FACTORIZATION RESTORATION THROUGH GLAUBER GLUONS

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based on:

Thomas Becher, Patrick Hager, Sebastian Jaskiewicz, MN, Dominik Schwienbacher [2408.10308]

Thomas Becher, MN, Dingyu Shao, Michel Stillger [2307.06359] (JHEP)

INTRODUCTION

- Factorization, the separation of physics effects associated with different scales, is a fundamental property of QFT
- Factorization of cross sections into high-energy (short-distance) parton cross sections convoluted with non-perturbative (long-distance) parton distribution functions is the basis for all calculations of hadron collider processes – **"PDF factorization"**
- This entails the absence of low-energy interactions between the colliding hadrons in the high-energy limit

INTRODUCTION

- Formal proof of PDF factorization has only been given for inclusive Drell-Yan processes (e.g. Higgs production) [Collins, Super, Sterman (1985)]
- Several authors have expressed doubts that it will be valid in general [e.g.: Collins, Qiu (2007); Gaunt (2014); Zeng (2015)]
- Observed **breakdown of collinear factorization** for space-like collinear splittings is often taken as indication that PDF factorization may be violated in higher orders of perturbation theory
  [e.g.: Catani, de Florian, Rodrigo (2011); Forshaw, Seymour, Siodmok (2012); Schwartz, Yan, Zhu (2017); Dixon, Hermann, Yan, Zhu (2019, Erratum: 2024); Cieri, Dhani, Rodrigo (2024); Henn, Ma, Xu, Yan, Zhang, Zhu (2024); Guan, Herzog, Ma, Mistlberger, Suresh (2024)]

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Soft gluon emission at two loops in full color

Lance J. Dixon,[a] Enrico Herrmann,[a] Kai Yan[b] and Hua Xing Zhu[c]

Abstract: The soft emission factor is a central ingredient in the factorization of generic n-particle gauge theory amplitudes with one soft gluon in the external state. We present

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In the limit where the outgoing soft gluon is also collinear with an incoming hard parton, potentially dangerous factorization-violating terms can arise.

We speculate that at next-to-next-to-next-to-leading order (NNNLO) in QCD, integrating over the phase space of the collinear splitting can give rise to soft-collinear poles which depend on the color charge of non-collinear partons entering the process. Such poles cannot be canceled by the conventional counterterms associated with renormalization of the parton distribution functions (PDFs), which by definition are process independent.

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Erratum 23 May 2024: In the limit where the outgoing soft gluon is also collinear with an incoming hard parton, potentially dangerous factorization-violating terms can arise, but they cancel after summing over colors.

INTRODUCTION

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- Collinear factorization:

$$\lim_{p_1 \| p_2} |\mathcal{M}_n(\{p_1, p_2, p_3, \dots, p_n\})\rangle = \mathbf{Sp}(\{p_1, p_2\})\, |\mathcal{M}_{n-1}(\{P, p_3, \dots, p_n\})\rangle + \dots$$

  breakes down if particle 1 is in the initial and particle 2 in the final state

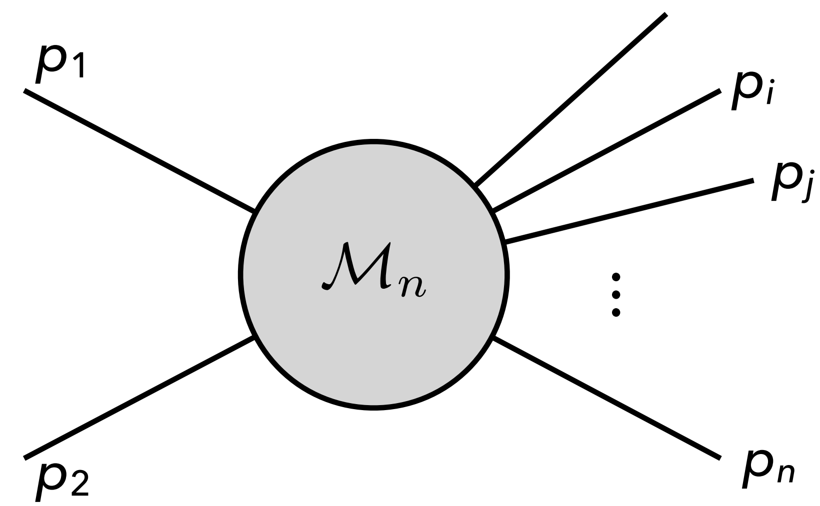
INTRODUCTION

Soft anomalous dimension of *n*-parton scattering amplitudes

- IR poles of scattering amplitudes can be renormalized in a way analogous to UV renormalization: [Becher, MN (2009); Gardi, Magnea (2009)]

$$|\mathcal{M}_n(\{\underline{p}\},\mu)\rangle = \lim_{\epsilon\to 0} \boldsymbol{Z}^{-1}(\epsilon,\{\underline{p}\},\mu)\,|\mathcal{M}_n(\epsilon,\{\underline{p}\})\rangle$$

$$\frac{d}{d\ln\mu}\,|\mathcal{M}_n(\{\underline{p}\},\mu)\rangle = \boldsymbol{\Gamma}(\{\underline{p}\},\mu)\,|\mathcal{M}_n(\{\underline{p}\},\mu)\rangle$$



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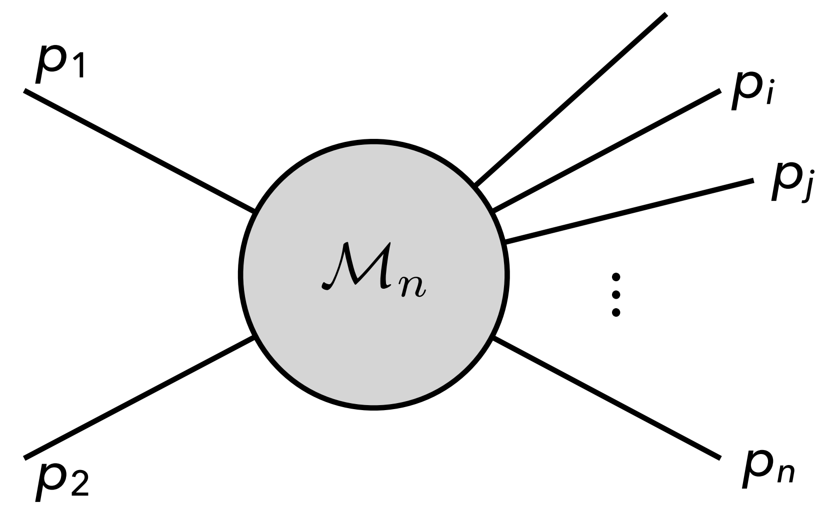


with:

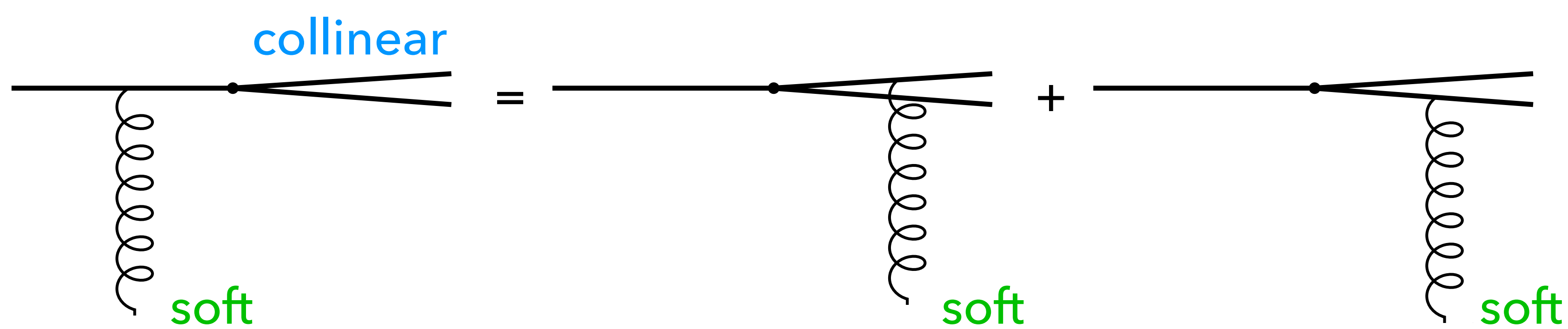
$$\boldsymbol{\Gamma}(\{\underline{p}\},\mu) = -\boldsymbol{Z}^{-1}(\epsilon,\{\underline{p}\},\mu)\,\frac{d}{d\ln\mu}\,\boldsymbol{Z}(\epsilon,\{\underline{p}\},\mu)$$

$$= \sum_{(ij)} \frac{\boldsymbol{T}_i\cdot\boldsymbol{T}_j}{2}\,\gamma_{\rm cusp}(\alpha_s)\,\ln\frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i(\alpha_s) + \mathcal{O}(\alpha_s^3)$$

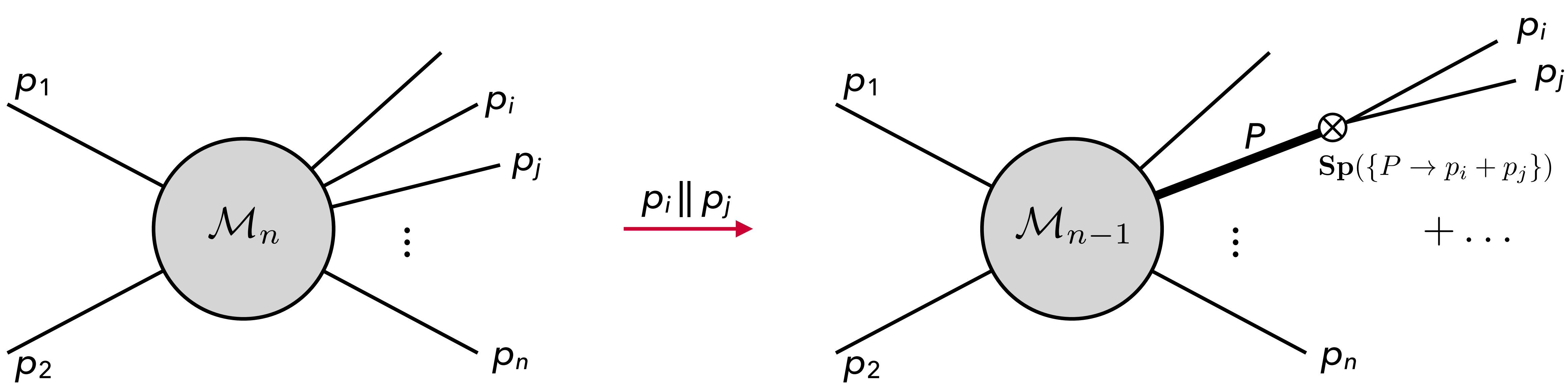
and $s_{ij} = (p_i+p_j)^2 > 0$ if particles i, j are both in the initial or final state

INTRODUCTION

- **Color coherence holds** if all three particles are incoming or outgoing (time-like splitting)

