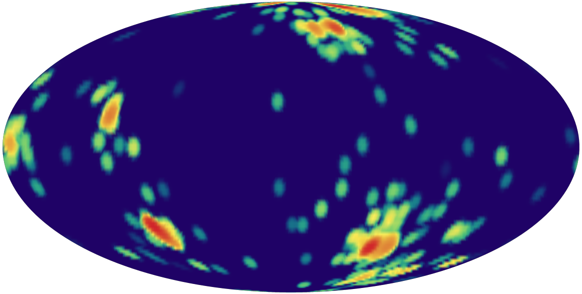
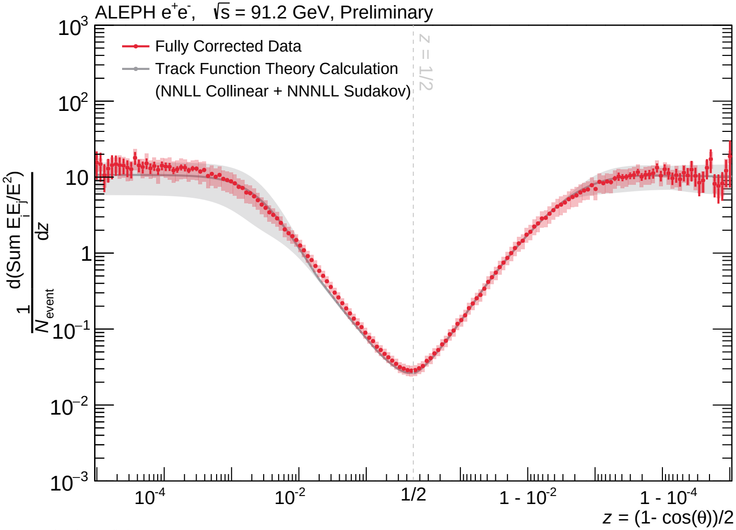
Energy Correlators on Tracks

Max Jaarsma



What to expect from this talk?

- Intuition for EEC
- Basics of resummation
- Theory vs. data!



Outline

What are energy correlators?

- Motivation
- Definition

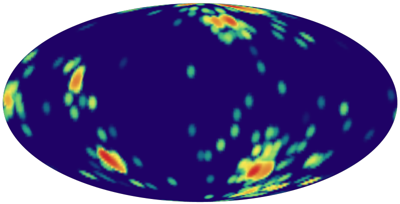
Why measure on tracks?

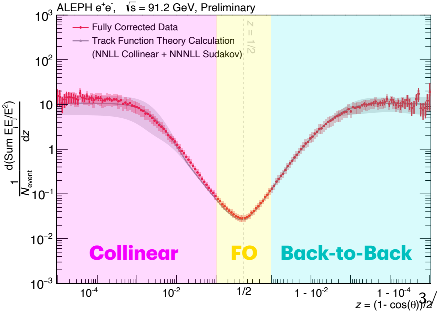
- Motivation
- Track functions

How to make a prediction?

- Split in three
- Re-sum large logs
- Glue back together

Outlook





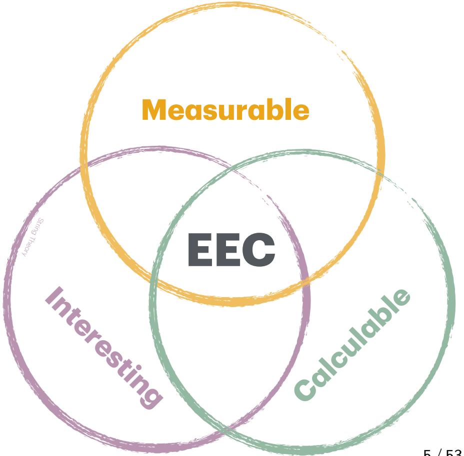
Energy correlators

EEC Motivation

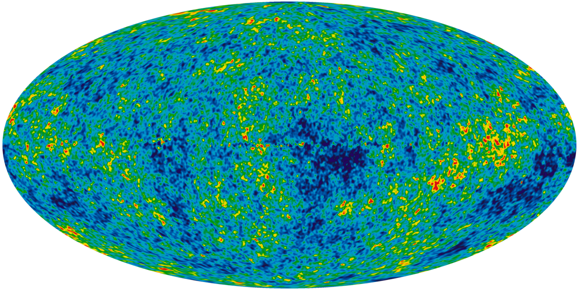
Desirable properties of an observable

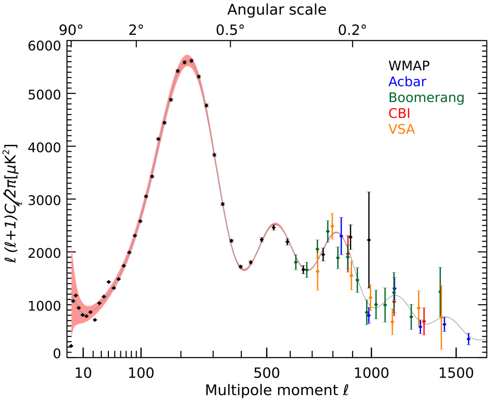
For an observable to be considered interesting it has to satisfy 3 criteria:

1. Accessible in an experiment
2. Calculable to high theoretical precision
3. Connected to some notable aspect of the theory



