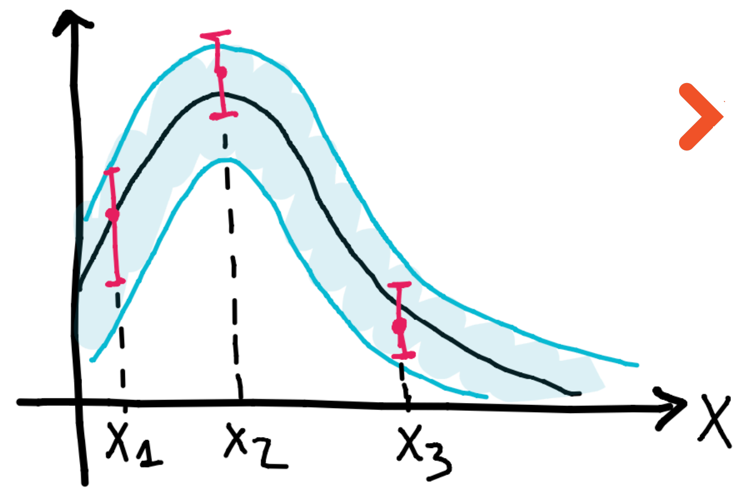

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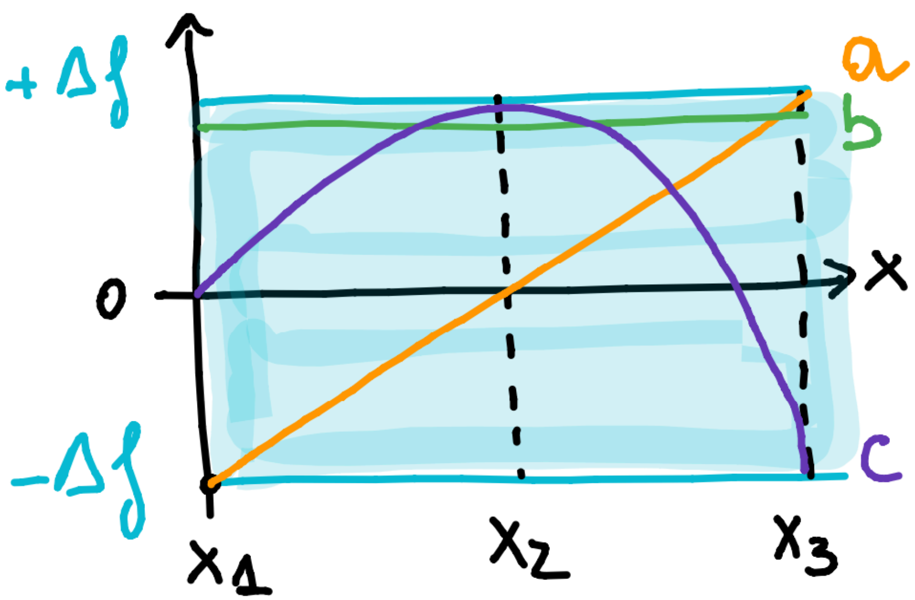

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every line (a, b, c) is a 100 % (anti-) correlated assumption

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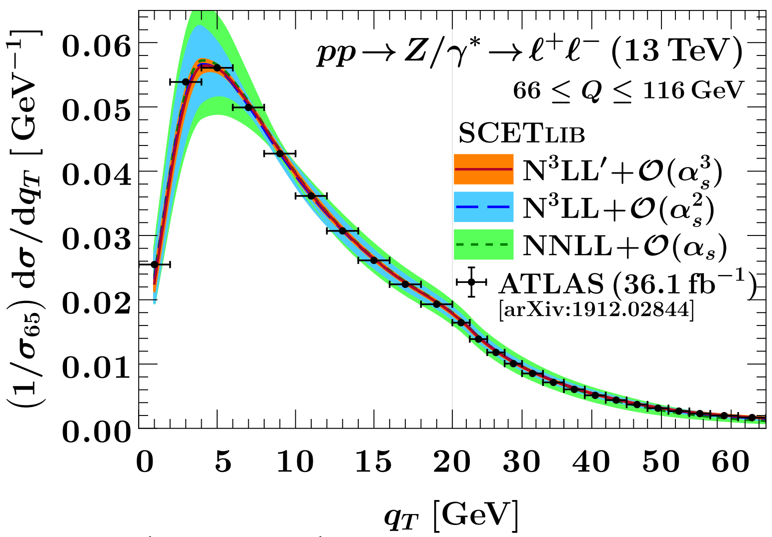

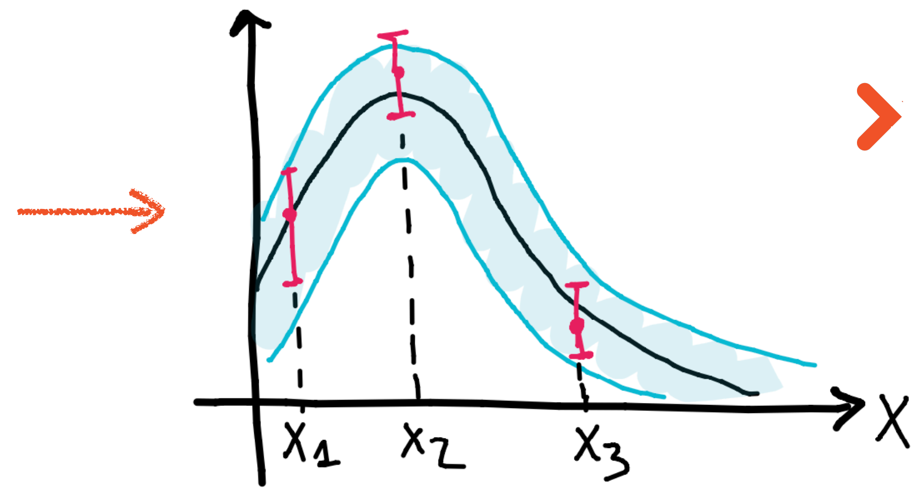

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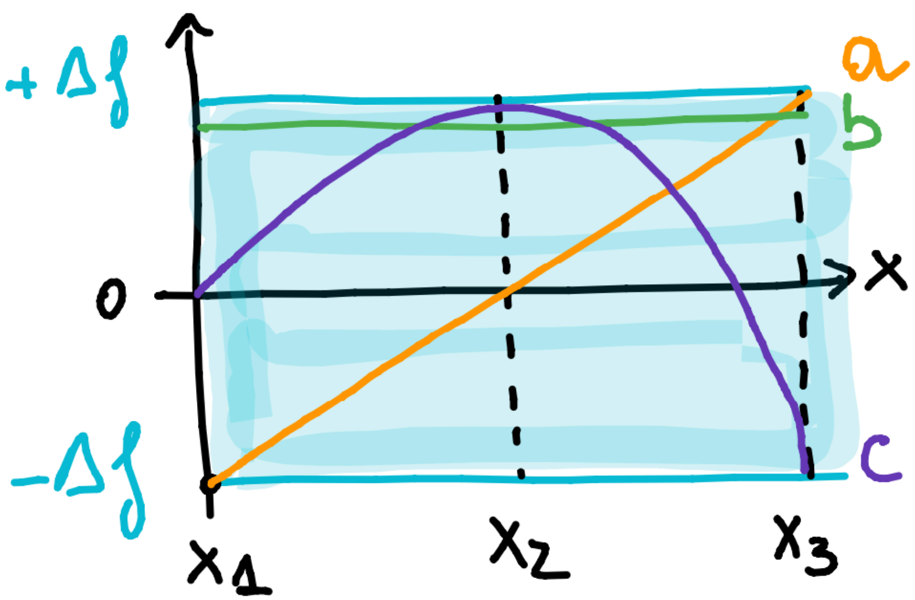

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every line (a, b, c) is a 100 % (anti-) correlated assumption

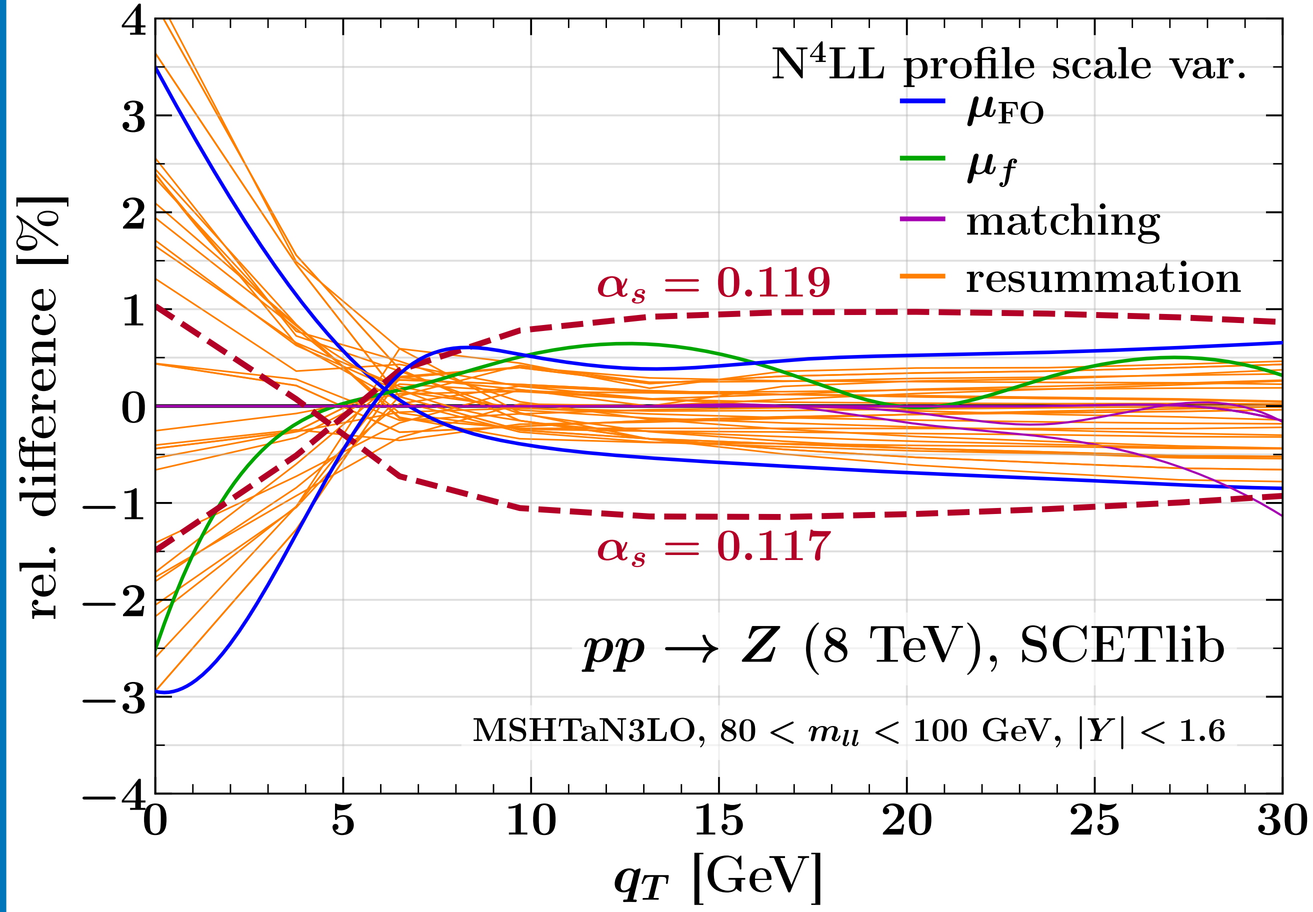
- no idea about the correct shape of scale variations (and therefore correlation): that's why we take envelopes!
- to get correct correlation: breakdown into *independent uncertainty components* required

Extraction of $\Delta\alpha_s$ with scale variations

In the q_T spectrum each bin has its own theory prediction

» point-by-point correlation crucial for the determination of the α_s uncertainty

What are we used to do? Scale variations!



Each variation is a 100 % (anti-) correlated correlation model, strongly impacts the result!

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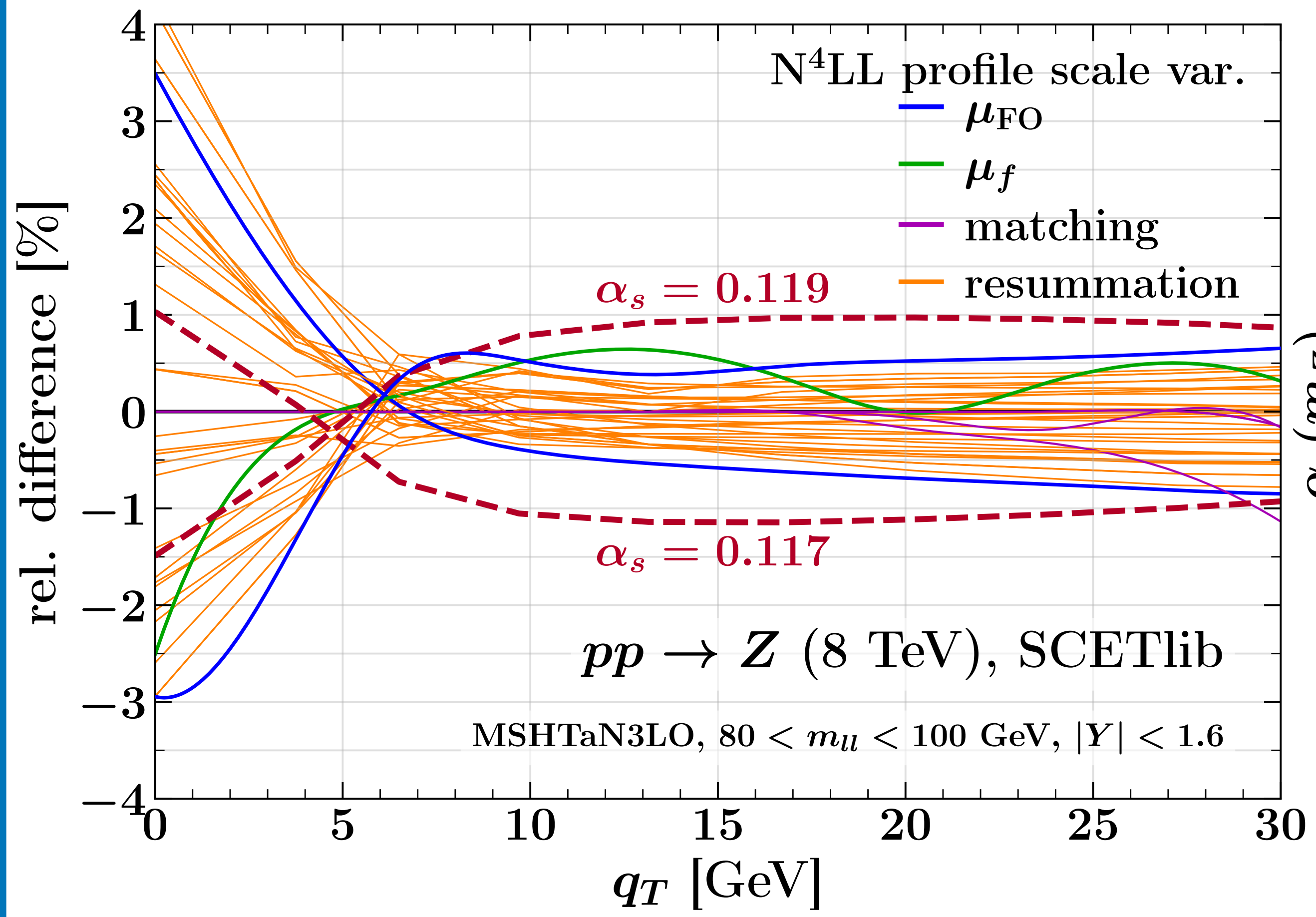
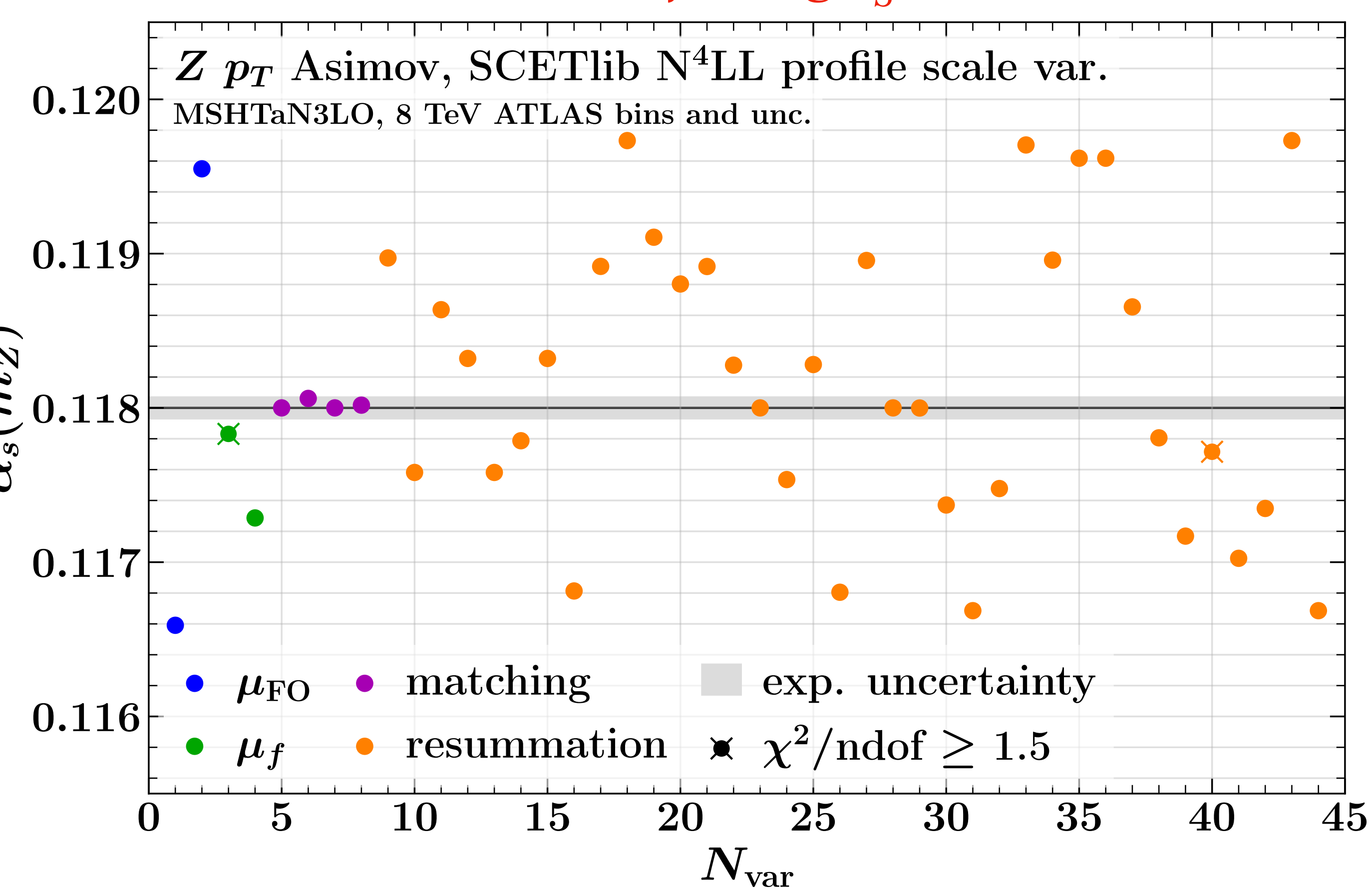


Only fitting α_S*



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Sum envelopes of different "sources": $\Delta_{\mathrm{scale}} = 2.3 \times 10^{-3}$

Naive envelope: $\Delta_{\mathrm{scale}} = 1.9 \times 10^{-3}$

❌

* explain later the setup of these fits

SCETlib scales from Billis, Michel, Tackmann '25

Theory Nuisance Parameters
(TNPs)

Theory Nuisance Parameters

all details here Tackmann '24!

Consider the same series expansion:

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$$\text{NNLO}: \; f(\alpha) = \hat{f}_0 + \alpha \hat{f}_1 + \alpha^2 \hat{f}_2 \pm \Delta f$$

2 Parametrize and include the leading source of uncertainty:

$$\text{N}^{2+1}\text{LO}: \; f^{\text{pred}}(\alpha, \theta_3) = \hat{f}_0 + \alpha \hat{f}_1 + \alpha^2 \hat{f}_2 + \alpha^3 f_3(\theta_3)$$

using theory nuisance parameters θ_n ;

- θ_n have physical true value $\hat{\theta}_n$, such that $\boxed{\hat{f}_n = f_n(\hat{\theta}_n)}$
  ... and therefore encode correct theory correlations
- TNPs well-defined parameters with true but unknown value

Theory Nuisance Parameters

3 How to ***define*** these θ_n?

- simplest case: $f_3(\theta_3) \equiv \theta_3$
- better: $f_n \equiv f_n(x)$ in general functions of different x dependencies

  account for the internal structure of f_3

Which dependencies to consider?

- in which we need correlations
- those helping to obtain better theory constraints

✳ discrete dependence : partonic channels, color factors, ...