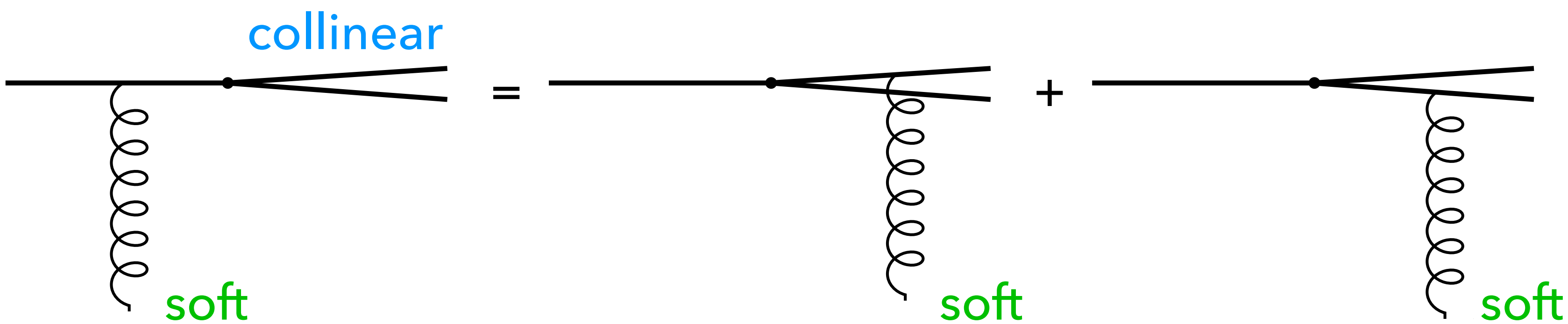


- **Collinear factorization holds:**

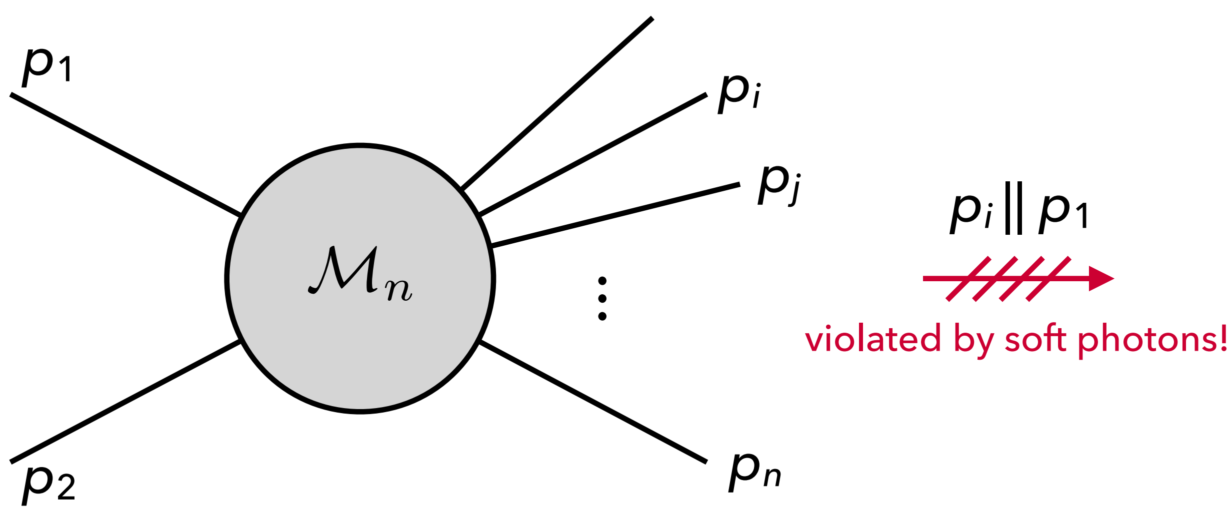


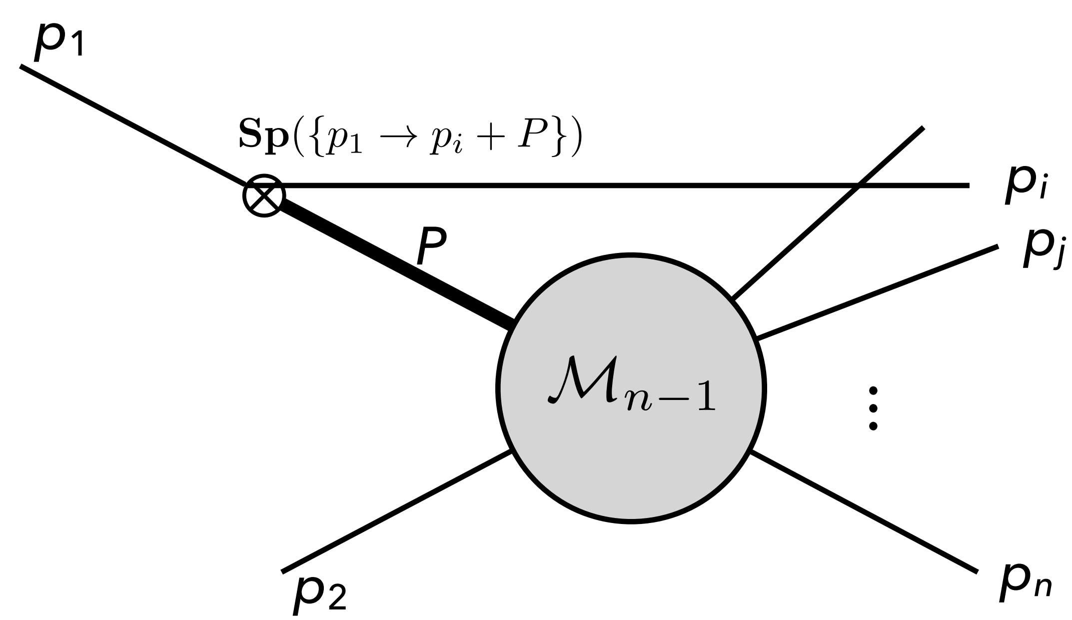
INTRODUCTION

- **Color coherence is broken** if not all particles are incoming/outgoing (space-like splitting), since both sides receive different phase factors

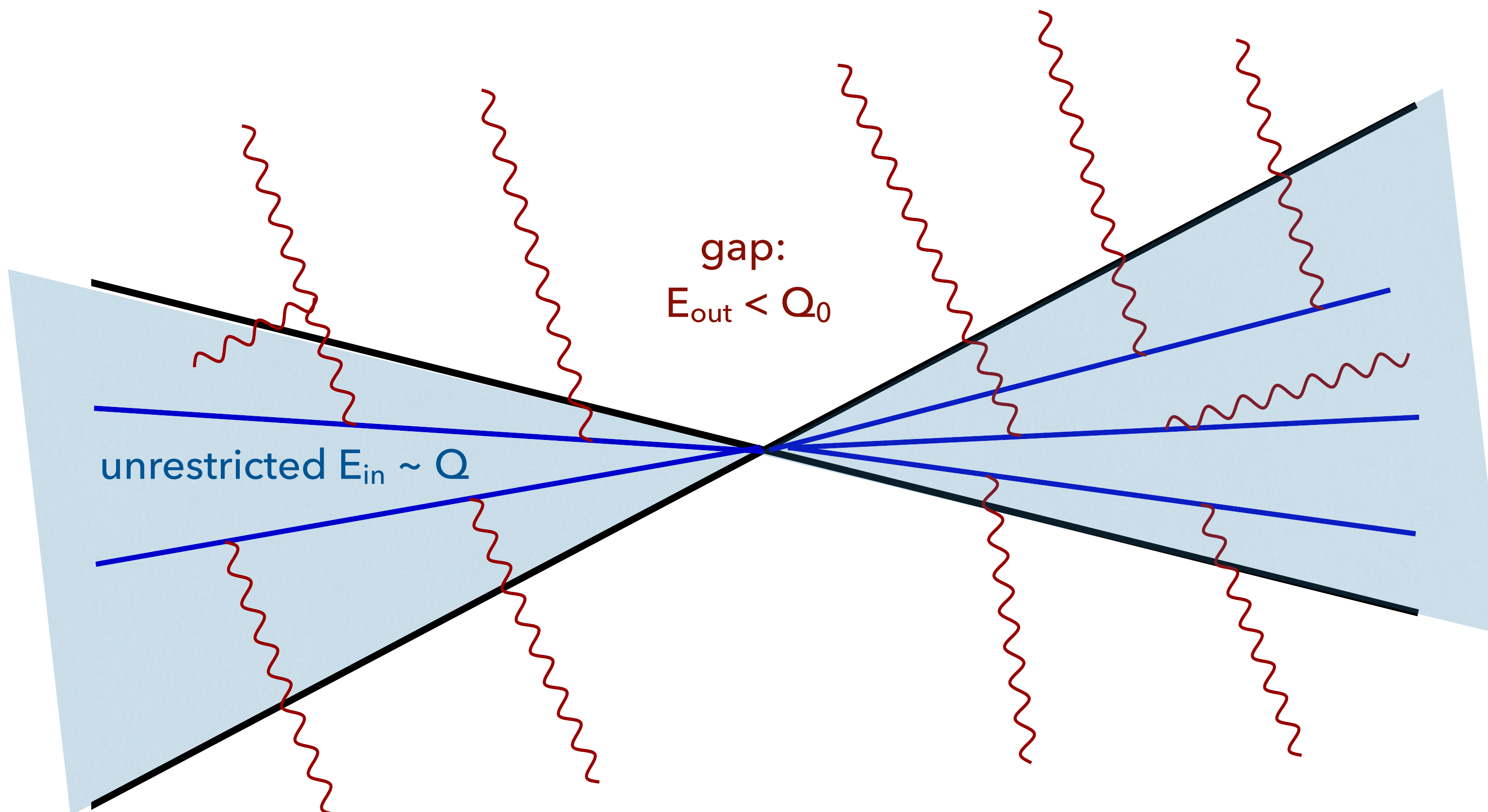


- **Collinear factorization is violated:**





LARGE LOGARITHMS IN LHC JET PROCESSES



$$L = \ln(Q/Q_0) \gg 1$$

Perturbative expansion includes "super-leading" logarithms:

$$\sigma \sim \sigma_{\text{Born}} \times \Big\{ 1 + \alpha_s L + \alpha_s^2 L^2 + \alpha_s^3 L^3 + \underbrace{\alpha_s^4 \, L^5 + \alpha_s^5 \, L^7 + \dots}_{\text{formally larger than } O(1)} \Big\}$$

state-of-the-art

[Forshaw, Kyrieleis, Seymour (2006)]

LARGE LOGARITHMS IN LHC JET PROCESSES

- "Super-leading logarithms" (SLLs) in exclusive jet cross sections have the same origin; they are double-logarithmic effects arising from complex phases in hard-scattering amplitudes that break color coherence [Forshaw, Kyrieleis, Seymour (2006)]
- Since PDF evolution is single logarithmic, the presence of SLLs necessitates the existence of low-energy interactions between the incoming partons, and the **key questions is whether this is a perturbative effect**