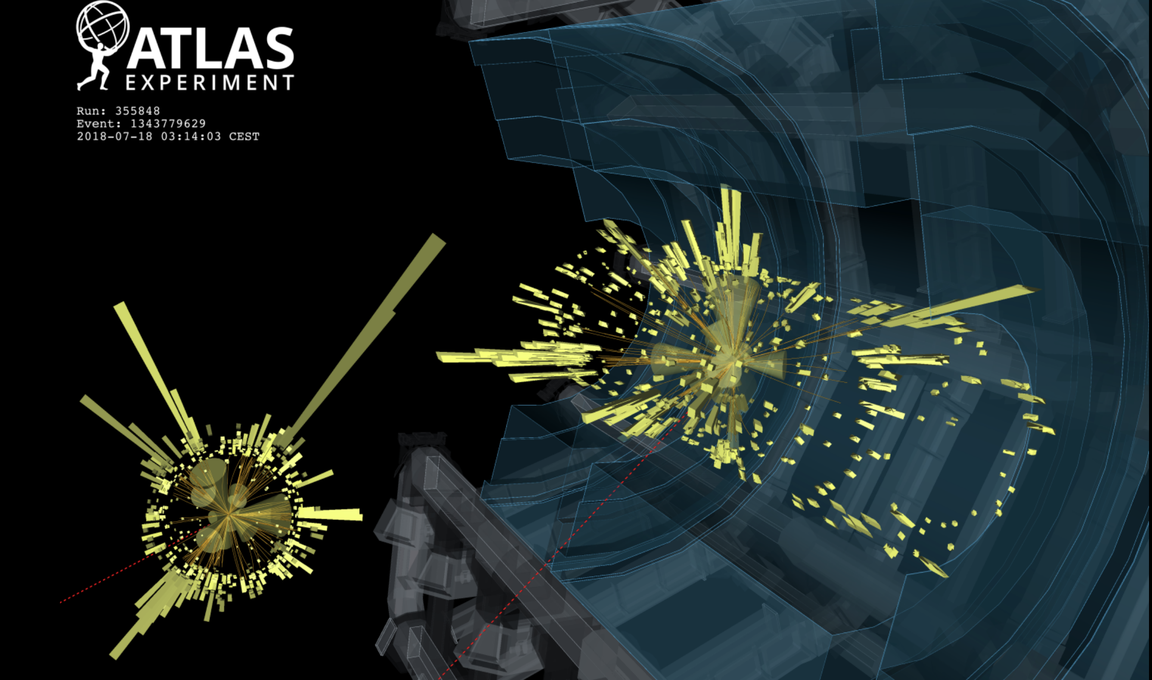
EEC: For the cosmologist(s) in the audience



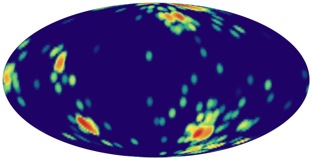


EEC for QCD = CMB for cosmology

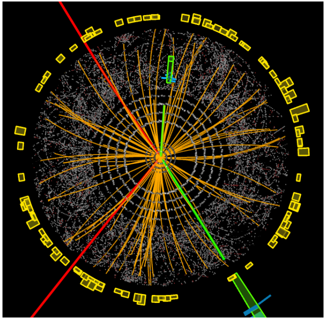
Secrets of the workings of the universe → fingerprints energy correlations


ATLAS
EXPERIMENT
Run: 355848
Event: 1343779629
2018-07-18 03:14:03 CEST

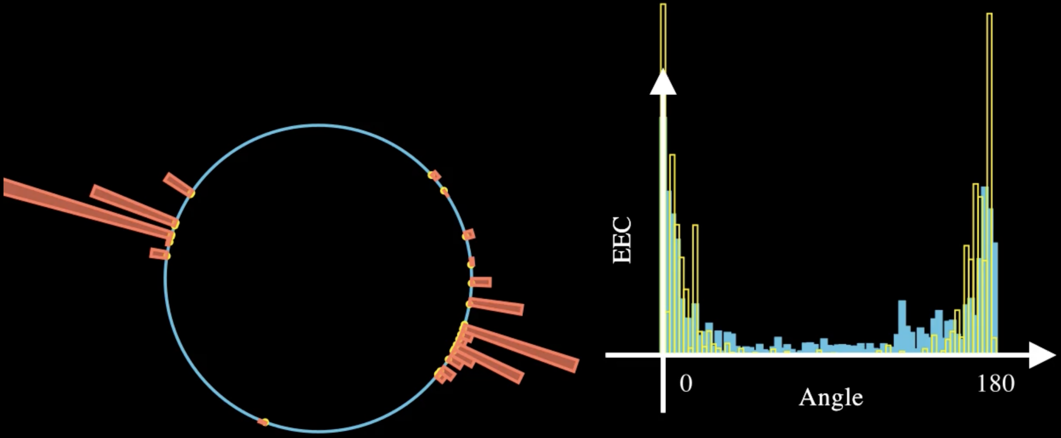
Definition



$$\text{EEC}(z) = \sum_{i,j} \int d\sigma \frac{E_i E_j}{Q^2} \delta\left(z - \frac{1-\cos\theta_{ij}}{2}\right)$$



1. Measure the angle between two particles
2. Take their energies and multiply them
3. Sum over all combinations of particles



Credit: Hua Xing Zhu

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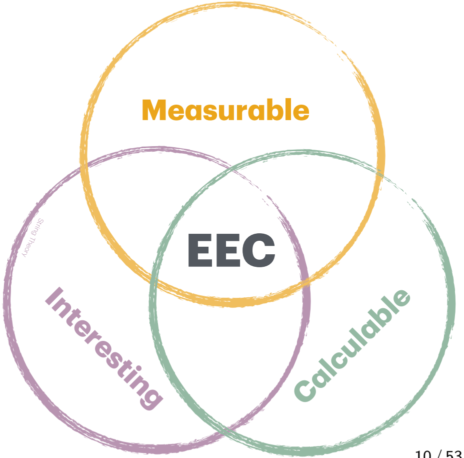


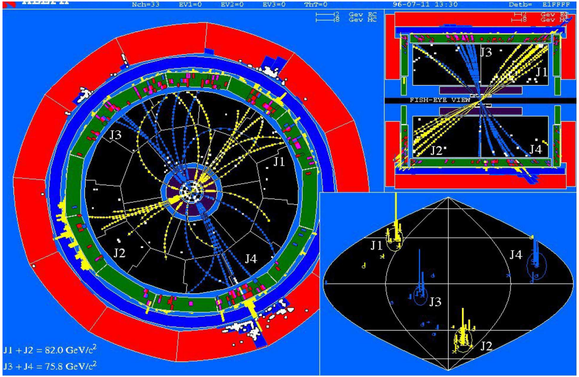
Motivation 1: Accessible in an experiment

Accessible in any detector with a tracker

- BELLE II: $e^+e^- \to$ jets
- ATLAS & CMS: $pp \to$ jets
- ALICE: Ion collisions

Even accessible in old LEP data

- OPAL
- **ALEPH**



Accessible in an experiment

So universal that they can be studied at any particle collider

Motivation 2: Calculable to high theoretical precision

- IRC safety: observable is insensitive to collinear and soft splittings

$$\mathcal{O}(p_1, p_2, p_3) \approx \mathcal{O}(p_1, p_{2+3}) \qquad \text{when} \quad p_2 \parallel p_3$$

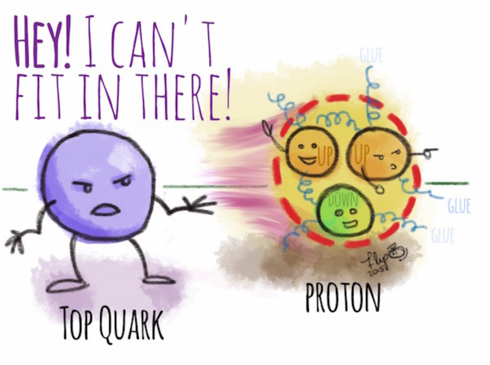
$$\mathcal{O}(p_1, p_2, p_3) \approx \mathcal{O}(p_1, p_2) \qquad \text{when} \quad p_3 \to 0$$