✳ continuous but discrete dependence : $E_{\rm cm}$, n_f, ...

✳ fully continuous dependence : p_T^Z , Y , q^2, masses

Strategy: break down internal structure until remaining unknowns $f_{n,i}$ are numbers

Theory Nuisance Parameters

3 Three parameterization strategies:

A. known functional form, for example $f_n(x)$ polynomial in $\ln x$

$$f_n(x) = \sum_{i=0}^{k} f_{n,i} \ln^i x$$

B. as point above, but only in some specific limit we can expand around

$$f_n(x) = f_{n0}(x) + f_{n1}(x)\epsilon + f_{n2}(x)\epsilon^2 + \mathcal{O}(\epsilon^3)$$

C. do not have enough information, perform an expansion in some complete functional basis $\{\phi_n\}$

$$f_n(x) = \sum_{i} f_{n,i}\, \phi_i(x)$$

strategy used for theoretical uncertainty from fixed-order Lim, Poncelet '24

Strongly depends on the case considered: resummation for Drell-Yan q_T spectrum

TNPs for Drell-Yan q_T spectrum

3 Considering $x \equiv q_T$

First apply strategy **B.** : expanding in $\epsilon = q_T^2/Q^2$

$$f(\alpha;x) \equiv \frac{\mathrm{d}\sigma}{\mathrm{d}q_T}(\alpha_s) = \underbrace{\frac{\mathrm{d}\sigma^{(0)}}{\mathrm{d}q_T}}_{\mathcal{O}(1)} + \underbrace{\frac{\mathrm{d}\sigma^{(1)}}{\mathrm{d}q_T}}_{\mathcal{O}(q_T^2/Q^2)} + \frac{\mathrm{d}\sigma^{(2)}}{\mathrm{d}q_T} + \ldots$$

And then strategy **A.**

$$\frac{\mathrm{d}\sigma^{(0)}}{\mathrm{d}q_T} = f_0\delta(q_T) + \sum_n \alpha_s^n \left\{ f_n\delta(q_T) + \sum_m f_{nm} \left[\frac{\ln^m(q_T/Q)}{q_T}\right]_+ \right\}$$

Use SCET factorization to resum the series for f_{nm}

$$\frac{\mathrm{d}\sigma^{(0)}}{\mathrm{d}q_T} = \left[\sum_{ab} H_{ab} \times B_a \otimes B_b \otimes S\right] (\alpha_s, L = \ln q_T/Q)$$

- Leading power q_T dependence is known to all orders

TNPs for Drell-Yan q_T spectrum

3 $F = \{H, B, S\}$ solution to RGE equations

$$F(\alpha_S, L) = F(\alpha_S) \exp \int_0^L \mathrm{d}L' \{\Gamma[\alpha_S(L')] L' + \gamma_F [\alpha_S(L')]\}$$

boundary conditions   anomalous dimensions

q_T dependence predicted by resummation in terms of several independent series!

- 5 scalar series: $H(\alpha_s)$, $S(\alpha_s)$ and $\Gamma(\alpha_s)$, $\gamma_\mu(\alpha_s)$, $\gamma_\nu(\alpha_s)$
- up to 5 one-dim functional series for beam functions* (+ DGLAP splitting function)

$$\tilde{b}_i(x, \alpha_s) = \sum_j \int \frac{\mathrm{d}z}{z} \left[\hat{I}_{ij,0}(z) + \hat{I}_{ij,1}(z) + I_{ij,2}\left(z, \theta_2^{B_{ij}}\right) \right] f_j\left(\frac{x}{z}\right)$$

* at the moment, using functional known form $I_{ij,n}(z, \theta_n^{B_{ij}}) = \frac{3}{2}\theta_n^{B_{ij}} \hat{I}_{ij,n}(z)$

to be changed in the future!

$$F(\alpha_s) = 1 + \sum_{n=1} \left(\frac{\alpha_s}{4\pi}\right)^n F_n \qquad \gamma(\alpha_s) = \sum_{n=0} \left(\frac{\alpha_s}{4\pi}\right)^{n+1} \gamma_n$$

TNPs for Drell-Yan q_T spectrum

3 Still considering N^{2+1}LL:

$$F(\alpha_s) = 1 + \frac{\alpha_s}{4\pi} F_1 + \left(\frac{\alpha_s}{4\pi}\right)^2 F_2(\theta_2^f) \longrightarrow F_n(\theta_n^f) = 4C_r(4C_A)^{n-1}(n-1)!\,\theta_n^f$$

$$\gamma(\alpha_s) = \frac{\alpha_s}{4\pi}\gamma_0 + \left(\frac{\alpha_s}{4\pi}\right)^2 \gamma_1 + \left(\frac{\alpha_s}{4\pi}\right)^3 \gamma_2(\theta_2^\gamma) \longrightarrow \gamma_n(\theta_n^\gamma) = 4C_r(4C_A)^n \theta_n^\gamma$$

(+ one order more for Γ)

C_r leading color factor, C_A^{n-1} leading $n-$loop color factor

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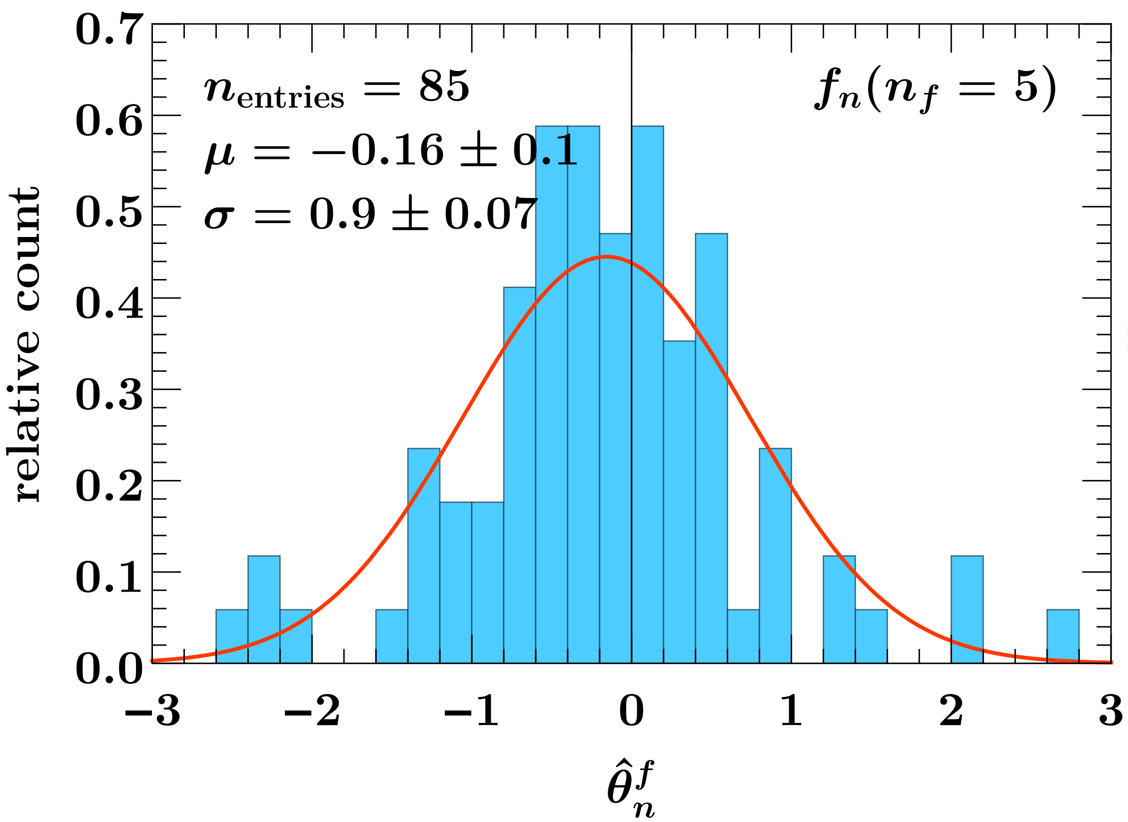
(+ one order more for Γ)

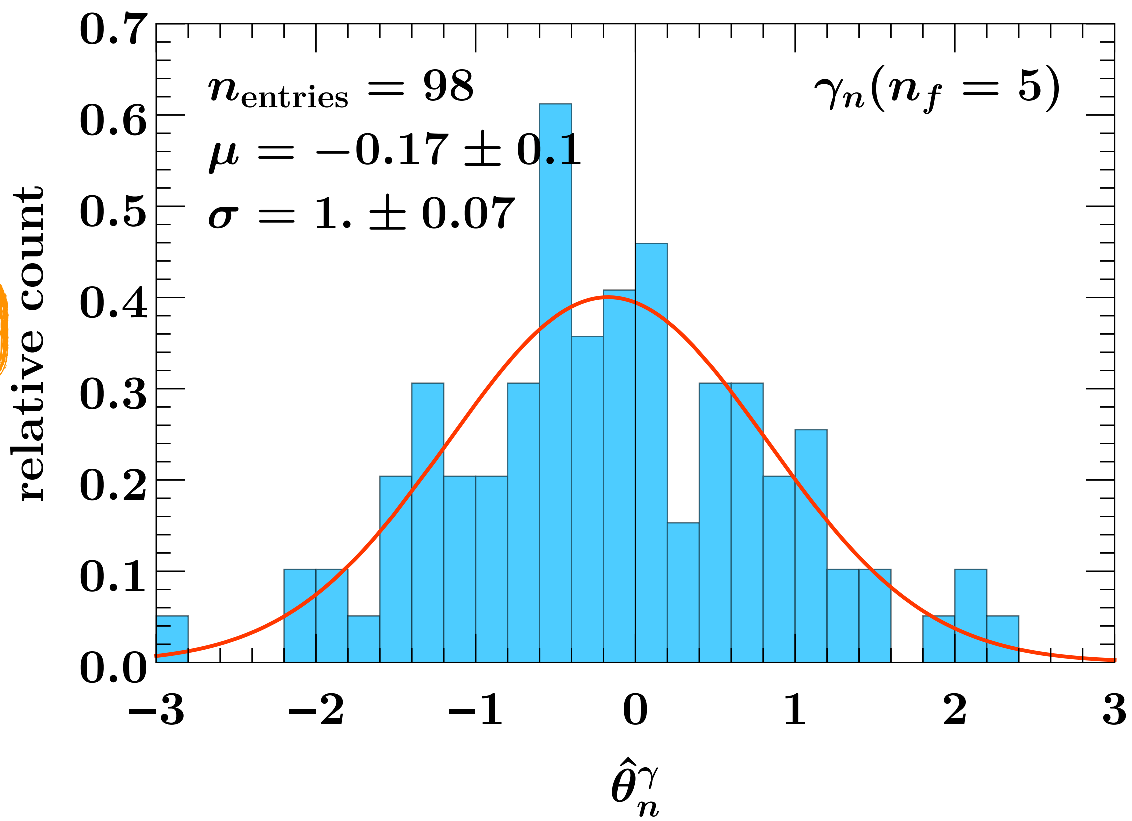
C_r leading color factor, C_A^{n-1} leading $n-$loop color factor

4 How to ***vary*** θ_n? With these normalizations, expected natural size $|\hat{\theta}_n| \lesssim 1$

look at other known $n-$loop coefficients from population sample and validation using known perturbative series here

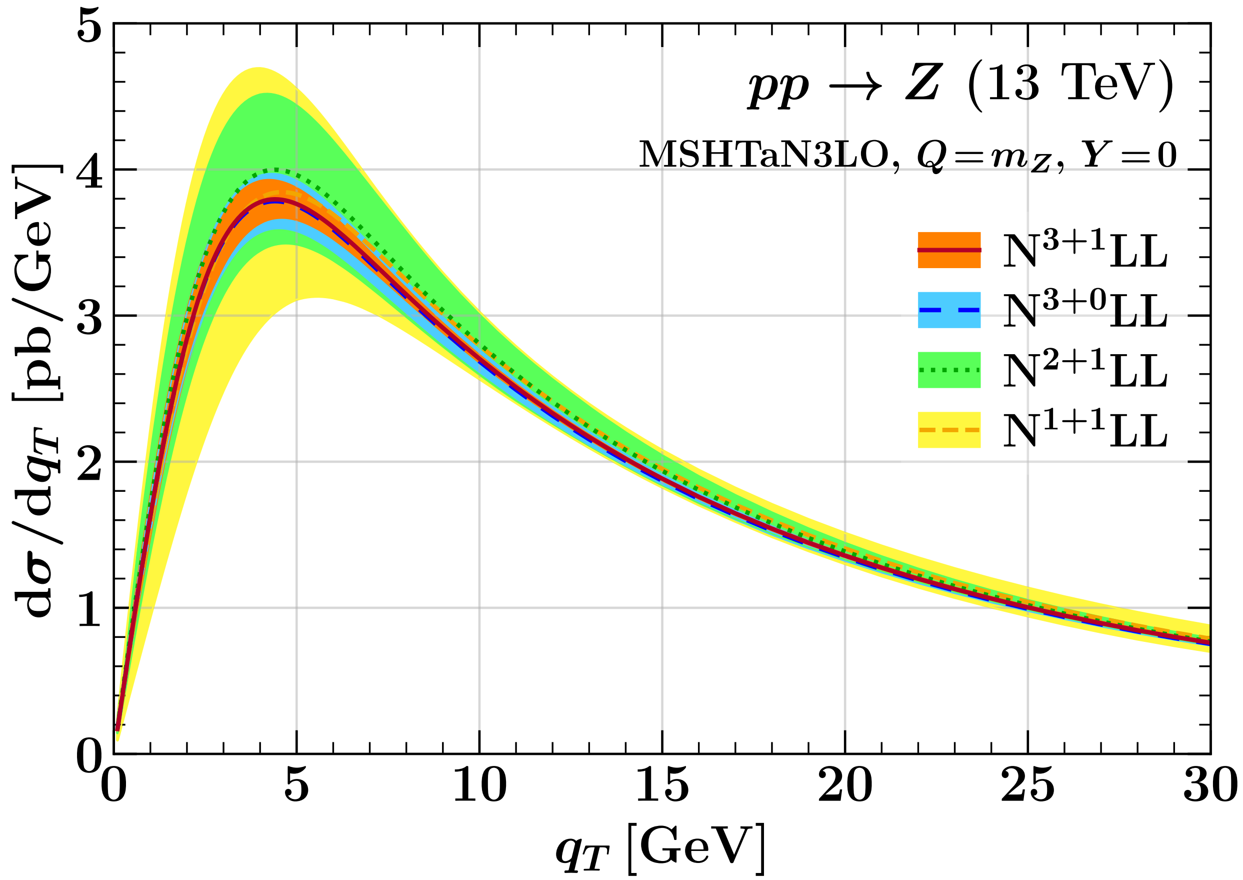


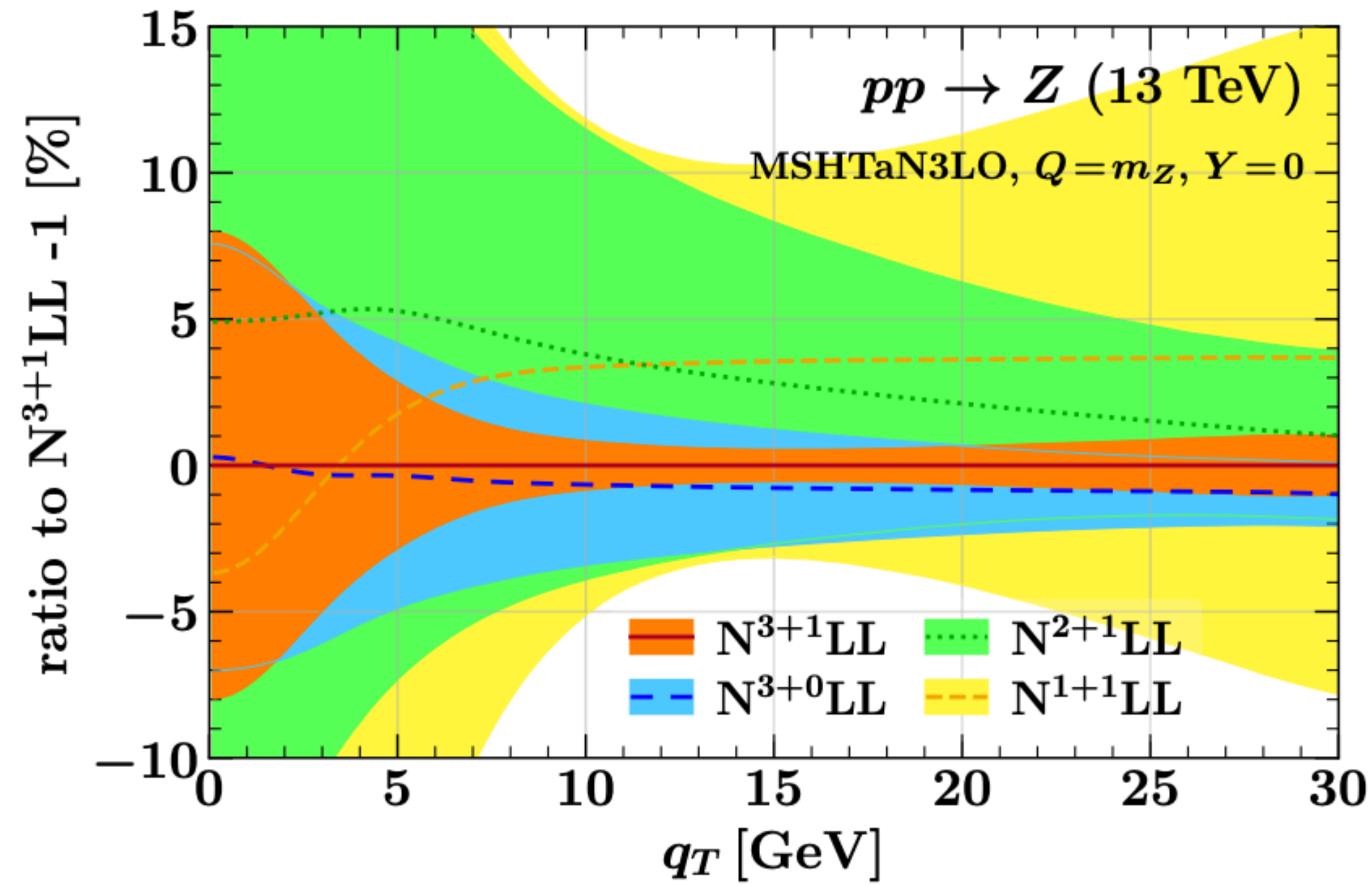
$\theta_n = 0 \pm 1$



TNPs breakdown for Drell-Yan q_T spectrum

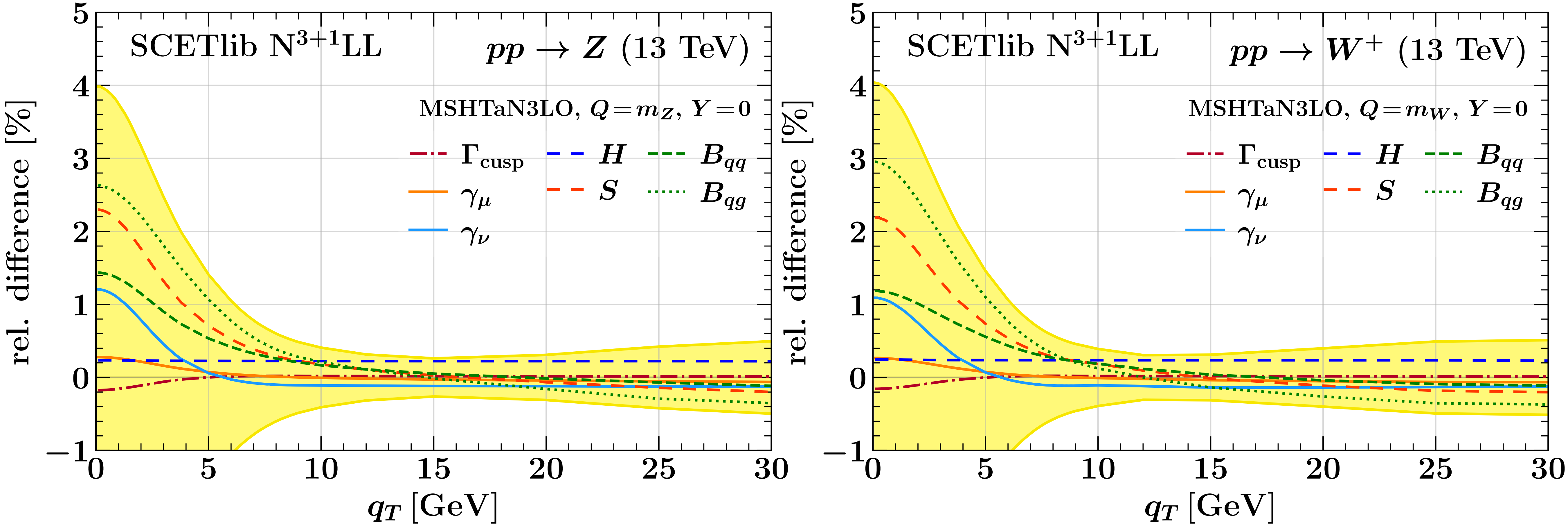
Comparing different orders at 95% theory CL ($\Delta\theta_n = \pm 2$)