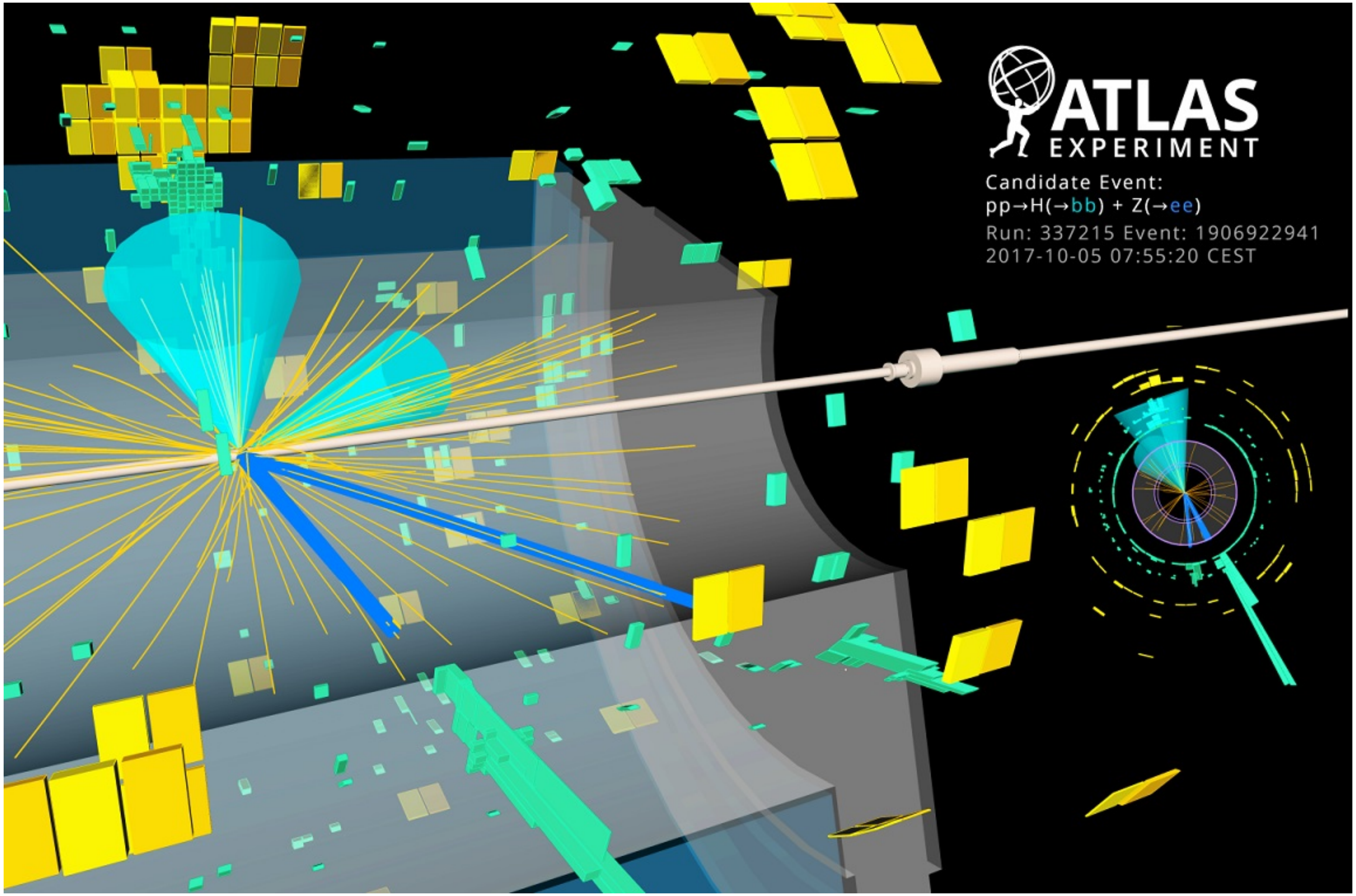
LARGE LOGARITHMS IN LHC JET PROCESSES

- Collinear factorization breaking and SLLs are associated with **Glauber dynamics,** whose cancellation was crucial in the factorization proof for the Drell-Yan process
- Both effects were discovered long ago, but an all-order understanding is still lacking
- Important progress was achieved using SCET to calculate the all-order structure of SLLs for arbitrary processes based on a new factorization theorem and an associated RG evolution equation
  [Becher, MN, Shao (2021); Becher, MN, Shao, Stillger (2023)]
- SLLs arise first at 4-loop order, and the restoration of PDF factorization requires an intricate interplay of high-energy and (perturbative) low-energy dynamics, whose mechanism has so far remained elusive

LARGE LOGARITHMS IN LHC JET PROCESSES

- Here, we identify for the first time a **genuine contribution of an active-active Glauber exchange** to a cross section, and show that in leading logarithmic approximation it has the required form to turn the double-logarithmic evolution back into single-logarithmic evolution
- Our analysis is not a proof of factorization, but it provides some evidence that PDF factorization may, indeed, be valid for non-global hadron collider observables
- It demonstrates that the breaking of collinear factorization does not necessarily translate into a breaking of PDF factorization

GAP-BETWEEN-JETS OBSERVABLES



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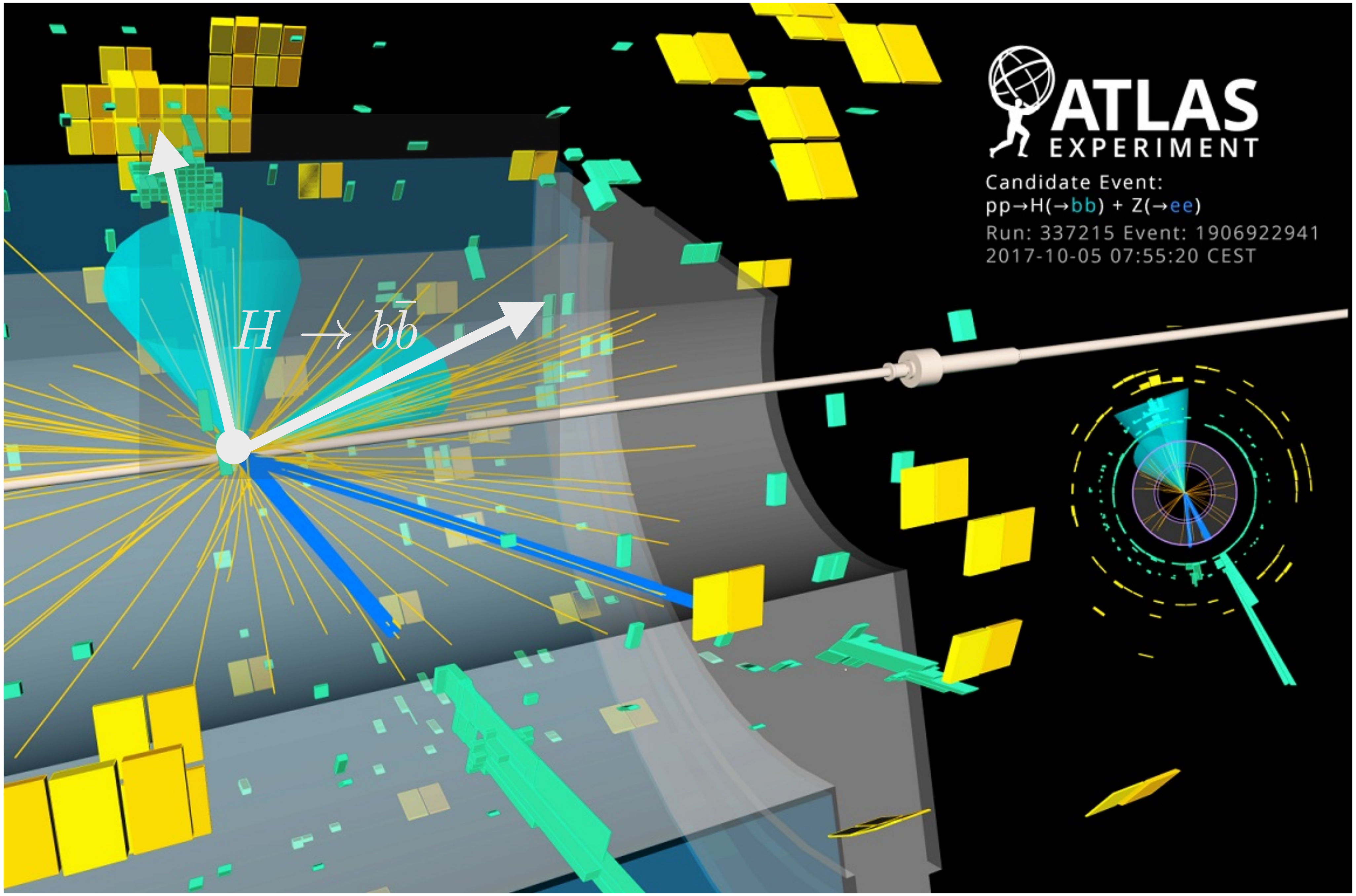

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GAP-BETWEEN-JETS OBSERVABLES

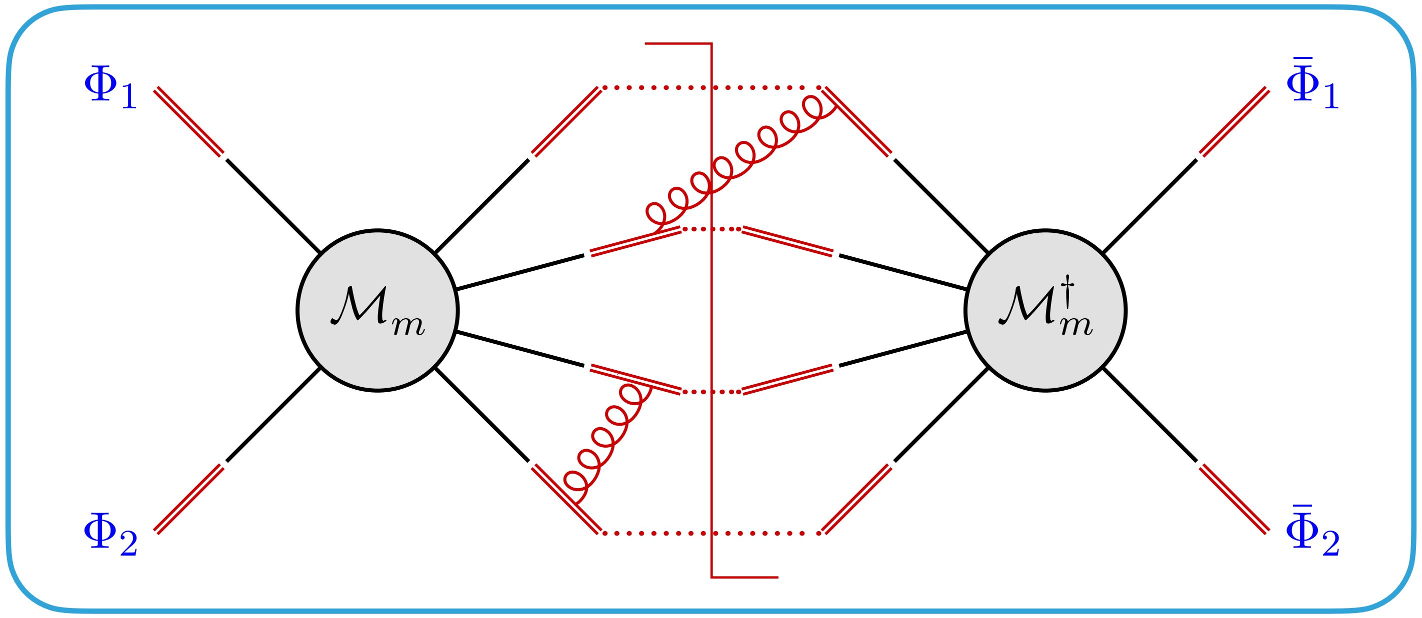
SCET factorization theorem for *M*-jet production at the LHC

$$\sigma(Q_0) = \sum_{m=m_0}^{\infty} \int d\xi_1 d\xi_2 \, \big\langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\}, Q, \xi_1, \xi_2, \mu) \otimes \boldsymbol{\mathcal{W}}_m(\{\underline{n}\}, Q_0, \xi_1, \xi_2, \mu) \big\rangle$$

[Becher, MN, Shao (2021);
Becher, MN, Rothen, Shao (2015, 2016)]

high scale

low scale



⇒ new perspective to think about non-global observables!

