The Quality/Cosmology Tension for a Post-Inflation QCD Axion

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Massachusetts Institute of Technology

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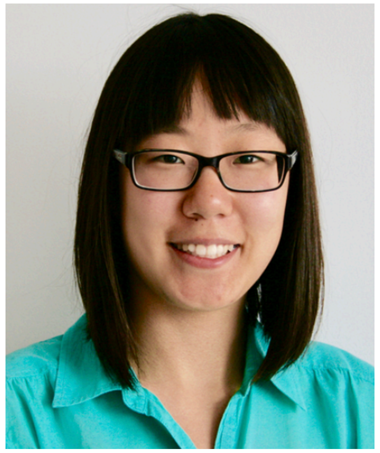
Collaboration with

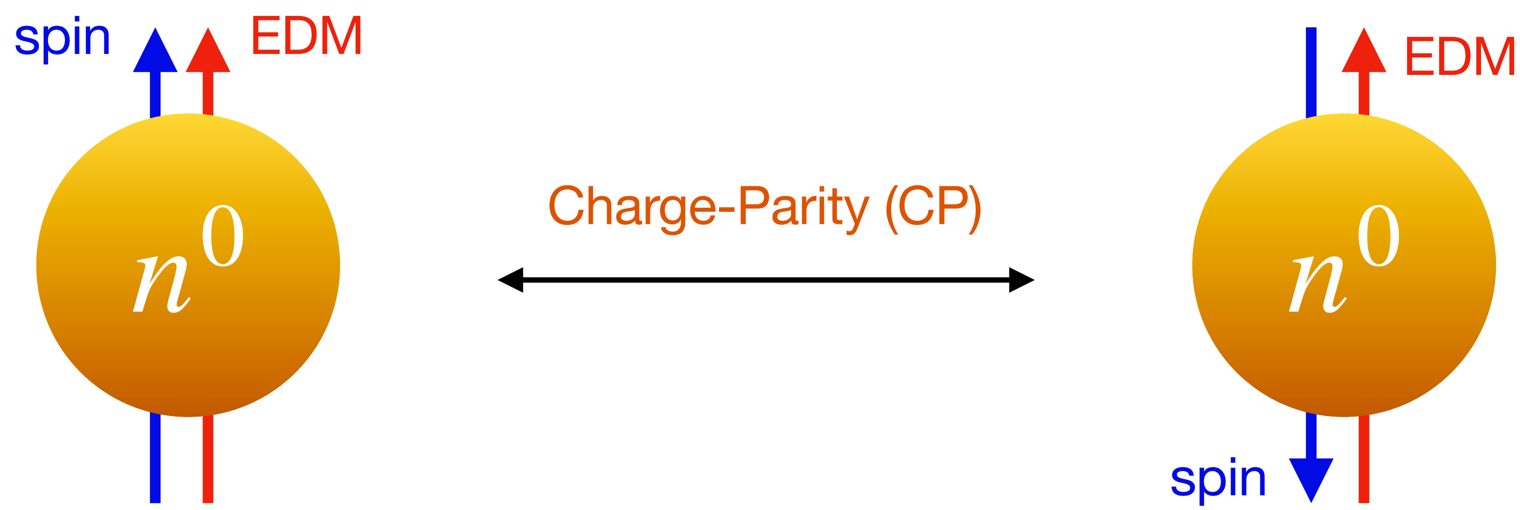


Qianshu Lu (IAS)

Matt Reece (Harvard)

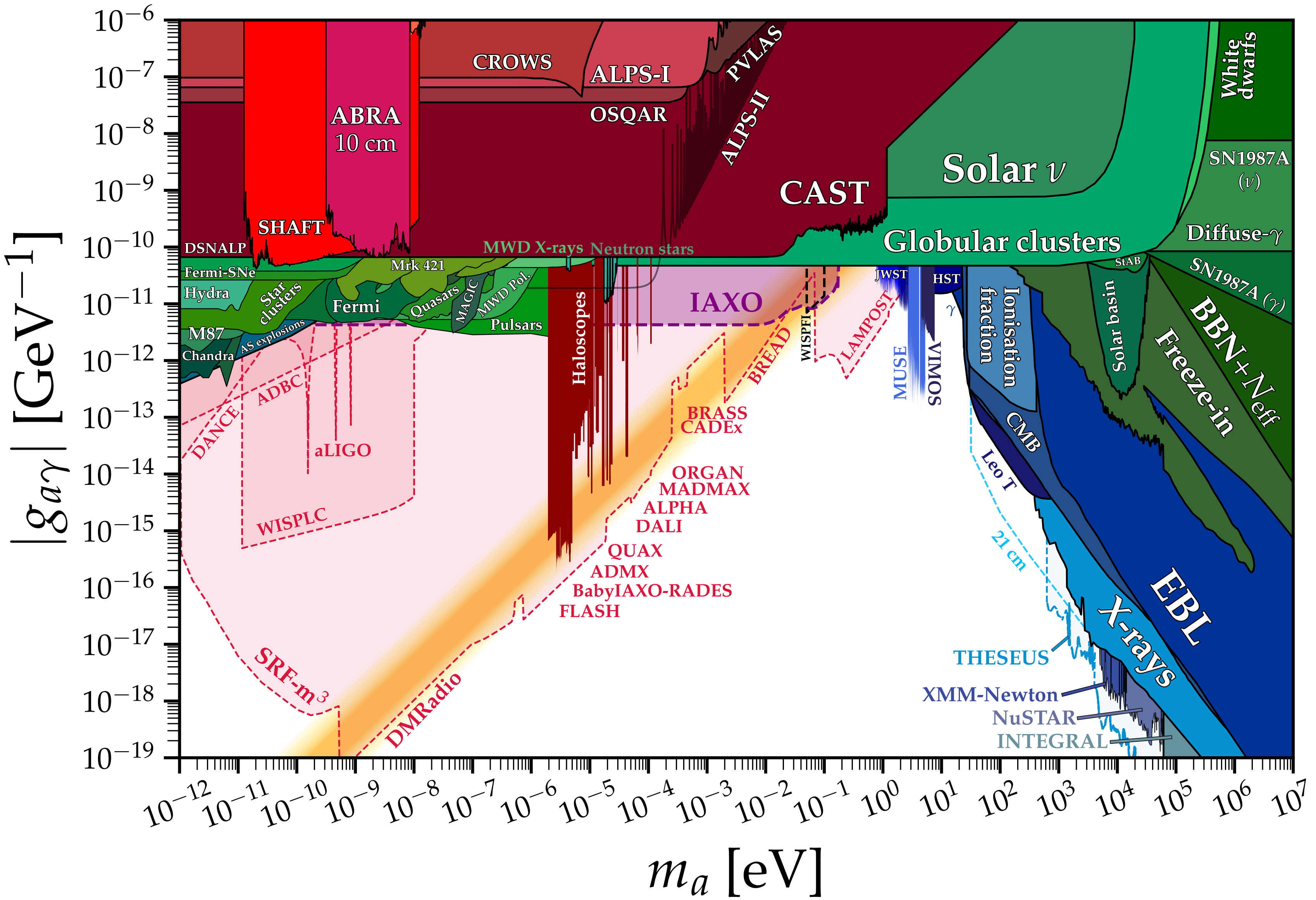
Why QCD Axions?

- Solves the strong CP problem: unnaturally small neutron EDM



Why QCD Axions?

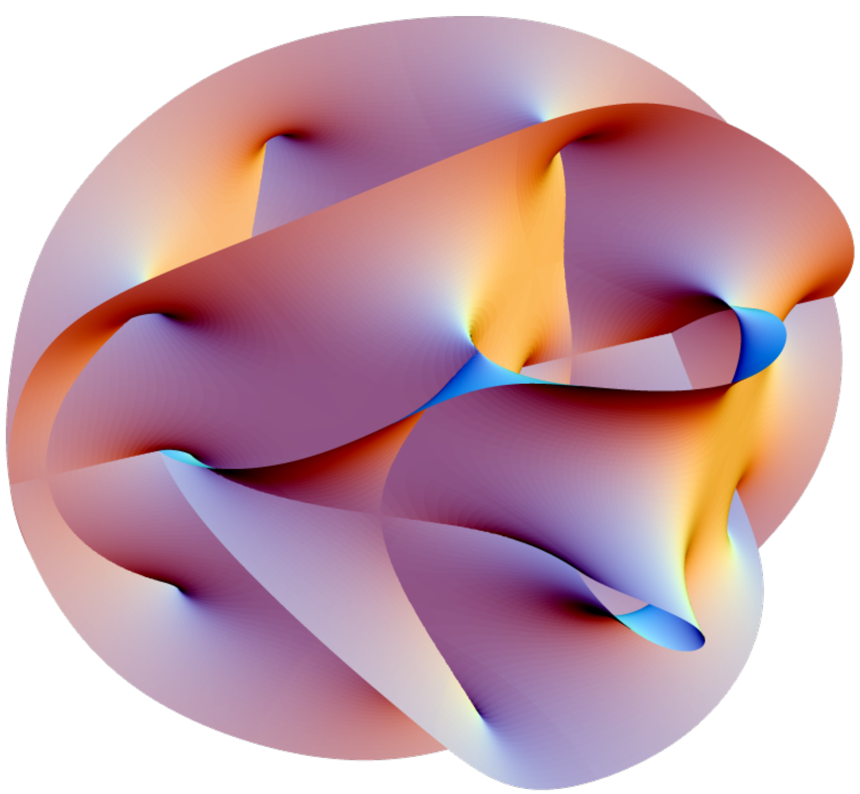
- Solves the strong CP problem
- Promising dark matter candidate



C. O'Hare, GitHub

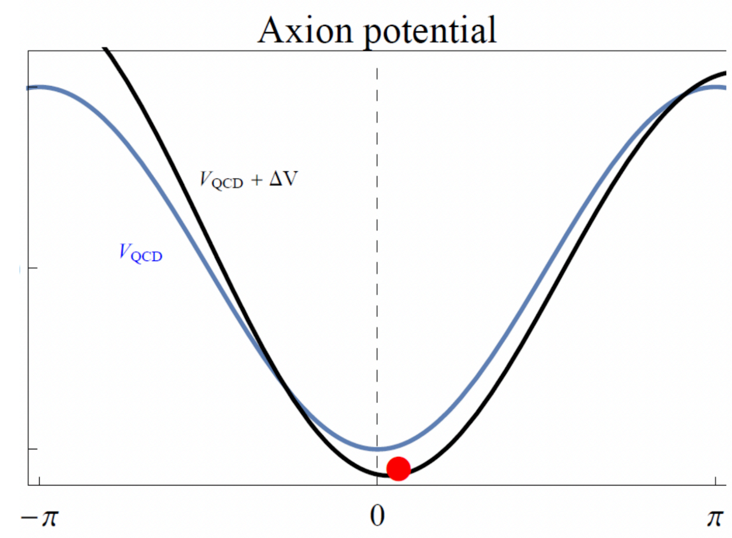
Why QCD Axions?

- Solves the strong CP problem
- Promising dark matter candidate
- Ubiquitous in string theory