TNPs breakdown for Drell-Yan q_T spectrum

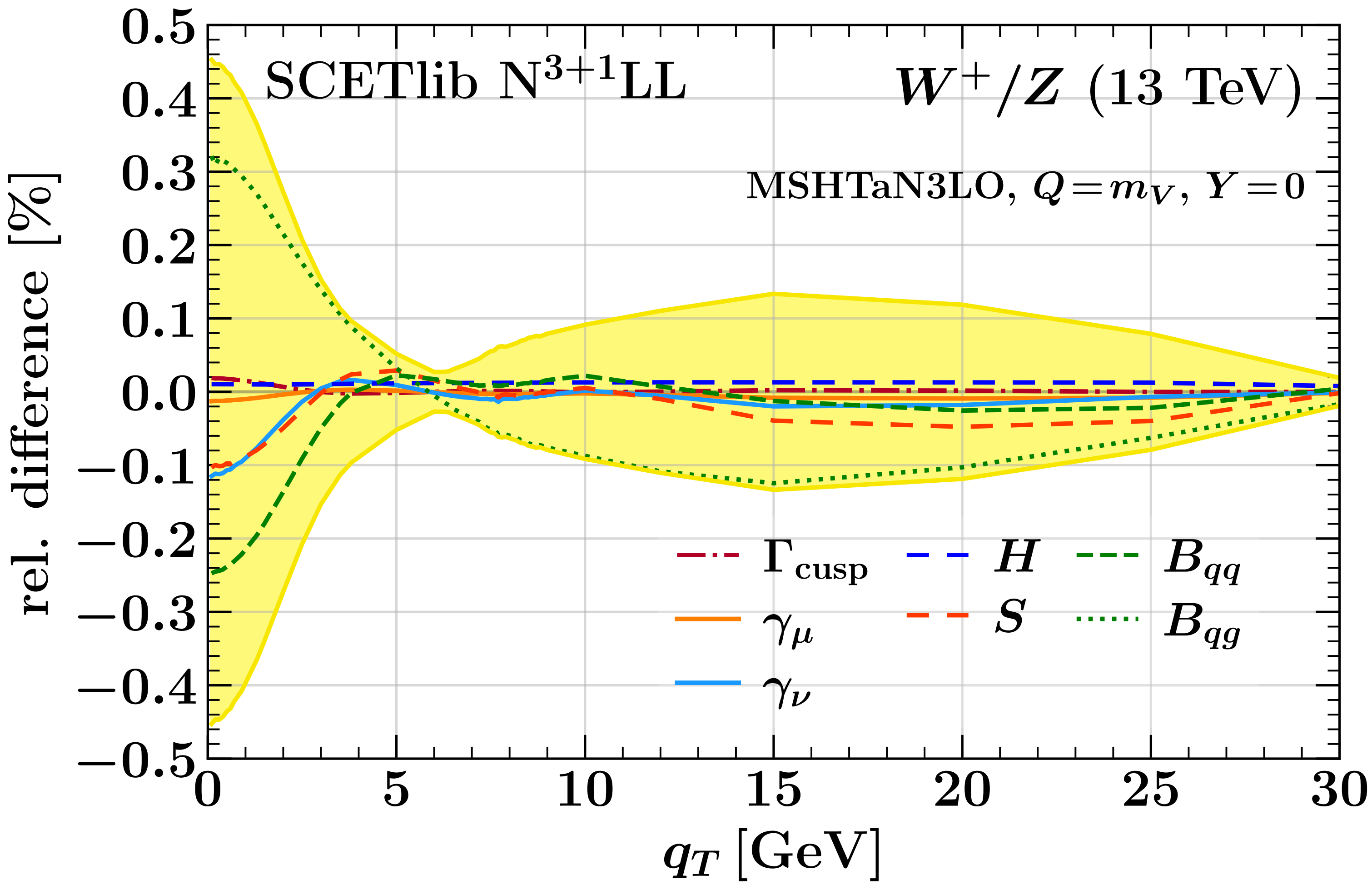
Breakdown of all the TNPs at $\mathrm{N}^{3+1}\mathrm{LL}$:



- varying each TNP by $\Delta\theta_n = \pm 1$(68% CL)
- providing breakdown into independent sources of uncertainty $\longrightarrow$ sum in quadrature!
- encoding correct point-by-point correlations

TNPs correlation for Drell-Yan q_T spectrum

Relative impacts on W/Z^*:



- uncertainties very similar for Z and W processes: same TNPs for both

  ⟶ each TNP impacts are 100% correlated between the processes:

  nice cancellation in the ratio!

*just for illustration: only leading **massless** contribution

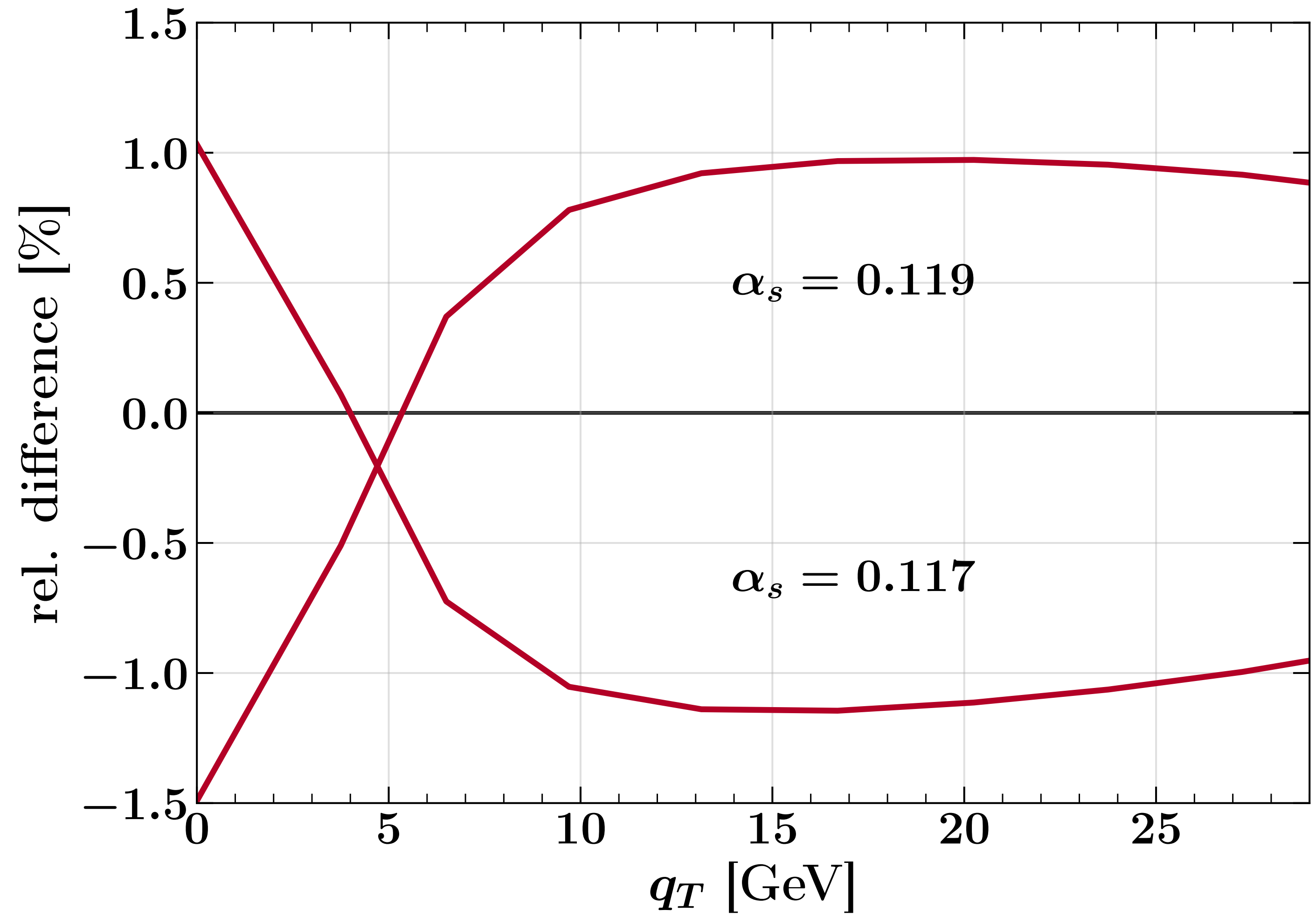
Towards α_s extraction from the Z q_T spectrum

WIP Cridge, Marinelli, Tackmann

Why $\alpha_s(m_Z)$?

For the extraction of α_s from the $Z\, q_T$ spectrum:

- *super* precise ATLAS q_T spectrum measurement [arXiv:2309.09318]
- many sources of theory uncertainties, major ones: **perturbative**, **nonperturbative**, PDFs
- correlations are fundamental:



using a differential spectrum to extract a parameter that **is** a *shape effect*

↓

shape uncertainty is equivalent to how the uncert. at different points in the spectrum are correlated

Asimov fits

Asimov fits: standard procedure to estimate expected uncertainties in a fully controlled setting

- using pseudodata (or Asimov data, or toy-data)

- results of the fits *not affected* by statistical fluctuations and possible subleading/higher-order effects present in the real data
  - theory model correctly describes pseudodata with a minimum $\chi^2 = 0$ (or very close)

- study the *dominant* sources of uncertainty and their impact on the extracted α_s
  - not concerned with subleading effects:

→ affecting the small q_T spectrum at few-% level, their associated uncert. is subdominant wrt the dominant ones

→ still necessary for fitting real data

neglected in our pseudodata and theory model

Asimov fits for $\alpha_S(m_Z)$ from $Z\ q_T$

Our theory inputs:

- SCETlib N^{3+1}LL and N^{4}LL only resummed contribution

  [default central scales and variations, no mass corrections and nonsingular power corrections]

Our toy data:

- Data defined as central theory prediction [$\alpha_S = 0.118$]

  [fixed nonp. params, MSHT20aN3LO PDF set]

- Using ATLAS exp. uncertainties and complete correlations for all 72 bins

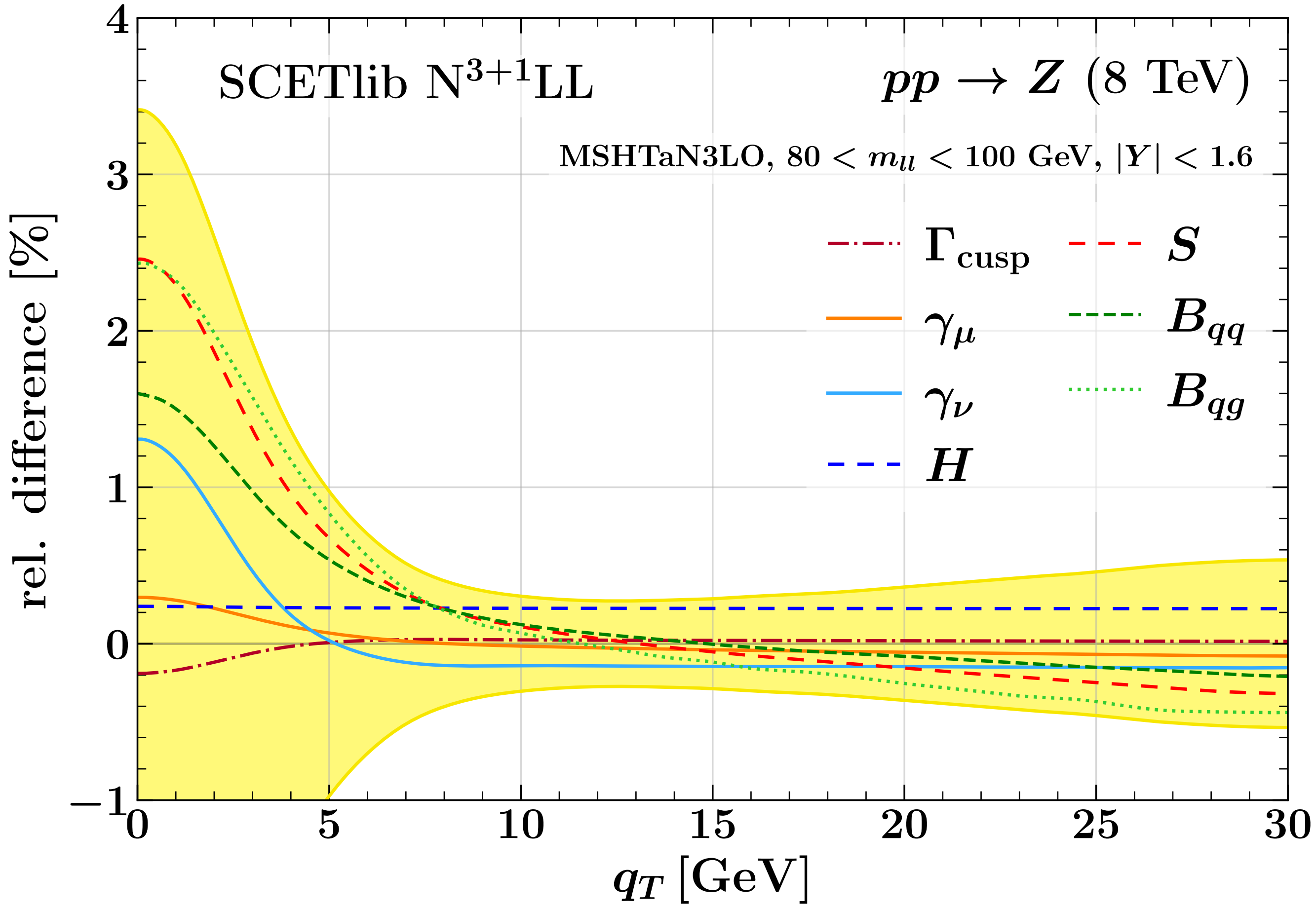
- 72 data points in ATLAS binning,

  9 q_T bins in [0,29] GeV for each 8 Y bin in [0.0,3.6]

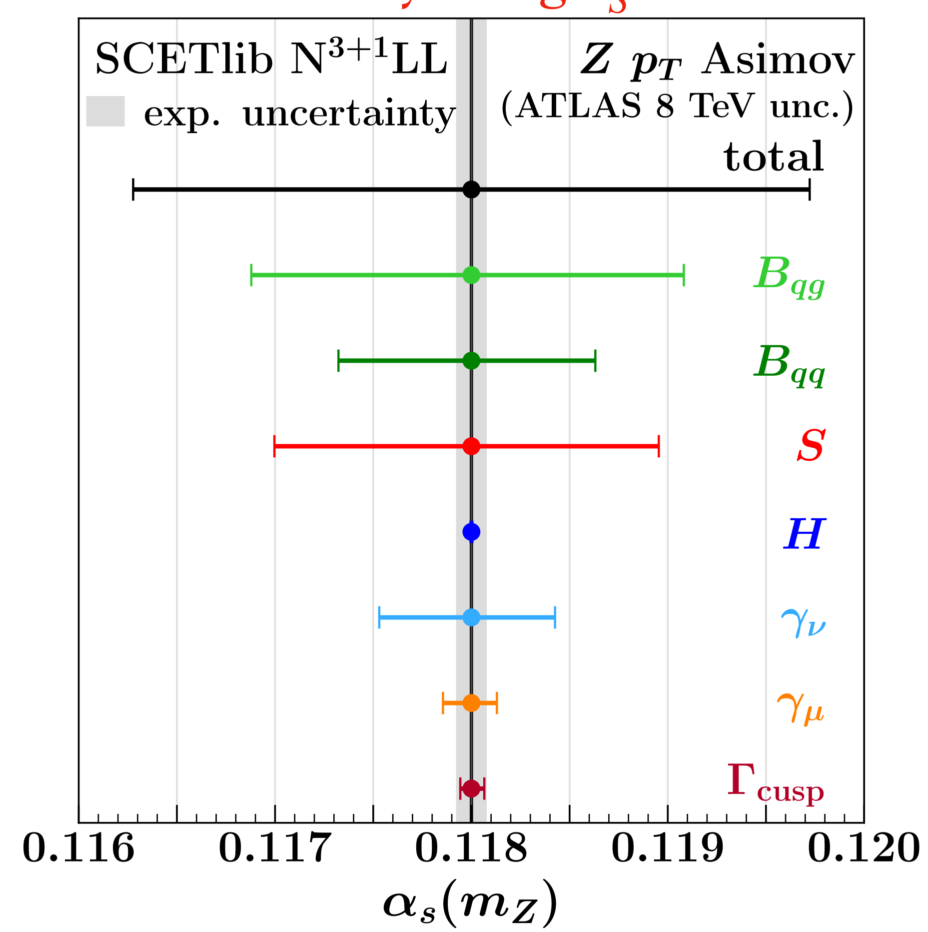
  [integrated in q_T, Y and Q]

- Using Minuit (and Minos) as minimizer for the fit

Scanning TNPs



Only fitting α_S



Repeat fit separately varying each TNP by $\Delta\theta_n = \pm 1$

- providing breakdown into independent sources of uncertainty
- encoding correct point-by-point correlations
- can now sum in quadrature $\Delta_{\mathrm{total}} = 1.7$

still does not let the fit decide between moving α_S or theory

↓

rather profiling

* uncertainties in units of 10^{-3}

Perturbative uncertainty with profiling TNPs

Profiling: fitting α_S *together* with all TNPs (allows the fit to decide what to do)

- TNPs are proper parameters, included in the fit with Gaussian constraint $\theta_n = 0 \pm 1$
- allows data to constrain TNPs and thereby reduce theory uncertainty

Pseudodata: central [$\alpha_s = 0.118$] N^4LL prediction

- simulates the fit to real data, which contains the all-order result

Perturbative uncertainty with profiling TNPs

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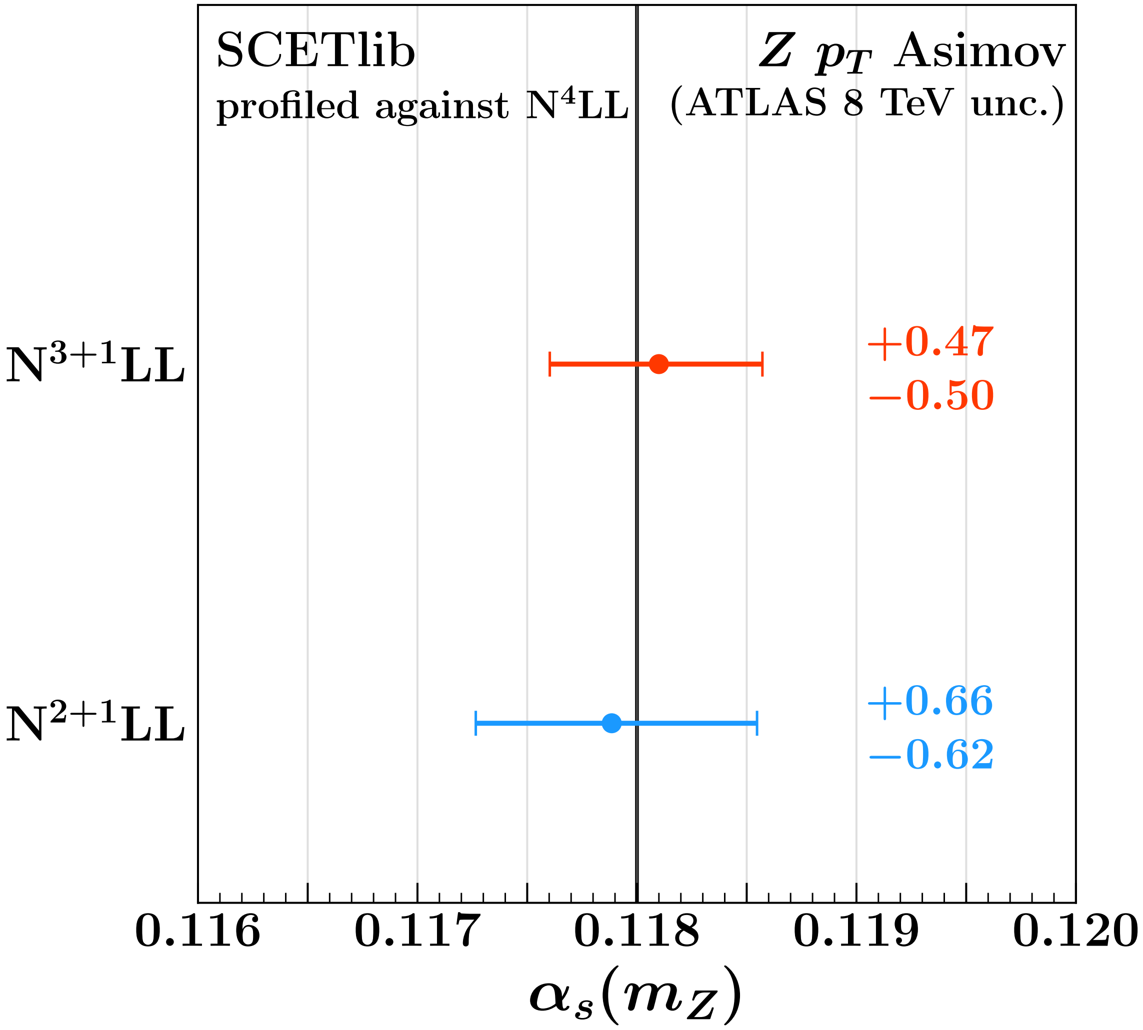
Pseudodata: central [$\alpha_s = 0.118$] N^{4}LL prediction