[see also: Martínez, De Angelis, Forshaw, Plätzer, Seymour (2018);
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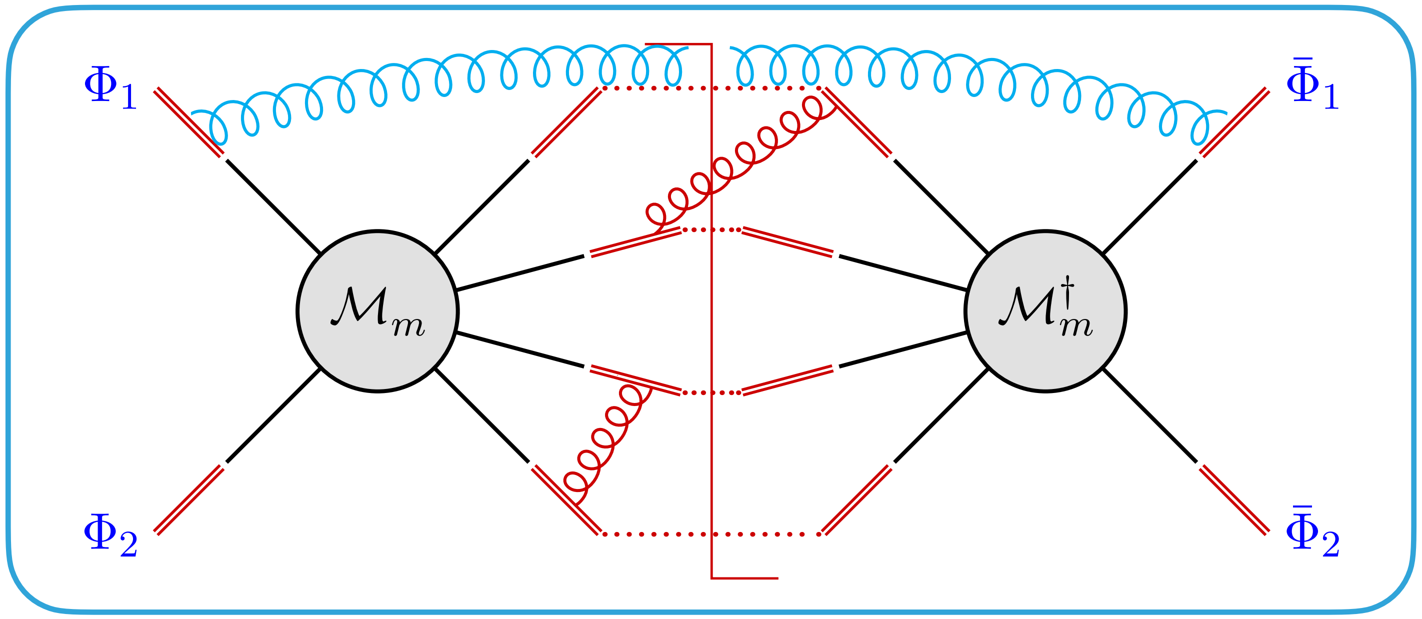

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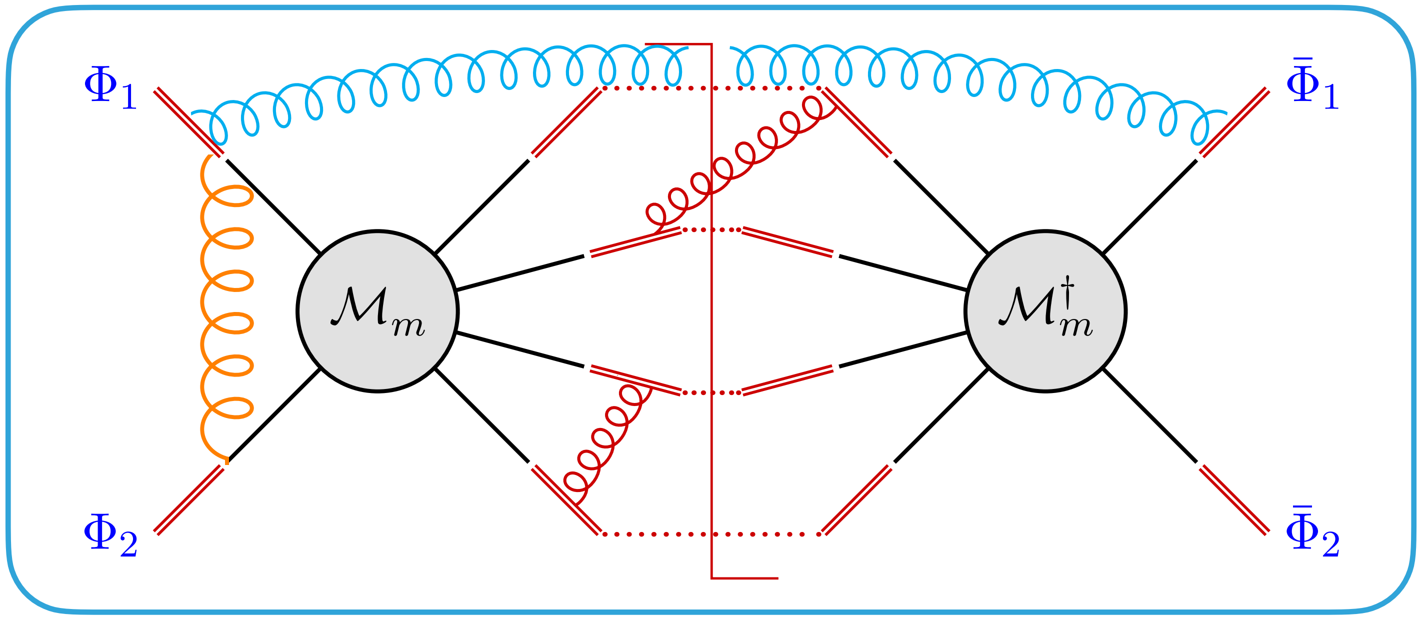

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[Becher, MN, Shao (2021);
Becher, MN, Rothen, Shao (2015, 2016)]

- Sum includes all partonic channels with multiplicity *m*
- Hard functions: $\boldsymbol{\mathcal{H}}_m(\{\underline{n}\}, Q, \xi_1, \xi_2, \mu) = \int d\mathcal{E}_m \, |\mathcal{M}_m(\{\underline{p}\})\rangle\langle\mathcal{M}_m(\{\underline{p}\})|$
- Functions $\boldsymbol{\mathcal{W}}_m$ contain the low-energy soft and collinear dynamics
- $\otimes$ indicates integration over the parton directions $\{n_i\}$

GAP-BETWEEN-JETS OBSERVABLES

SCET factorization theorem for *M*-jet production at the LHC

$$\sigma(Q_0) = \sum_{m=m_0}^{\infty} \int d\xi_1 d\xi_2 \left\langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\}, Q, \xi_1, \xi_2, \mu) \otimes \boldsymbol{\mathcal{W}}_m(\{\underline{n}\}, Q_0, \xi_1, \xi_2, \mu) \right\rangle$$

[Becher, MN, Shao (2021);
Becher, MN, Rothen, Shao (2015, 2016)]

high scale

low scale

- RG evolution of the hard functions:

$$\mu \frac{d}{d\mu} \boldsymbol{\mathcal{H}}_l \, (\{\underline{n}\}, Q, \mu) = -\sum_{m \le l} \boldsymbol{\mathcal{H}}_m(\{\underline{n}\}, Q, \mu) \, \boldsymbol{\Gamma}^H_{ml}(\{\underline{n}\}, Q, \mu)$$

operator in color space and in the infinite space of parton multiplicities

- Basis for the **resummation of large logarithmic corrections**

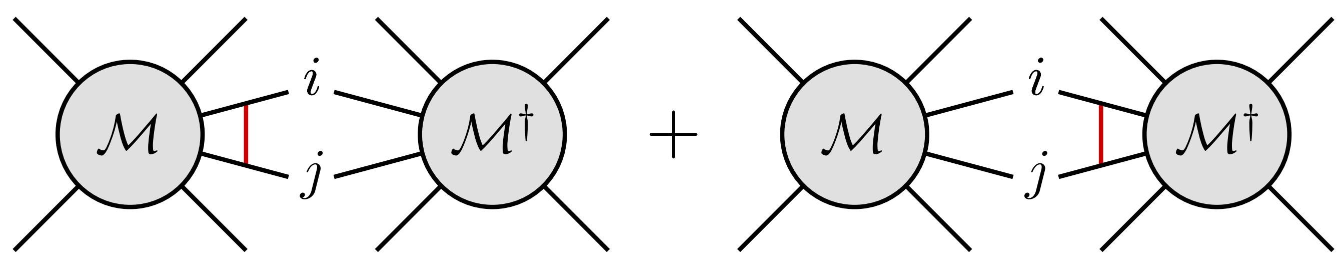
GAP-BETWEEN-JETS OBSERVABLES

Structure of the anomalous dimension

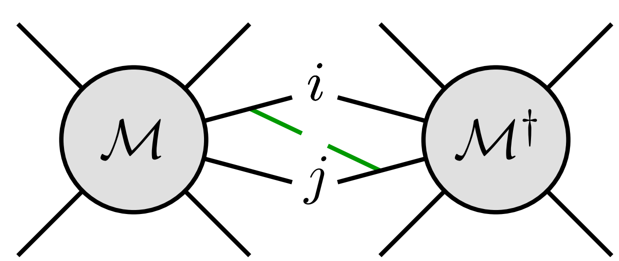
$$\mathbf{\Gamma}^H = \frac{\alpha_s}{4\pi} \begin{pmatrix} \boldsymbol{V}_{2+M} & \boldsymbol{R}_{2+M} & 0 & 0 & \dots \\ 0 & \boldsymbol{V}_{2+M+1} & \boldsymbol{R}_{2+M+1} & 0 & \dots \\ 0 & 0 & \boldsymbol{V}_{2+M+2} & \boldsymbol{R}_{2+M+2} & \dots \\ 0 & 0 & 0 & \boldsymbol{V}_{2+M+3} & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} + \mathcal{O}(\alpha_s^2)$$

- Action on hard functions:

$$\boldsymbol{\mathcal{H}}_m \, \boldsymbol{V}_m = \sum_{(ij)}$$



$$\boldsymbol{\mathcal{H}}_m \, \boldsymbol{R}_m = \sum_{(ij)}$$



GAP-BETWEEN-JETS OBSERVABLES

Structure of the anomalous dimension

$$\boldsymbol{\Gamma}^H = \gamma_{\rm cusp}(\alpha_s)\left(\boldsymbol{\Gamma}^c \ln\frac{\mu^2}{Q^2} + \boldsymbol{V}^G\right) + \frac{\alpha_s}{4\pi}\,\overline{\boldsymbol{\Gamma}} + \boldsymbol{\Gamma}^C$$

[Becher, MN, Shao, Stillger (2023)]

- $\gamma_{\rm cusp}$: light-like cusp anomalous dimension
- $\boldsymbol{\Gamma}^c$: soft & collinear emissions (real/virtual gluons)
- $\mathbf{V}^G$: virtual Glauber-gluon exchange (purely imaginary)
- $\overline{\boldsymbol{\Gamma}}$: real/virtual emissions after collinear subtractions (into the gap)
- $\boldsymbol{\Gamma}^C$: purely collinear emissions (DGLAP and more)

$\Rightarrow$ all double logarithms arise from $\boldsymbol{\Gamma}^c$ (source of the SLLs)

RESUMMATION OF SUPER-LEADING LOGARITHMS

Evaluate factorization theorem at low scale $\mu_s \sim Q_0$

- Low-energy matrix element:

$$\mathcal{W}_m(\{\underline{n}\}, Q_0, x_1, x_2, \mu_s) = f_{a/p}(x_1)\, f_{b/p}(x_2)\, \mathbf{1} + \mathcal{O}(\alpha_s)$$

- Hard-scattering functions:

$$\mathcal{H}_m(\{\underline{n}\}, Q, \mu_s) = \sum_{l \le m} \mathcal{H}_l(\{\underline{n}\}, Q, Q)\, \mathbf{P} \exp\left[\int_{\mu_s}^{Q} \frac{d\mu}{\mu}\, \mathbf{\Gamma}^H(\{\underline{n}\}, Q, \mu)\right]_{lm}$$

- Expanding the solution in a power series generates arbitrarily high parton multiplicities starting from the $2 \to M$ Born process

RESUMMATION OF SUPER-LEADING LOGARITHMS

SLLs arise from the terms in $\mathbf{P}\exp\left[\int_{\mu_s}^{Q}\frac{d\mu}{\mu}\,\mathbf{\Gamma}^H(\{\underline{n}\},Q,\mu)\right]_{lm}$ **with the highest number of insertions of** $\mathbf{\Gamma}^c$

- Three properties simplify the calculation:

- color coherence in the absence of Glauber phases: $\boldsymbol{\mathcal{H}}_m\,\mathbf{\Gamma}^c\,\overline{\mathbf{\Gamma}} = \boldsymbol{\mathcal{H}}_m\,\overline{\mathbf{\Gamma}}\,\mathbf{\Gamma}^c$
- collinear safety: $\langle\boldsymbol{\mathcal{H}}_m\,\mathbf{\Gamma}^c\otimes\mathbf{1}\rangle = 0$
- cyclicity of the trace: $\langle\boldsymbol{\mathcal{H}}_m\,\boldsymbol{V}^G\otimes\mathbf{1}\rangle = 0$