- IRC safe $\rightarrow$ Can be reliably calculated in perturbation theory
- The only event-shape known analytically to order α_s^2

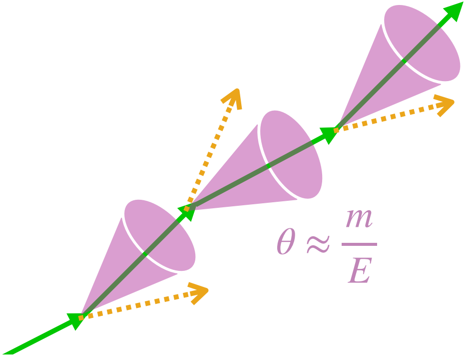
Calculable to high precision

The EEC is IRC safe and can therefore be predicted from perturbative QCD

Motivation 3: Clearly connected to theory of interest



- Top quark mass measurement
  Holguin, Moult, Pathak, Procura, Schöfbeck, Schwarz (2024)



- Dead-cone effect
  Craft, Lee Meçaj, Moult (2024)

Connection to something interesting

Many interesting phenomena leave their fingerprint on energy correlators

Track-Based observables

ATLAS
EXPERIMENT

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Event: 1358542809
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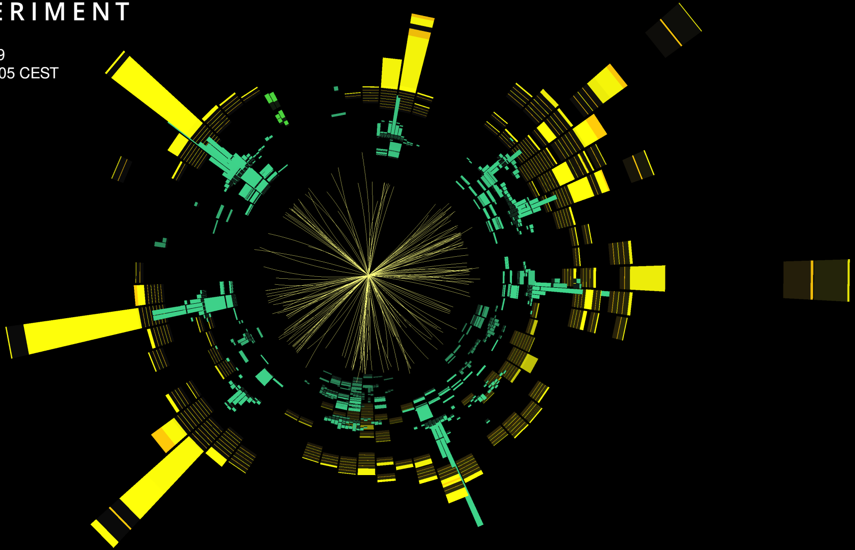
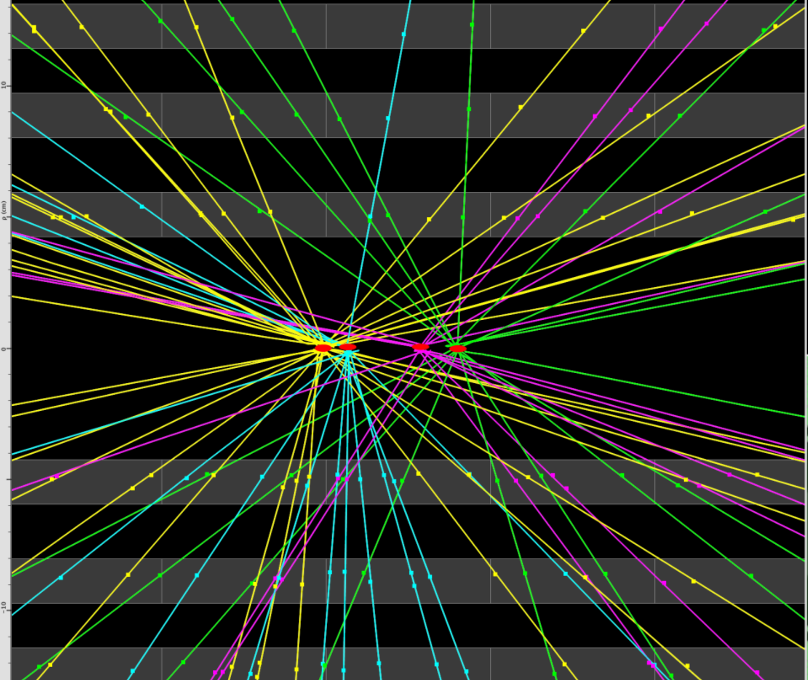


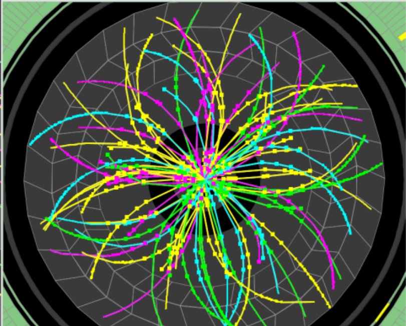


ATLAS EXPERIMENT

Run Number: 153565, Event Number: 4487360

Date: 2010-04-24 04:18:53 CEST

Event with 4 Pileup Vertices in 7 TeV Collisions



Track functions: Implementation

Cross section for a general observable $\mathcal{O}$

$$\frac{d\sigma}{d\mathcal{O}} = \sum_N \int d\Pi_N \, \frac{d\sigma_N}{d\Pi_N} \, \delta[\mathcal{O} - \hat{\mathcal{O}}(\{p_i\})]$$

- N final state partons
- Partonic cross section
- Measurement of $\mathcal{O}$ on partons

Track function formalism

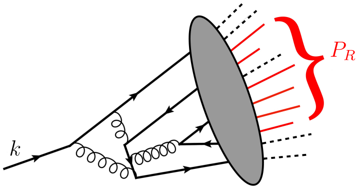
Measure on tracks $\Rightarrow$ attach track function to each parton