- simulates the fit to real data, which contains the all-order result

1 N^{2+1}LL theory model

2 N^{3+1}LL theory model

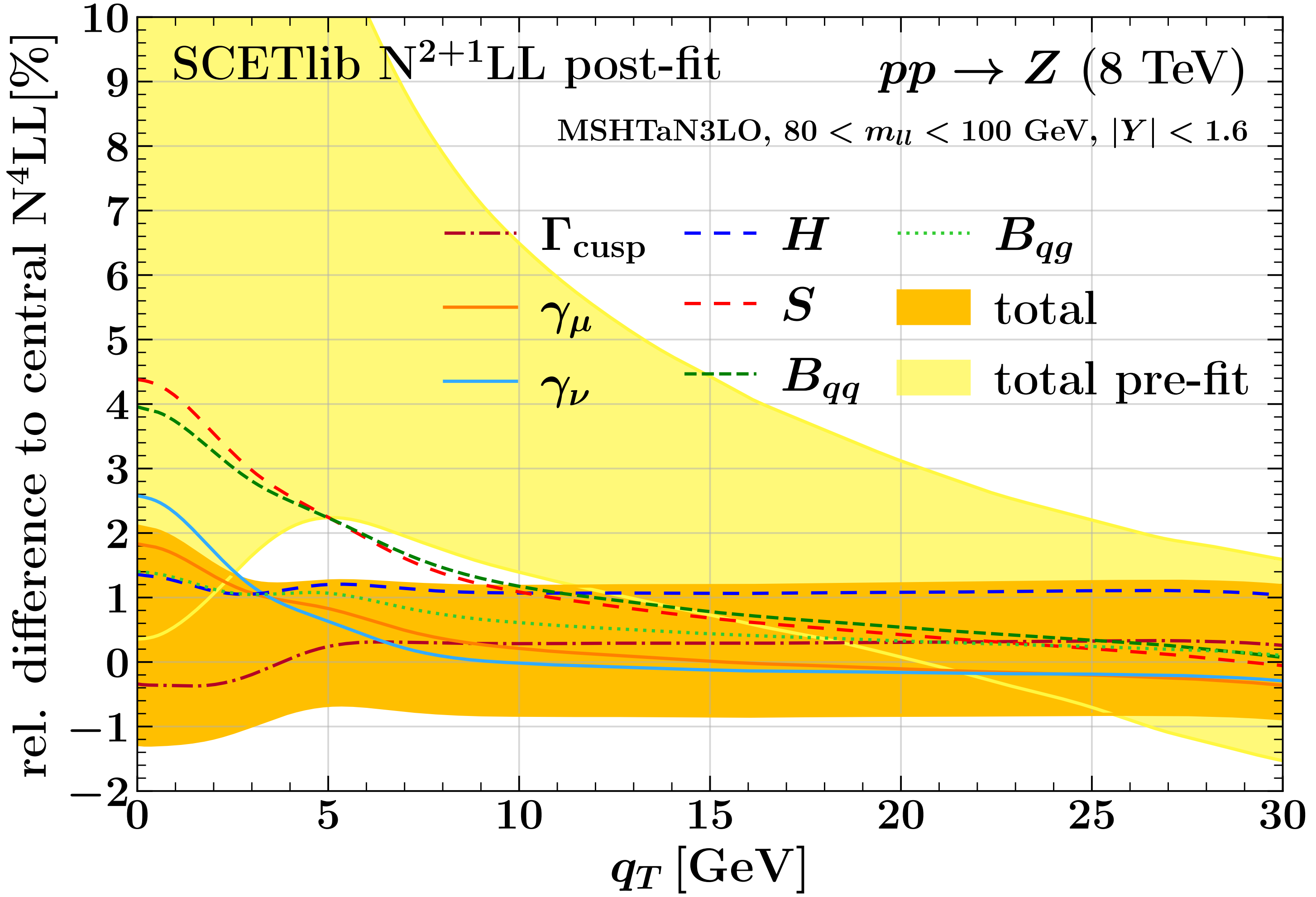
→ look at the post-fit constraints on TNPs

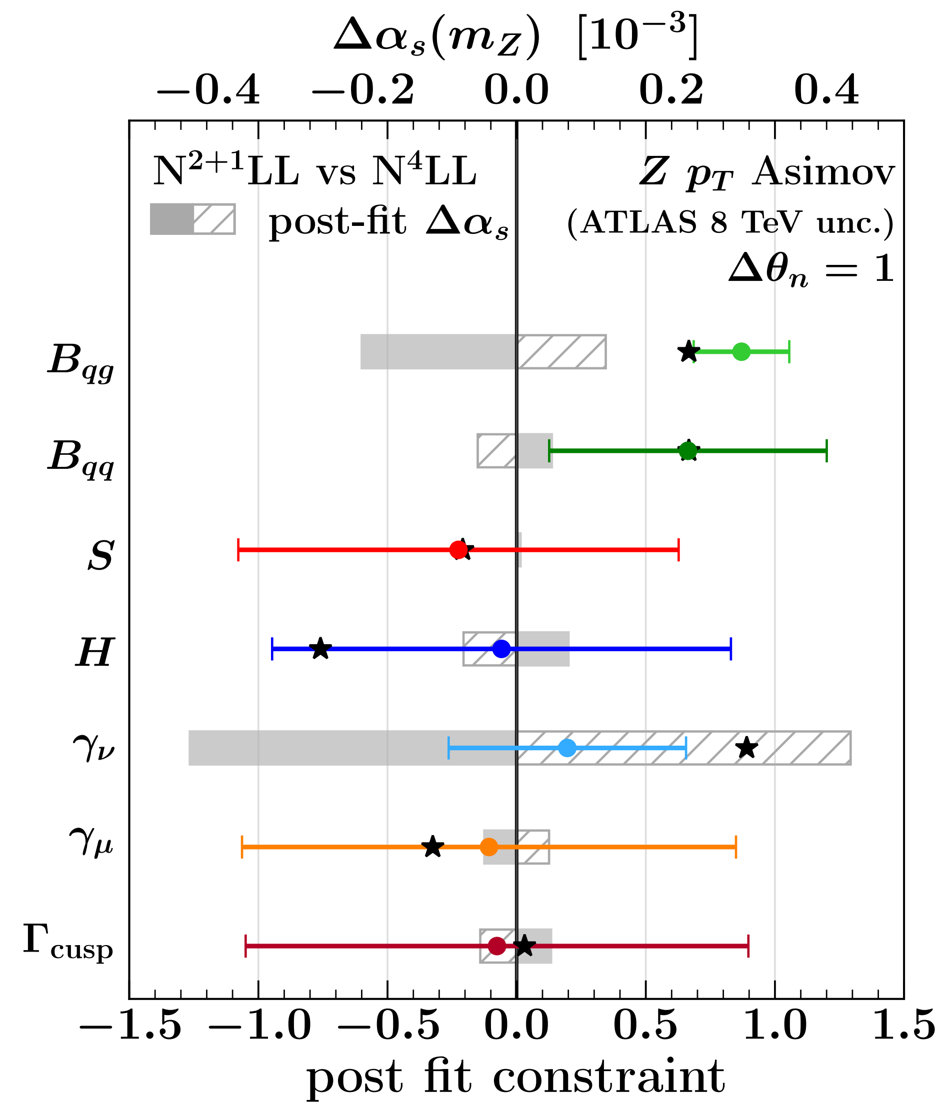


* uncertainties in units of 10^{-3}

Post-fit constraints on $N^{2+1}LL$

Profiling lower order against higher order: $N^{2+1}LL$



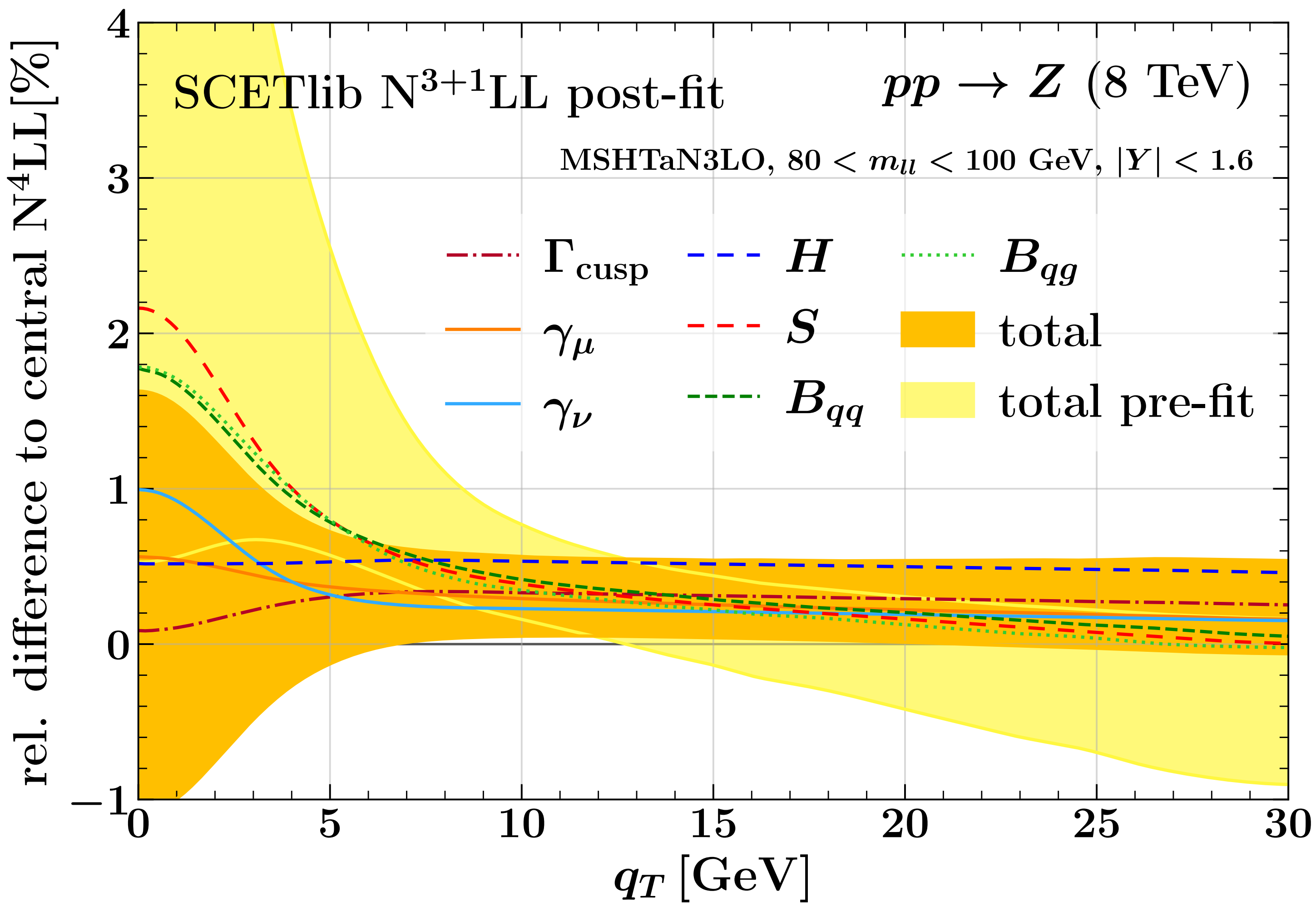


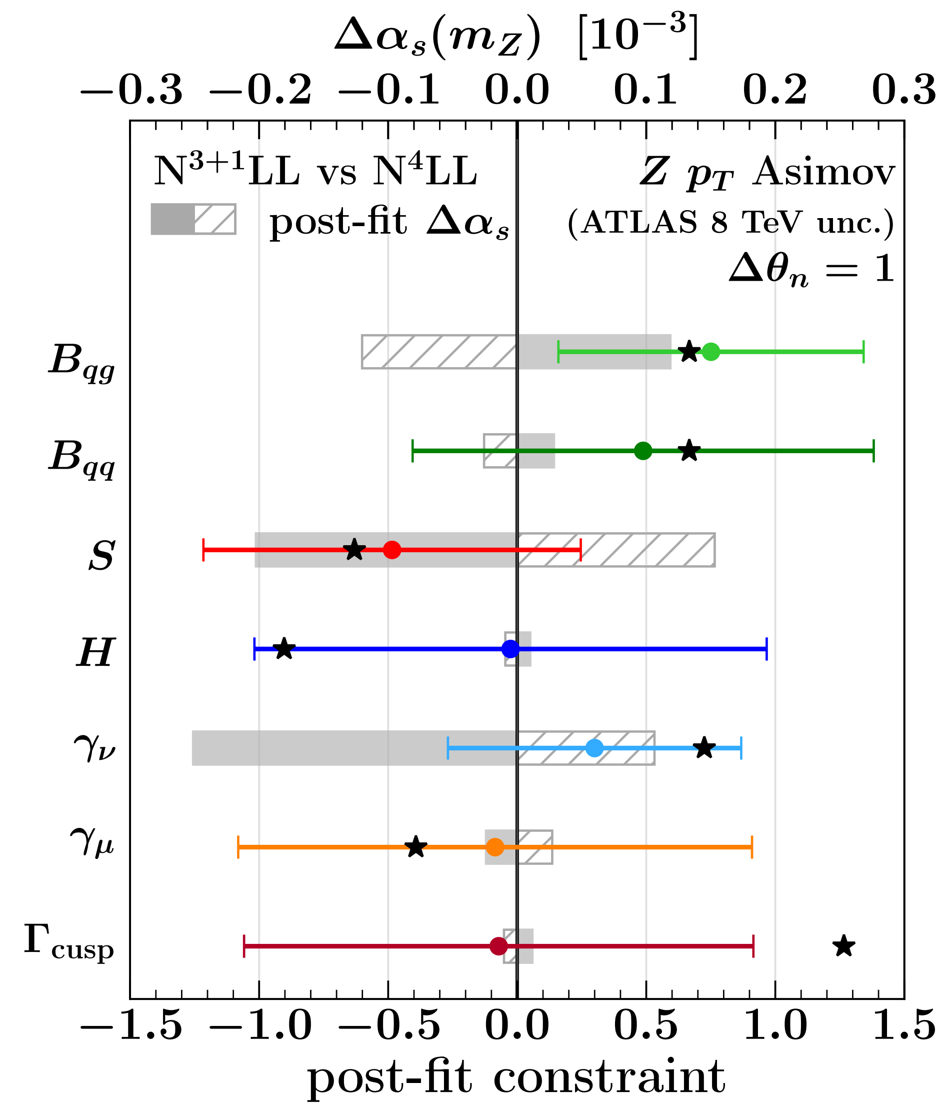
- N^{2+1}LL *strongly pulled*, toward correct true values [★]
  - indication that the order is not enough for the data
- post-fit prediction for q_T spectrum driven by constraints from data
- grey → TNP down variation, dashed grey → TNP up variation

  γ_ν and B_{qg} have the largest remaining impact on $\alpha_s(m_Z)$ after profiling

Post-fit constraints on $N^{3+1}LL$

Profiling lower order against higher order: $N^{3+1}LL$





- $N^{3+1}LL$ pulled, toward correct true values [★]
- post-fit prediction for q_T spectrum driven by constraints from data
- γ_ν, S and B_{qg} have the largest remaining impact on $\alpha_s(m_Z)$ after profiling

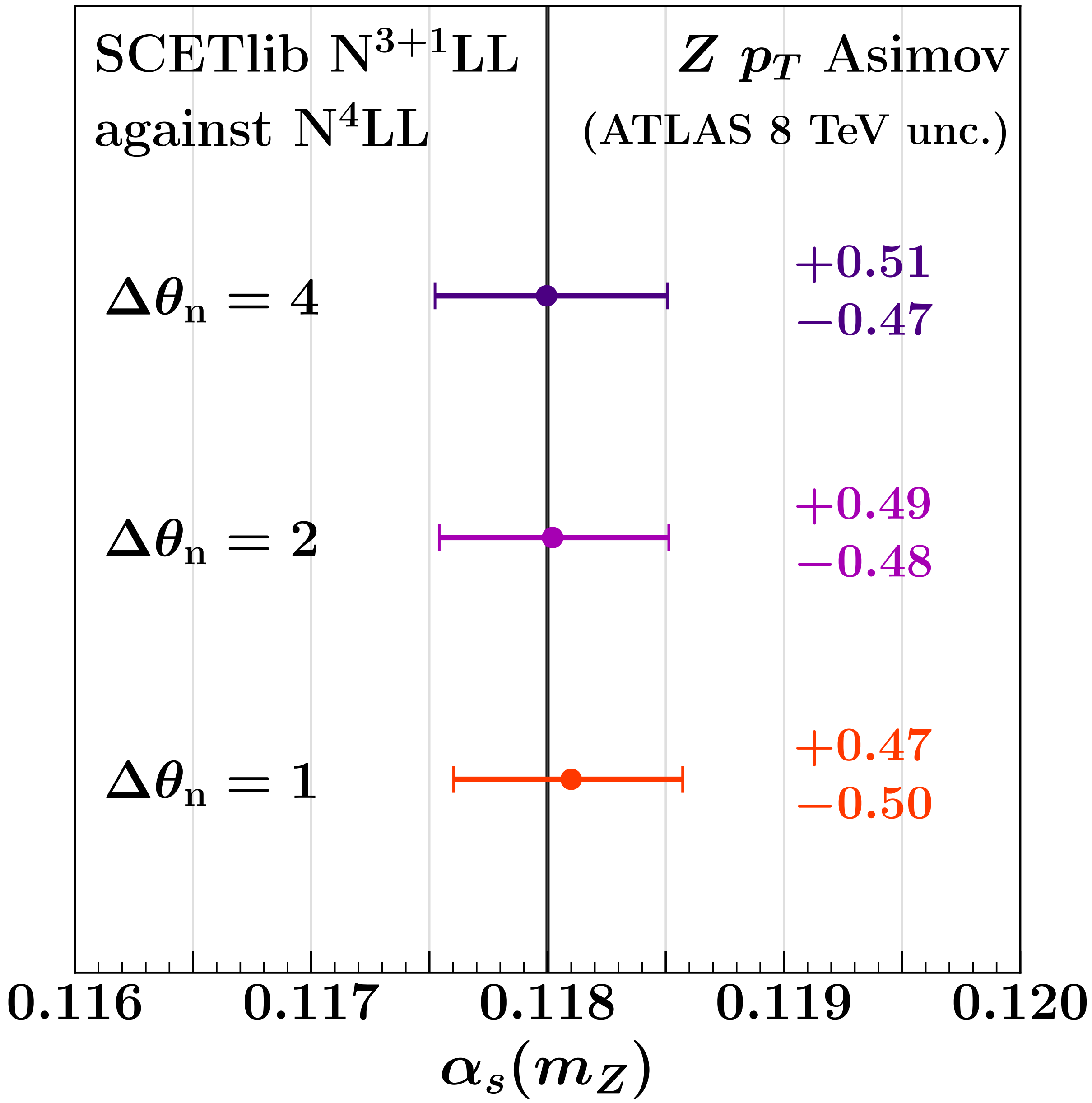
⟶ for the exact correlation between parameters, look at the post-fit covariance matrix!

Different theory constraints on TNPs

What happens by changing the prior theory constraint?