RESUMMATION OF SUPER-LEADING LOGARITHMS

SLLs arise from the terms in $\mathbf{P}\exp\left[\int_{\mu_s}^{Q}\frac{d\mu}{\mu}\,\mathbf{\Gamma}^H(\{\underline{n}\},Q,\mu)\right]_{lm}$ **with the highest number of insertions of Γ^c**

- Under the color trace, insertions of Γ_c are non-zero only if they come in conjunction with (at least) two Glauber phases and one $\overline{\Gamma}$
- Relevant color traces at $\mathcal{O}(\alpha_s^{n+3}L^{2n+3})$: [Becher, MN, Shao (2021)]

$$C_{rn} = \left\langle \boldsymbol{\mathcal{H}}_{2\to M}\,(\mathbf{\Gamma}^c)^r\,\boldsymbol{V}^G\,(\mathbf{\Gamma}^c)^{n-r}\,\boldsymbol{V}^G\,\overline{\mathbf{\Gamma}}\otimes\mathbf{1}\right\rangle$$

- First SLL appears at 4-loop order and involves the color traces:

$$C_{11} = \left\langle \boldsymbol{\mathcal{H}}_{2\to M}\,\mathbf{\Gamma}^c\boldsymbol{V}^G\boldsymbol{V}^G\,\overline{\mathbf{\Gamma}}\otimes\mathbf{1}\right\rangle, \quad C_{01} = \left\langle \boldsymbol{\mathcal{H}}_{2\to M}\,\boldsymbol{V}^G\mathbf{\Gamma}^c\boldsymbol{V}^G\,\overline{\mathbf{\Gamma}}\otimes\mathbf{1}\right\rangle$$

RESUMMATION OF SUPER-LEADING LOGARITHMS

UV poles of the low-energy matrix element

- RG invariance of the cross section implies that the $1/\varepsilon$ poles of the bare functions $\boldsymbol{\mathcal{W}}_m^{\rm bare}$ must be of the form:

$$\boldsymbol{\mathcal{W}}_m^{\rm bare} = \mathbf{1} + \frac{\alpha_s}{4\pi}\,\frac{\overline{\boldsymbol{\Gamma}}}{2\varepsilon} + \left(\frac{\alpha_s}{4\pi}\right)^2 \left(\frac{\boldsymbol{V}^G\,\overline{\boldsymbol{\Gamma}}}{2\varepsilon^2} + \dots\right) + \left(\frac{\alpha_s}{4\pi}\right)^3 \left(\frac{\boldsymbol{V}^G\boldsymbol{V}^G\,\overline{\boldsymbol{\Gamma}}}{3\varepsilon^3} - \frac{\boldsymbol{\Gamma}^c\boldsymbol{V}^G\,\overline{\boldsymbol{\Gamma}}}{3\varepsilon^3}\,\ln\frac{Q^2}{\mu_s^2} + \dots\right) + \mathcal{O}(\alpha_s^4)$$

collinear anomaly
[Becher, MN (2010); Chiu, Jain, Neill, Rothstein (2011)]

- Log-enhanced term $\boldsymbol{\Gamma}^c$ appears first at **3-loop order** and gives rise to **large rapidity logarithms** $\ln^n(Q/\mu_s)$ in the low-energy theory
- How is this pole term reproduced in the low-energy theory? Is it of perturbative or nonperturbative nature?

STRUCTURE OF THE FACTORIZATION THEOREM ?

hard scale $Q \sim \sqrt{\hat{s}}$ — $\mathcal{H}_m$

double-logarithmic evolution
(super-leading logs)

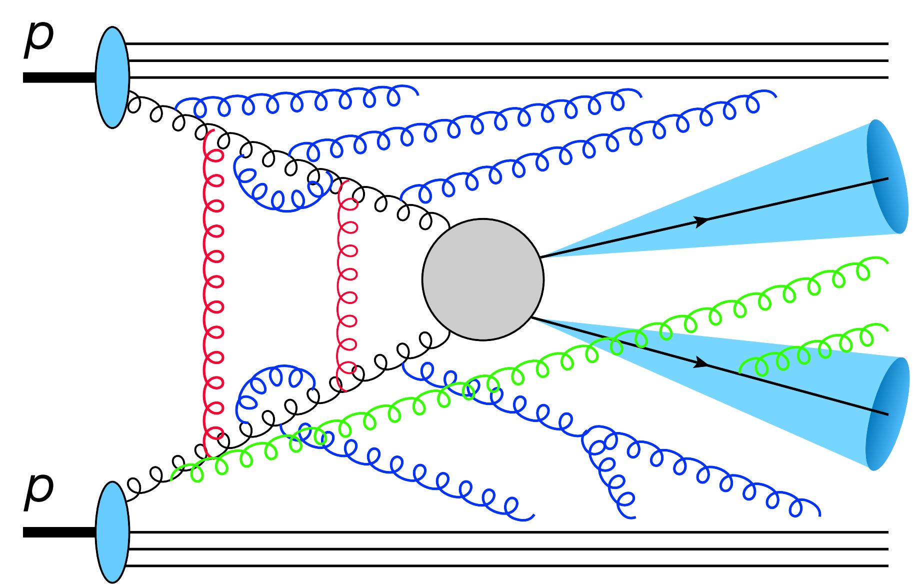
jet-veto scale Q_0 — $\mathcal{W}_m^{\rm pert}$

single-logarithmic PDF evolution

OR: *double-logarithmic evolution and breaking of PDF factorization?*

hadronic scale $\Lambda_{\rm QCD}$ — $f_{a/p}(\xi_1)\, f_{b/p}(\xi_2)$

OR: *something more complicated, e.g. a combined two-proton distribution?*



$$\sigma \overset{?}{\sim} \sum_m \mathcal{H}_m \otimes \mathcal{W}_m^{\rm pert} \otimes f_{a/p}\, f_{b/p}$$

PERTURBATIVE ANALYSIS OF THE LOW-ENERGY DYNAMICS

UV poles of the low-energy matrix element

- RG invariance of the cross section implies that the $1/\varepsilon$ poles of the bare functions $\boldsymbol{\mathcal{W}}_m^{\rm bare}$ must be of the form:

$$\boldsymbol{\mathcal{W}}_m^{\rm bare} = \mathbf{1} + \frac{\alpha_s}{4\pi}\,\frac{\overline{\boldsymbol{\Gamma}}}{2\varepsilon} + \left(\frac{\alpha_s}{4\pi}\right)^2 \left(\frac{\boldsymbol{V}^G\,\overline{\boldsymbol{\Gamma}}}{2\varepsilon^2} + \dots\right) + \left(\frac{\alpha_s}{4\pi}\right)^3 \left(\frac{\boldsymbol{V}^G\boldsymbol{V}^G\,\overline{\boldsymbol{\Gamma}}}{3\varepsilon^3} - \frac{\boldsymbol{\Gamma}^c\,\boldsymbol{V}^G\,\overline{\boldsymbol{\Gamma}}}{3\varepsilon^3}\ln\frac{Q^2}{\mu_s^2} + \dots\right) + \mathcal{O}(\alpha_s^4)$$

- We have reproduced the pole terms involving only $\overline{\boldsymbol{\Gamma}}$ and $\boldsymbol{V}^G$ using the known 1-loop and 2-loop results for the soft current

[Catani, Grazzini (2000); Duhr, Gehrmann (2013); Dixon, Hermann, Yan, Zhu (2019)]

- The Glauber phases are contained in the soft current and do not need additional Glauber contributions in SCET

PERTURBATIVE ANALYSIS OF THE LOW-ENERGY DYNAMICS

UV poles of the low-energy matrix element

- To reproduce the last term, we have considered 3-loop graphs featuring a **soft gluon emitted into the gap** (dependence on Q_0), a **collinear emission** (rapidity log), and a **virtual gluon** connecting the two incoming partons (Glauber phase)
- There exist three such diagrams (plus mirror copies)

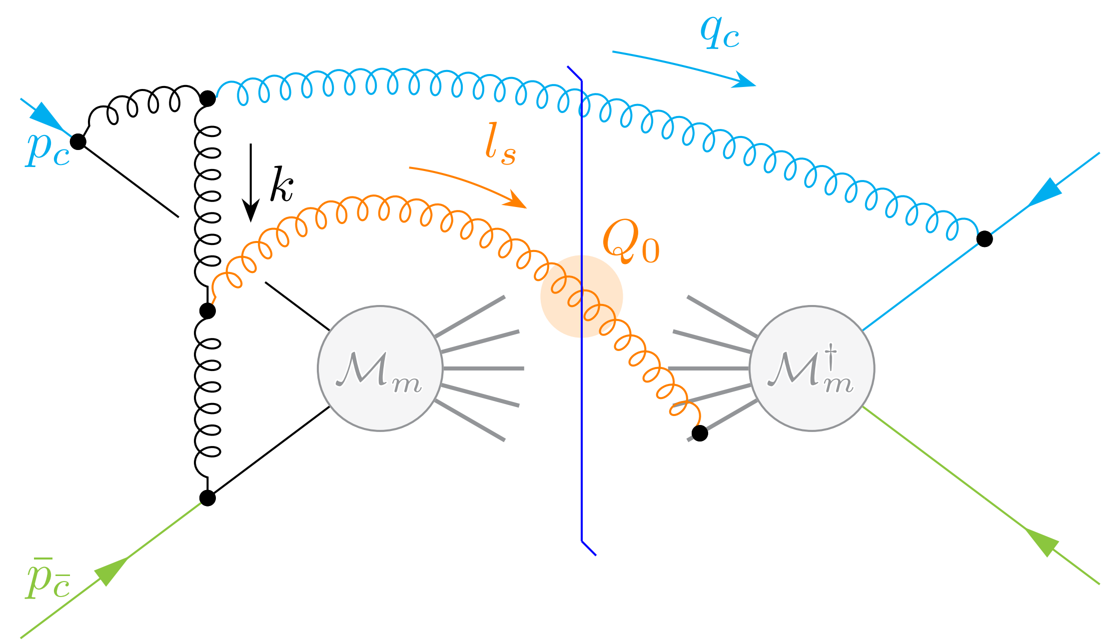


FIG. 1. Sample perturbative contribution to the gap-between-jets cross section. The gray inner subdiagrams make up the hard function $\boldsymbol{\mathcal{H}}_m$, while the remainder is part of $\boldsymbol{\mathcal{W}}_m$. The orange gluon is soft and enters the veto region, the blue and green partons are collinear to the beams. Possible scalings of the virtual gluon momentum k will be analyzed below.