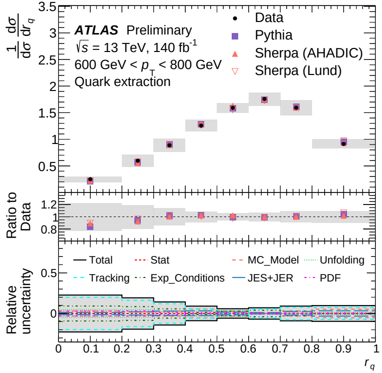
Cross section for a track-based observable $\mathcal{O}$

$$\frac{d\sigma}{d\mathcal{O}} = \sum_N \int d\Pi_N \, \frac{d\sigma_N}{d\Pi_N} \int \left(\prod_{i=1}^{N} dx_i \, T_i(x_i) \right) \delta[\mathcal{O} - \hat{\mathcal{O}}(\{x_i p_i\})]$$

Chang, Procura, Thaler, Waalewijn (2013)

Basic properties





Track function interpretation

Probability density for subset of fragments

- Support for $x \in [0, 1]$
- Normalised to 1

$$\int_0^1 \mathrm{d}x\, T_i(x, \mu) = 1$$

- Calculable scale dependence

$$\frac{\mathrm{d}}{\mathrm{d}\mu} T_i(x, \mu) = \ldots$$

- Recently extracted from data

Energy Correlators on Tracks

Regimes

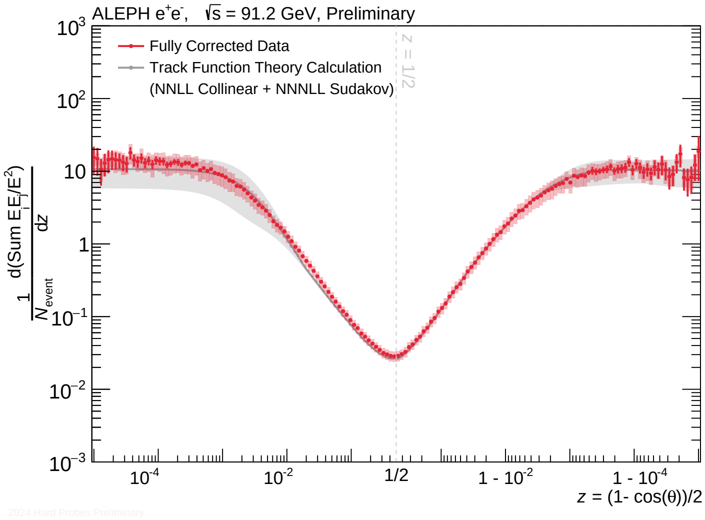
Splitting the plot in three

- Collinear
- Fixed-Order
- Back-to-Back

Resummation

- General idea
- Collinear
- Back-to-Back(Sudakov)

Stitch them back together



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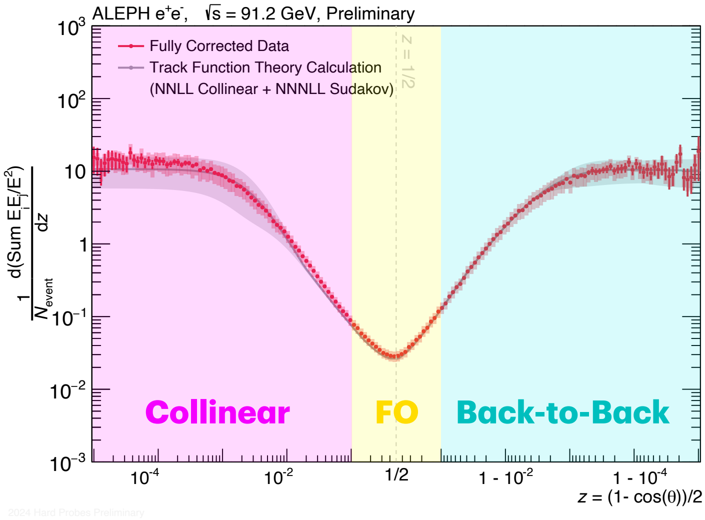
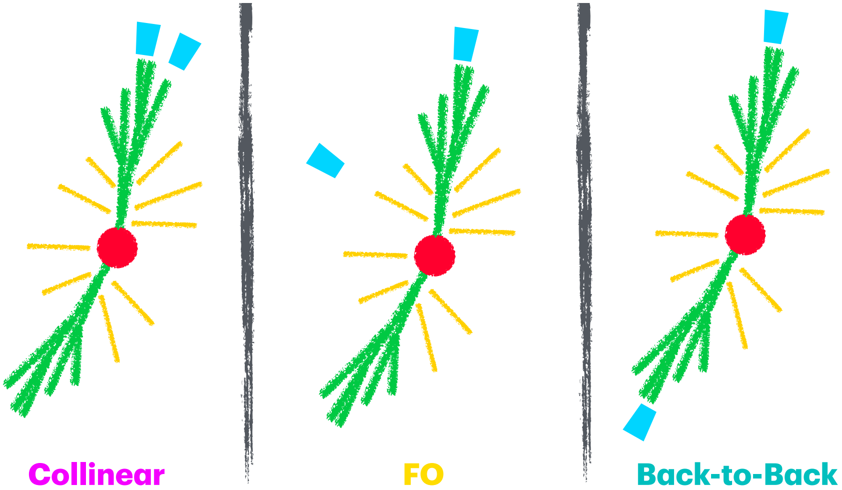


1. Identify Logs
2. Count powers
3. Factorize
4. Derive RGE
5. Solve RGE



Resummation

Resummation - General idea

$$\sigma = 1 \quad \text{LO}$$
$$+ C_{11} a_s^1 \log^1 + C_{10} a_s \quad \text{NLO}$$
$$+ C_{22} a_s^2 \log^2 + C_{21} a_s^2 \log^1 + C_{20} a_s^2 \quad \text{NNLO}$$
$$+ \underbrace{C_{33} a_s^3 \log^3}_{\text{LL}} + \underbrace{C_{32} a_s^3 \log^2}_{\text{NLL}} + \underbrace{C_{31} a_s^3 \log^1}_{\text{NNLL}} + \underbrace{C_{30} a_s^3}_{\text{NNNLL}} \quad \text{NNNLO}$$

$$\log \sim \frac{1}{a_s}$$

Resummation - General idea

- Large Logs: fixed-order calculation

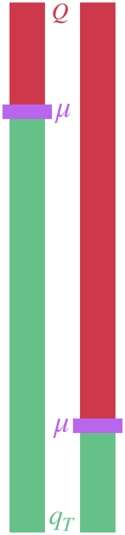
$$\sigma = 1 + C_{11} a_s \log\left(\frac{Q^2}{q_T^2}\right) + \ldots + C_{22} a_s^2 \log^2\left(\frac{Q^2}{q_T^2}\right) + \ldots$$