Using now $\theta_n = 0 \pm \Delta\theta_n$ with $\Delta\theta_n = 1, 2, 4$

Fit N^{3+1}LL against N^4LL data

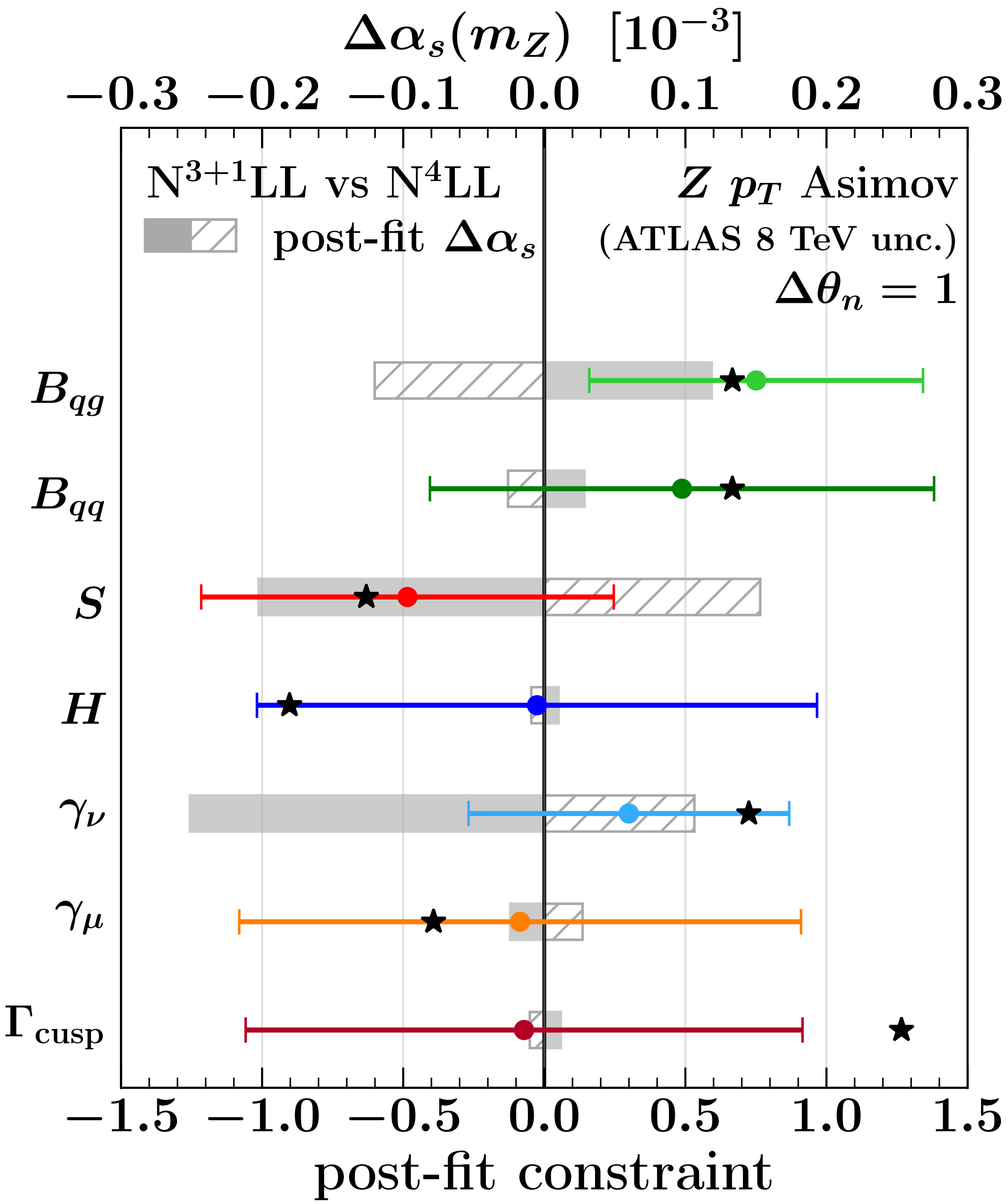


- profiling substantially reduces the dependence on theory constraint (with scanning, α_s unc. directly depends on choice of $\Delta\theta_n$)
- the effect relative to the theory constraint strongly depends on the power of the experimental constraint
- only slight difference in the uncertainties when relaxing the TNP constraint

* uncertainties in units of 10^{-3}

Different theory constraints on TNPs

Using now $\theta_n = 0 \pm \Delta\theta_n$ with $\Delta\theta_n = 1$



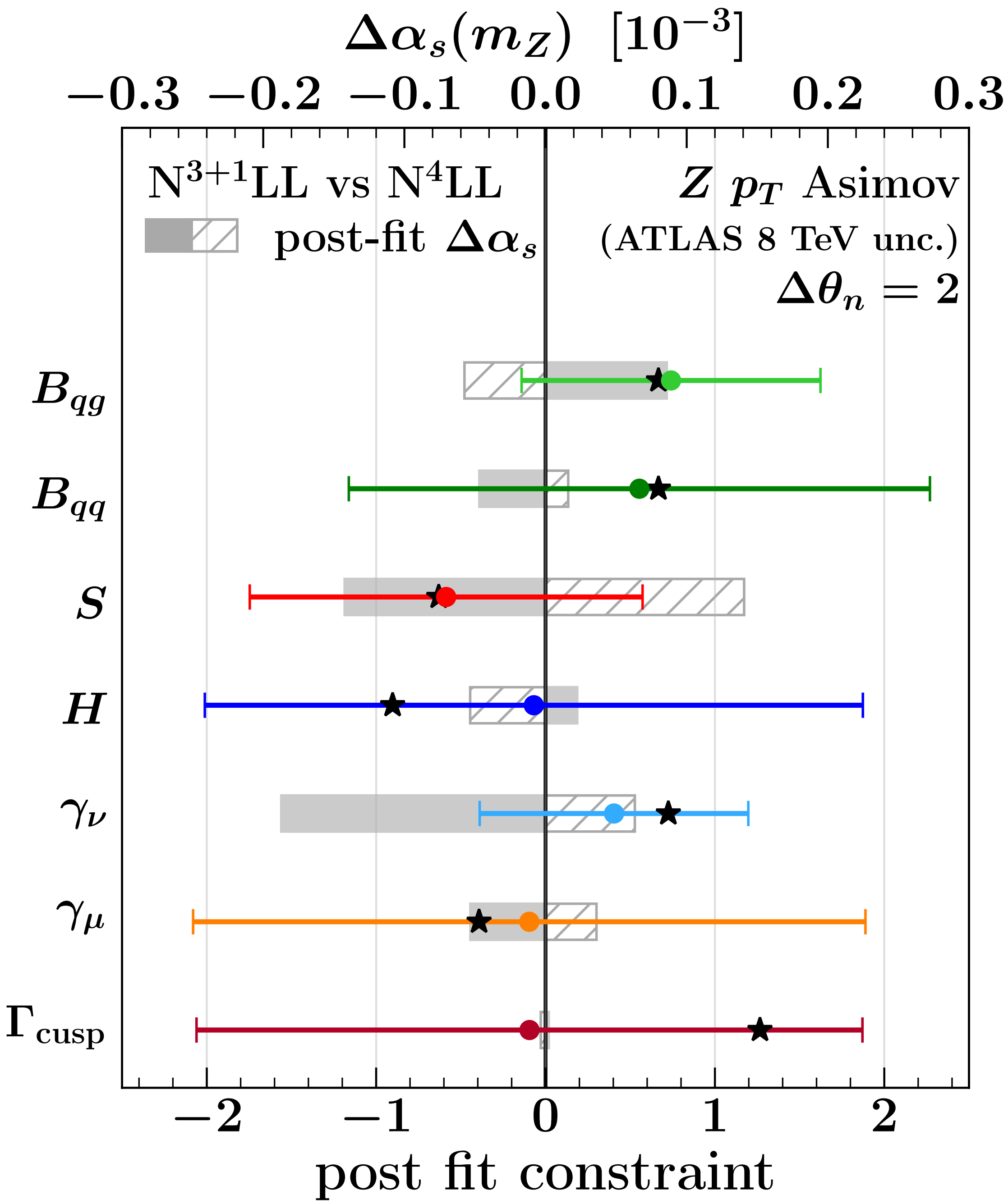
[don't be fooled by the different x-range!]

Fit $N^{3+1}LL$ against N^4LL data

1 $\Delta\theta_n = 1$ start seeing the exp. constraint

Different theory constraints on TNPs

Using now $\theta_n = 0 \pm \Delta\theta_n$ with $\Delta\theta_n = 2$



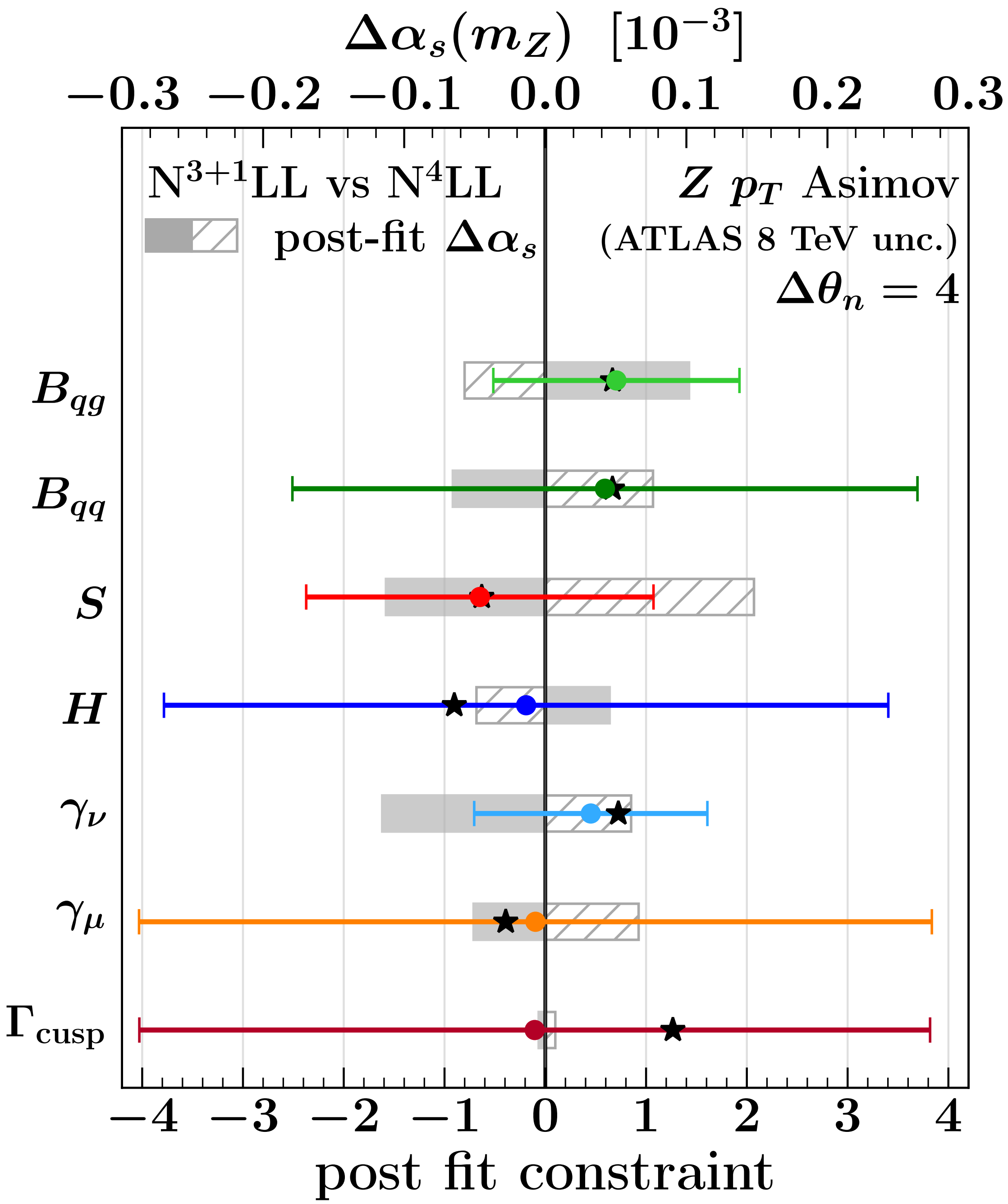
[don't be fooled by the different x-range!]

Fit N^{3+1}LL against N^{4}LL data

1. $\Delta\theta_n = 1$ start seeing the exp. constraint
2. $\Delta\theta_n = 2$ it's basically a factor 2 w.r.t $\Delta\theta_n = 1$

Different theory constraints on TNPs

Using now $\theta_n = 0 \pm \Delta\theta_n$ with $\Delta\theta_n = 4$



[don't be fooled by the different x-range!]

Fit $N^{3+1}LL$ against N^4LL data

1. $\Delta\theta_n = 1$ start seeing the exp. constraint
2. $\Delta\theta_n = 2$ it's basically a factor 2 w.r.t $\Delta\theta_n = 1$
3. $\Delta\theta_n = 4$ data can constrain TNPs more

Nonperturbative uncertainty in Asimov fit

1 Collins-Soper (CS) kernel [∼ rapidity anomalous dimensions]:

$$\tilde{\gamma}_\nu(b_T) = \tilde{\gamma}_\nu^{\text{pert}}\left(b_6^*(b_T)\right) + \tilde{\gamma}_\nu^{\text{nonp}}(b_T) \qquad \tilde{\gamma}_\nu^{\text{nonp}}(b_T) = -\lambda_\infty f_\nu\left(\frac{\lambda_2}{\lambda_\infty}b_T^2 + \frac{\lambda_4}{\lambda_\infty}b_T^4\right)$$

2 Transverse Momentum Distributions (TMDs)[∼ intrinsic k_T of the partons inside the protons]:

$$\tilde{f}(b_T) = \tilde{f}_{\text{pert}}(b_T)\,\tilde{f}_{\text{nonp}}(b_T)$$

$$\ln\left(\tilde{f}_{\text{nonp}}(b_T)\right) = -\Lambda_\infty\, b_T\, f\left[\frac{\Lambda_2}{\Lambda_\infty}\,b_T + \left(\frac{\Lambda_4}{\Lambda_\infty} + \frac{1}{3}\frac{\Lambda_2^3}{\Lambda_\infty^3}\right)b_T^3\right]$$

λ_2, λ_4 and Λ_2, Λ_4 quadratic/quartic small b_T coefficients

$\lambda_\infty, \Lambda_\infty$ determine $b_T \to \infty$ behavior

From Collins and Rogers '14, $f_\nu(x)$ and $f(x)$ behavior

$$\tilde{\gamma}_\nu^{\text{nonp}}(b_T \to 0) \sim b_T^2, \quad \tilde{\gamma}_\nu^{\text{nonp}}(b_T \to \infty) \sim \text{const}$$

$$\log\left(\tilde{f}_{\text{nonp}}(b_T \to 0)\right) \sim b_T^2, \quad \log\left(\tilde{f}_{\text{nonp}}(b_T \to \infty)\right) \sim b_T$$

Nonperturbative uncertainty in Asimov fit

What is used in our fits:

$$f_\nu\left(\frac{\lambda_2}{\lambda_\infty}b_T^2 + \frac{\lambda_4}{\lambda_\infty}b_T^4\right) = \tanh\left(\frac{\lambda_2}{\lambda_\infty}b_T^2 + \frac{\lambda_4}{\lambda_\infty}b_T^4\right)$$

$$f\left(\frac{\Lambda_2}{\Lambda_\infty}b_T + \frac{\Lambda_4}{\Lambda_\infty}b_T^3\right) = 2\tanh\left(\frac{\Lambda_2}{\Lambda_\infty}b_T + \frac{\Lambda_4}{\Lambda_\infty}b_T^3\right)$$

Also using inputs from lattice QCD for the CS kernel [some details in SCET24 Ploessl's talk]:

- exploit lattice QCD calculations of the CS kernel to obtain good constraints on λ_∞, λ_2 and λ_4

representative values:

$$\lambda_\infty = 1.7 \pm 0.5$$
$$\lambda_2 = 0.09 \pm 0.03$$
$$\lambda_4 = 0.007 \pm 0.007$$

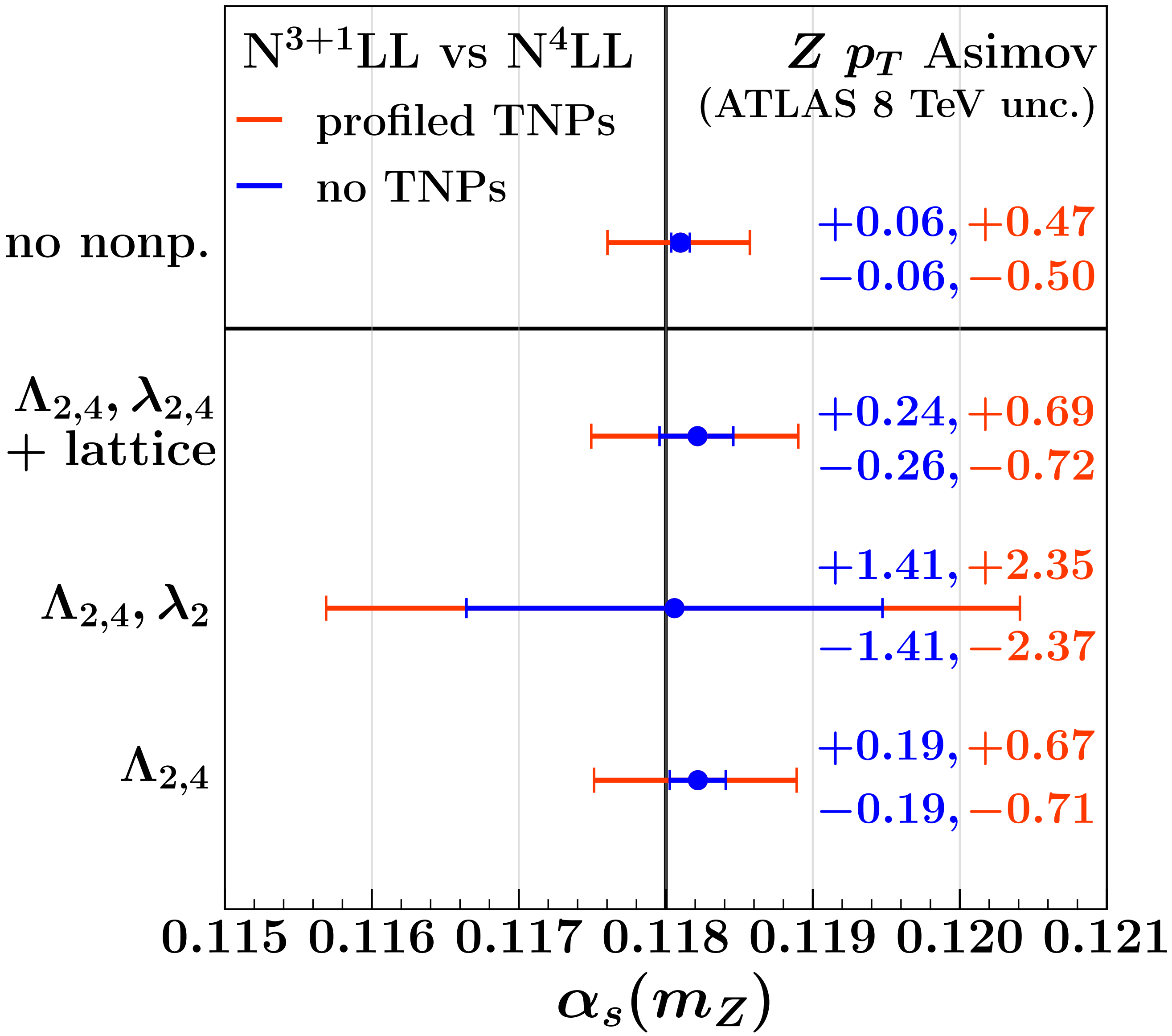
+ full covariance matrix from lattice fit

Nonperturbative uncertainty in Asimov fit

fit unc. only: fitting *only* α_S and nonp.