PERTURBATIVE ANALYSIS OF THE LOW-ENERGY DYNAMICS

UV poles of the low-energy matrix element

- To reproduce the last term, we have considered 3-loop graphs featuring a **soft gluon emitted into the gap** (dependence on Q_0), a **collinear emission** (rapidity log), and a **virtual gluon** connecting the two incoming partons (Glauber phase)
- There exist three such diagrams (plus mirror copies)

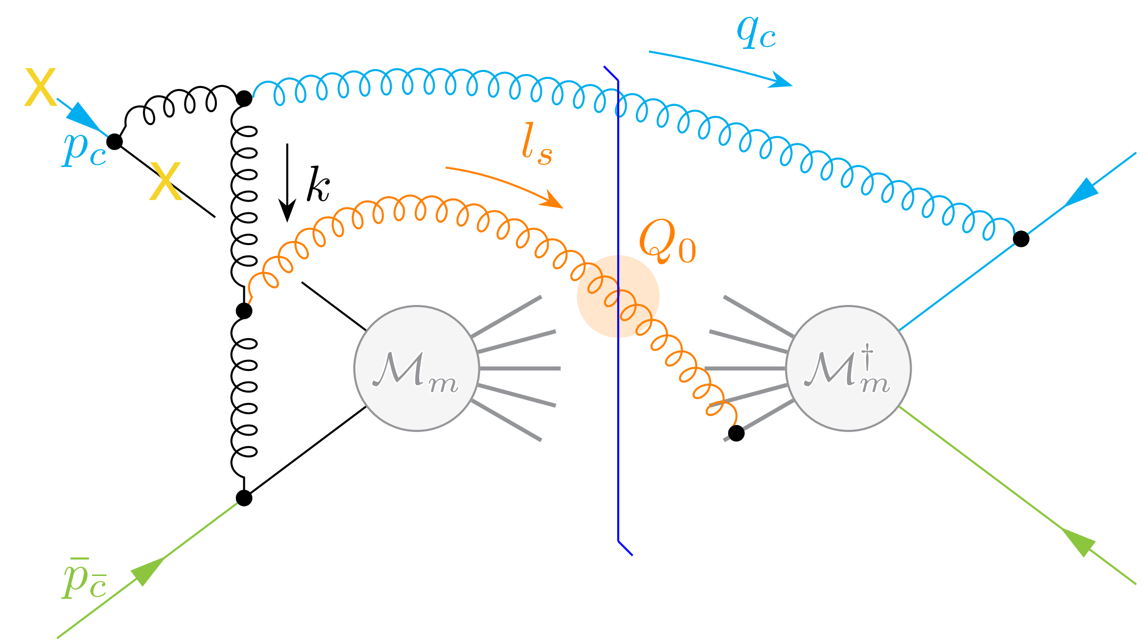


FIG. 1. Sample perturbative contribution to the gap-between-jets cross section. The gray inner subdiagrams make up the hard function $\boldsymbol{\mathcal{H}}_m$, while the remainder is part of $\boldsymbol{\mathcal{W}}_m$. The orange gluon is soft and enters the veto region, the blue and green partons are collinear to the beams. Possible scalings of the virtual gluon momentum k will be analyzed below.

PERTURBATIVE ANALYSIS OF THE LOW-ENERGY DYNAMICS

UV poles of the low-energy matrix element

- Stripp off the numerator structure and consider the related pentagon (box) integrals for fixed external momenta
- Comparison with the exact results ensures that all regions are considered [Bern, Dixon, Kosower (1993)]
- Introducing a power-counting parameter $\lambda = Q_0/Q$, the external momenta scale as $p_c, q_c \sim Q(\lambda^2, 1, \lambda)$, $\bar{p}_{\bar{c}} \sim Q(1, \lambda^2, \lambda)$, and $l_2 \sim Q(\lambda, \lambda, \lambda)$
- Introduce variables $s_{i,i+1} = (p_i + p_{i+1})^2$ and $m^2 = p_5^2$

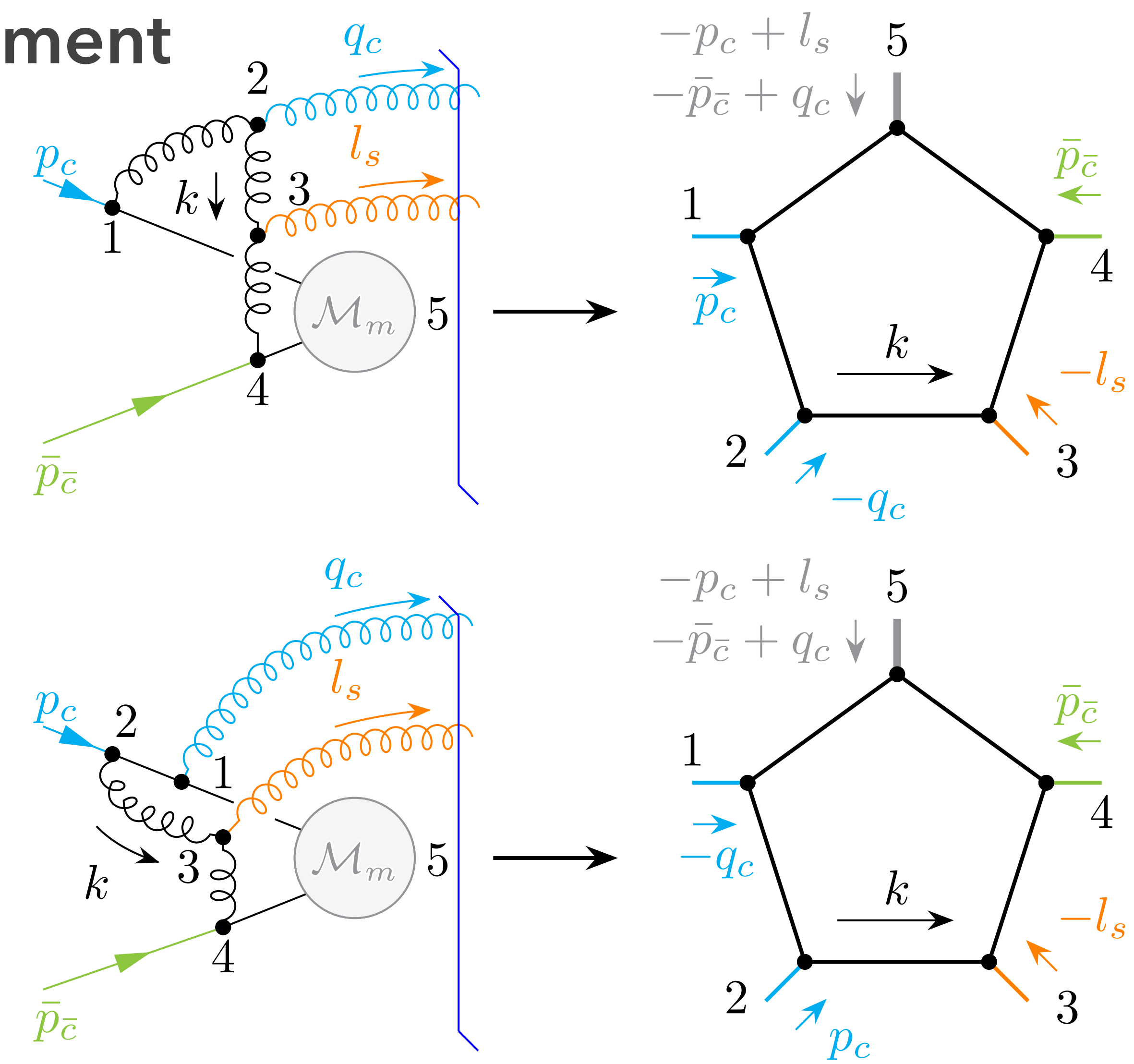
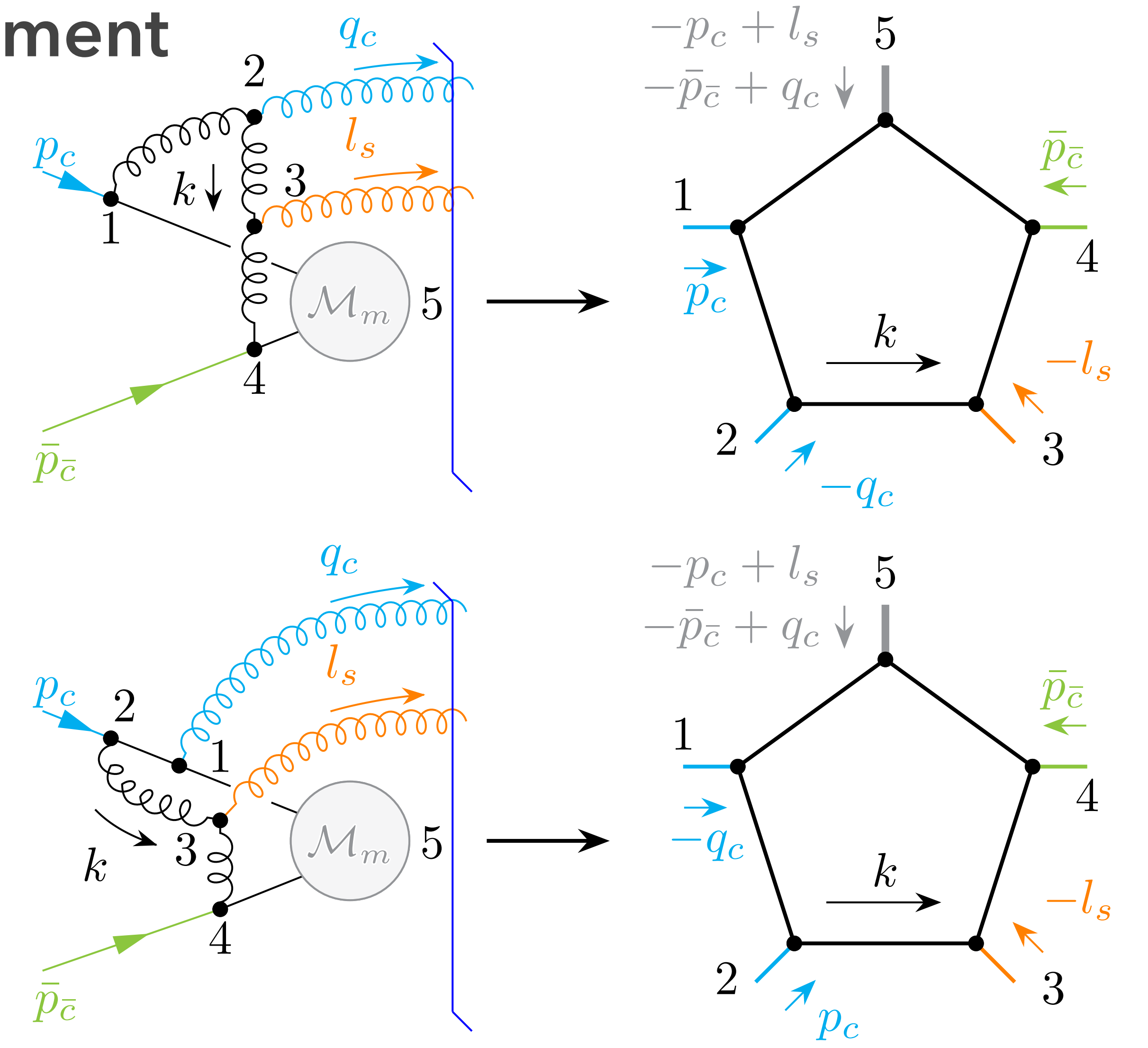


FIG. 2. Mapping of low-energy contributions to $\boldsymbol{\mathcal{W}}_m$ onto pentagon diagrams. The external momentum $p_5^2 \neq 0$ flows into the hard amplitude $\mathcal{M}_m$.