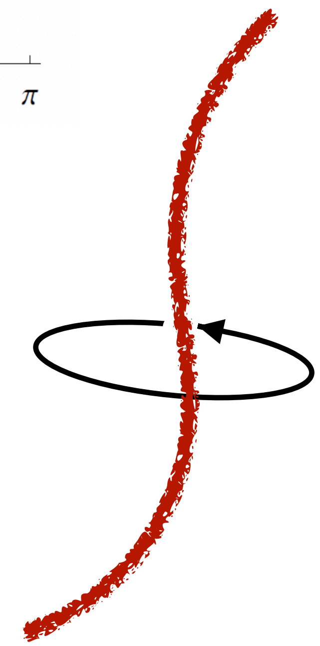


→ Axiverse

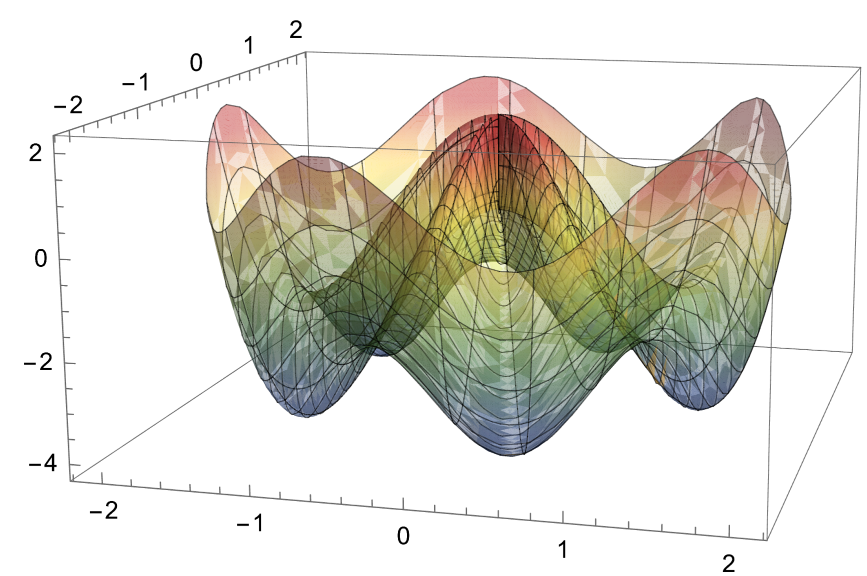
Outline



- Introduction
  - Axion quality problem
  - Axion in cosmology



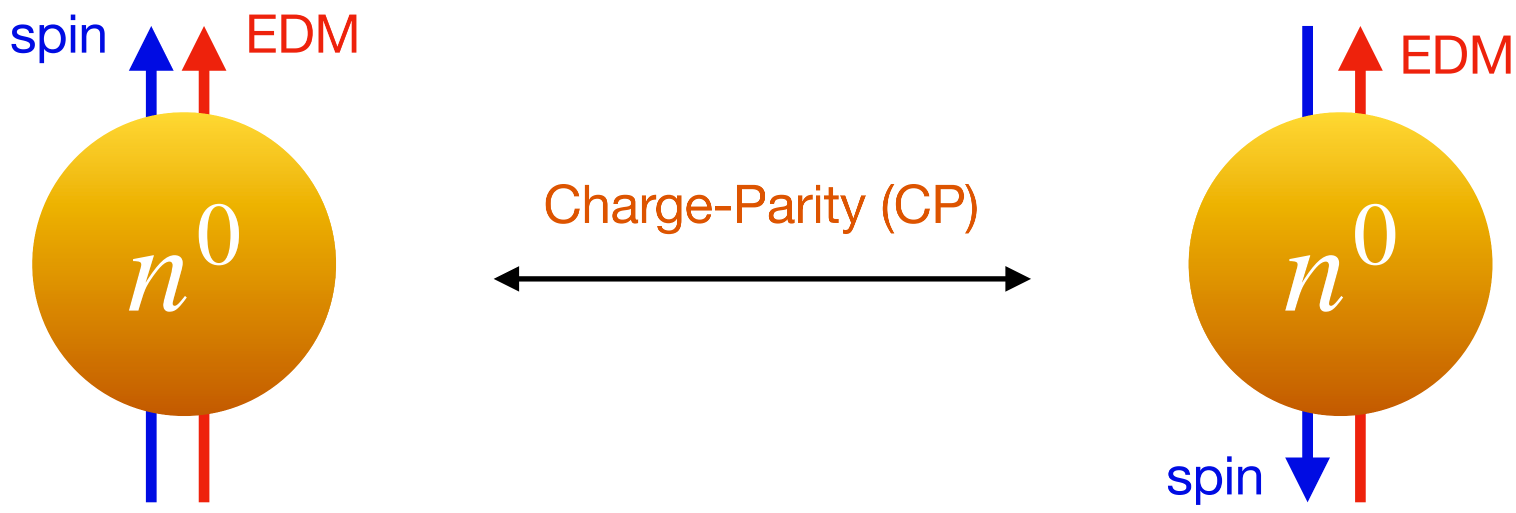
- $\mathbb{Z}_p$ model: canonical solution revisited
- Barr-Seckel Model: cosmological tension
- Conclusions



Axion Quality Problem

Strong CP Problem

- θ term gives rise to neutron electric dipole moment (EDM)
- Write down the neutron EDM term $-\mathrm{i}\frac{d_n}{2}\overline{N}\sigma_{\mu\nu}\gamma^5 N F^{\mu\nu}$
- We can calculate $d_n \sim 10^{-16}\,\theta\, e\,\mathrm{cm}$



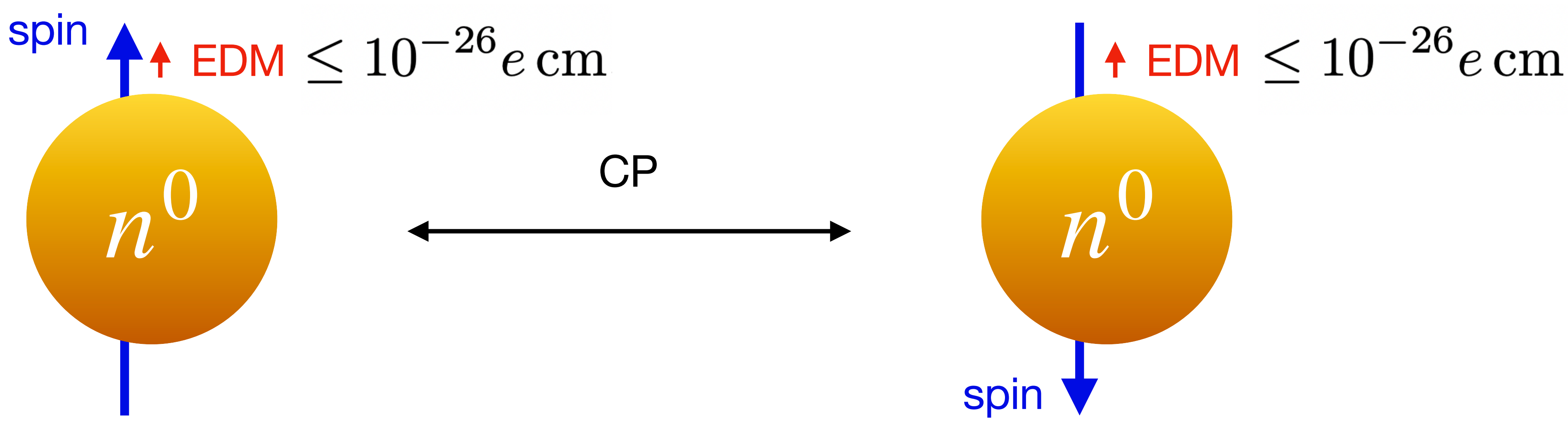
Strong CP Problem

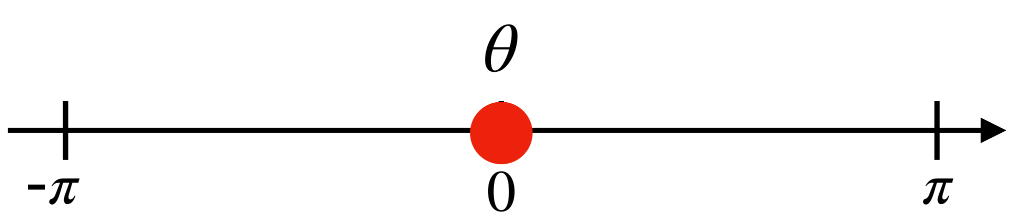
- We don't observe a nonzero neutron EDM!

$$d_n \sim 10^{-16}\,\theta\, e\,\mathrm{cm} \quad \xrightarrow{|d_n| \le 10^{-26} e\,\mathrm{cm}} \quad |\theta| \lesssim 10^{-10}$$

Phys. Rev. Lett. ***124***, *081803*

Experimental measurements





θ is "unnaturally" close to 0

Axion Solution to Strong CP

[Phys.Rev.Lett. 38 (1977) 1440-1443, Phys.Rev.D 16 (1977) 1791-1797] R. Peccei, H. Quinn
[Phys.Rev.Lett. 40 (1978) 223-226] S. Weinberg
[Phys.Rev.Lett. 40 (1978) 279-282] F. Wilczek
...

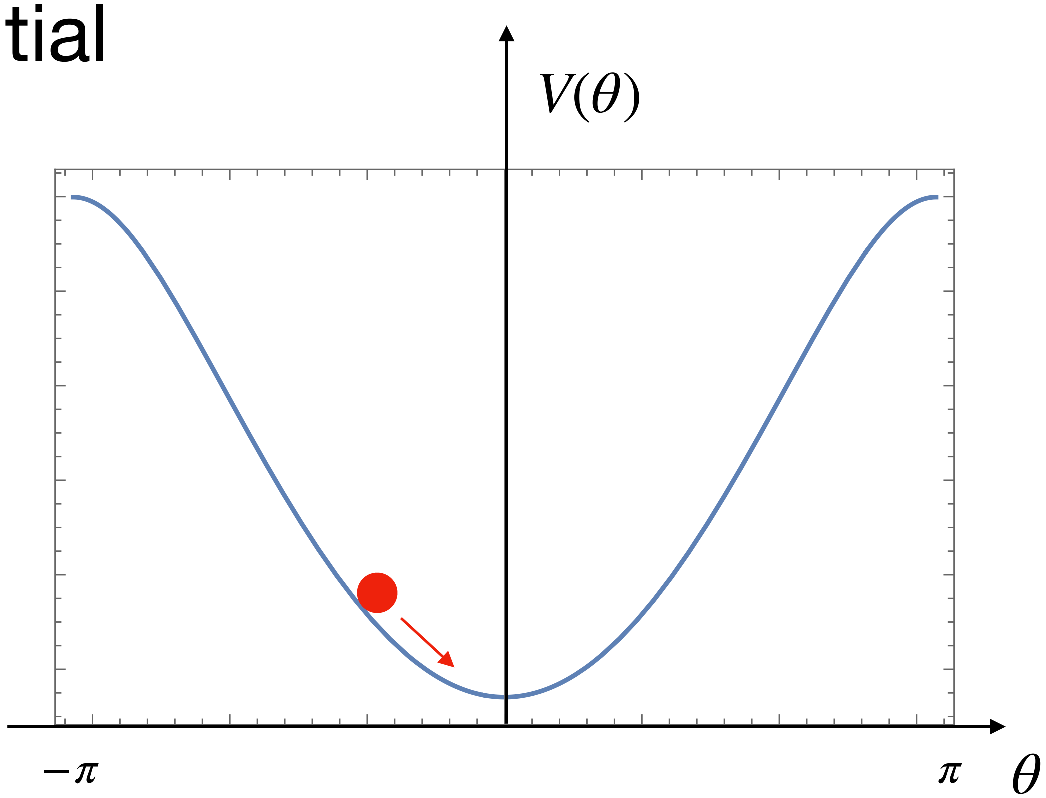
- We can make θ dynamical: **axion** field

$$\theta\, G^a_{\mu\nu}\tilde{G}^{a\mu\nu} \rightarrow \left(\theta_0 + \frac{a(x)}{f_a}\right) G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$$

- QCD generates a nonperturbative potential

$$V_{\mathrm{QCD}}(\theta) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4 m_u m_d}{(m_u + m_d)^2} \sin^2 \frac{N\theta}{2}}$$

$$\sim \cos(N\theta)$$



Axion quality problem

- Canonical 4D* axion models: $U(1)_{\rm PQ}$ global symmetry

$$\frac{\mathcal{L}_{\rm KSVZ}}{\sqrt{-g}} = \frac{1}{2}\partial_\mu\phi^*\partial^\mu\phi + \mathrm{i}Q^\dagger\overline{\sigma}^\mu D_\mu Q + \mathrm{i}\overline{Q}^\dagger\overline{\sigma}^\mu D_\mu\overline{Q} + \left(y\phi Q\overline{Q} + \mathrm{h.c.}\right) - V(\phi^*\phi).$$

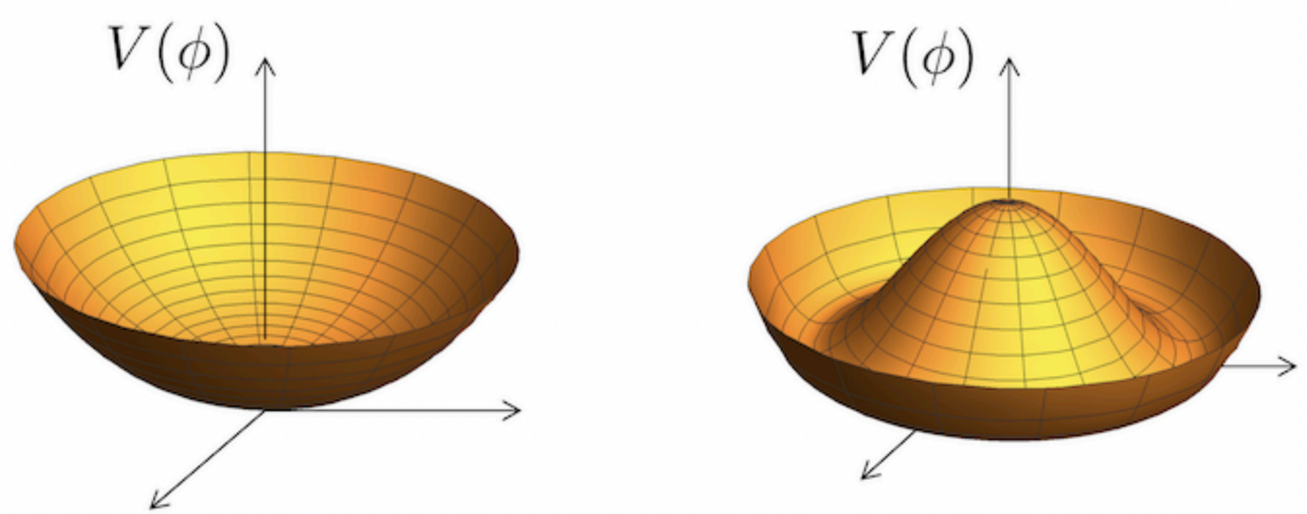
e.g. KSVZ type model

$$\mathrm{U}(1)_{\mathrm{PQ}}: \quad \phi \mapsto \mathrm{e}^{\mathrm{i}\alpha}\phi, \quad Q \mapsto \mathrm{e}^{-\mathrm{i}\alpha}Q;$$
$$\mathrm{U}(1)_{\mathrm{Q}}: \quad Q \mapsto \mathrm{e}^{\mathrm{i}\beta}Q, \quad \overline{Q} \mapsto \mathrm{e}^{-\mathrm{i}\beta}\overline{Q}.$$

- Spontaneous breaking of PQ symmetry gives axion

$$\phi(x) = \frac{1}{\sqrt{2}}\Big(\underbrace{f}_{\text{decay constant}} + \underbrace{s(x)}_{\text{saxion}}\Big)\exp(\mathrm{i}\,\underbrace{\theta(x)}_{\text{axion}}).$$