- Logarithmic tower can be captured by factorization

$$\sigma = H(Q, \mu) \times J(q_T, \mu)$$

- μ acts as border between hard and collinear
- Large Logs: factorized calculation

$$\sigma = \left[1 + C_{11} a_s \log\left(\frac{Q^2}{\mu^2}\right) + \ldots\right]\left[1 + C_{11} a_s \log\left(\frac{\mu^2}{q_T^2}\right) + \ldots\right]$$



Resummation - General idea

Key to resummation

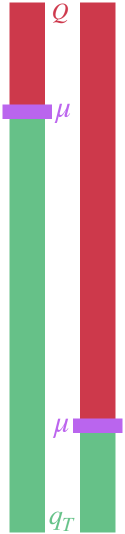
Factorization + RGE constrains the coefficients of large logs

- Large Logs: full vs. factorized

$$\sigma = 1 + C_{11} a_s \log\left(\frac{Q^2}{q_T^2}\right) + \ldots + C_{22} a_s^2 \log^2\left(\frac{Q^2}{q_T^2}\right) + \ldots$$
$$= \left[1 + C_{11} a_s \log\left(\frac{Q^2}{\mu^2}\right) + \ldots\right]\left[1 + C_{11} a_s \log\left(\frac{\mu^2}{q_T^2}\right) + \ldots\right]$$

- μ independence $\rightarrow$ tower of logs captured

$$\frac{d\sigma}{d\mu} = 0 \qquad \rightarrow \qquad C_{22} = \frac{C_{11}^2}{2}\,, \quad C_{33} = \frac{C_{11}^3}{6}\,, \ldots$$



Resummation - General idea

Key to resummation

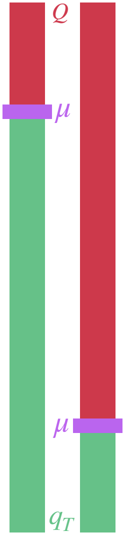
Solving RGE resums logs

- Factorization → Renormalization Group Equations

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2} H(Q,\mu) = -\gamma(\mu)\, H(Q,\mu)$$
$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2} J(q_T,\mu) = +\gamma(\mu)\, J(q_T,\mu)$$

- Solve RGE → exponentiating logs

$$J(q_T,\mu) = \exp\left[\int_{\mu_J}^{\mu} \mathrm{d}\ln\mu\, \gamma(\mu)\right] J(q_T,\mu_J)$$



Key to resummation

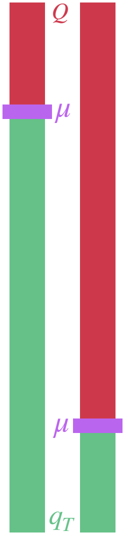
Exponentiate the large logs by evolving from one scale to the other

- Start from factorized formula

$$\sigma = H(Q,\mu) \times J(q_T,\mu)$$

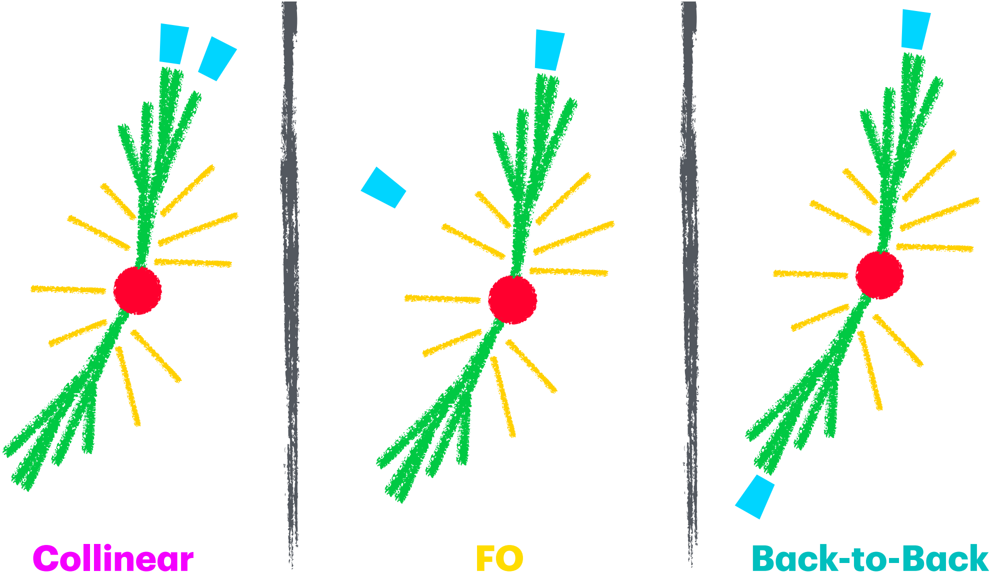
- Pick scales μ_H and μ_J that keep the logs small
- Introduce evolution kernel where the logs are exponentiated

$$\sigma = H(Q,\mu_H) \times U(\mu_H,\mu_J) \times J(q_T,\mu_J)$$

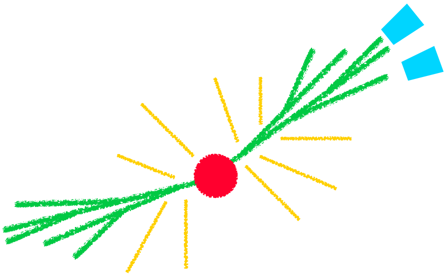


Our prediction

Regimes



Resummation - Collinear regime



$$\frac{\mathrm{d}\sigma}{\mathrm{d}z} \approx H \otimes J$$

- Small angle limit

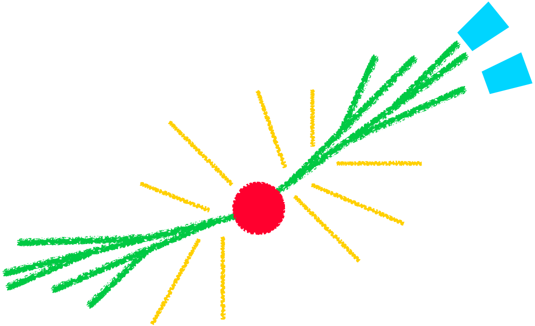
$$\theta \to 0 \qquad z \to 0$$

- Large logarithms as $z \to 0$:

$$\frac{1}{z} \underbrace{C_{kk} a_s^k \log^k z}_{\text{LL}} + \frac{1}{z} \underbrace{C_{kk-1} a_s^k \log^{k-1} z}_{\text{NLL}} + \dots$$