profiled TNPs: α_s + nonp. + TNPs

Fit $N^{3+1}LL$ against N^4LL data

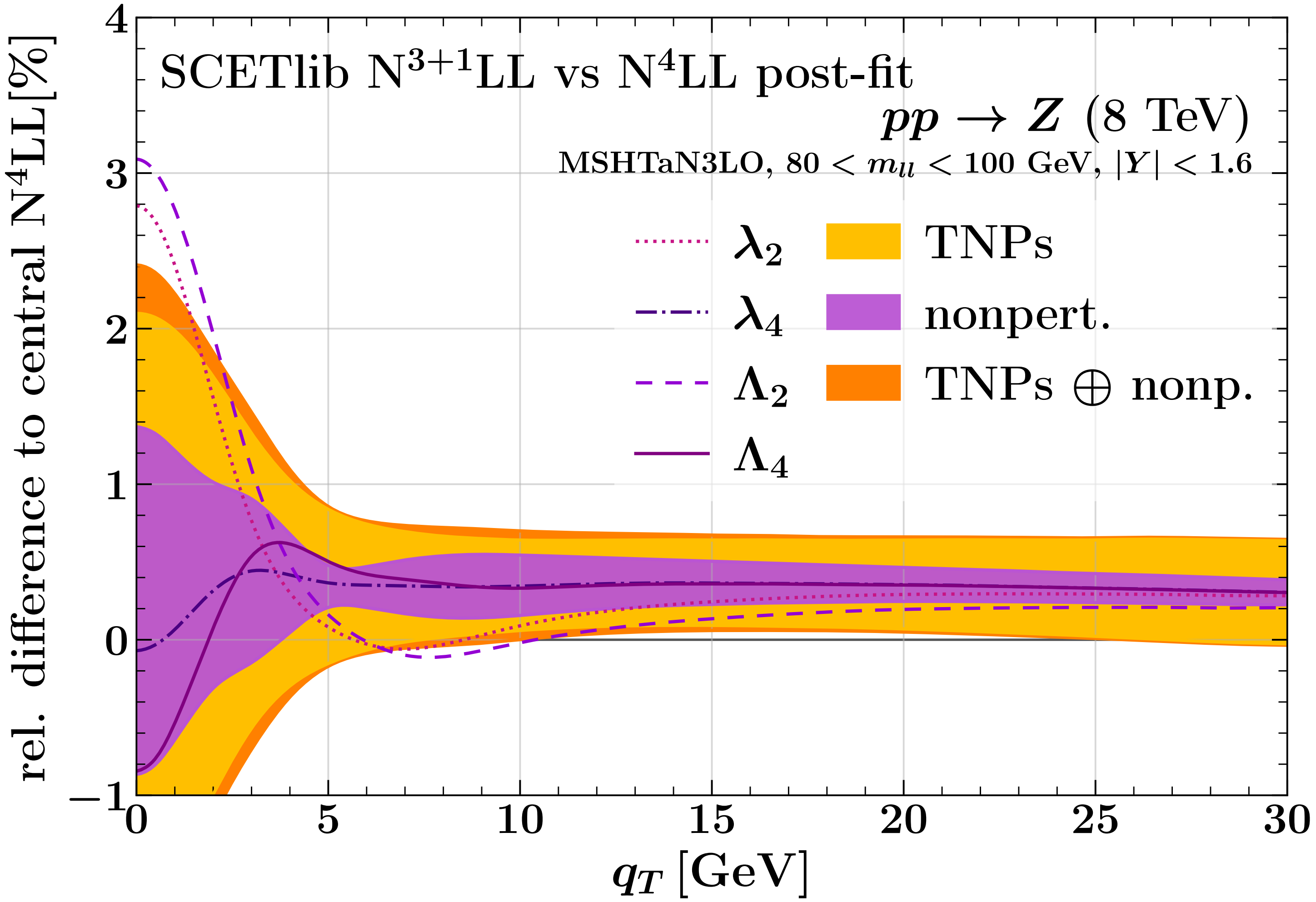


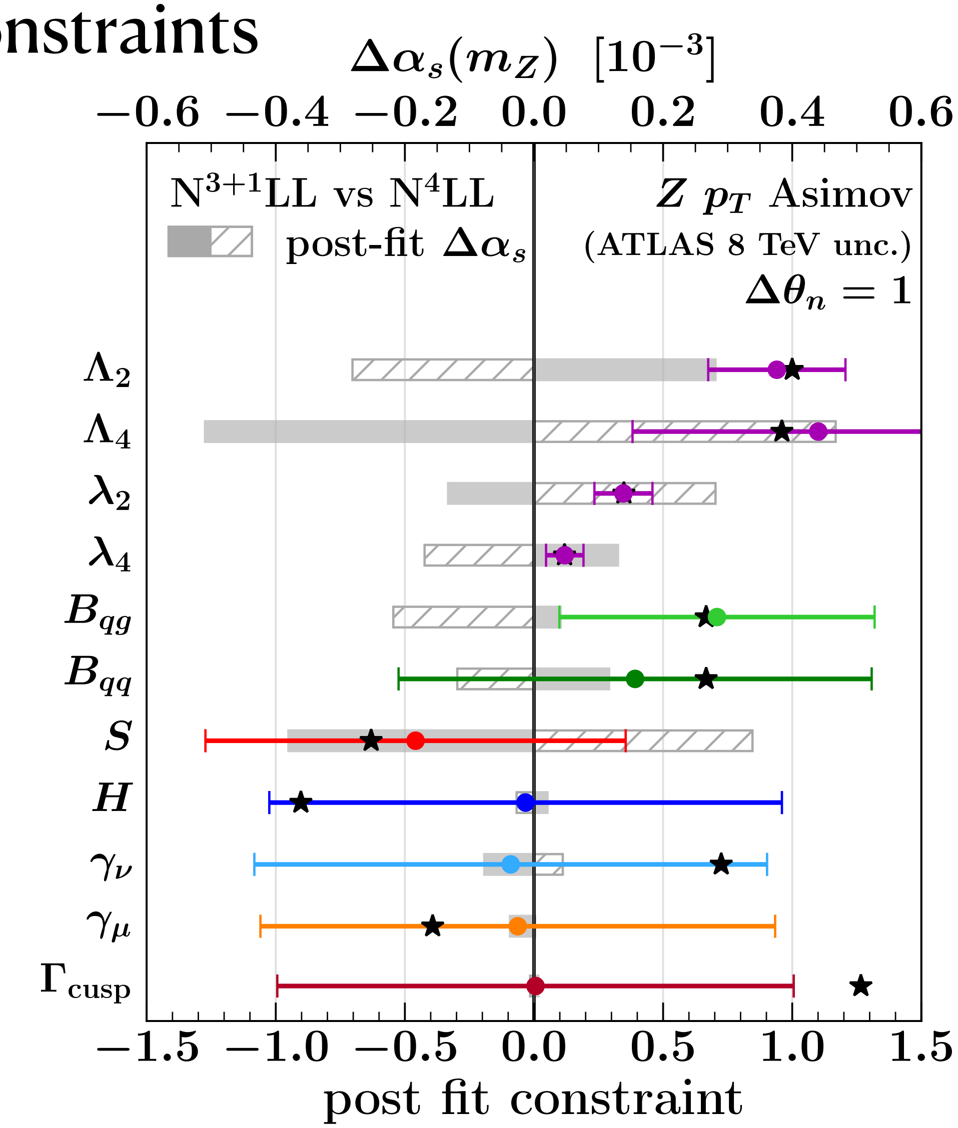
- Fit only α_S , Λ_2 and Λ_4 (fixed $\tilde{\gamma}_\nu^{\text{nonp}}$)
- Not using lattice constraints parameters fitted: λ_2, Λ_2 , Λ_4
- Using lattice constraints parameters fitted: λ_2, λ_4, Λ_2, Λ_4

* uncertainties in units of 10^{-3}

Post-fit constraints on $N^{3+1}LL$

$N^{3+1}LL$ vs N^4LL fitting nonp. and including lattice constraints





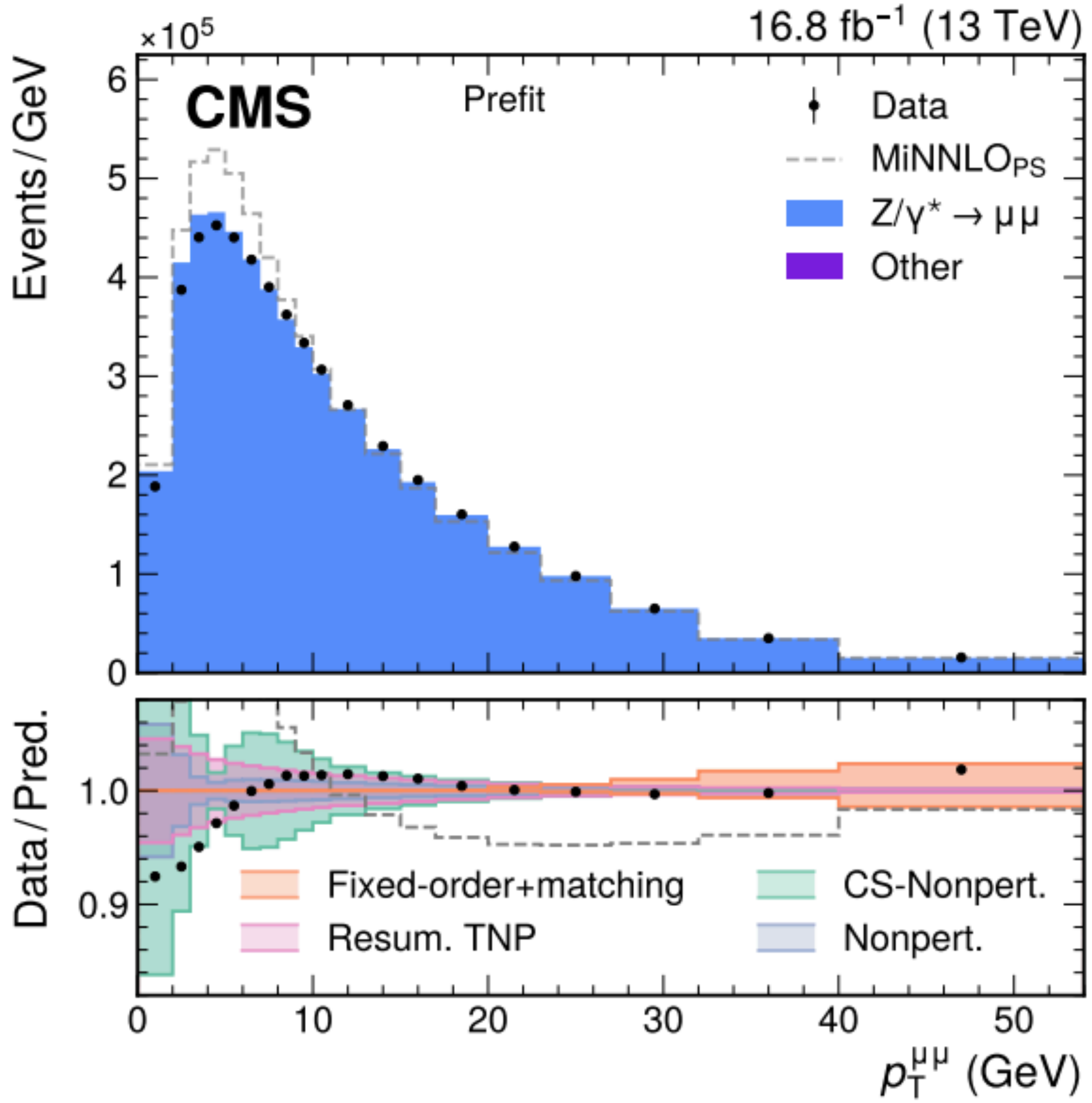
- $N^{3+1}LL$ pulled, toward correct true values [★]
- Data now also constrain nonp. params., therefor less constraint on TNPs
- S, Λ_4 and Λ_2 have the largest remaining impact on $\alpha_s(m_Z)$ after profiling

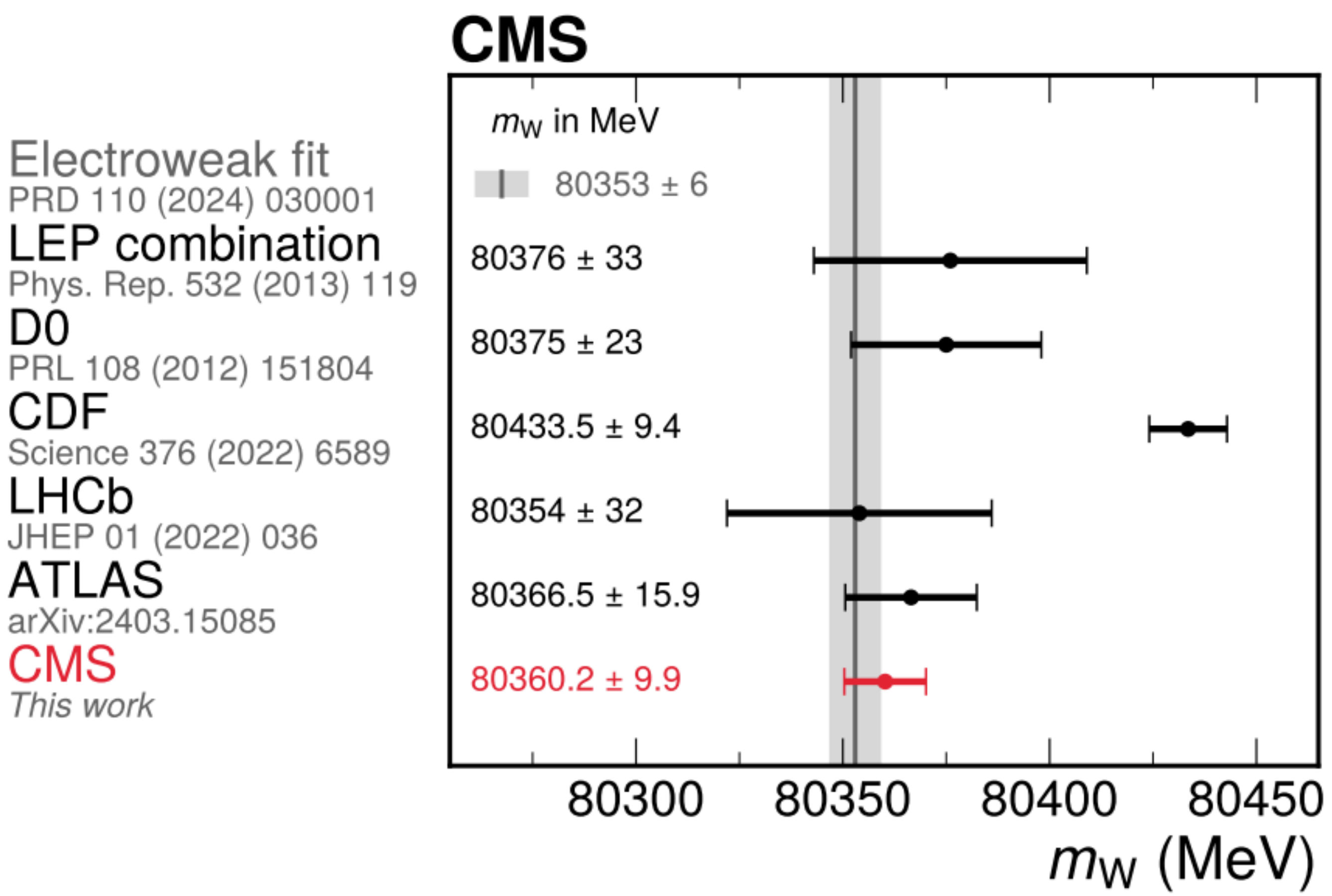
m_W determination from CMS

CMS W mass measurement

Recent CMS W mass measurement arXiv:2412.13872

Theory input: $N^{3+0}LL$+NNLO (SCETlib and DYturbo)



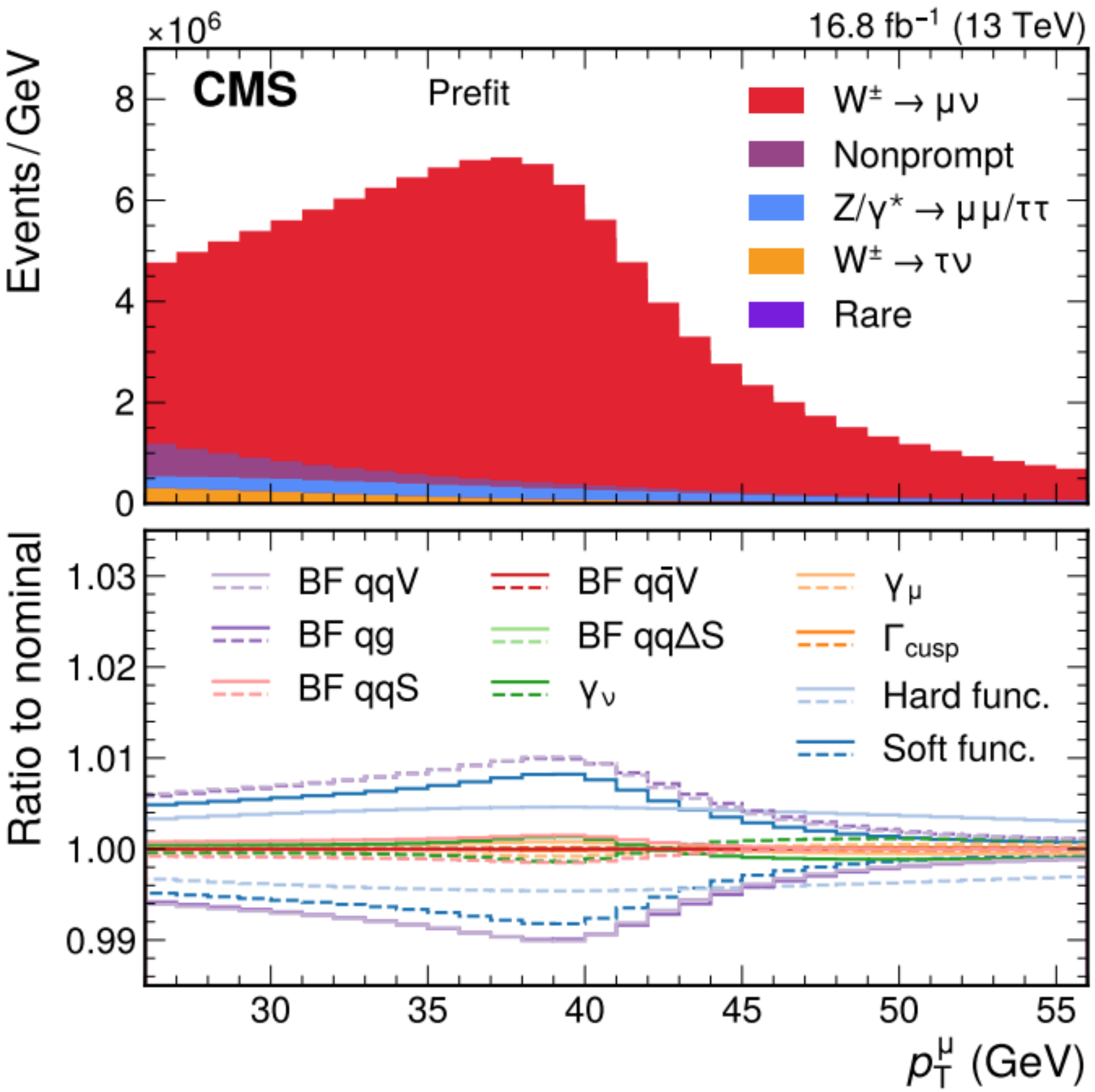


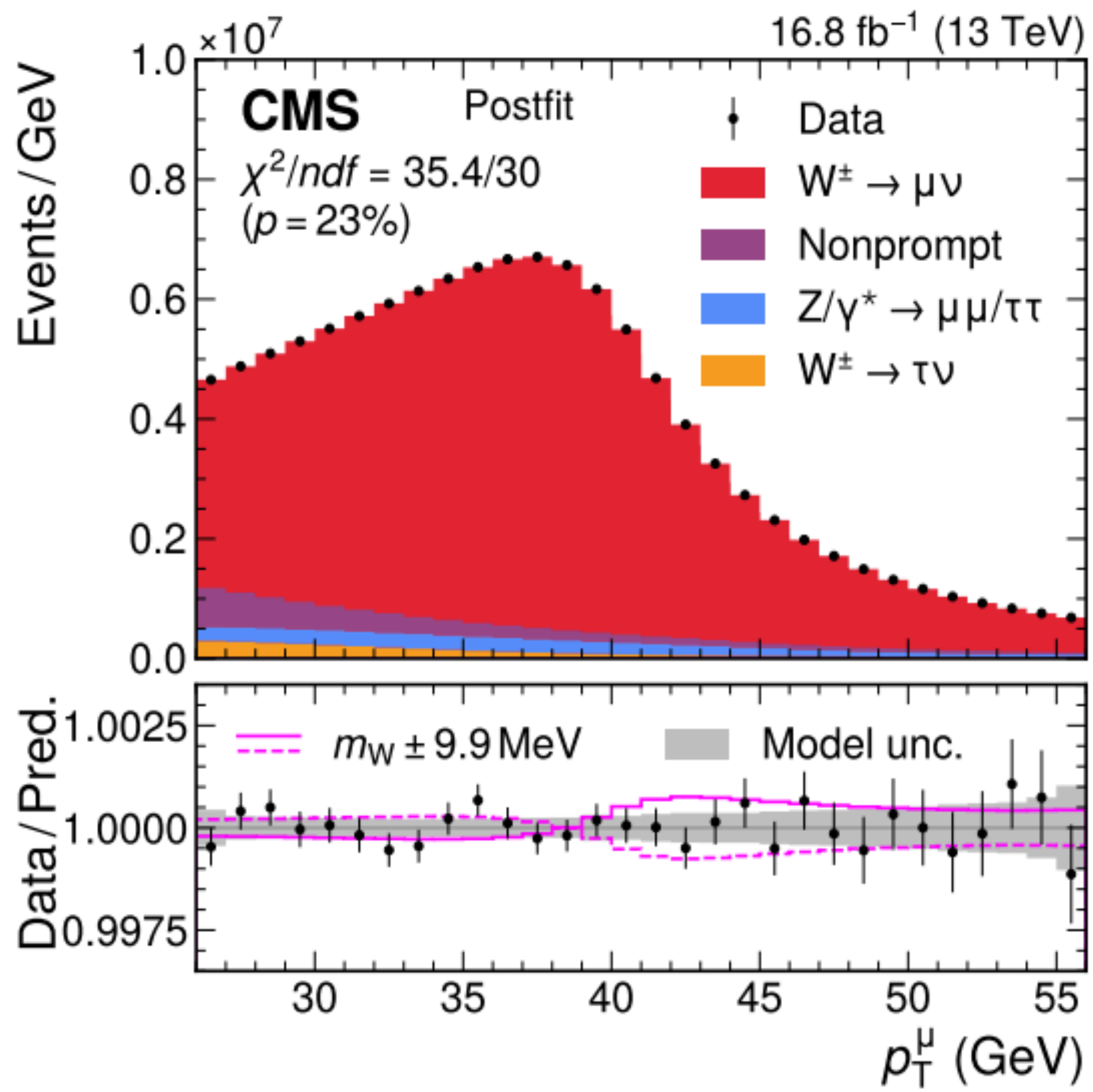
- p_T^W modeling fundamental: uncertainties in the low p_T region affect the shape as m_W variation
- theory correlations are crucial: uncertainty propagated from p_T^W to p_T^μ to m_W!