*Needs axion period 2π



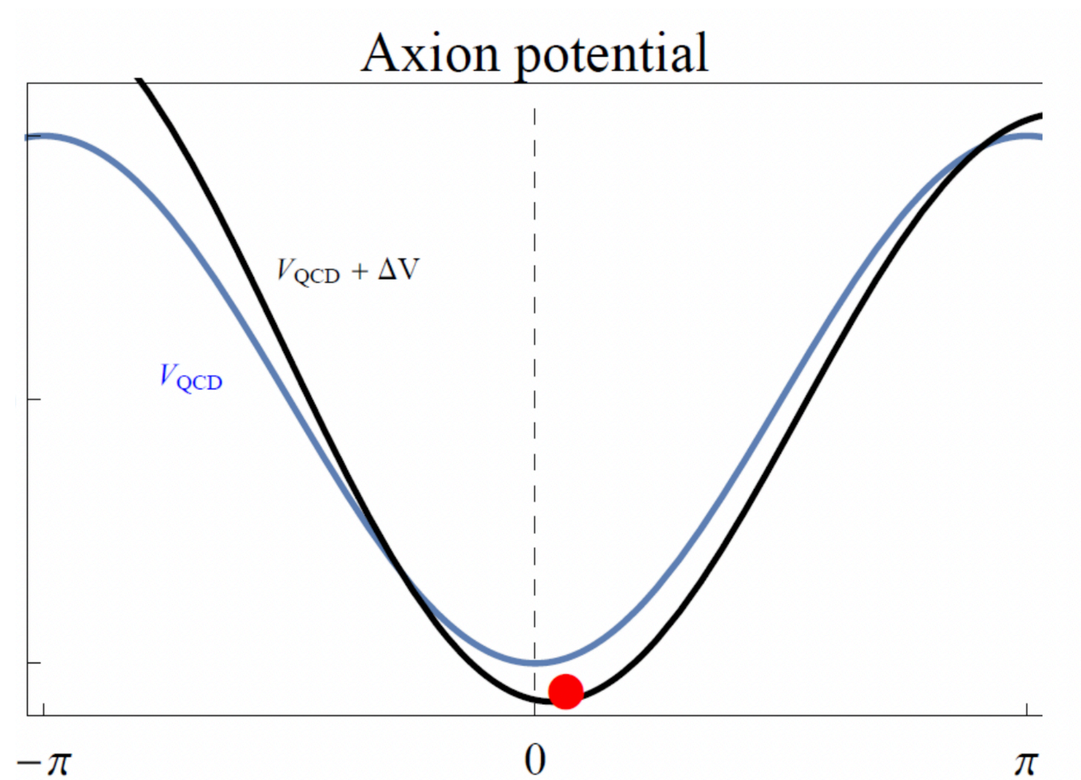
Axion quality problem

- Global symmetry doesn't exist in quantum gravity!
- PQ breaking terms spoil the axion solution to the strong CP problem!

$$\frac{c}{M_{\rm Pl}^{n-4}}\phi^n + \text{h.c.} \longrightarrow 2|c|M_{\rm Pl}^4\left(\frac{f}{\sqrt{2}M_{\rm Pl}}\right)^n \cos\left(n\theta+\varphi\right)$$

PQ breaking term

Correction to axion potential



G. Landini, PASCOS22

Axion quality problem

- Typical solution: impose gauge symmetry to forbid operators

  e.g. $\mathbb{Z}_p$ symmetry forbids $\frac{c}{M_{\rm Pl}^{n-4}}\phi^n + \text{h.c.}$ up to $n = p$

  G. Lazarides, C. Panagiotakopoulos, and Q. Shafi, 1986

  - Generally needs $p \geq 14$ still

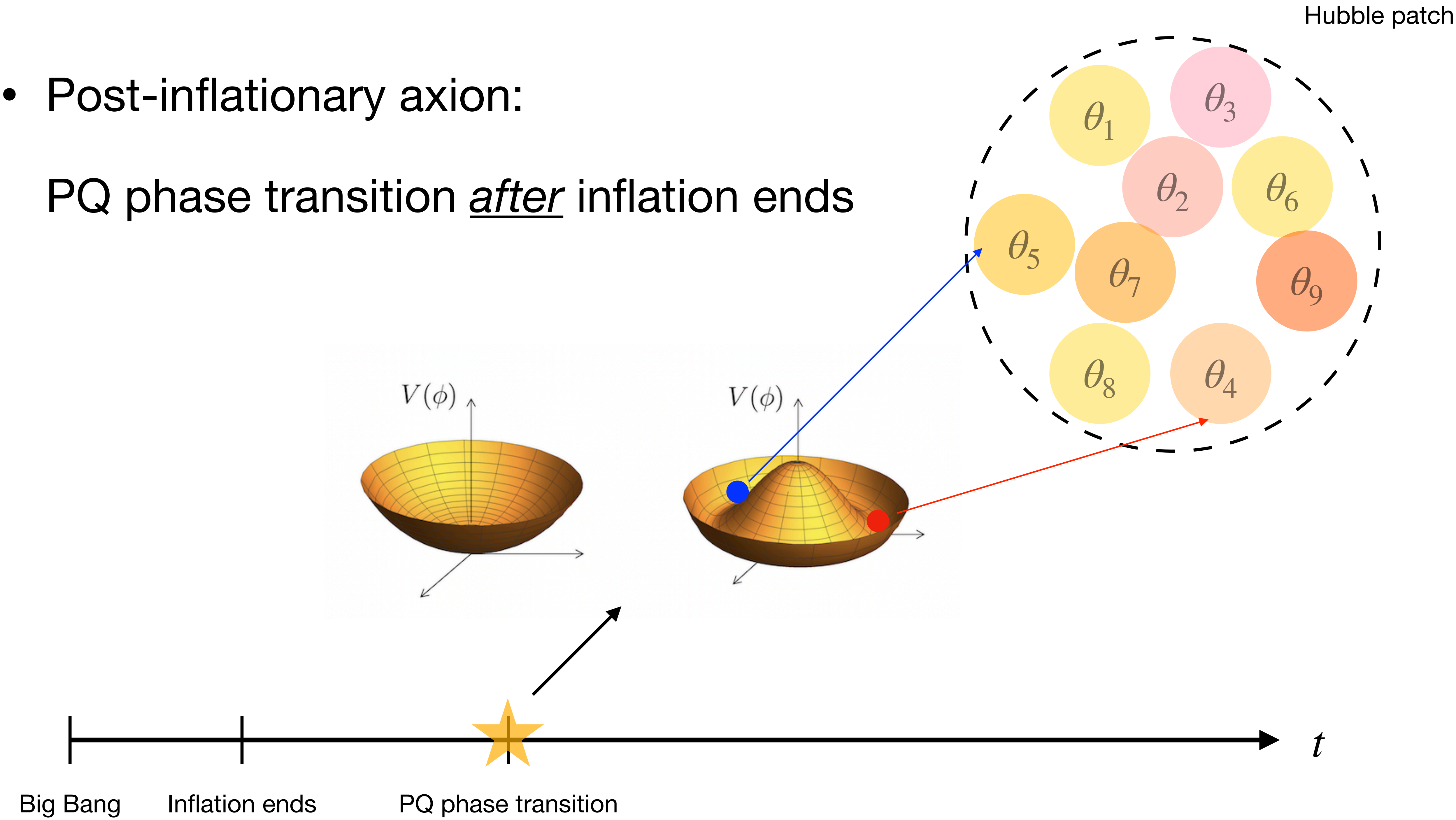
- Other solutions: extra-dimension models

Axion in cosmology

Axion in cosmology

- Post-inflationary axion:

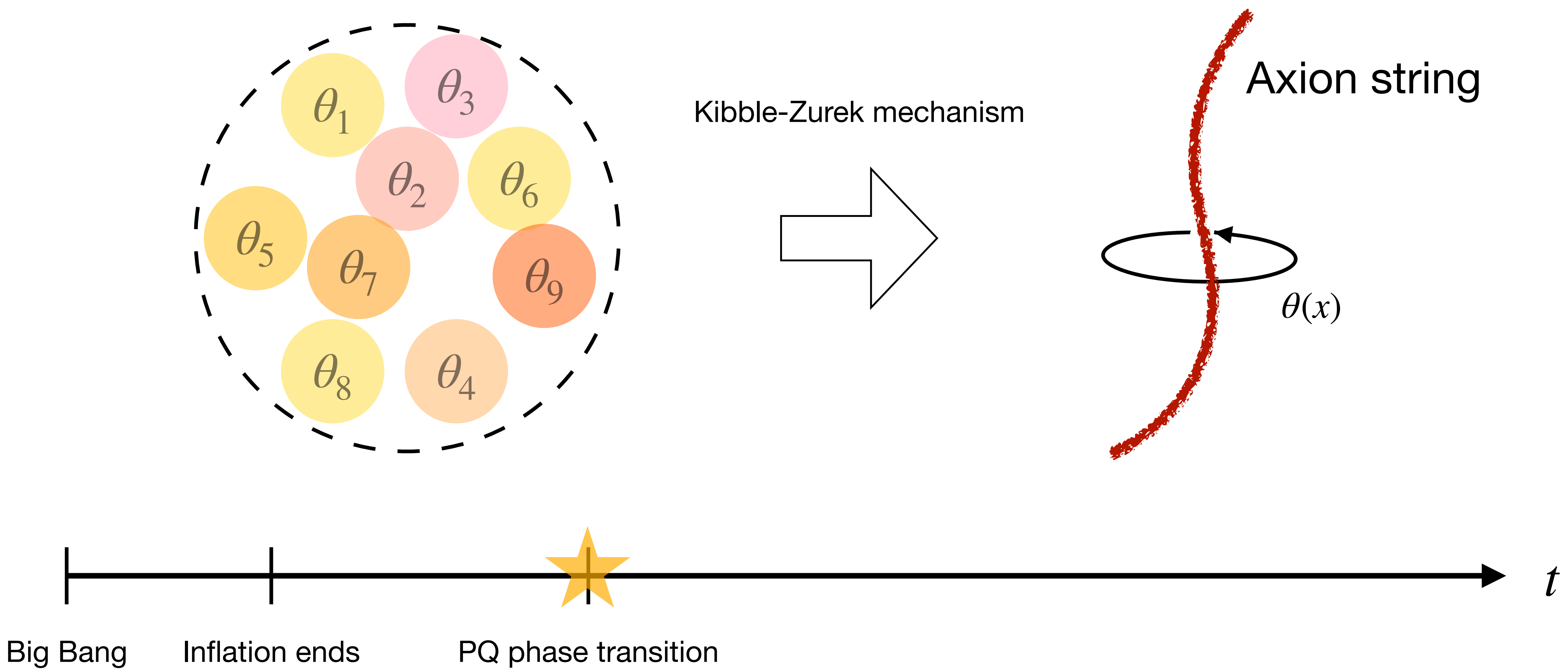
  PQ phase transition *after* inflation ends



Axion in cosmology

- Post-inflationary axion:

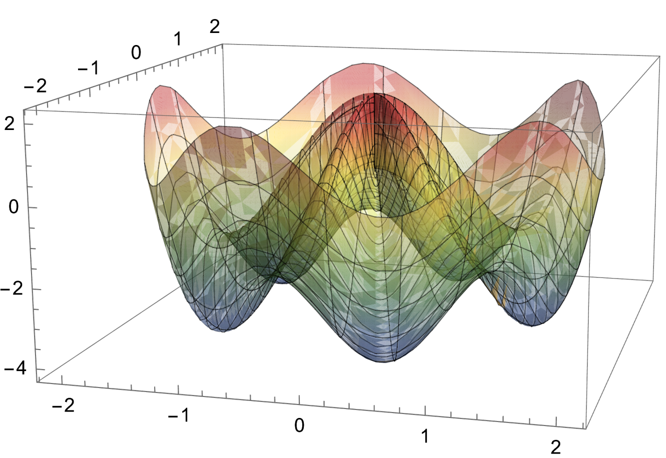
  axion strings form after PQ transition

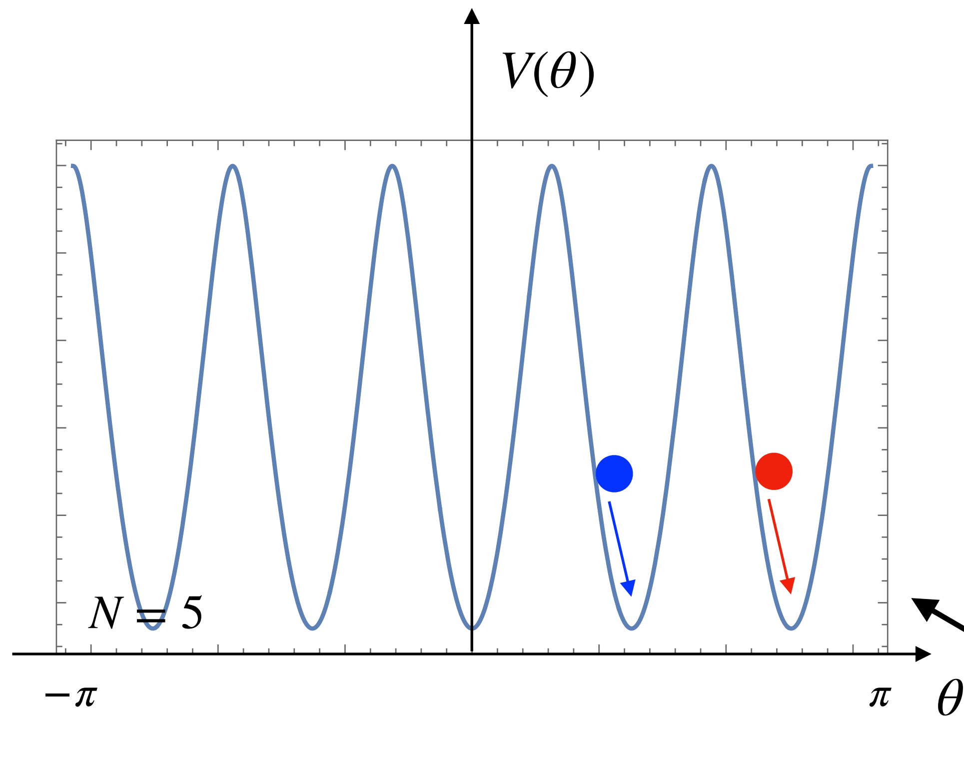


Axion in cosmology

- Post-inflationary axion:

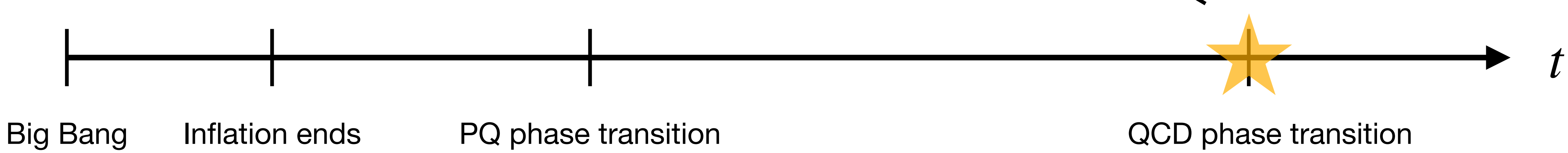
  domain walls may form after QCD transition





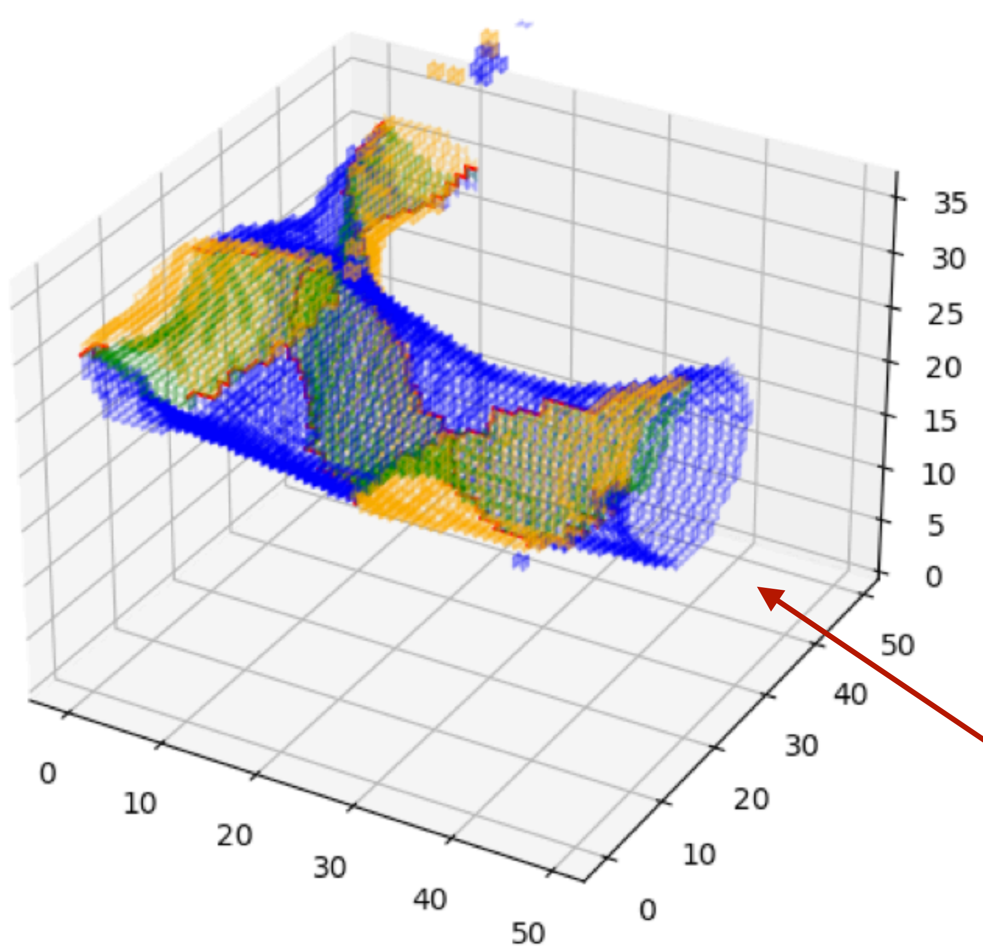
Domain wall number

$$V_{\rm QCD}(\theta) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4 m_u m_d}{(m_u + m_d)^2} \sin^2 \frac{N\theta}{2}}$$



Domain wall problem

- Domain walls need to annihilate
  - Otherwise their energy density would dominate
- Domain walls end on strings



$N > 1$: may have stable domain wall-string network

Each string has N domain walls attached to it or domain wall can form a closed bubble

Domain wall problem

- Domain walls need to annihilate
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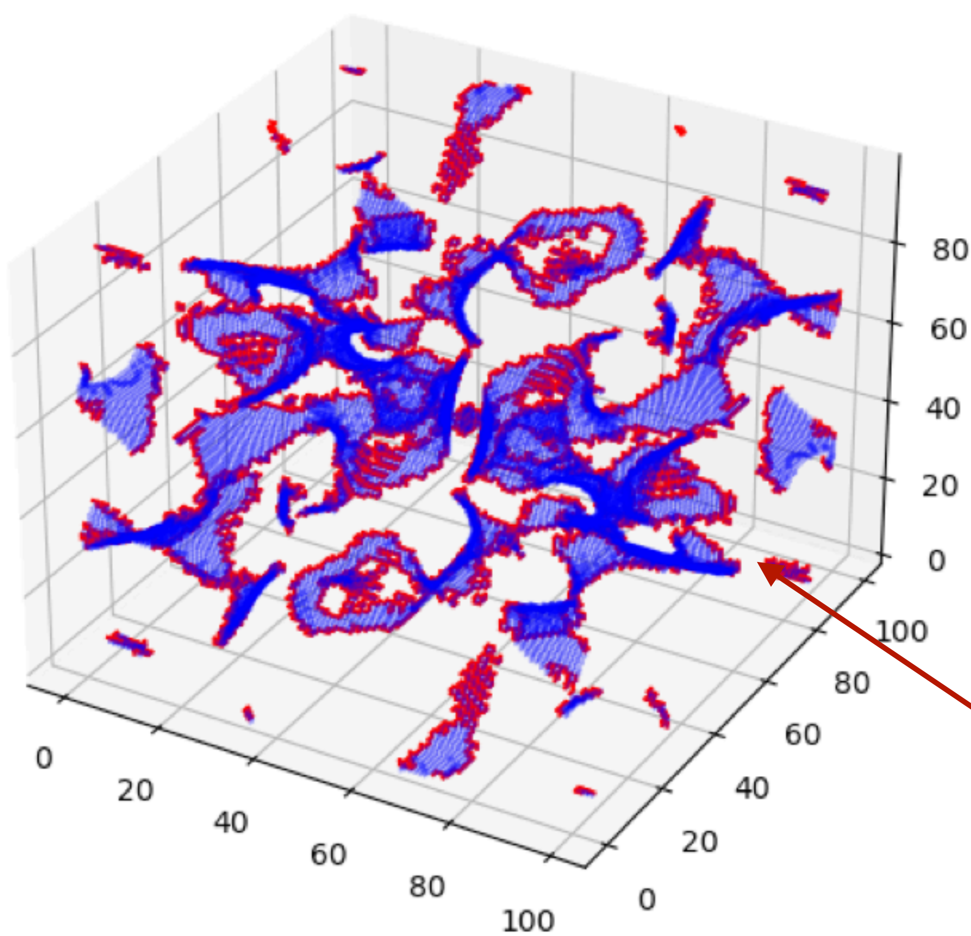


Needs $N = 1$ to destroy domain wall-string network to achieve observed cosmology

Network is unstable if $N = 1$ and will collapse

Basic tension

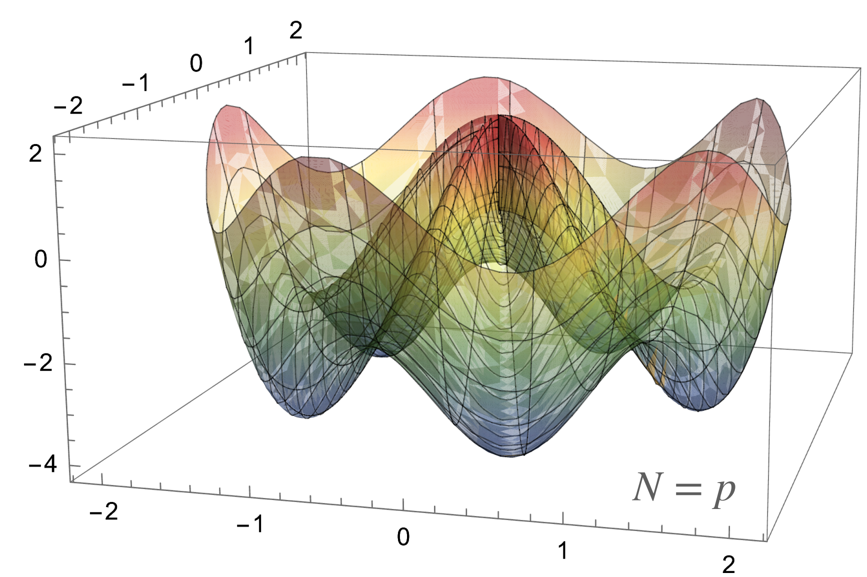
- $\mathbb{Z}_p$ symmetry solution to the quality problem

  $\Rightarrow$ domain wall number (at least) $N = p > 1$

- PQ scalar $$\Phi(x) = \frac{1}{\sqrt{2}}\left(f + s(x)\right) e^{i\varphi(x)}$$

  $\mathbb{Z}_p$ transformation $\Phi \mapsto e^{2\pi i/p}\Phi$ $\qquad \varphi(x) \mapsto \varphi(x) + \frac{2\pi}{p}$

  allows for potential $\cos(p\,\varphi)$



$\mathbb{Z}_p$ model revisited

General charge under $\mathbb{Z}_p$

- PQ scalar now has charge q under $\mathbb{Z}_p$ gauge symmetry

$$\Phi(x) = \frac{1}{\sqrt{2}}\left(f + s(x)\right) \mathrm{e}^{\mathrm{i}\varphi(x)}$$

generator of $\mathbb{Z}_p$ symmetry $\quad z: \quad \varphi(x) \mapsto \varphi(x) + \frac{2\pi q}{p}.$

- For simplicity we require q, p to be *relatively prime*

Concrete KSVZ type model

- PQ scalar has charge q under $\mathbb{Z}_p$
- Heavy quarks $Q^{(i)}, \tilde{Q}^{(i)}$ has charge $k, -(k+q)$ under $\mathbb{Z}_p$

  generation $i \in \{1, \ldots, r\}$
- Rephasing shifts the action through chiral anomaly

$$\Phi \mapsto \mathrm{e}^{\mathrm{i}q\alpha}\Phi, \quad Q^{(i)} \mapsto \mathrm{e}^{\mathrm{i}k\alpha}Q^{(i)}, \quad \widetilde{Q}^{(i)} \mapsto \mathrm{e}^{-\mathrm{i}(k+q)\alpha}\widetilde{Q}^{(i)},$$

$$\Delta I = \frac{1}{8\pi^2} r \left[k\alpha + (-k-q)\alpha\right] \int \mathrm{tr}(G \wedge G) = \boxed{-\frac{rq}{8\pi^2}\alpha \int \mathrm{tr}(G \wedge G).}$$

Concrete KSVZ type model

- Rephasing shifts the action through chiral anomaly

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- $\mathbb{Z}_p$ is valid gauge symmetry $\Rightarrow \exp(\mathrm{i}\Delta I) = 1$ when $\alpha = 2\pi/p$

$$\Rightarrow \quad \frac{rq}{8\pi^2}\frac{2\pi}{p}\int \mathrm{tr}\,(G \wedge G)$$

needs to be a multiple of 2π

Concrete KSVZ type model

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$$\Rightarrow \quad \frac{rq}{8\pi^2}\frac{2\pi}{p}\int \mathrm{tr}\,(G \wedge G)$$ needs to be a multiple of 2π

- Quantization of instanton number requires $\frac{1}{8\pi^2}\int \mathrm{tr}\,(G \wedge G)$ to be an integer

$$\Rightarrow \frac{rq}{p}$$ is integer

Concrete KSVZ type model

- Rephasing shifts the action through chiral anomaly

$$\Delta I = \frac{1}{8\pi^2} r \left[k\alpha + (-k-q)\alpha\right] \int \mathrm{tr}(G \wedge G) = -\frac{rq}{8\pi^2}\alpha \int \mathrm{tr}(G \wedge G).$$

- $\mathbb{Z}_p$ is valid gauge symmetry + quantization of instanton number

$\Rightarrow \dfrac{rq}{p}$ is integer

Number of generation | r is a multiple of p | Number of vacua

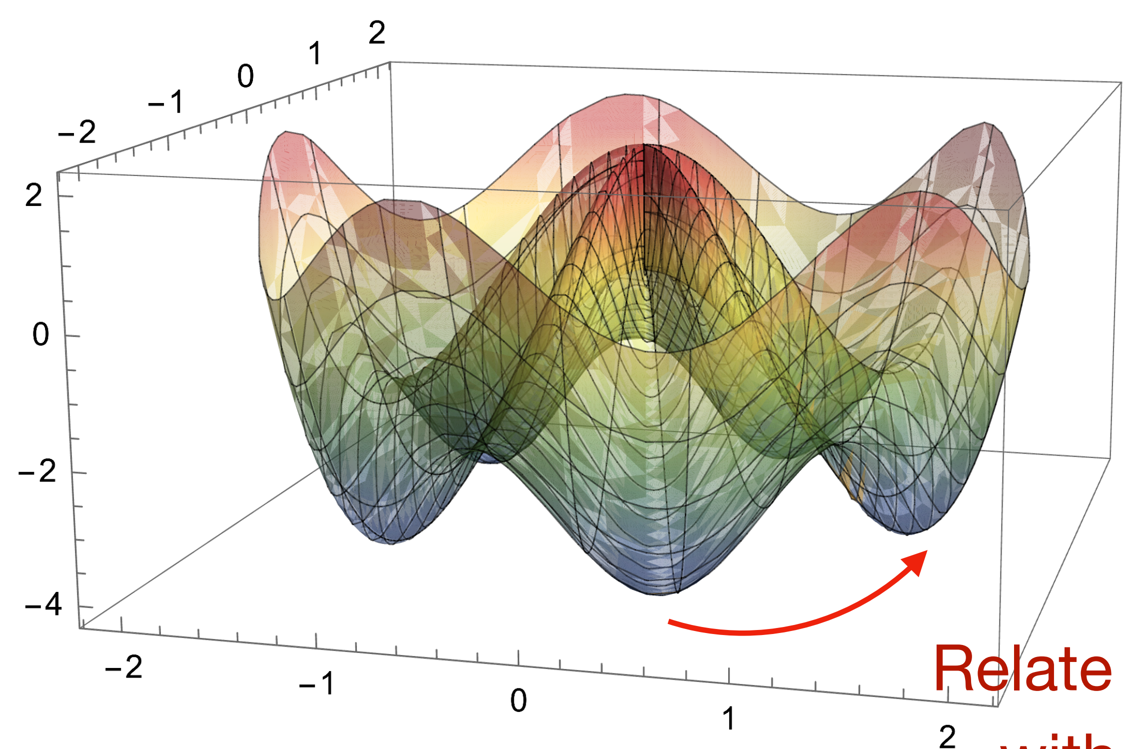
$\Rightarrow$ effective coupling $\dfrac{r}{8\pi^2} \int \varphi(x)\mathrm{tr}(G \wedge G)$ gives $N_{DW} > 1$

Gauge equivalence?