- Insensitive to soft
- Insensitive to other jet

Resummation - Collinear regime



- In collinear limit

$$p_i \cdot p_j \sim z x^2 Q^2$$

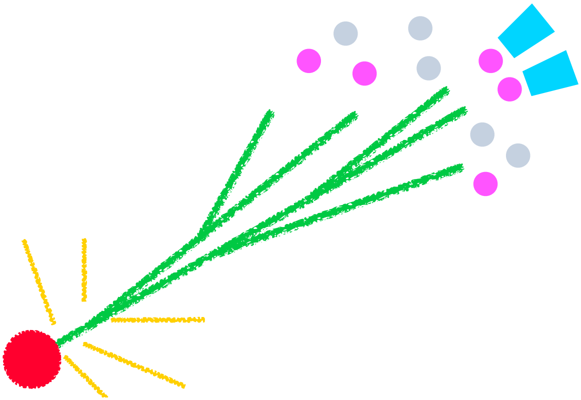
- Hard-Collinear factorization

$$\Lambda^2_{\mathsf{QCD}} \ll z x^2 Q^2 \ll Q^2$$

- DGLAP-like evolution
- Natural scales

$$\mu_H^2 \sim Q^2 \qquad \mu_J^2 \sim z x^2 Q^2$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z} \approx \sum_{\mathsf{f}} \frac{\mathrm{d}}{\mathrm{d}z} \int_0^1 \mathrm{d}x\, x^2 H_f(x, Q^2, \mu^2) J_f(z x^2 Q^2, \mu^2)$$

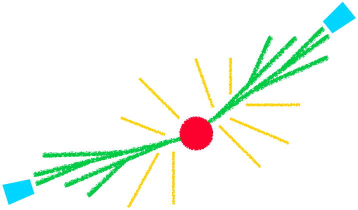


- Restricting to charged particles
- Jet on tracks becomes

$$J_i = \mathcal{J}_{i\to j}\, T_j(2) + \mathcal{J}_{i\to jk}\, T_j(1)\, T_k(1)$$

- $T(2)$ contact term
- $T(1)\,T(1)$ non-contact term
- mix under evolution

Resummation - Back-to-Back regime



$$\frac{d\sigma}{dz} \approx H \otimes J \otimes J \otimes S$$

- Large angle limit

$$\theta \to 180^\circ \qquad z \to 1$$

- Double Logs (Sudakov)

$$C_{24}\, \underbrace{a_s^2 \frac{\log^4(1-z)}{1-z}}_{\text{LL}} + C_{23}\, \underbrace{a_s^2 \frac{\log^3(1-z)}{1-z}}_{\text{NLL}} + \dots$$

- Sensitive to both jets
- Recoil from soft radiation

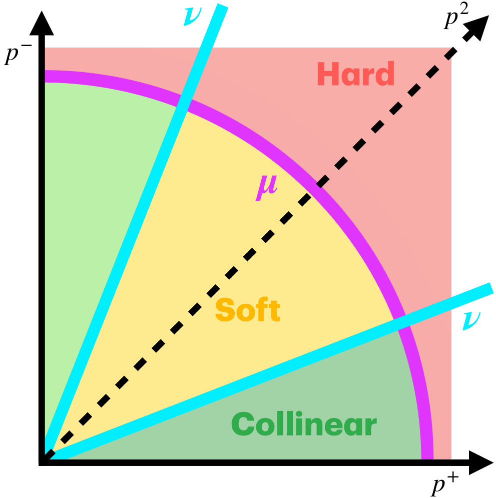
- Collinear and Soft overlap $\rightarrow$ double logs
- Collinear and Soft overlap $\rightarrow$ rapidity scale

Two sets of sliders:

- μ: virtuality
- ν: rapidity

Sudakov logarithms

Overlap of soft and collinear introduces double logs, which are resummed by a combination of rapidity RGE and virtuality RGE



RGE for virtuality

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2} H = \left[\gamma_H(\mu) + \Gamma_{\mathsf{cusp}}(\mu)\,\ln\left(\frac{Q^2}{\mu^2}\right)\right] H$$

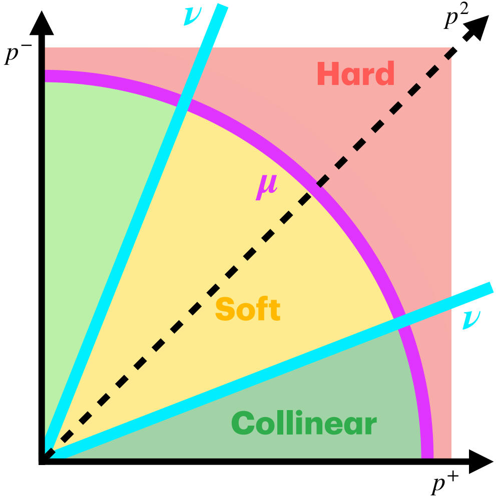
$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2} J = \left[\gamma_J(\mu) + \Gamma_{\mathsf{cusp}}(\mu)\,\ln\left(\frac{\nu}{Q}\right)\right] J$$

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2} S = \left[\gamma_S(\mu) + \Gamma_{\mathsf{cusp}}(\mu)\,\ln\left(\frac{\mu^2}{\nu^2}\right)\right] S$$

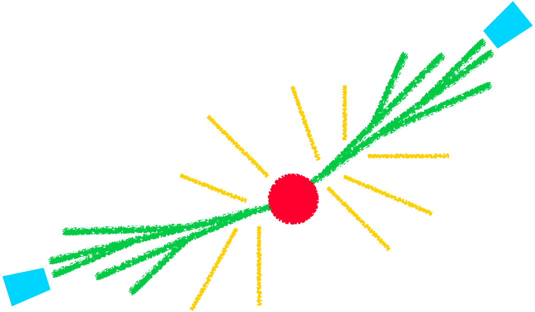
RGE for rapidity

$$\frac{\mathrm{d}}{\mathrm{d}\ln\nu} J = -\tfrac{1}{2}\gamma_\nu(b_\perp,\mu)\, J$$

$$\frac{\mathrm{d}}{\mathrm{d}\ln\nu} S = \gamma_\nu(b_\perp,\mu)\, S$$



Resummation - Back-to-Back regime



- TMD-like factorization

$$q_T^2 = (1-z)Q^2$$

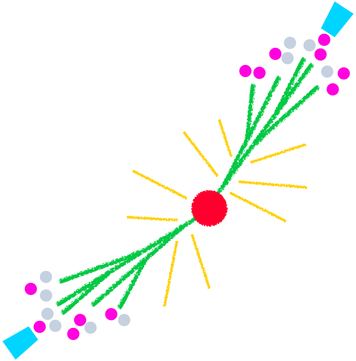
- Virtuality scales:

$$\mu_H \sim Q \qquad \mu_J \sim \mu_S \sim b_\perp^{-1}$$

- Rapidity scales:

$$\nu_J \sim Q \qquad \nu_S \sim b_\perp^{-1}$$

$$\text{EEC}(z) \approx \sum_q \int_0^\infty \mathrm{d}b_\perp\, b_\perp J_0\big(\sqrt{1-z}\,b_\perp Q\big) H(Q,\mu) \times J_q(b_\perp,Q,\mu,\nu) J_{\bar{q}}(b_\perp,Q,\mu,\nu) S(b_\perp,\mu,\nu)$$