PERTURBATIVE ANALYSIS OF THE LOW-ENERGY DYNAMICS

UV poles of the low-energy matrix element

- In Euclidean kinematics ($s_{i,i+1} < 0$, $m^2 < 0$) we find that only the soft-collinear region $k_{sc} \sim Q(\lambda, \lambda^2, \lambda^{\frac{3}{2}})$ contributes at leading order in λ; it agrees with the λ expansion of the exact result
- After analytic continuation to the physical region and integration over the collinear momentum q_c, one obtains a scaleless integral that vanishes
- Good news, since $k_{sc}^2 \sim Q_0^3/Q \ll Q_0^2$



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PERTURBATIVE ANALYSIS OF THE LOW-ENERGY DYNAMICS

UV poles of the low-energy matrix element

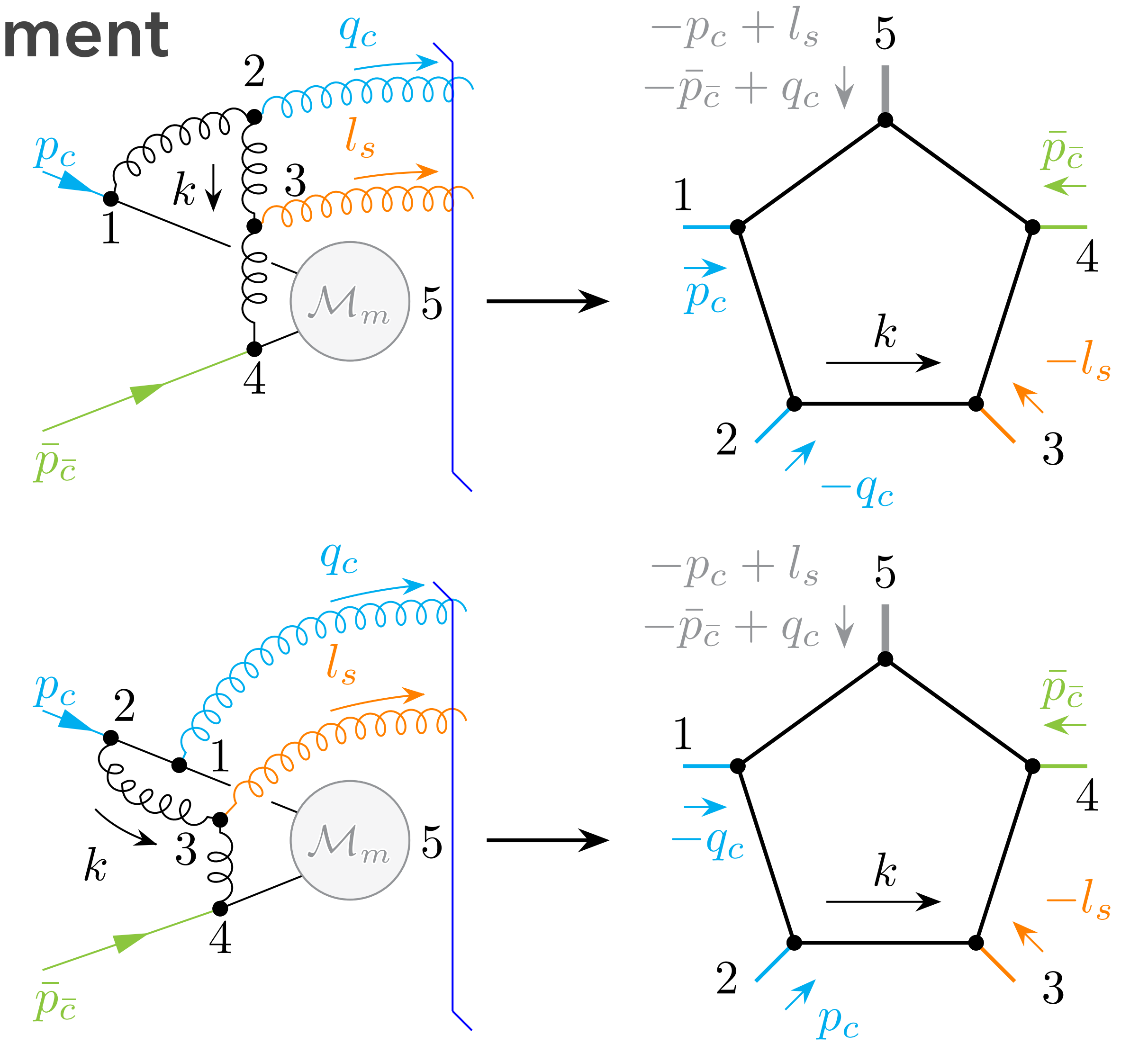
- In the physical region, some previously power-suppressed $\mathcal{O}(\lambda)$ terms get promoted to leading order, since they multiply the factor:

$$\underbrace{\frac{s_{45}\,s_{51}}{s_{45}\,s_{51}-m^2 s_{23}}}_{\lambda^{-1}}\left[1-e^{i\pi\varepsilon\,\Theta}\left(1+\underbrace{\frac{m^2 s_{23}-s_{45}\,s_{51}}{s_{45}\,s_{51}}}_{\lambda}\right)^{-\varepsilon}\right]$$

with:

$$\Theta \equiv \theta(m^2)+\theta(s_{23})-\theta(s_{45})-\theta(s_{51})$$

- In the physical region, one finds $m^2, s_{23} > 0$ and $s_{45}, s_{51} < 0$ for the upper graph (but not for the lower one)



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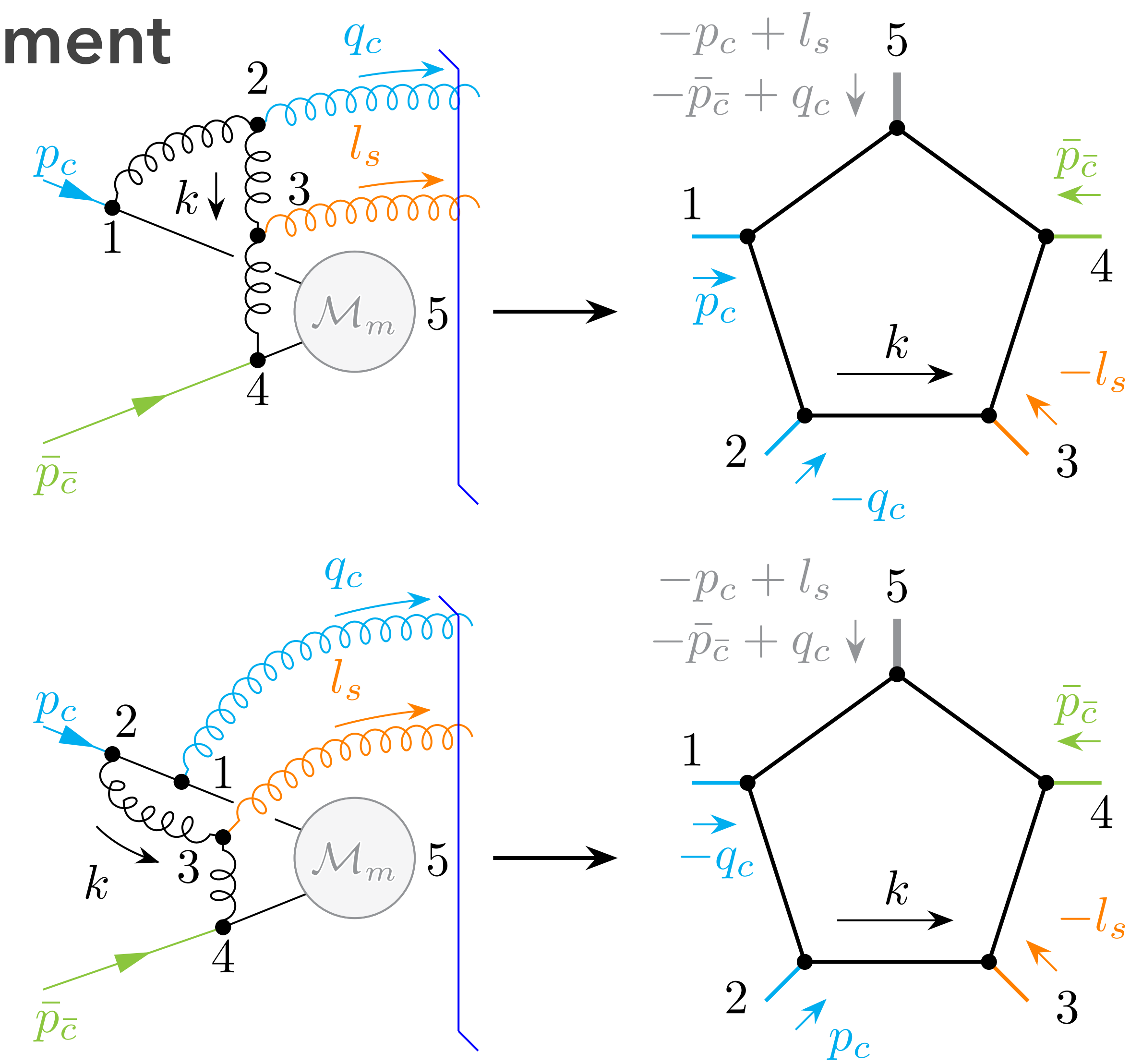
PERTURBATIVE ANALYSIS OF THE LOW-ENERGY DYNAMICS

UV poles of the low-energy matrix element

- The extra terms correspond to a new region with scaling $k_g \sim Q(\lambda^2, \lambda, \lambda)$
- This **Glauber region** is missed in all publicly available region-finder codes, such as pySecDec and Asy2.1
- The relevant integral is well defined in dimensional regularization:

$$I^{\rm g} = i(4\pi)^{2-\varepsilon} \int \frac{d^dk}{(2\pi)^d} \frac{1}{-k_T^2} \frac{1}{k^+ q_c^- - k_T^2 - 2k_T\cdot q_{cT}} \times \frac{1}{\left[-k^+(p_{\bar c}^- - q_c^-) - q_c^+ p_{\bar c}^- - k_T^2 - 2k_T\cdot q_{cT}\right]} \times \frac{1}{\bar p_{\bar c}^+ (k^- - l_s^-)} \frac{1}{-l_s^+ k^- - k_T^2 + 2k_T\cdot l_{sT}}$$



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PERTURBATIVE ANALYSIS OF THE LOW-ENERGY DYNAMICS

Glauber-gluon contribution in SCET

- The Glauber region gives the only non-zero contribution after integration over the collinear momentum q_c
- Calculating it using SCET with Glauber gluons [Rothstein, Stewart (2016)], but **without an additional Glauber regulator** we find:

$$\mathcal{W}_m^{\text{bare}} \ni \frac{N_c\,\alpha_s^3}{12\pi^2\varepsilon^3}\,\ln\frac{Q^2}{Q_0^2}\; f^{abc}\sum_{j>2} J_j\,\boldsymbol{T}_1^a\,\boldsymbol{T}_2^b\,\boldsymbol{T}_j^c$$

- Complete agreement with theoretical prediction derived assuming PDF factorization!

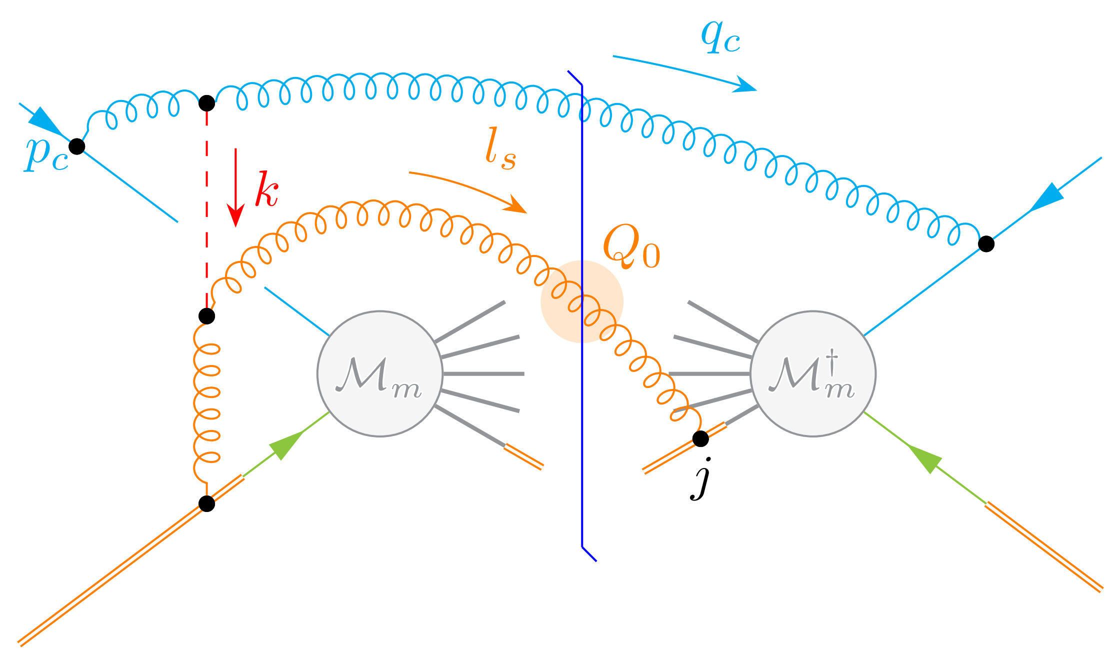


FIG. 3. Example of a collinear space-like splitting with a genuine Glauber mode (red) contributing to the low-energy matrix elements. The soft gluon is emitted into the gap with constraint Q_0 and attaches to leg j on the right-hand side. Soft Wilson lines are drawn in orange, where relevant.