- If all vacua separated by domain walls are gauge equivalent under $\mathbb{Z}_p$ transformation, then *physical* domain wall number is 1

G. Lazarides and Q. Shafi, 1982

$$\varphi(x) \mapsto \varphi(x) + \frac{2\pi q}{p}$$ is a gauge transformation



Relate neighboring minima with transformation z^k

Gauge equivalence?

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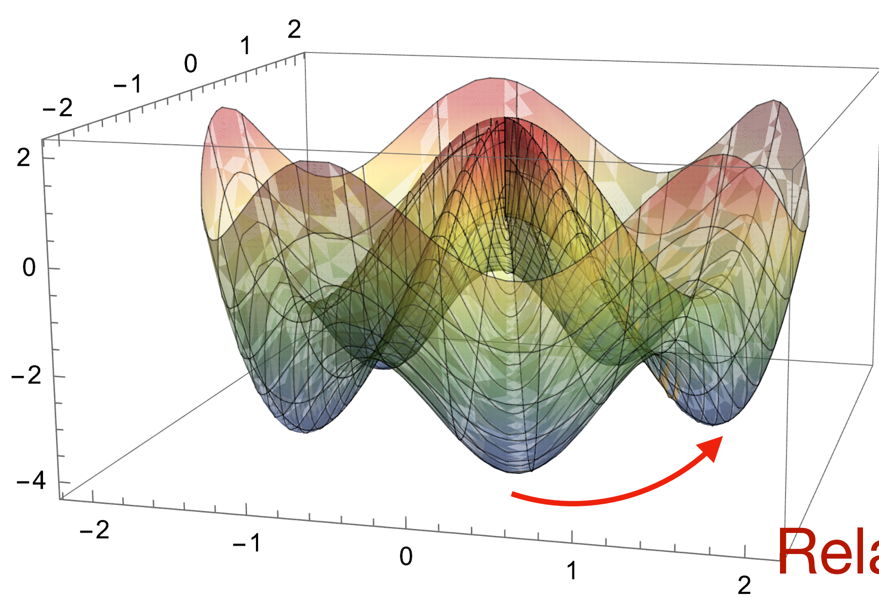
G. Lazarides and Q. Shafi, 1982

- Physical N_{DW} in our KSVZ model should've been $|s| = |r|/p$

*neighboring minima are gauge equivalent iff $|s| = 1$

Number of distinct gauge orbits of minima



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*neighboring minima are gauge equivalent iff $|s| = 1$

Number of distinct gauge orbits of minima

- Can also see this through defining 2π-periodic axion

$$\theta(x) = p\varphi(x) = \arg(\Phi(x)^p)$$

Lowest-dim $\mathbb{Z}_p$-invariant operator that carries PQ charge and obtains VEV

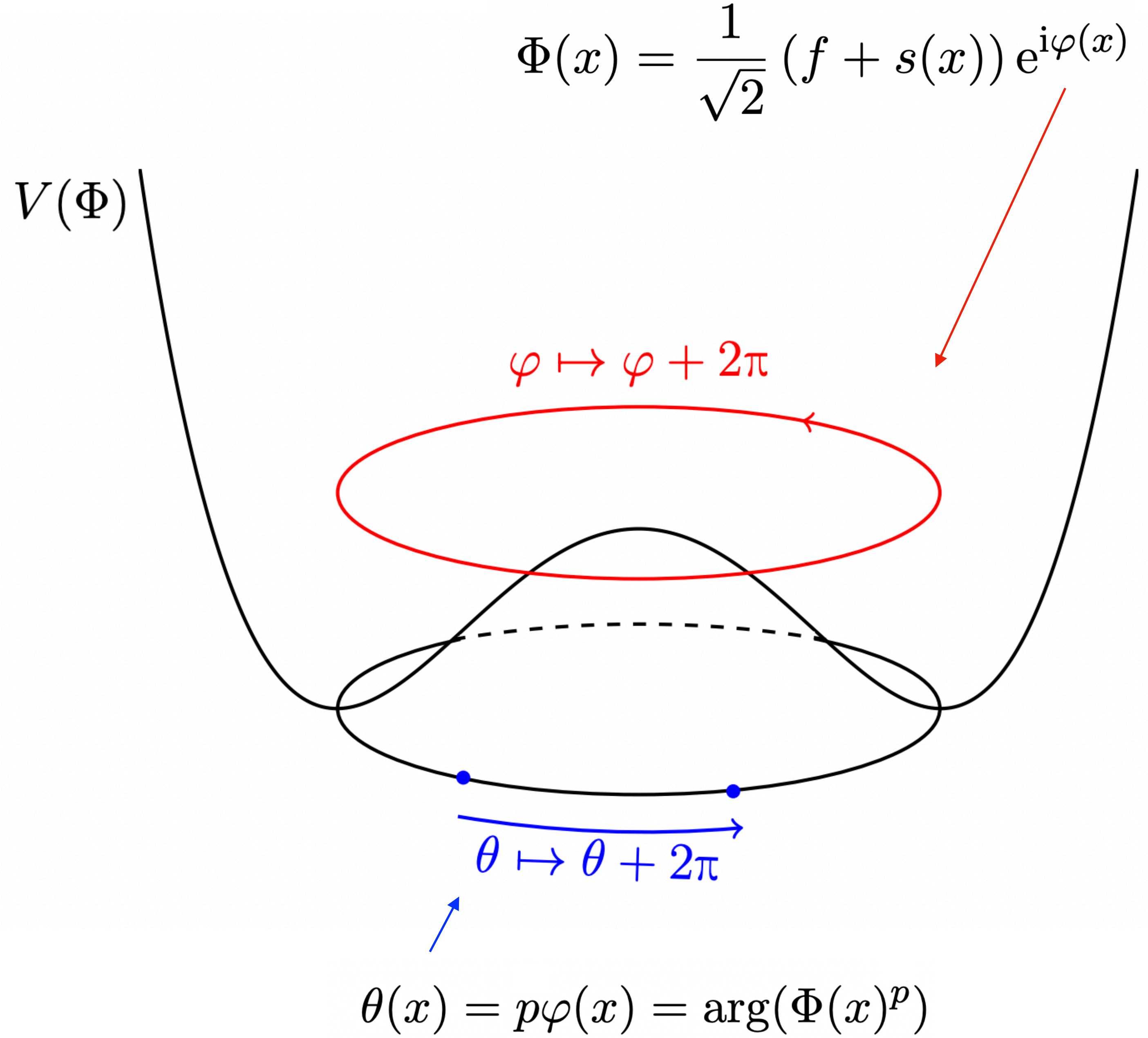
$$\frac{r}{8\pi^2}\int \varphi(x)\mathrm{tr}(G \wedge G) \longrightarrow \frac{s}{8\pi^2}\int \theta(x)\mathrm{tr}(G \wedge G)$$

Gauge equivalence!

- We could have physical domain wall number $N_{DW} = 1$ by having $s = 1$

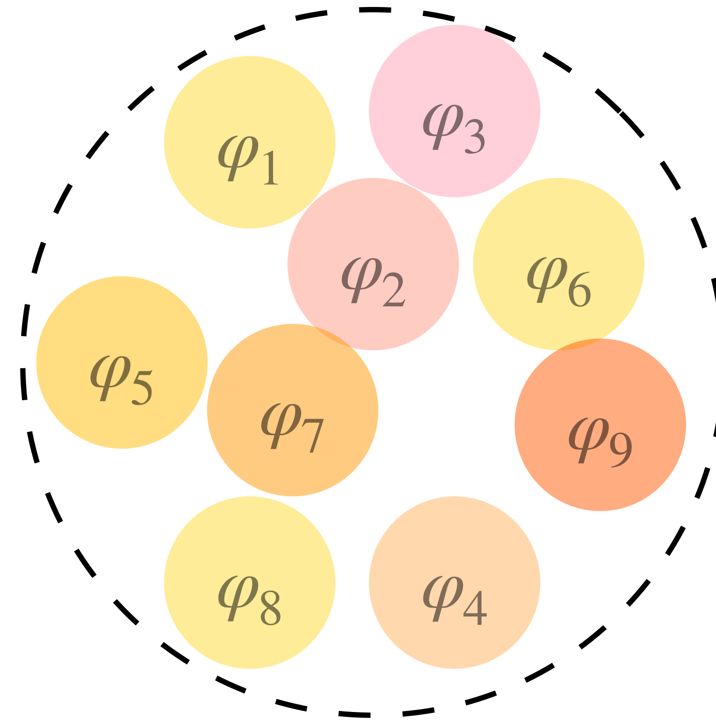
$$\frac{s}{8\pi^2} \int \theta(x) \mathrm{tr}(G \wedge G)$$

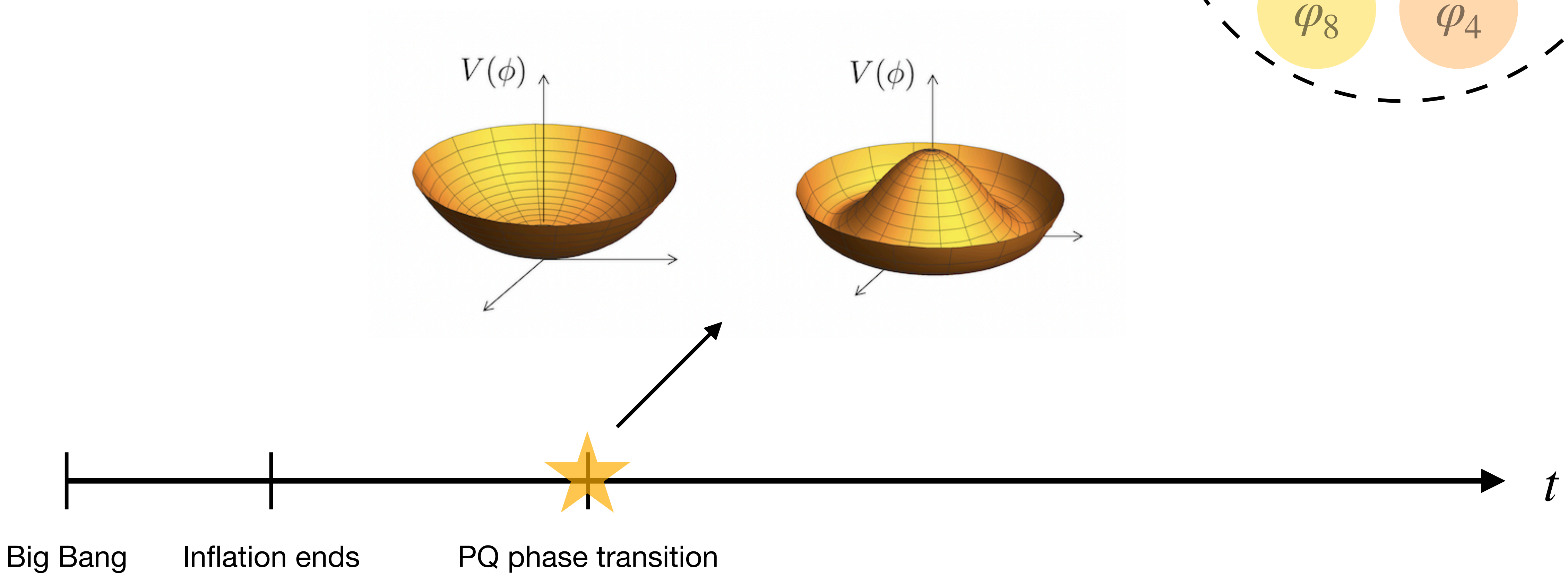
- Axion field is identified with the phase of the *lowest-dimensional gauge invariant operator that is constructed from the PQ scalar*



What type of string forms?