CMS *W* mass measurement

Perturbative uncertainties in the resummed prediction: $N^{3+0}LL^*$ SCETlib

- contribution of all theoretical and experimental uncert. before and after profiling





* used in the CMS m_W determination

Summary

Theory uncertainties including correct correlation are crucial for the interpretation of precision measurements:

having meaningful theory uncert. is as important as meaningful exp. uncert.!

1 Theory Nuisance Parameters perfect candidate

- include correct point-by-point correlations across the q_T spectrum, different processes ,...
- can be constrained by data reducing theory uncertainty

2 Perturbative uncertainty with TNPs

- perturbative uncertainty can be correctly profiled
- TNPs not "easy and cheap" as scale variation, but worth it!

3 First applications to Drell-Yan work as advertised

- enabled recent precision m_W measurement by CMS
- very promising extraction of α_s from Z q_T spectrum

THANK YOU!

Backup slides

Resummation details

Leading power cross section: $VV' = \{\gamma\gamma, \gamma Z, Z\gamma, ZZ, W^+W^+, W^-W^-\}$ $\qquad x_{a,b} = \frac{Q}{E_{\rm cm}} e^{\pm Y}$

$$\frac{d\sigma^{(0)}}{d^4q} = \frac{1}{2E_{\rm cm}^2} L_{VV'}(q^2) \sum_{a,b} H_{VV'\,ab}(q^2,\mu) \times \int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{b}_T\cdot\vec{q}_T} \tilde{B}_a(x_a, b_T, \mu, \nu/Q)\, \tilde{B}_b(x_b, b_T, \mu, \nu/Q) \tilde{S}(b_T, \mu, \nu)$$

leptonic tensor: $L_{VV'}(q^2)$

hard function: $H_{VV'\,ab}(q^2,\mu)$

beam functions: $\tilde{B}_a(x_a, b_T, \mu, \nu/Q)\, \tilde{B}_b(x_b, b_T, \mu, \nu/Q)$

soft function: $\tilde{S}(b_T, \mu, \nu)$

Unprimed vs primed counting:

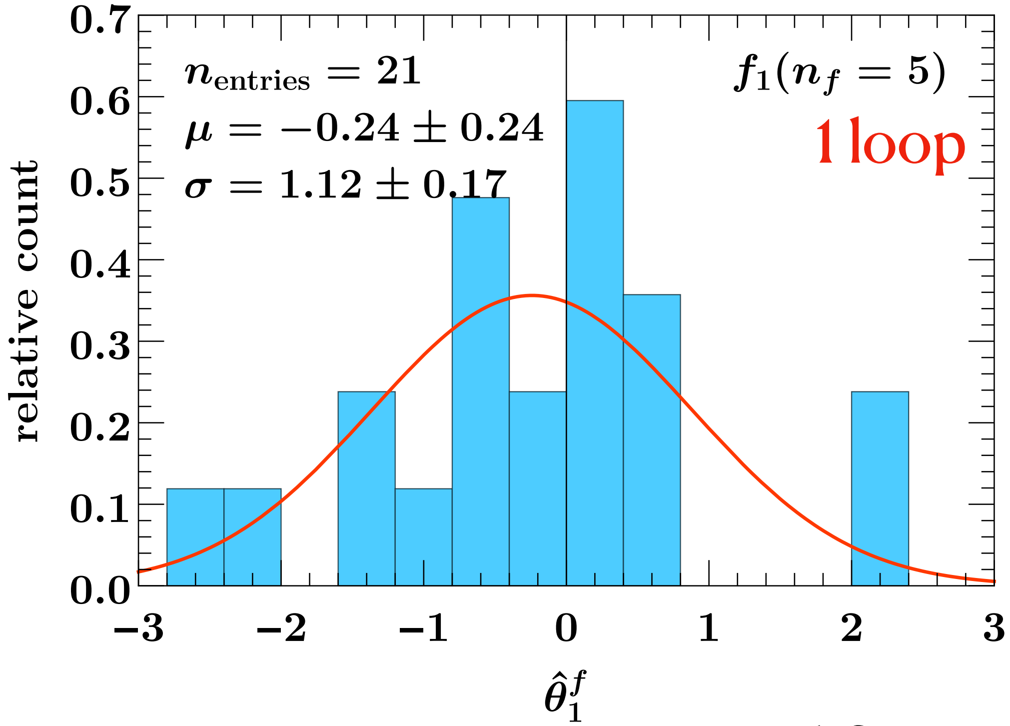
- primed orders boundary condition added to α_s^n higher

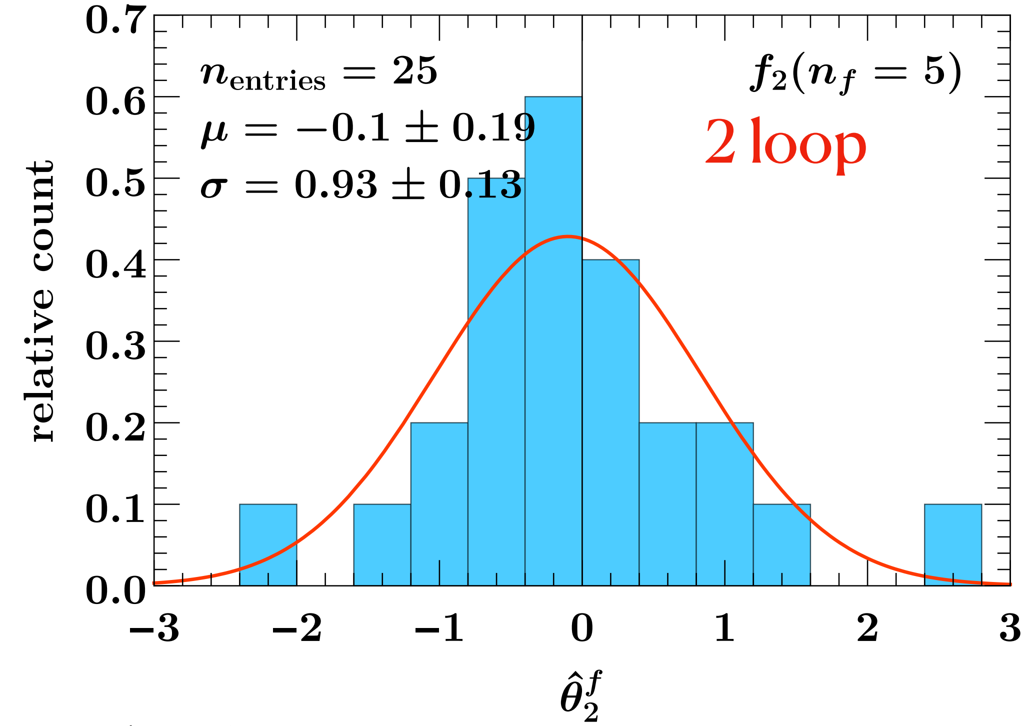
Order	Boundary cond. (FO singular)	Anomalous dimensions γ_i (noncusp)	$\Gamma_{\rm cusp}$, β	FO matching (nonsingular)
LL	1	-	1-loop	-
NLL	1	1-loop	2-loop	-
NLL′ (+NLO$_0$)	α_s	1-loop	2-loop	α_s
NNLL (+NLO$_0$)	α_s	2-loop	3-loop	α_s
NNLL′ (+NNLO$_0$)	α_s^2	2-loop	3-loop	α_s^2
N^{3}LL (+NNLO$_0$)	α_s^2	3-loop	4-loop	α_s^2
N^{3}LL′ (+N^{3}LO$_0$)	α_s^3	3-loop	4-loop	α_s^3
N^{4}LL (+N^{3}LO$_0$)	α_s^3	4-loop	5-loop	α_s^3

[from Michel's talk]

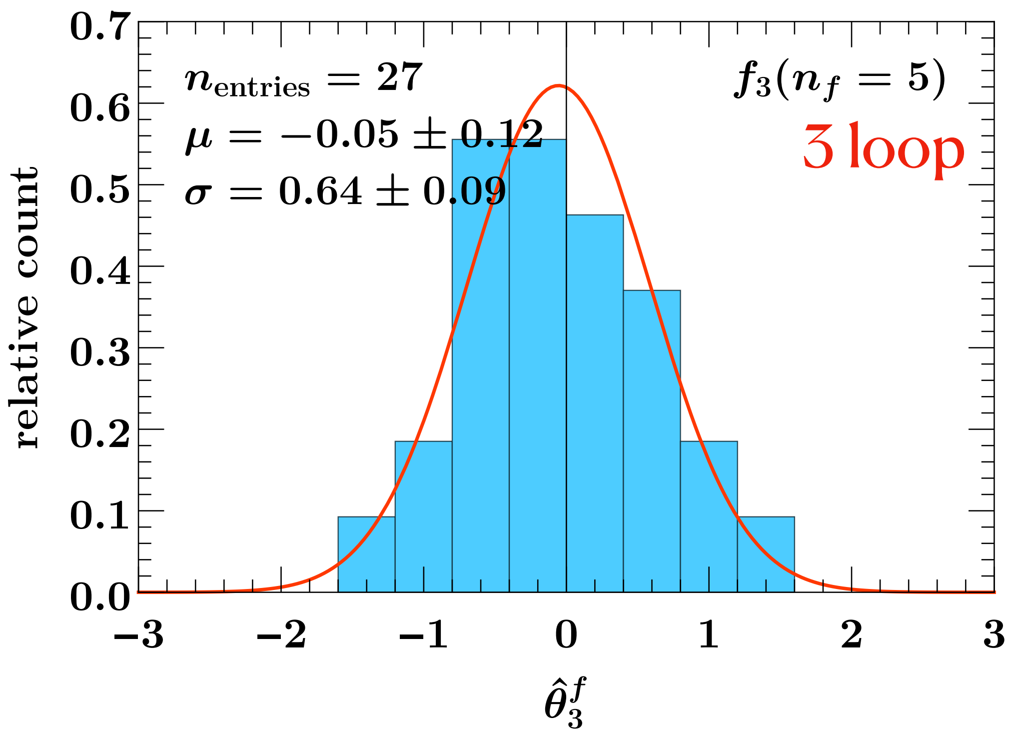
TNPs for Boundary Conditions

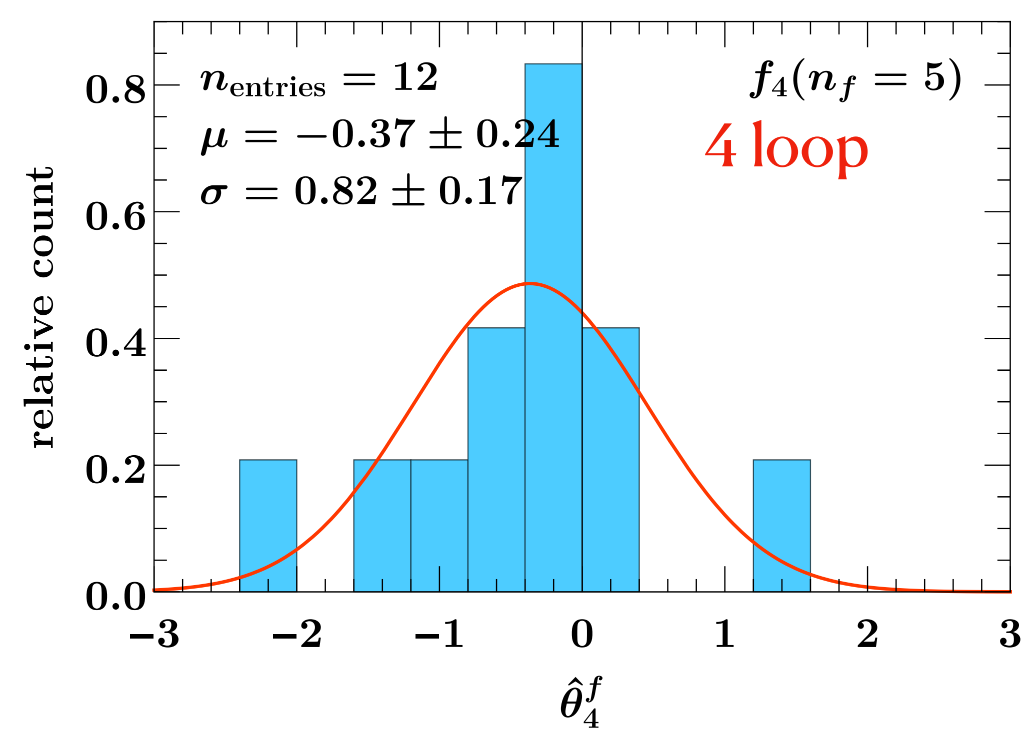
$$F_n(\theta_n) = 4C_r(4C_A)^{n-1}(n-1)!\,\theta_n^f$$





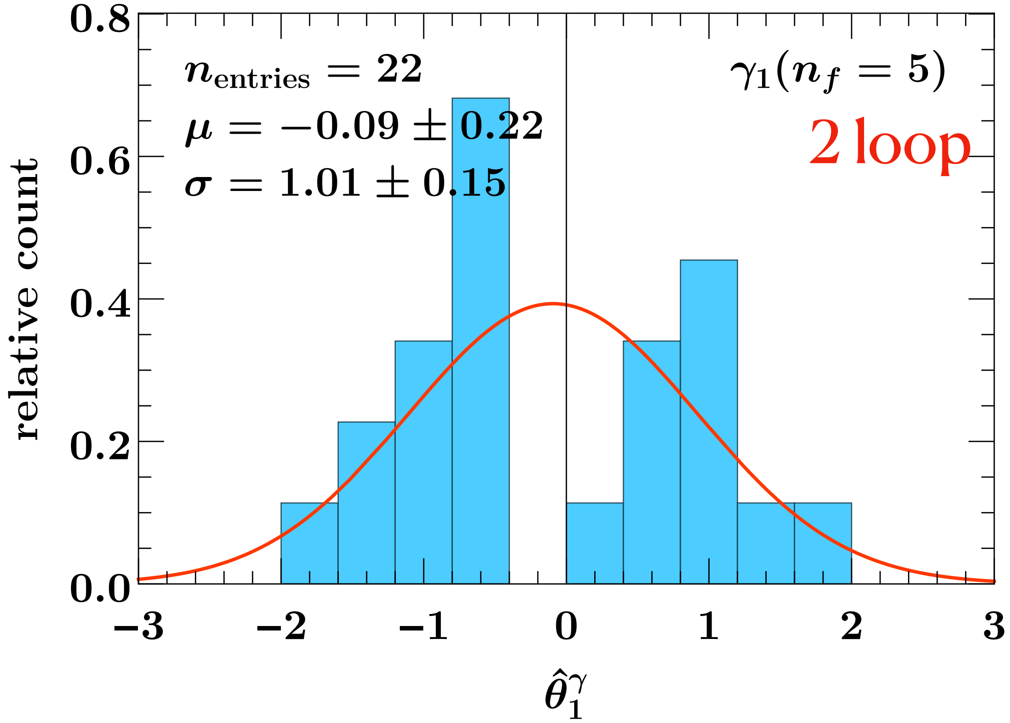
Good fit to a Gaussian with $\theta_n \approx 0$ and $\Delta\theta_n \approx 1$

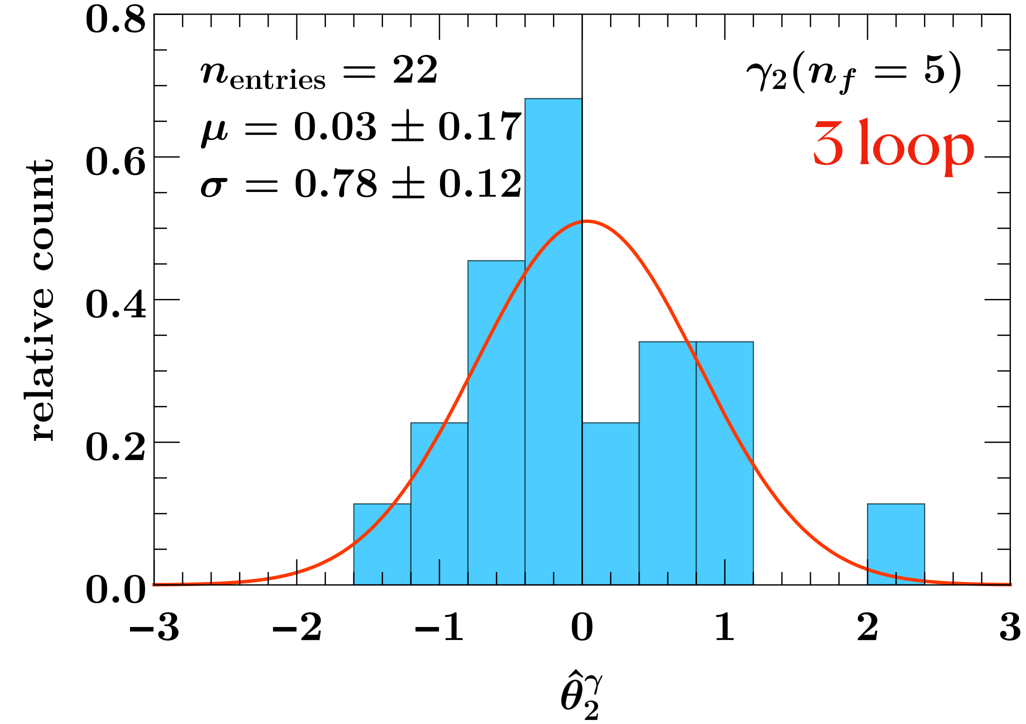




TNPs for Anomalous Dimensions

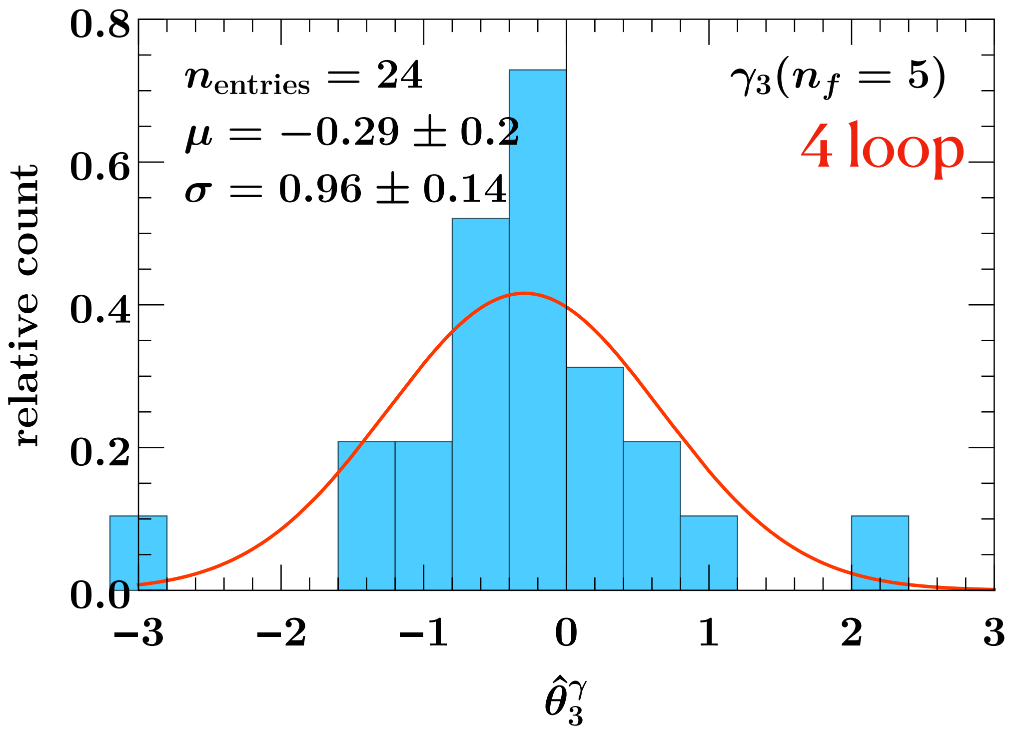
$$\gamma_n(\theta_n) = 4C_r(4C_A)^n\,\theta_n^\gamma$$