CONCLUSIONS

- We have uncovered a **new mechanism that reconciles the breaking of collinear factorization with PDF factorization**
- In an interplay of space-like collinear splittings and soft emissions, the contribution of perturbative Glauber gluons **restores the factorization of the cross section** by converting double-logarithmic into single-logarithmic evolution below the jet-veto scale Q_0
- In the future, it will be important to understand the all-order structure of these effects, probably using a **more suitable implementation of Glauber effects in SCET**
- This would pave the way for a **proof of PDF factorization** for a much wider class of observables!

Backup Slides

RESUMMATION OF SUPER-LEADING LOGARITHMS

Detailed structure of the soft anomalous-dimension coefficients

$$\left.\begin{aligned} \boldsymbol{V}_m &= \overline{\boldsymbol{V}}_m + \boldsymbol{V}^G + \sum_{i=1,2} \boldsymbol{V}_i^c \ln\frac{\mu^2}{\hat{s}} \\ \boldsymbol{R}_m &= \overline{\boldsymbol{R}}_m + \sum_{i=1,2} \boldsymbol{R}_i^c \ln\frac{\mu^2}{\hat{s}} \end{aligned}\right\} \quad \boldsymbol{\Gamma} = \overline{\boldsymbol{\Gamma}} + \boxed{\boldsymbol{V}^G + \boldsymbol{\Gamma}^c} \ln\frac{\mu^2}{\hat{s}}$$

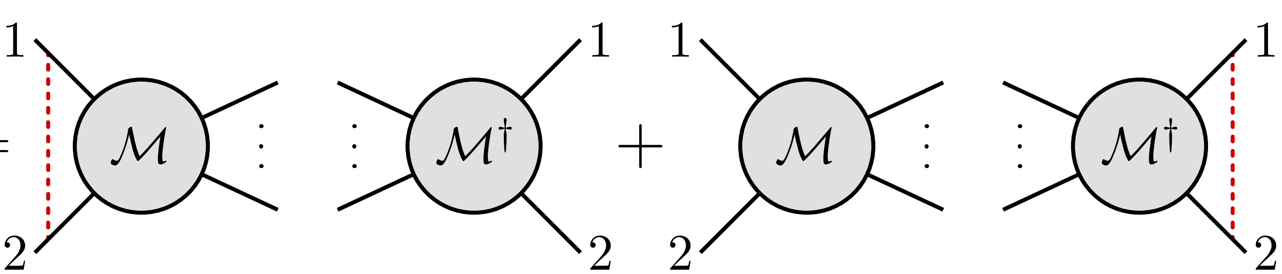
Glauber phase

soft emission (collinear div. subtracted)

collinear emission

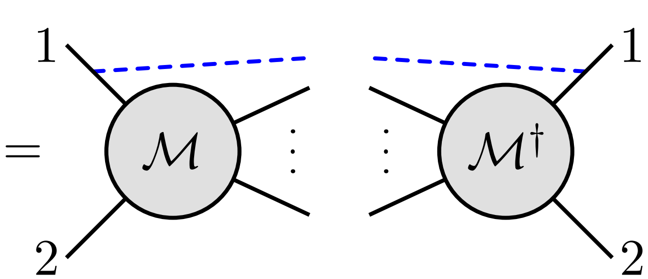
where:

$\boldsymbol{\mathcal{H}}_m\, \boldsymbol{V}^G =$



$$\boldsymbol{V}^G = -2i\pi \left(\boldsymbol{T}_{1,L} \cdot \boldsymbol{T}_{2,L} - \boldsymbol{T}_{1,R} \cdot \boldsymbol{T}_{2,R} \right)$$

$\boldsymbol{\mathcal{H}}_m\, \boldsymbol{R}_1^c =$



new color space of emitted gluon

$$\boldsymbol{\Gamma}^c = \sum_{i=1,2} \left[C_i\, \boldsymbol{1} - \boldsymbol{T}_{i,L} \circ \boldsymbol{T}_{i,R}\, \delta(n_k - n_i) \right]$$

RESUMMATION OF SUPER-LEADING LOGARITHMS

Detailed structure of the soft anomalous-dimension coefficients

Glauber phase

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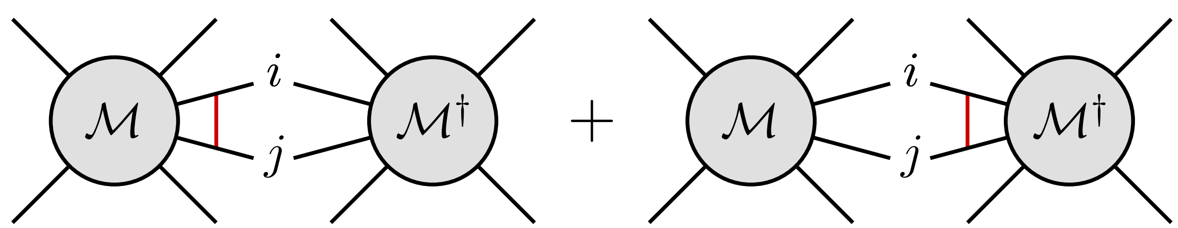
soft emission (collinear div. subtracted)

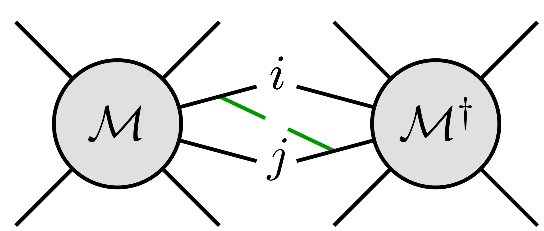
collinear emission

where:

$$\mathcal{H}_m\,\overline{\boldsymbol{V}}_m = \sum_{(ij)}$$



$$\mathcal{H}_m\,\overline{\boldsymbol{R}}_m = \sum_{(ij)}$$



RESUMMATION OF SUPER-LEADING LOGARITHMS

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soft emission (collinear div. subtracted)

collinear emission

where:

$$\overline{\boldsymbol{\Gamma}} = 2\sum_{(ij)} \left(\boldsymbol{T}_{i,L}\cdot\boldsymbol{T}_{j,L} + \boldsymbol{T}_{i,R}\cdot\boldsymbol{T}_{j,R}\right) \int \frac{d\Omega(n_k)}{4\pi}\, \overline{W}_{ij}^k - 4\sum_{(ij)} \boldsymbol{T}_{i,L}\circ\boldsymbol{T}_{j,R}\, \overline{W}_{ij}^k\, \Theta_{\text{hard}}(n_k)$$

$$\overline{W}_{ij}^k = W_{ij}^k - \frac{1}{n_i\cdot n_k}\,\delta(n_i - n_k) - \frac{1}{n_j\cdot n_k}\,\delta(n_j - n_k)\,; \qquad W_{ij}^k = \frac{n_i\cdot n_j}{n_i\cdot n_k\, n_j\cdot n_k}$$

subtracted dipole emitter

dipole emitter

RESUMMATION OF SUPER-LEADING LOGARITHMS

SLLs arise from the terms in $\mathbf{P}\exp\left[\int_{\mu_s}^{Q}\frac{d\mu}{\mu}\,\boldsymbol{\Gamma}^H(\{\underline{n}\},Q,\mu)\right]_{lm}$ **with the highest number of insertions of Γ^c^**

- Under the color trace, insertions of Γ_c are non-zero only if they come in conjunction with (at least) two Glauber phases and one $\overline{\Gamma}$
- Relevant color traces at $\mathcal{O}(\alpha_s^{n+3}L^{2n+3})$: [Becher, MN, Shao (2021)]

$$C_{rn} = \left\langle \boldsymbol{\mathcal{H}}_{2\to M}\,(\boldsymbol{\Gamma}^c)^r\,\boldsymbol{V}^G\,(\boldsymbol{\Gamma}^c)^{n-r}\,\boldsymbol{V}^G\,\overline{\boldsymbol{\Gamma}}\otimes\mathbf{1}\right\rangle$$

- Kinematic information contained in $(M+1)$ angular integrals from $\overline{\Gamma}$:

$$J_j = \int\frac{d\Omega(n_k)}{4\pi}\left(W_{1j}^k - W_{2j}^k\right)\Theta_{\text{veto}}(n_k)\,;\quad \text{with}\quad W_{ij}^k = \frac{n_i\cdot n_j}{n_i\cdot n_k\,n_j\cdot n_k}$$

RESUMMATION OF SUPER-LEADING LOGARITHMS

General result for $2 \to M$ hard processes

$$C_{rn} = -256\pi^2 \, (4N_c)^{n-r} \left[\sum_{j=3}^{M+2} J_j \sum_{i=1}^{4} c_i^{(r)} \left\langle \boldsymbol{\mathcal{H}}_{2\to M} \, \boldsymbol{O}_i^{(j)} \right\rangle - J_2 \sum_{i=1}^{6} d_i^{(r)} \left\langle \boldsymbol{\mathcal{H}}_{2\to M} \, \boldsymbol{S}_i \right\rangle \right]$$

[Becher, MN, Shao, Stillger (2023)]

Basis of color structures:

$$\boldsymbol{O}_1^{(j)} = f_{abe} f_{cde} \, \boldsymbol{T}_2^a \, \{\boldsymbol{T}_1^b, \boldsymbol{T}_1^c\} \, \boldsymbol{T}_j^d - (1 \leftrightarrow 2)$$

$$\boldsymbol{O}_2^{(j)} = d_{ade} d_{bce} \, \boldsymbol{T}_2^a \, \{\boldsymbol{T}_1^b, \boldsymbol{T}_1^c\} \, \boldsymbol{T}_j^d - (1 \leftrightarrow 2)$$

$$\boldsymbol{O}_3^{(j)} = \boldsymbol{T}_2^a \, \{\boldsymbol{T}_1^a, \boldsymbol{T}_1^b\} \, \boldsymbol{T}_j^b - (1 \leftrightarrow 2)$$

$$\boldsymbol{O}_4^{(j)} = 2C_1 \, \boldsymbol{T}_2 \cdot \boldsymbol{T}_j - 2C_2 \, \boldsymbol{T}_1 \cdot \boldsymbol{T}_j$$

$$\boldsymbol{S}_1 = f_{abe} f_{cde} \, \{\boldsymbol{T}_1^b, \boldsymbol{T}_1^c\} \, \{\boldsymbol{T}_2^a, \boldsymbol{T}_2^d\}$$

$$\boldsymbol{S}_2 = d_{ade} d_{bce} \, \{\boldsymbol{T}_1^b, \boldsymbol{T}_1^c\} \, \{\boldsymbol{T}_2^a, \boldsymbol{T}_2^d\}$$

$$\boldsymbol{S}_3 = d_{ade} d_{bce} \left[\boldsymbol{T}_2^a \left(\boldsymbol{T}_1^b \boldsymbol{T}_1^c \boldsymbol{T}_1^d \right)_+ + (1 \leftrightarrow 2) \right]$$

$$\boldsymbol{S}_4 = \{\boldsymbol{T}_1^a, \boldsymbol{T}_1^b\} \, \{\boldsymbol{T}_2^a, \boldsymbol{T}_2^b\}$$

$$\boldsymbol{S}_5 = \boldsymbol{T}_1 \cdot \boldsymbol{T}_2$$

$$\boldsymbol{S}_6 = \boldsymbol{1}$$

RESUMMATION OF SUPER-LEADING LOGARITHMS

General result for $2 \to M$ hard processes

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- Series of SLLs, starting at 3-loop order:

$$\sigma_{\rm SLL} = \sigma_{\rm Born} \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{4\pi} \right)^{n+3} L^{2n+3} \, \frac{(-4)^n \, n!}{(2n+3)!} \sum_{r=0}^{n} \frac{(2r)!}{4^r \, (r!)^2} \, C_{rn}$$

from scale integrals (at fixed coupling)