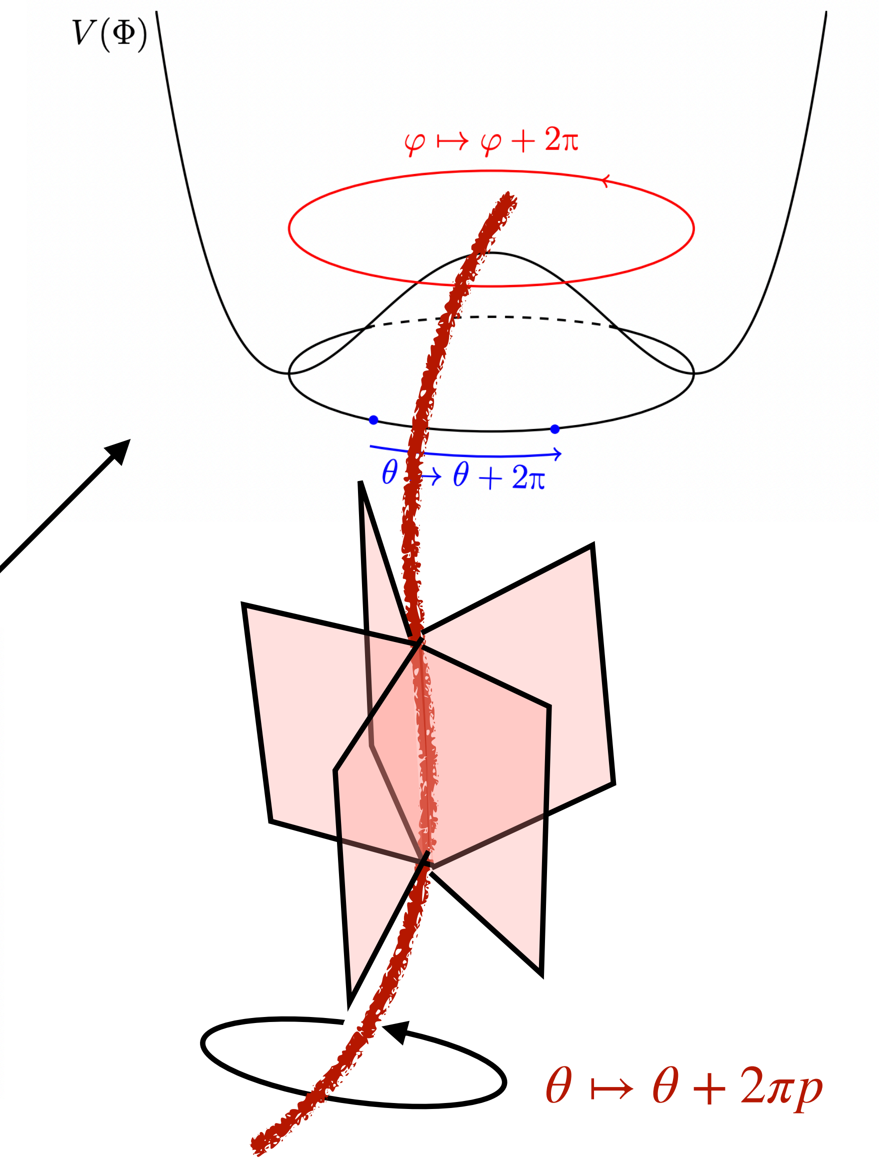
- At PQ phase transition,
  angular modes φ gets random value



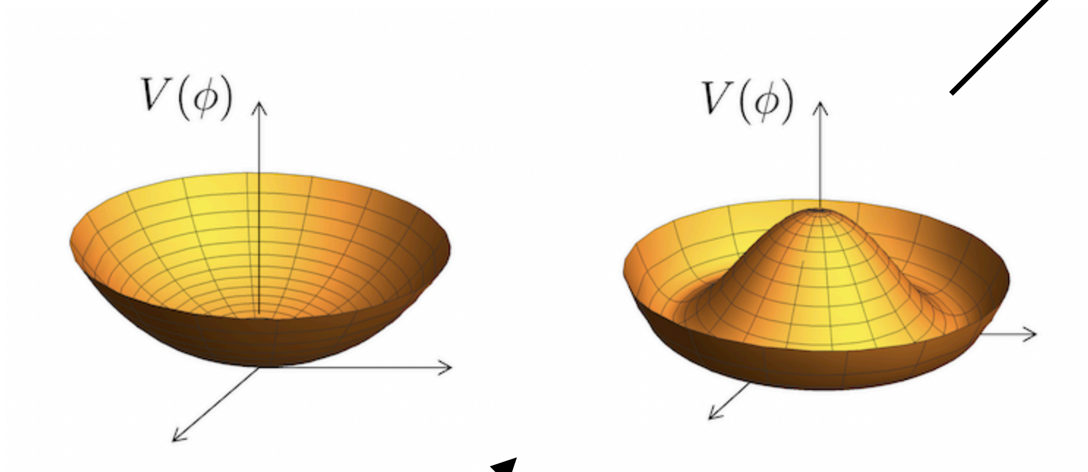


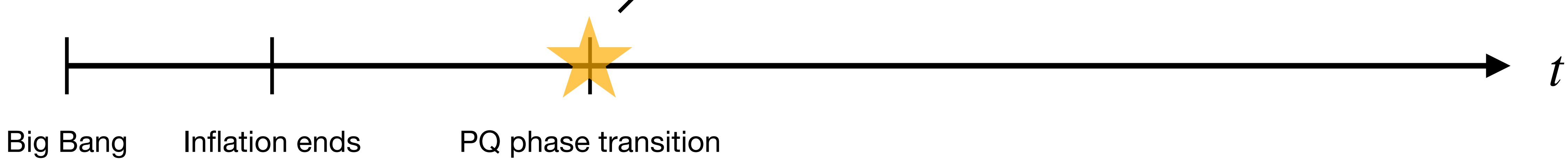
What type of string forms?

- Naturally form strings where φ, NOT θ, winds around by 2π !



Kibble-Zurek mechanism



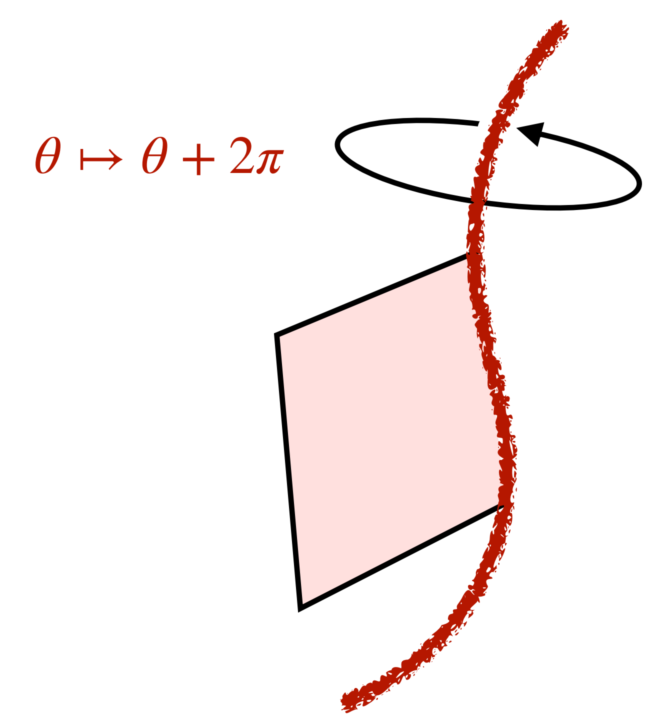


$\mathbb{Z}_p$ strings

- Gauge symmetry allows the existence of z^k strings

Winding of φ is
$$\oint_{z^k} \mathrm{d}\varphi = 2\pi \left(\frac{kq}{p} + n \right)$$

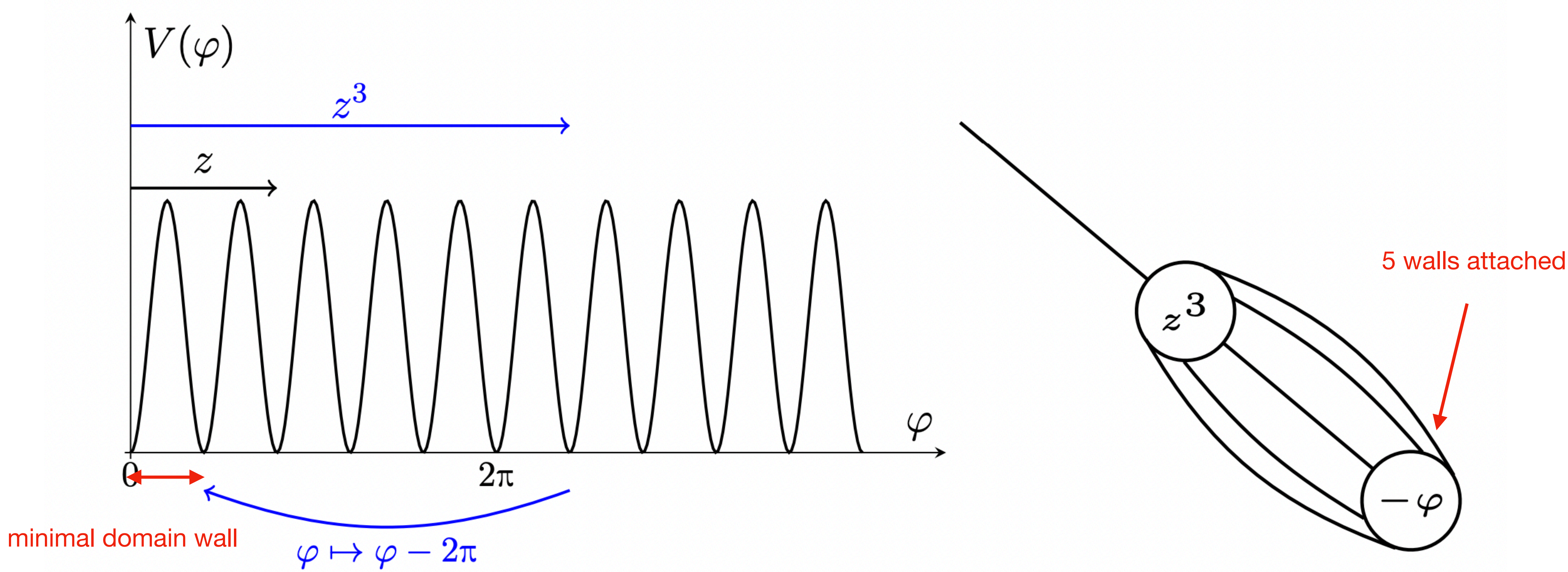
- To annihilate a minimal domain wall, we need θ winding 2π (i.e. φ winding $2\pi/p$), can be achieved by some choice of n



- We technically can produce the strings that a *single minimal domain* walls can end on

Dynamics…

- Concrete example: $\mathbb{Z}_5$ symmetry, PQ scalar has charge $q = 2$
- Need complicated composite object to annihilate domain walls…
- *Also modifies prediction of DM abundance



Dynamics…

- Concrete example: $\mathbb{Z}_5$ symmetry, PQ scalar has charge $q = 2$
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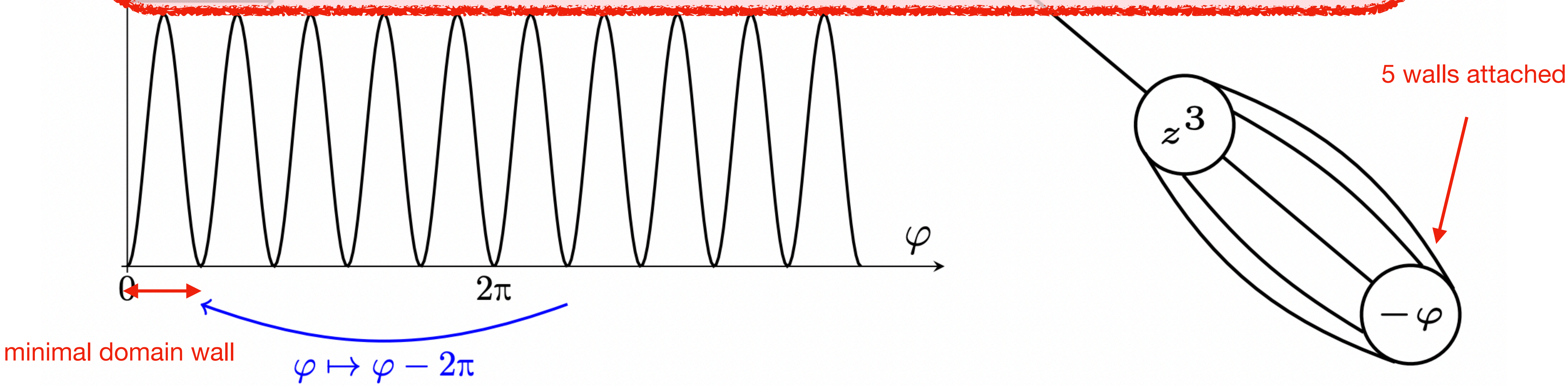
Domain wall number = 1 is not sufficient to solve the domain wall problem;
Often needs nontrivial dynamics!