- Jet function on tracks

$$J_i(b_\perp, Q, \mu, \nu) = T_j(1, \mu)\, \mathcal{C}_{ji}(1, b_\perp, Q, \mu, \nu)$$

- TMD-matching coefficients known to α_s^3

Gluing the pieces back together

Glue back together

To stitch the three parts together we:

- Smoothly turn off resummation with z
- This is done using profile scales:

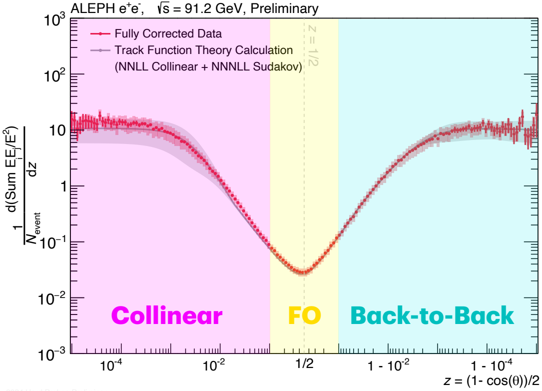
$$\mu_{H,J,S} \to \mu_{H,J,S}(z)$$

- At some point

$$\mu_H(z_{\text{FO}}) = \mu_J(z_{\text{FO}}) = \mu_S(z_{\text{FO}}) = \mu_{\text{FO}}$$

Stitching the parts together

Stitching together is done by smoothly turning off resummation as a function of z. This is done by using profile scales



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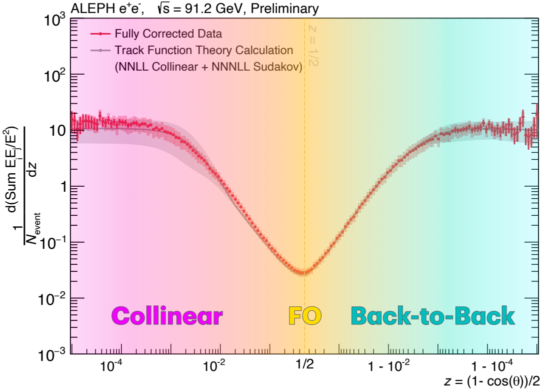


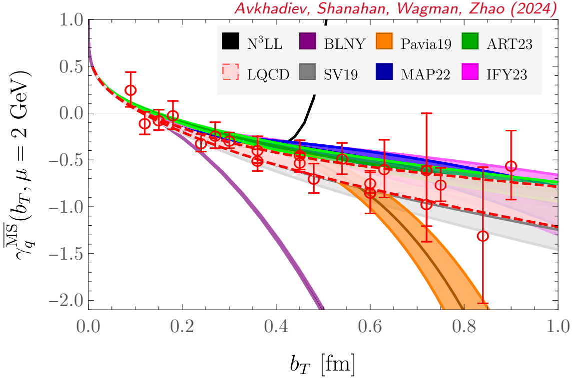
6 Technical Slides: Non-Perturbative Effects

Non-perturbative effects: Collins-Soper kernel

- Rapidity evolution: CS kernel

$$\frac{\mathrm{d}}{\mathrm{d}\ln\nu}\ln J(b_\perp, Q, \mu, \nu) = -\frac{1}{2}\gamma_\nu(b_\perp, \mu)$$

$$\frac{\mathrm{d}}{\mathrm{d}\ln\nu}\ln S(b_\perp, \mu, \nu) = \gamma_\nu(b_\perp, \mu)$$



- Contains a non-perturbative piece

$$\gamma_\nu(b_\perp, \mu) = \underbrace{\gamma_\nu(b_\perp, \mu_0)}_{\text{perturbative: FO}} + \underbrace{\int_{\mu_0}^{\mu} \mathrm{d}\ln\mu^2\, \Gamma_{\mathsf{cusp}}}_{\text{perturbative: resummed}} + \underbrace{\gamma_\nu^{\mathsf{NP}}(b_\perp)}_{\text{non-perturbative}}$$

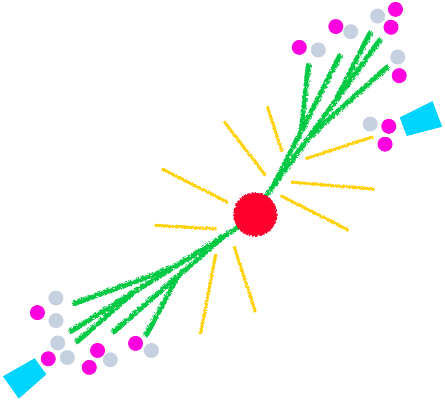
- NP-part extracted from lattice and data

Non-perturbative effects: Power correction

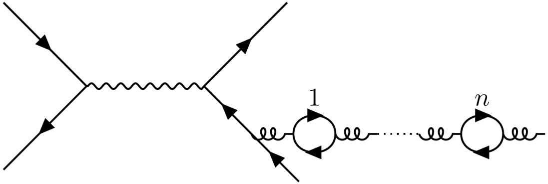
The most dominant power correction

The detector triggers on hadrons fragmenting from a soft gluon

- Present for all event shapes
- Non-perturbative



Non-perturbative effects: Power correction

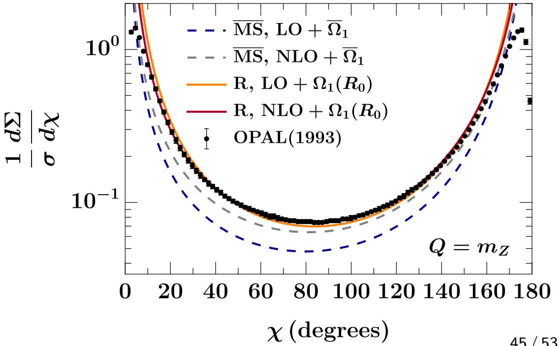


- Probed by renormalons

$$\text{EEC}(z)\big|_{\text{NP}} \propto \frac{\Omega_1}{Q}\frac{1}{[z(1-z)]^{\frac{3}{2}}}$$