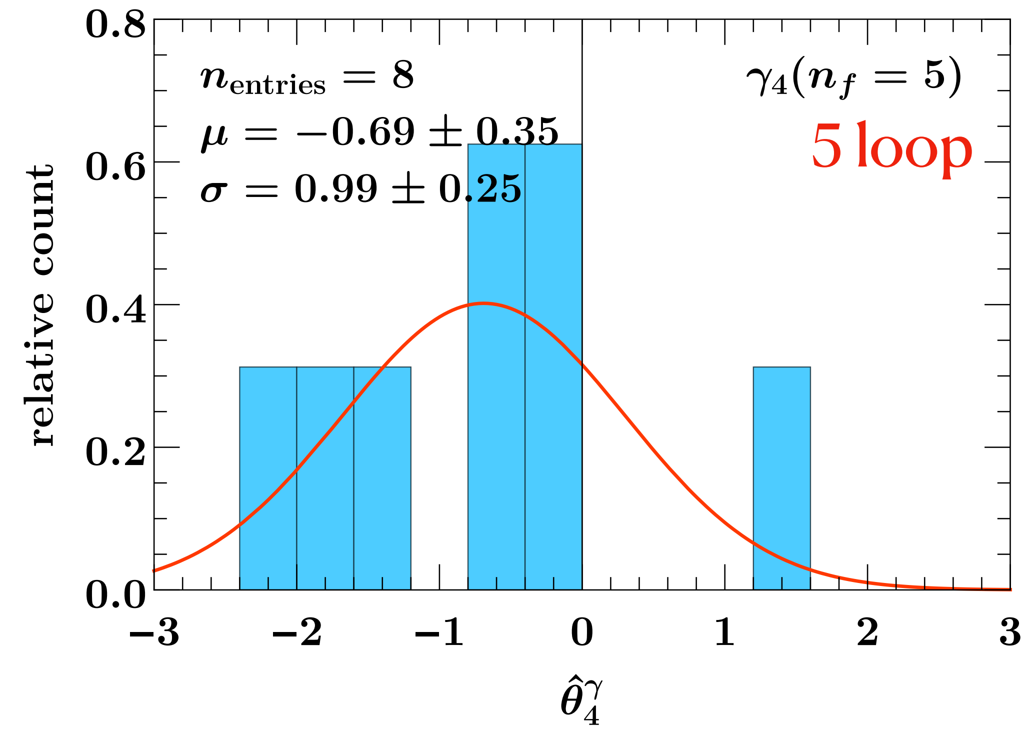




Good fit to a Gaussian with $\theta_n \approx 0$ and $\Delta\theta_n \approx 1$

✓





TNPs for Drell-Yan q_T spectrum

4 How to ***vary*** θ_n?

$$F_n(\theta_n) = 4C_r(4C_A)^{n-1}(n-1)!\,\theta_n^F$$

Validated using known perturbative series:

$$\text{BC:} \quad F(\alpha_S) = 1 + \frac{\alpha_S}{4\pi}F_1 + \left(\frac{\alpha_S}{4\pi}\right)^2 F_2 + \left(\frac{\alpha_S}{4\pi}\right)^3 F_3 + \left(\frac{\alpha_S}{4\pi}\right)^4 F_4 + \mathcal{O}\left(\alpha_S^5\right)$$

factorizing out $\left(\frac{\alpha_S}{4\pi}\right)^n$	1	+4.9	−24.0	−4065.5	−123979.0	C_{gg}
	1	−8.5	−48.6	−1386.7	−42014.9	$C^V_{q\bar{q}}$
factorizing out 4^n	1	+1.2	−1.5	−63.5	−484.3	
	1	−2.1	−3.0	−21.7	−164.1	
factorizing out $C_r C_A^{n-1}$	1	+0.4	−0.2	−2.4	−5.9	
	1	−1.6	−0.8	−1.8	−4.6	
factorizing out $(n-1)!$	1	+0.4	−0.2	−1.2	−1.0	
	1	−1.6	−0.8	−0.9	−0.8	

→ $\theta_n = 0 \pm \mathcal{O}(1)$

TNPs for Drell-Yan q_T spectrum

4 How to ***vary*** θ_n?

$$\gamma_n(\theta_n) = 4C_r(4C_A)^n\,\theta_n^\gamma$$

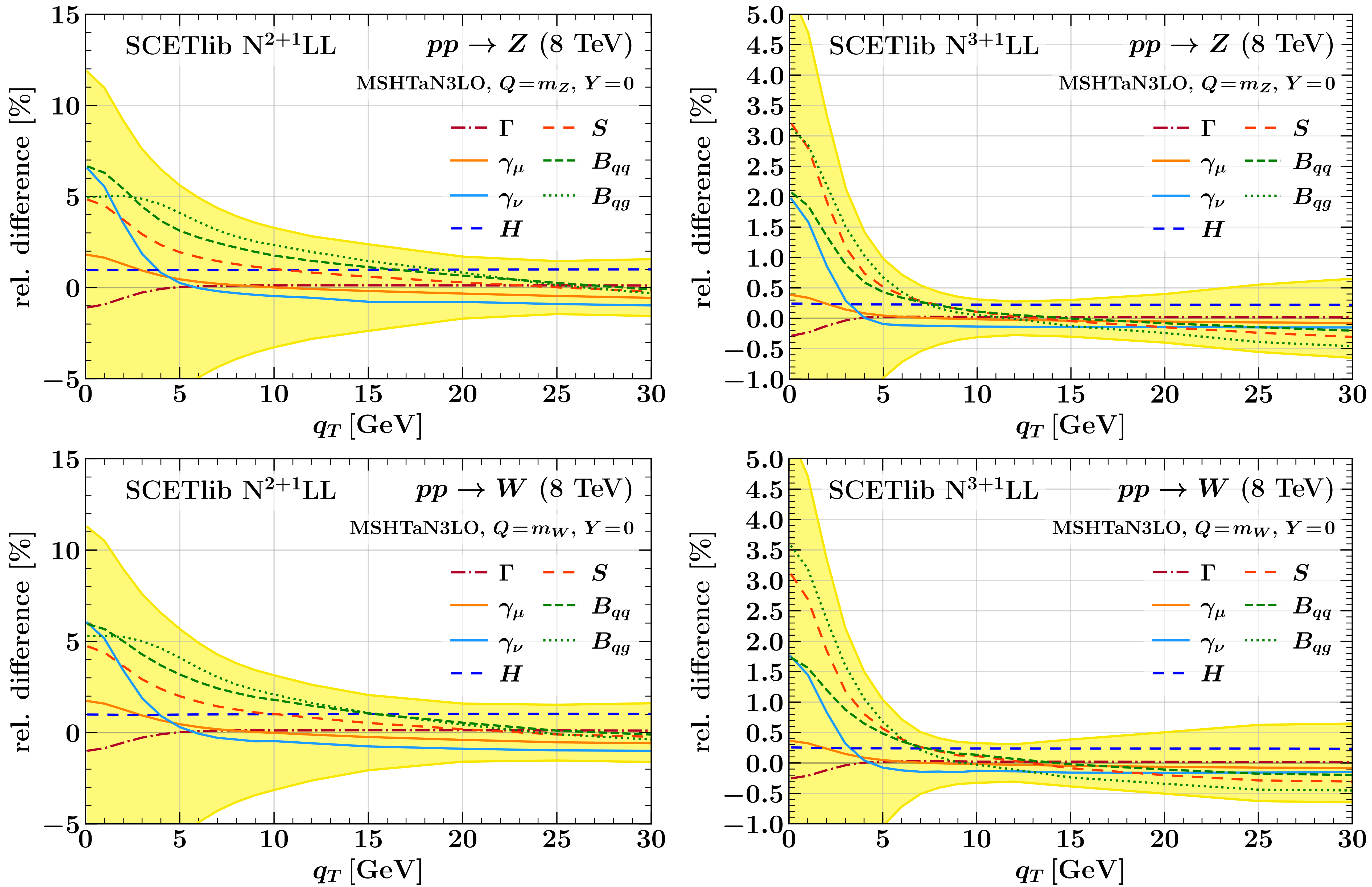
Validated using known perturbative series:

$$\text{AD:}\quad \gamma(\alpha_S) = \frac{\alpha_S}{4\pi}\gamma_1 + \left(\frac{\alpha_S}{4\pi}\right)^2\gamma_2 + \left(\frac{\alpha_S}{4\pi}\right)^3\gamma_3 + \left(\frac{\alpha_S}{4\pi}\right)^4\gamma_4 + \left(\frac{\alpha_S}{4\pi}\right)^5\gamma_5 + \mathcal{O}\left(\alpha_S^6\right)$$

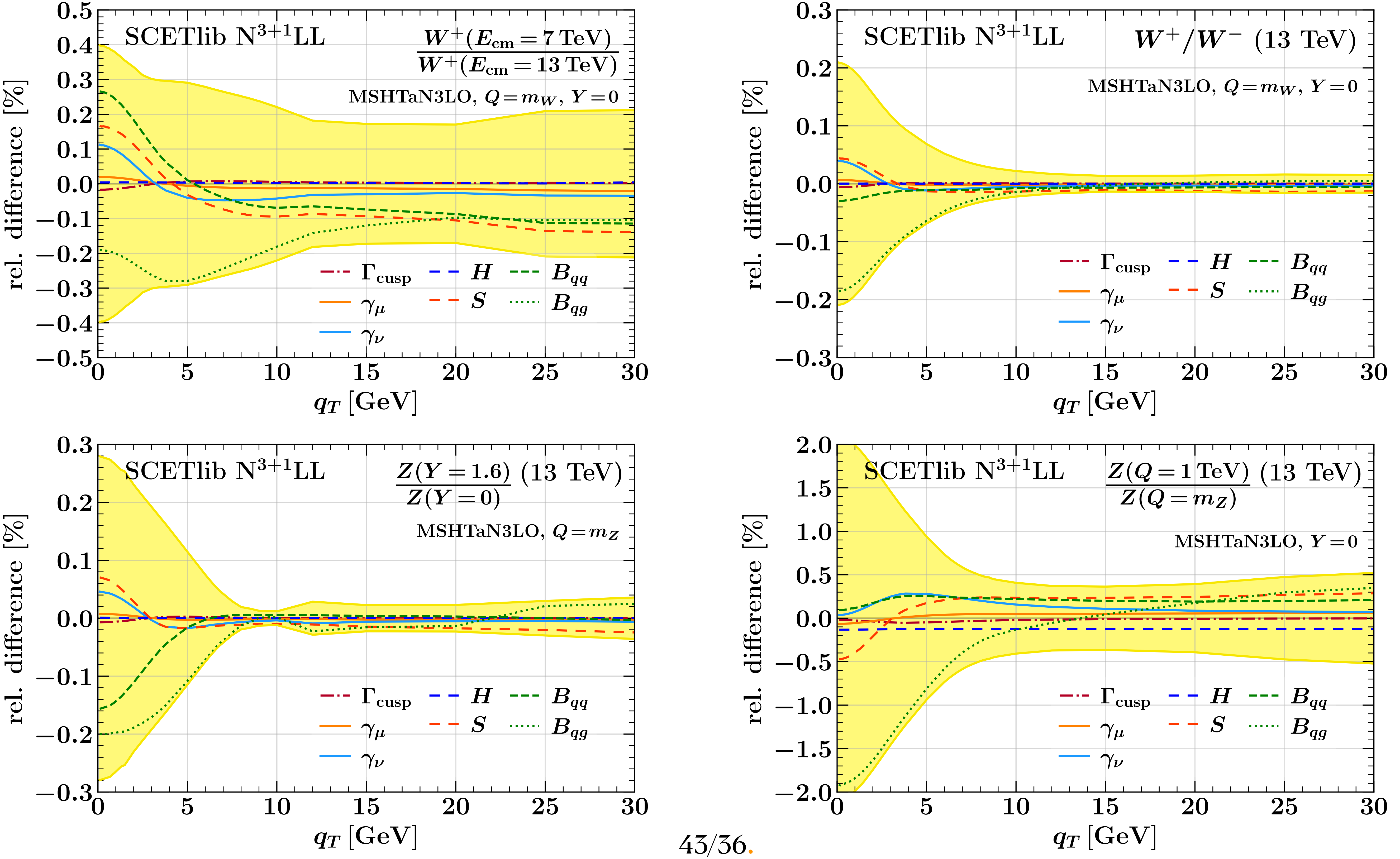
factorizing out $\left(\frac{\alpha_S}{4\pi}\right)^n$	4.0	56.2	474.9	2824.8	42824.1	$-\gamma_m/2$
	5.3	36.8	239.2	141.2	70000.0	Γ_{cusp}
factorizing out $4 \cdot 4^n$	2.0	7.0	14.8	22.1	83.6	
	2.7	4.6	7.5	1.1	136.7	
factorizing out $C_r C_A^n$	1.5	1.8	1.2	0.6	0.8	
	2.0	1.5	0.6	0.03	1.3	

$\longrightarrow$ $\theta_n = 0 \pm 1$

Application to Drell-Yan q_T spectrum



Application to Drell-Yan q_T spectrum



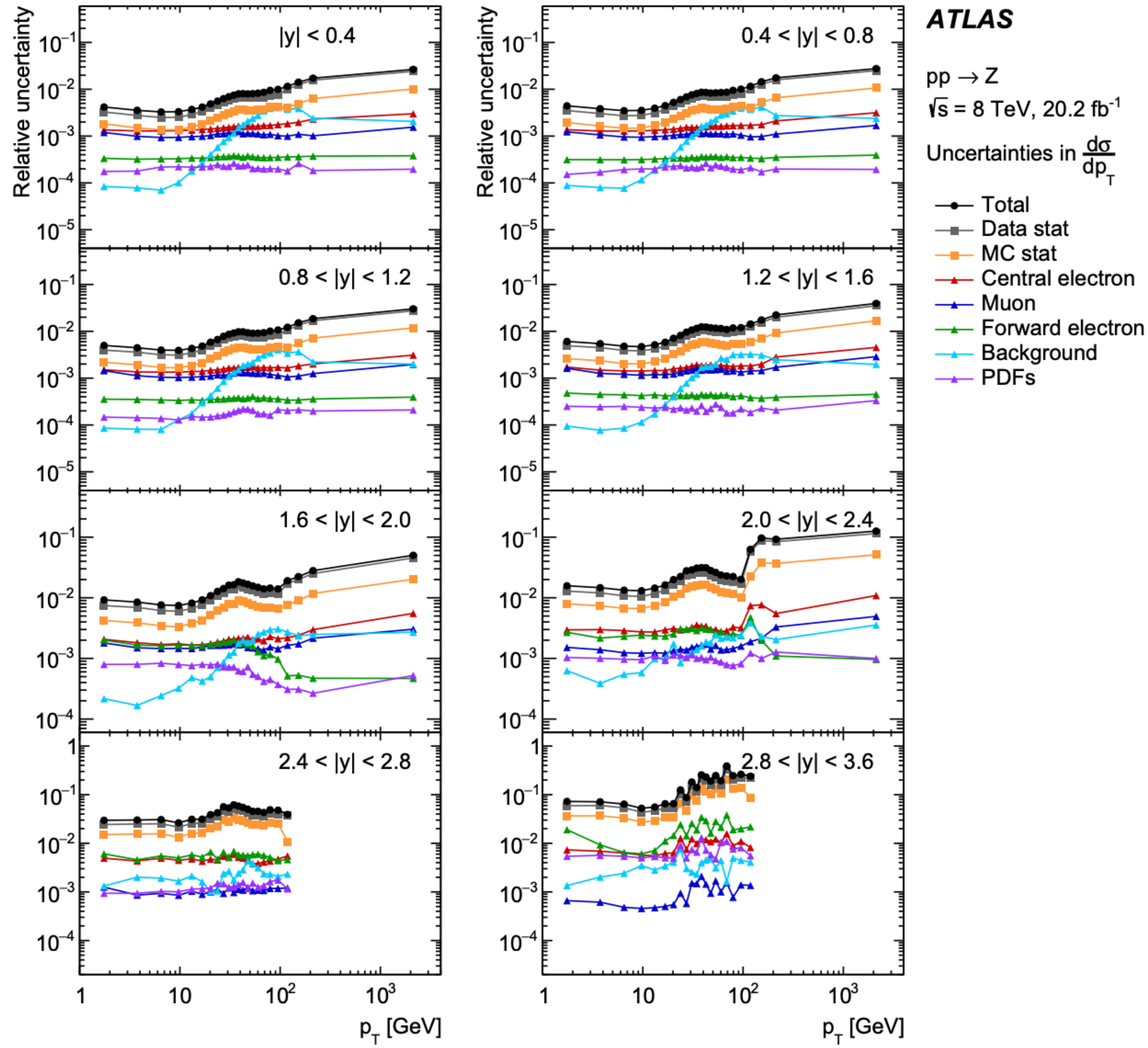
$\alpha_s(m_Z)$ determination from ATLAS [arXiv:2309.12986]

Fit of strong coupling constant using q_T in [0,29] GeV based on N^{3}LO+N^{4}LLa from DYTurbo

$\alpha_S(m_Z) = 0.1183 \pm 0.0009$

Breakdown of uncertainties on α_s in units of 10^{-3}:

Experimental uncertainty	±0.44	
PDF uncertainty	±0.51	
Scale variation uncertainties	±0.42	
Matching to fixed order	0	−0.08
Non-perturbative model	+0.12	−0.20
Flavour model	+0.40	−0.29
QED ISR	±0.14	
N^{4}LL approximation	±0.04	
Total	+0.91	−0.88



CMS W mass measurement

N^{3+0}LL is an approximation of N^{3+1}LL:

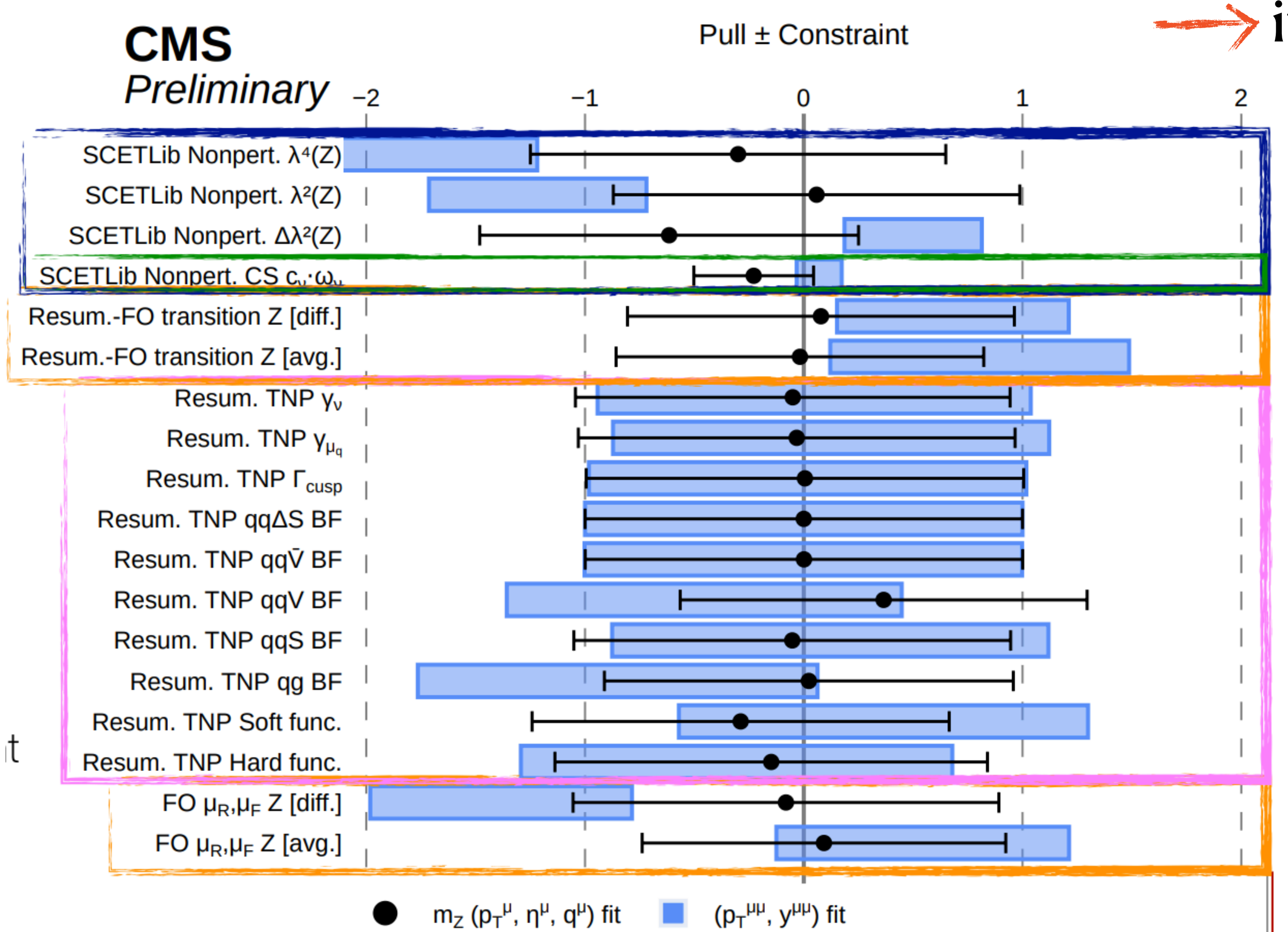
$$m_W = 80360.2 \pm 9.9\,\text{MeV}$$

$$f(\alpha, \theta_4) = \hat{f}_0 + \hat{f}_1\alpha + \hat{f}_2\alpha^2 + [\hat{f}_3 + \alpha_0 f_4(\theta_4)]\alpha^3$$

consider the N^{3}LL structure but absorb the N^{3+1}LL TNPs uncert. term into the N^{3}LL structure

- limited effect on the overall size of theory uncert. but correlation approximated by lower order structure

→ if possible prefer the N^{m+1}LL prescription!



Source of uncertainty	Impact (MeV) Nominal	Global
Muon momentum scale	4.8	4.4
Muon reco. efficiency	3.0	2.3
W and Z angular coeffs.	3.3	3.0
Higher-order EW	2.0	1.9
p_T^V modeling	2.0	0.8
PDF	4.4	2.8
Nonprompt background	3.2	1.7
Integrated luminosity	0.1	0.1
MC sample size	1.5	3.8
Data sample size	2.4	6.0
Total uncertainty	9.9	9.9

from Long's talk

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