Barr-Seckel Model

Barr-Seckel model review

S.M. Barr and D. Seckel, 1992

- KSVZ type model, two scalars Φ_p and Φ_q
- Additional $U(1)'$ gauge symmetry
- Minimal field content to cancel gauge anomalies

	spin	$\mathrm{SU}(3)_\mathrm{C}$	$\mathrm{U}(1)_\mathrm{Y}$	$\mathrm{U}(1)'$	copies	$\mathrm{U}(1)_\mathrm{PQ}$
Q_0	1/2	$\mathbf{3}$	0	0	$(p+q)$	c
$\widetilde{Q}_p$	1/2	$\overline{\mathbf{3}}$	0	p	q	$a-c$
$\widetilde{Q}_{-q}$	1/2	$\overline{\mathbf{3}}$	0	$-q$	p	$-b-c$
Φ_p	0	$\mathbf{1}$	0	p	1	a
Φ_q	0	$\mathbf{1}$	0	q	1	b

Solving axion quality

S.M. Barr and D. Seckel, 1992

- Identify axion as a combination of θ_p and θ_q

$$\mathcal{L}_{\rm BS} \supset \frac{q\theta_p - p\theta_q}{8\pi^2} \int \mathrm{tr}(G \wedge G) \equiv \frac{\theta}{8\pi^2} \int \mathrm{tr}(G \wedge G)$$

- Assume p, q are co-prime numbers, then $(\Phi_p^\dagger)^q(\Phi_q)^p$ is the lowest dimensional PQ violating operator
  - Modifies potential $\Delta V \sim v_p^q v_q^p / \Lambda^{p+q-4}$
- Axion has high quality when $p + q$ is sufficiently large
  - e.g. $v_p \sim v_q \sim f_a \sim 10^{12}\,\mathrm{GeV} \Rightarrow p + q \geq 10$

Composite strings

- Axion string translates to nontrivial winding of θ_p and θ_q

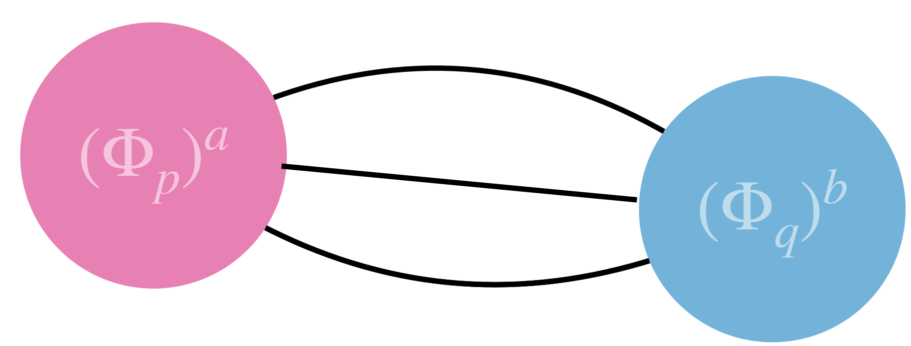
$$\theta = q\theta_p - p\theta_q$$

$$\theta_p \mapsto \theta_p + 2\pi a, \theta_q \mapsto \theta_q + 2\pi b$$

$$\theta \mapsto \theta + (aq - bp)\, 2\pi$$

- Since p, q are co-prime, we can always have a, b integers such that $aq - bp = 1$

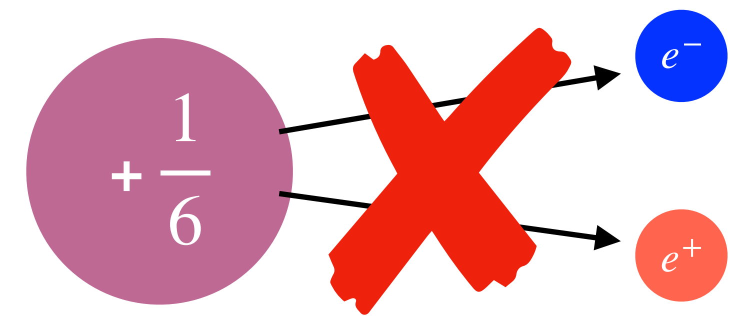
- But composite string with exact winding need not form…



Fractionally charged relics

L. Di Luzio, F. Mescia, and E. Nardi, 2017

- New heavy quarks in KSVZ type models need to not form fractionally (electrically) charged particles
  - Bounds on exotic charged hadrons
  - Observations have stringent constraints



- Only specific representations work
  - Specifically, for Q to be fundamental in $SU(3)_C$ and singlet in $SU(2)_L$, we need nonzero $U(1)_Y$ charge $-\frac{1}{3}+n$
- We need to assign $U(1)_Y$ charges to the Barr-Seckel matter contents

Barr-Seckel without relics

- Assign $U(1)_Y$ charges and require gauge anomaly cancellation

$$Y_{Q_0} = -\frac{1}{3} + n_1 = x,$$
$$Y_{\widetilde{Q}_p} = \frac{1}{3} + n_2 = y,$$
$$Y_{\widetilde{Q}_{-q}} = \frac{1}{3} + n_3 = z,$$

Charge assignment

$$\begin{aligned}
\mathrm{SU(3)_C^2U(1)_Y}: &\quad (p+q)x + qy + pz = 0;\\
\mathrm{U(1)_Y^3}: &\quad (p+q)x^3 + qy^3 + pz^3 = 0;\\
\mathrm{U(1)_Y^2U(1)'}: &\quad qy^2p + pz^2(-q) = 0;\\
\mathrm{U(1)_YU(1)'^2}: &\quad qyp^2 + pzq^2 = 0.
\end{aligned}$$

Anomaly cancellation

- Minimal matter content with 3 generations

	$\mathrm{SU(3)_C}$	$\mathrm{U(1)_Y}$	$\mathrm{U(1)'}$	copies
Q_0^i	$\mathbf{3}$	$x^i = (-\frac{1}{3}, -\frac{1}{3}, \frac{2}{3})$	$(0,0,0)$	$(p+q)\times 3$
$\widetilde{Q}_p^i$	$\overline{\mathbf{3}}$	$y^i = (\frac{1}{3}, \frac{1}{3}, -\frac{2}{3})$	$(p,p,-p)$	$q\times 3$
$\widetilde{Q}_{-q}^i$	$\overline{\mathbf{3}}$	$z^i = (\frac{1}{3}, \frac{1}{3}, -\frac{2}{3})$	$(-q,-q,q)$	$p\times 3$

Landau poles

- Compute $SU(3)_c$ and $U(1)_Y$ Landau poles at 1-loop

$$-7\ln\frac{m_t}{\Lambda}+2\left(q\ln\frac{v_p}{\Lambda}+p\ln\frac{v_q}{\Lambda}\right)+\frac{2\pi}{\alpha_s(m_t)}>0$$

$$\frac{41}{6}\ln\frac{m_t}{\Lambda}+4\sum_i Q_{i,Y}^2\left(q\ln\frac{v_p}{\Lambda}+p\ln\frac{v_q}{\Lambda}\right)+\frac{2\pi}{\alpha_{\mathrm{Y}}(m_t)}>0$$

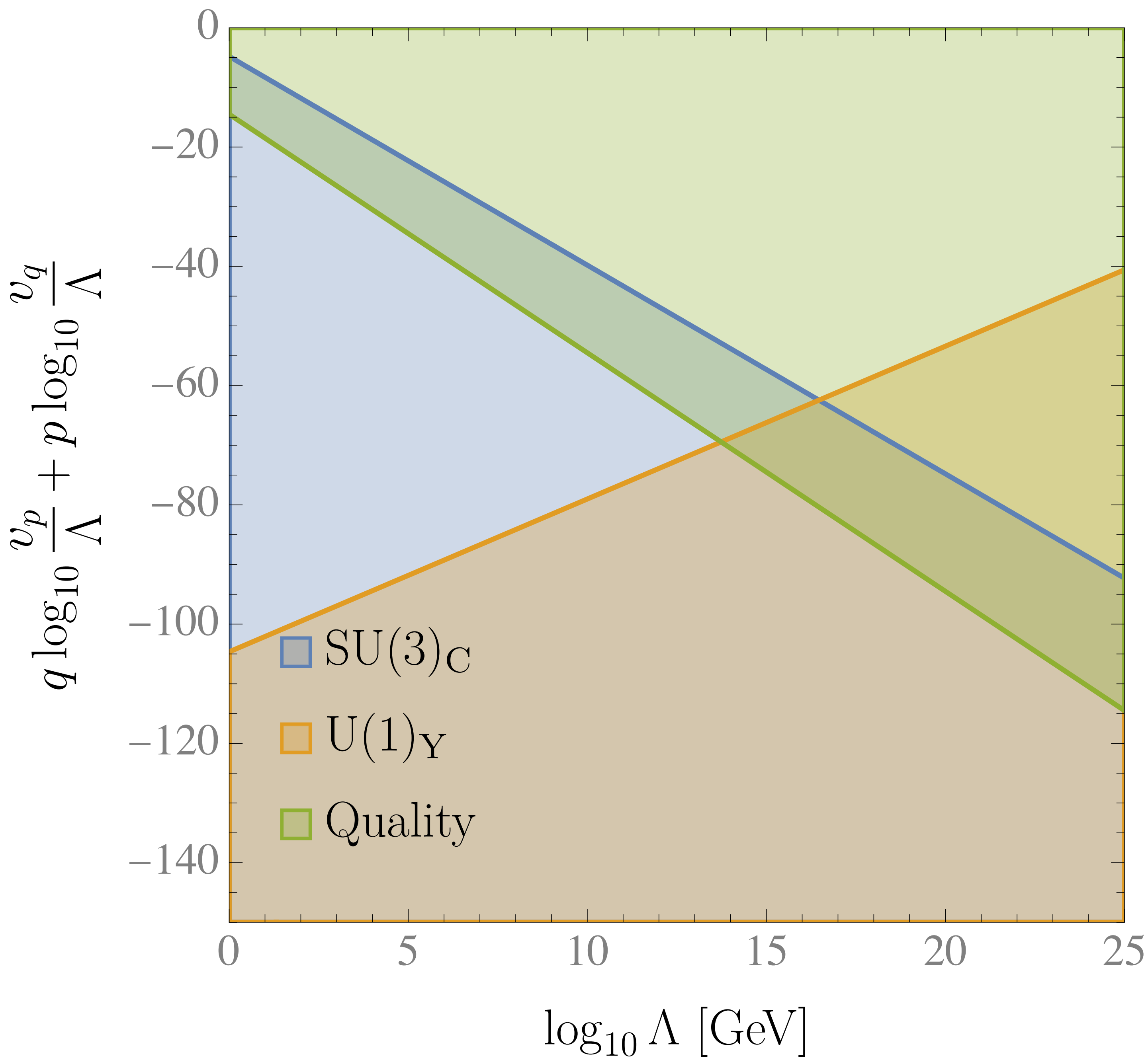
- Requiring good quality axion

$$\ln g+\left(q\ln\frac{v_p}{\Lambda}+p\ln\frac{v_q}{\Lambda}\right)-4\ln\frac{\Lambda_{\mathrm{IR}}}{\Lambda}<\ln 10^{-10}$$

*assuming gauge theory is consistent until gravity scale

Cosmology/Quality Tension

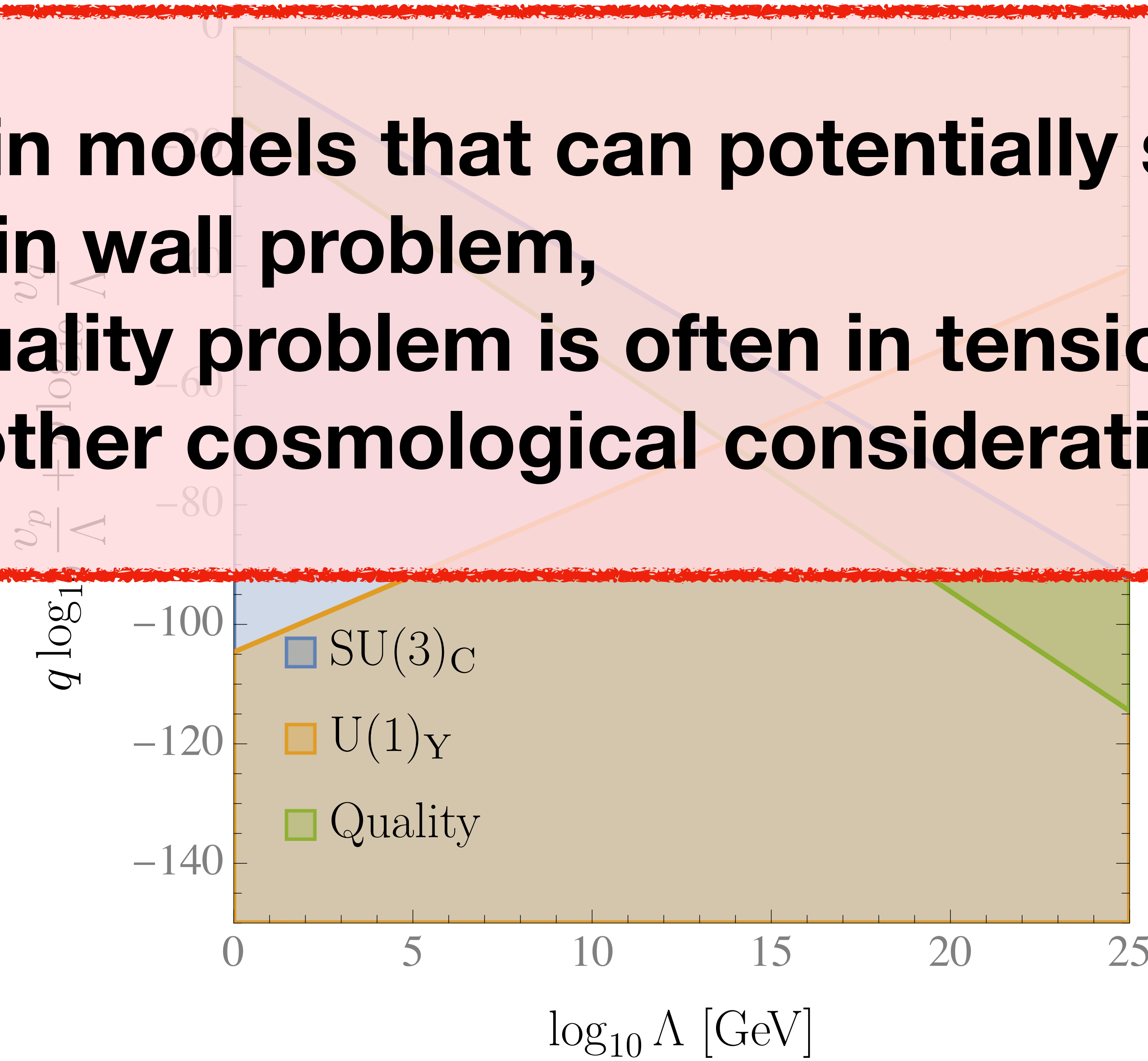
- Considering all constraints, no parameter space left!