- Ω_1 constrained from event-shapes

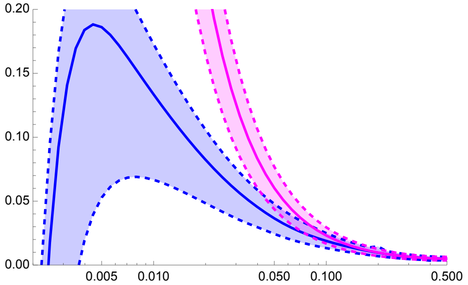
Shindler, Stewart, Sun (2024)

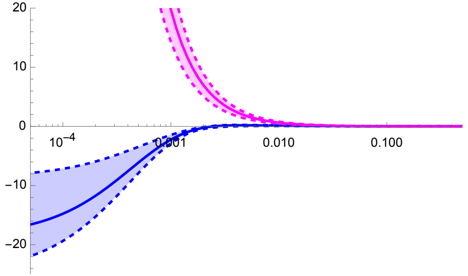


- Problematic as $z \to 0$ and $z \to 1$
- Resummation in back-to-back limit required
- In back-to-back limit PC $\sim \Omega_1 b_\perp$

$$\text{EEC}(z)\Big|_{\text{NP}}^{z\to 1} \propto \int \mathrm{d}b_\perp\, b_\perp J_0(\sqrt{1-z}\,b_\perp Q)\, \Omega_1 b_\perp$$

- resummed PC vs. "fixed-order" PC





Non-perturbative effects: Free hadron region

- Free hadron region (in collinear limit) is characterized by

$$zQ^2 \ll \Lambda^2_{\mathsf{QCD}}$$

- Resummed EEC keeps growing due to single-log structure

The collinear plateau

The EEC will eventually reach a plateau in the free hadron region

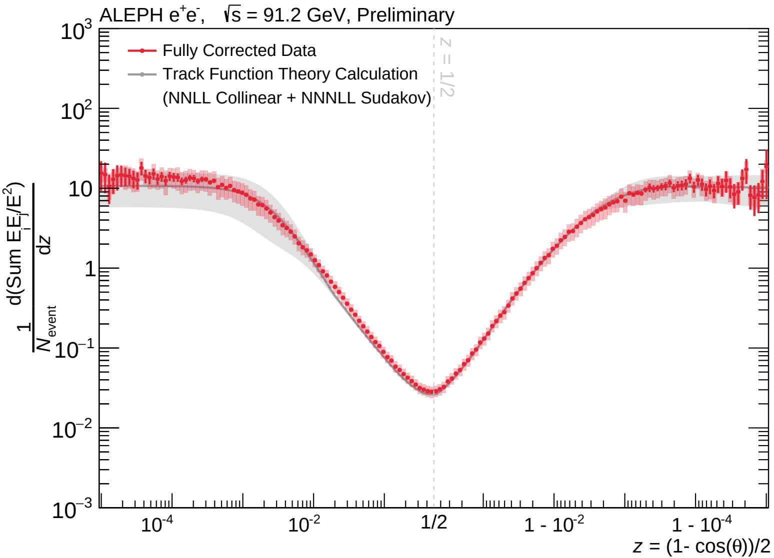
Non-perturbative effects: Free hadron region

1. The di-jet configuration dominates the EEC
2. Assume similar number of hadrons
3. Assume similar distribution of energy
4. Collinear limit: $N(N-1)$ contributions
5. back-to-back limit: N^2 contributions
6. For $Q \sim m_Z$ we have $N \sim 10 - 100$
7. Tri-jet configuration: collinear contribution is larger than back-to-back contribution.

The collinear plateau

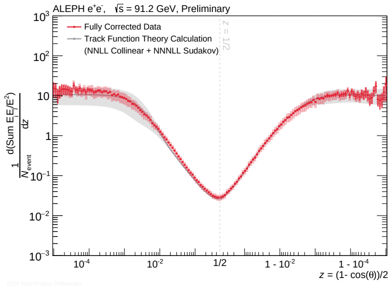
Height of plateau comparable to that of the back-to-back limit.

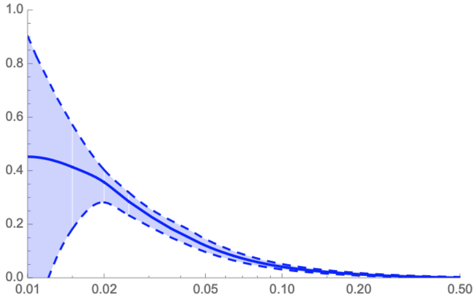
Result



ALEPH e+e-, √s = 91.2 GeV, Preliminary
Fully Corrected Data
Track Function Theory Calculation
(NNLL Collinear + NNNLL Sudakov)
z = 1/2
$\frac{1}{N_{event}} \frac{d(\text{Sum } E_i E_j / E^2)}{dz}$
10^3
10^2
10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-2}
1/2
$1 - 10^{-2}$
$1 - 10^{-4}$
z = (1- cos(θ))/2

Outlook





- Constraining non-perturbative parameters?
- Asymmetry in EEC is less sensitive to NP effects $\rightarrow \alpha_s$ measurement?

Summary

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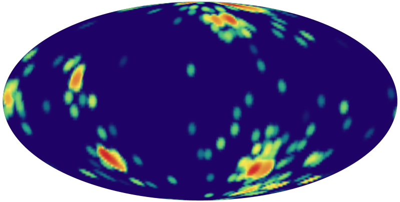
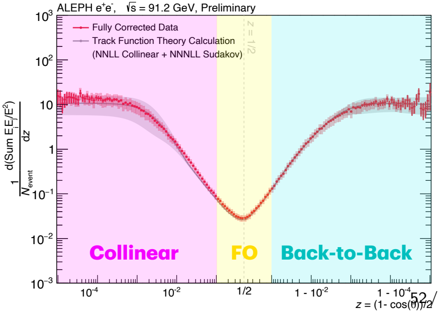
- Energy correlators are promising observables
- Track-based measurement allows for amazing angular resolution
- Precise theoretical predictions using resummation
- Excellent agreement between theory and experiment
- Opportunities for extraction of theory parameters





Thank you for your attention!

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