Cosmology/Quality Tension

- Considering all constraints, no parameter space left!

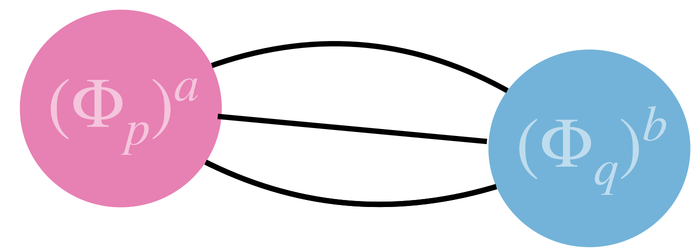
**Even in models that can potentially solve the domain wall problem,
the quality problem is often in tension
with other cosmological considerations!**

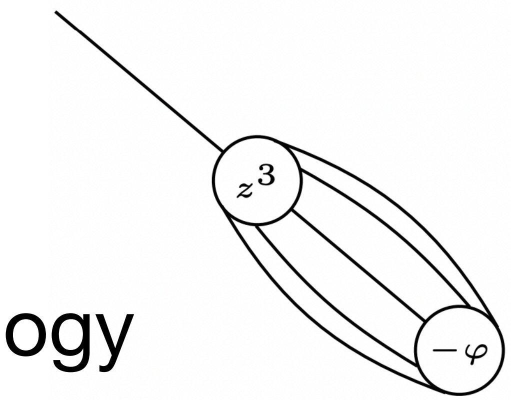


Comments on other models

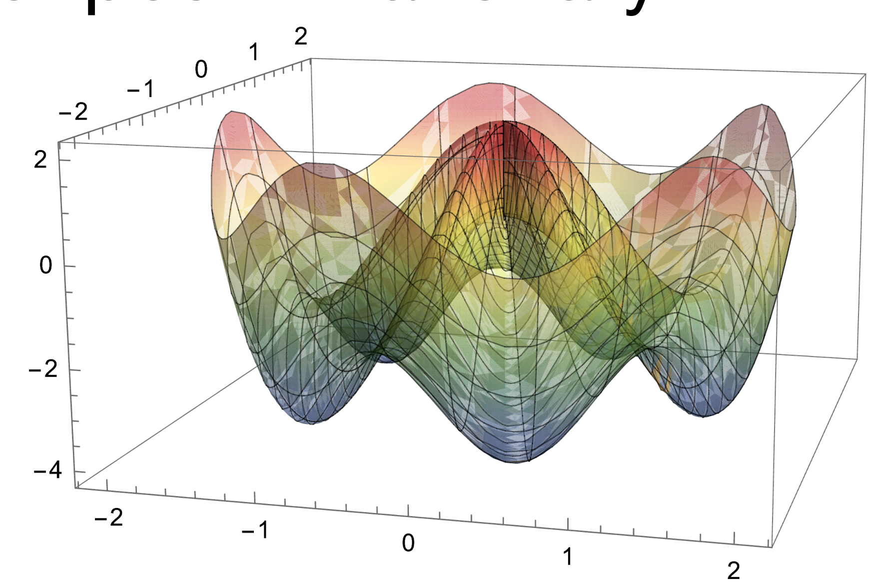
- Embedding $\mathbb{Z}_p$ in continuous $SU(N)$ *M. Ardu, L. Di Luzio, G. Landini, A. Strumia, D. Teresi, and J.-W. Wang, 2007.12663*
    - No exotic relics and no Landau pole problem
    - Problem with dynamical string formation
- Nonabelian gauge symmetry for quality *L. Di Luzio, E. Nardi, and L. Ubaldi, 1704.01122*
    - Better string formation scenario
    - Same exclusion as Barr-Seckel model
- Composite axion models *e.g. M. Redi and R. Sato, 1602.05427; L. Randall, Phys. Lett. B 284 (1992); ……*
    - Often features a $SU(n)$ gauge group
    - Domain wall number is a multiple of n

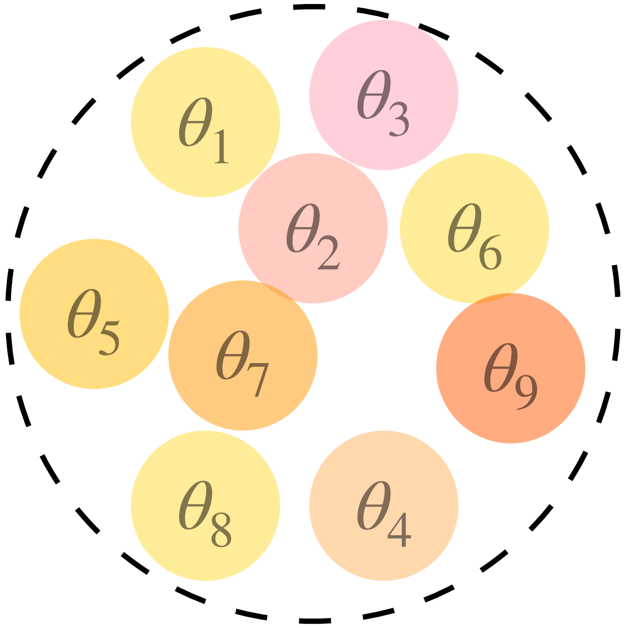
Conclusions





- Axion quality is at tension with standard axion cosmology
  - Domain wall number naively is not 1
  - Under gauge equivalence, no guarantee dynamics will produce the correct strings
  - Often face exotic charged relics and Landau pole problems
- Dynamics of axion string formation needs more careful study
- Worthwhile to investigate a wide variety of post-inflationary scenarios through simulations




θ_1
θ_3
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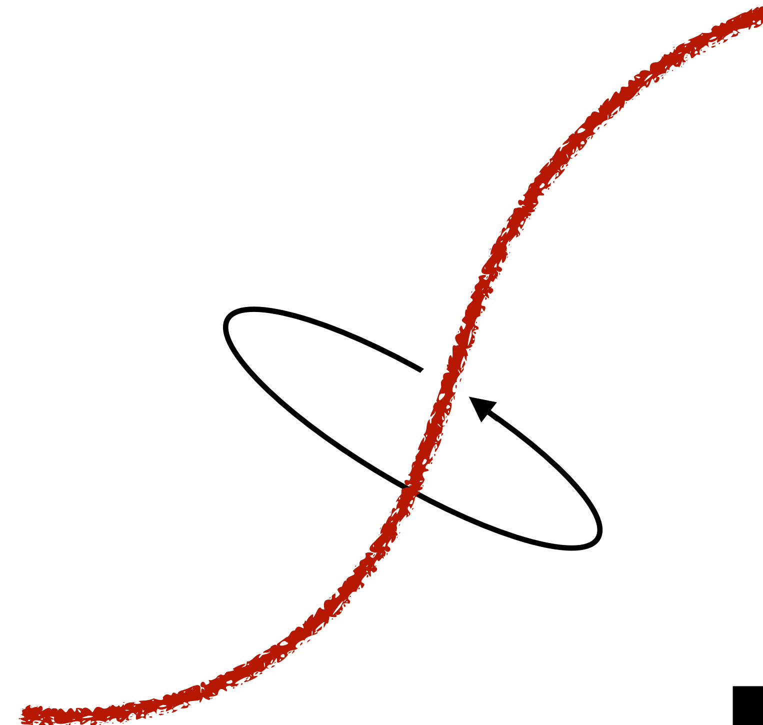
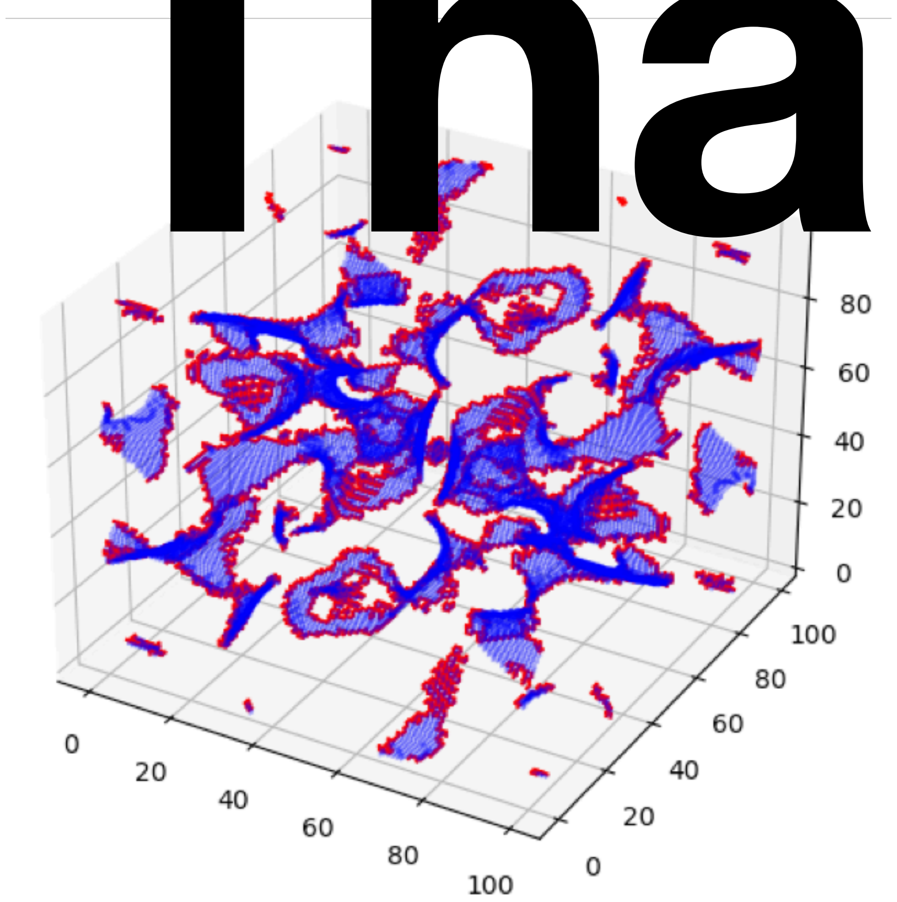
θ_6
θ_5
θ_7
θ_9
θ_8
θ_4



Thank you!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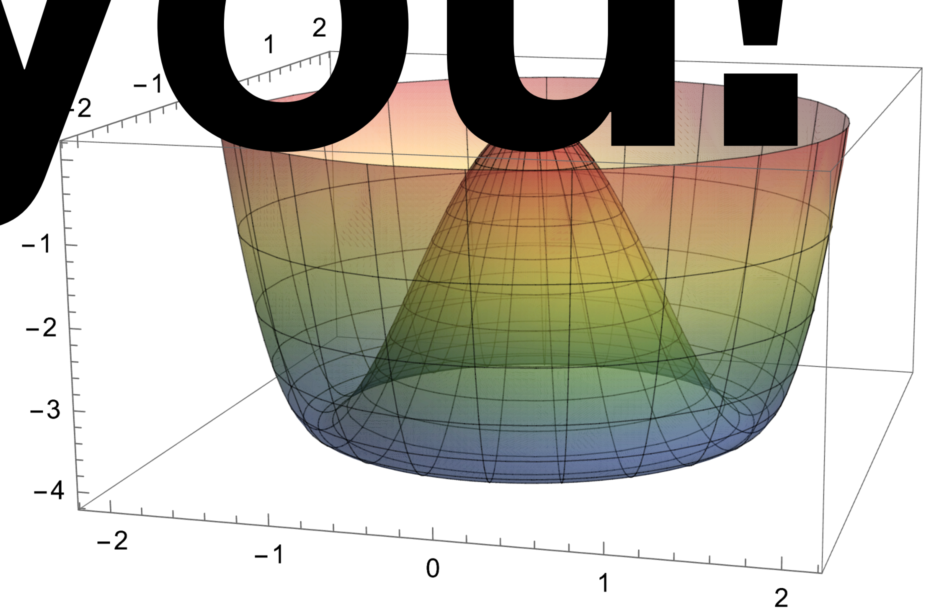
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−3
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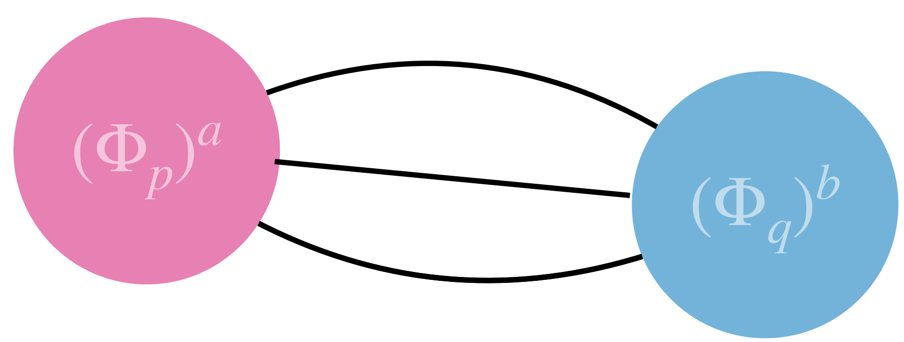
$(\Phi_p)^a$
$(\Phi_q)^b$